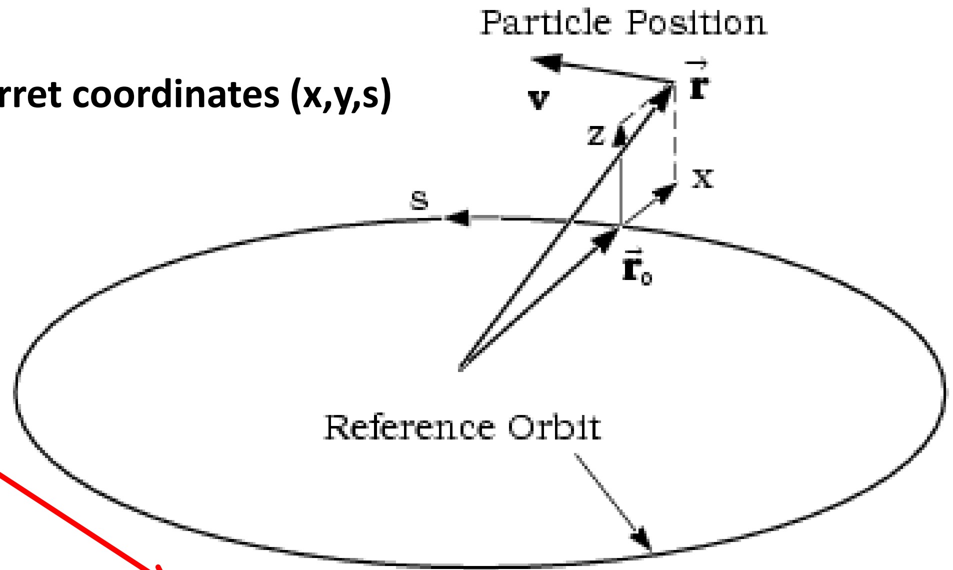
Transverse (Betatron) Motion

- Linear betatron motion
- Dispersion function of off momentum particle
- Simple Lattice design considerations
- Nonlinearities

Review

Frenet-Serret coordinates (x,y,s)



Hill's equations (derivatives w.r.t. s)

$$x'' + K_x(s)x = \pm\frac{\Delta B_y}{B\rho}, \quad y'' + K_y(s)y = \mp\frac{\Delta B_x}{B\rho}$$

$$K_x(s) = \frac{1}{\rho^2} \mp \frac{B_1}{B\rho}, \qquad K_y(s) = \pm\frac{B_1}{B\rho}$$

Natural focusing from dipoles (curvature)

Focusing from quadrupoles

Higher order magnet, usually field errors

$$\theta = \frac{s}{R} = \frac{\beta ct}{R}$$

Solution of Hill's equations X(s), X'(s) form a coordinate set and can be transformed thru matrix representation

$$\begin{pmatrix} X(s) \\ X'(s) \end{pmatrix} = M(s,s_0)\begin{pmatrix} X(s_0) \\ X'(s_0) \end{pmatrix}$$

$$|M(s,s_0)| = 1 \qquad\qquad |Trace(M(s,s_0))| \le 2$$

Stable solution conditions

Courant-Snyder parameterization

$$M(s)=\begin{pmatrix}\cos\Phi+\alpha\sin\Phi & \beta\sin\Phi \\ -\gamma\sin\Phi & \cos\Phi-\alpha\sin\Phi\end{pmatrix}=I\cos\Phi+J\sin\Phi$$

$$I=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix},\quad J=\begin{pmatrix}\alpha & \beta\\ -\gamma & -\alpha\end{pmatrix},\quad J^2=-I,\quad \text{or}\quad \beta\gamma=1+\alpha^2$$

Where α,β,γ,ϕ are functions of s and describes position dependent beam properties.

Focusing quadrupole:

$$M(s,s_0)=\begin{pmatrix}\cos\sqrt{K}\ell & \frac{1}{\sqrt{K}}\sin\sqrt{K}\ell\\ -\sqrt{K}\sin\sqrt{K}\ell & \cos\sqrt{K}\ell\end{pmatrix}\rightarrow\begin{pmatrix}1 & 0\\ -1/f & 1\end{pmatrix}$$

Defocusing quadrupole:

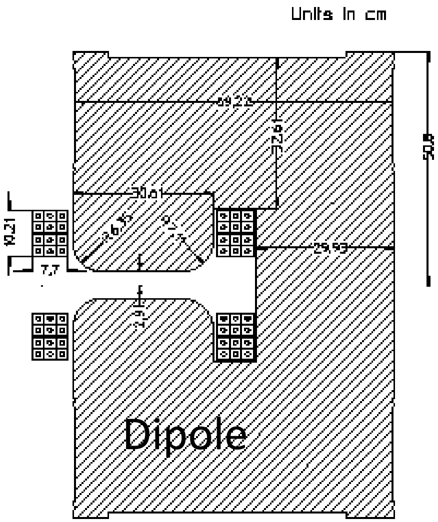
$$M(s,s_0)=\begin{pmatrix}\cosh\sqrt{|K|}\ell & \frac{1}{\sqrt{|K|}}\sinh\sqrt{|K|}\ell\\ \sqrt{|K|}\sinh\sqrt{|K|}\ell & \cosh\sqrt{|K|}\ell\end{pmatrix}\rightarrow\begin{pmatrix}1 & 0\\ 1/f & 1\end{pmatrix}$$

Dipole: $K=1/\rho^2$

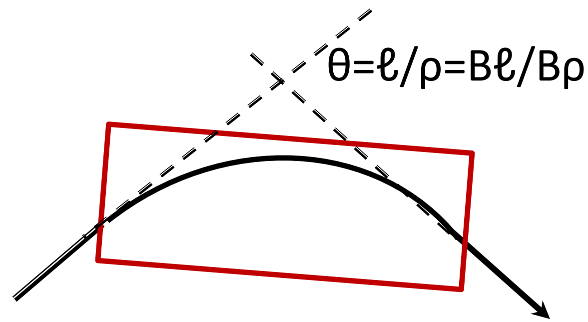
$$M(s,s_0)=\begin{pmatrix}\cos\frac{\ell}{\rho} & \rho\sin\frac{\ell}{\rho}\\ -\frac{1}{\rho}\sin\frac{\ell}{\rho} & \cos\frac{\ell}{\rho}\end{pmatrix}\rightarrow\begin{pmatrix}1 & \ell\\ 0 & 1\end{pmatrix}$$

Drift space: $K=0$

$$M(s,s_0)=\begin{pmatrix}1 & \ell\\ 0 & 1\end{pmatrix}$$



$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

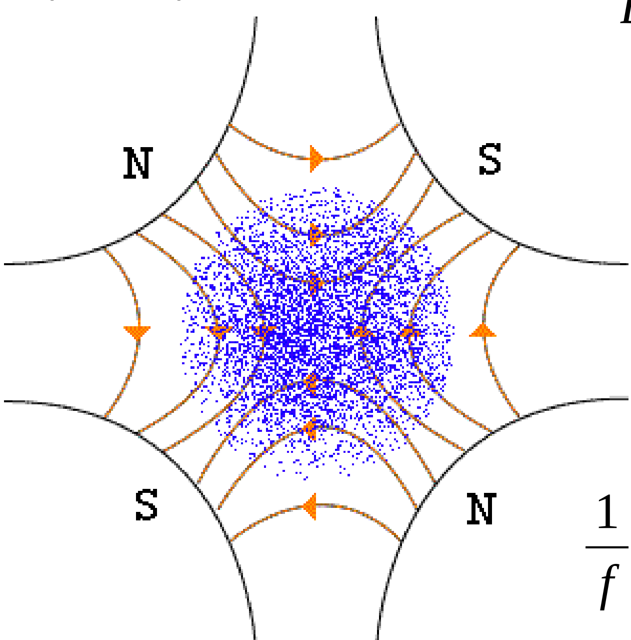


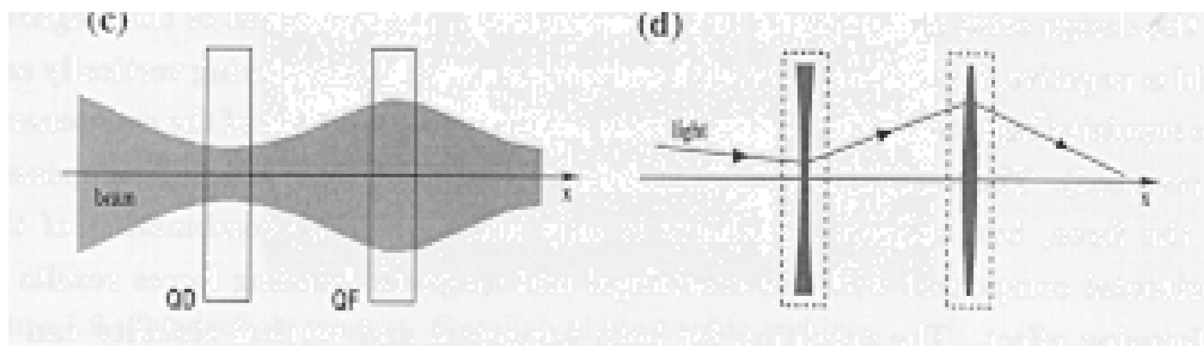
$$\gamma m \frac{\upsilon^2}{\rho} = q \upsilon B$$

$$\rho = \frac{\gamma m v}{qB} = \frac{p}{qB}$$

$$B\rho[T-m] = \frac{p}{q} = \frac{A}{Z} \times 3.33564 \times p[GeV/c/u]$$

quadrupole





$$\frac{1}{f} = \mp \frac{B_1 \ell}{B\rho}$$

f>0, if focusing, f<0 if defocusing

For two dimensional magnetic field, one can expand the magnetic field using **Beth representation**.

$$\vec{B} = B_x(x,y)\hat{x} + B_y(x,y)\hat{y}$$

$$B_x = -\frac{1}{h_s}\frac{\partial(h_s A_2)}{\partial y} = -\frac{1}{h_s}\frac{\partial A_s}{\partial y}, B_y = \frac{1}{h_s}\frac{\partial(h_s A_2)}{\partial x} = \frac{1}{h_s}\frac{\partial A_s}{\partial x}$$

For h_s=1 or ρ=∞, one obtains the multipole expansion:

$$B_y + jB_x = B_0\sum_n (b_n + ja_n)(x+jy)^n, \qquad A_s = \mathrm{Re}\left\{B_0\sum_n \frac{1}{n+1}(b_n + ja_n)(x+jy)^{n+1}\right\}$$

b_0 : dipole, a_0 : skew (vertical) dipole; $B_y = B_0 b_0$, $B_x = B_0 a_0$,

b_1 : quad, a_1 : skew quad; $B_y = B_0 b_1 x$, $B_x = B_0 b_1 y$, $B_y = -B_0 a_1 y$, $B_x = B_0 a_1 x$,

b_2 : sextupole, a_2 : skew sextupole;

$$\frac{1}{B\rho}(B_y + jB_x) = \mp\frac{1}{\rho}\sum_n (b_n + ja_n)(x+jy)^n$$

Floquet Theorem

$$X'' + K(s)X = 0 \qquad K(s) = K(s+L)$$

$$X(s) = a w(s) e^{j\psi(s)}, \quad w(s) = w(s+L), \quad \psi(s+L) - \psi(s) = 2\pi\mu$$

$$\beta(s) = w^2, \quad \alpha = -\frac{1}{2}\beta', \quad \gamma = \frac{1+\alpha^2}{\beta}, \qquad w(s) = \sqrt{\beta(s)}, \quad \psi(s) = \int_{s_0}^{s} \frac{1}{\beta} ds$$

$$\begin{pmatrix} X(s_2) \\ X'(s_2) \end{pmatrix} = M(s_2, s_1) \begin{pmatrix} X(s_1) \\ X'(s_1) \end{pmatrix}$$

$$M(s_2, s_1) = \begin{pmatrix} \sqrt{\frac{\beta_2}{\beta_1}}(\cos\mu + \alpha_1 \sin\mu) & \sqrt{\beta_1\beta_2}\sin\mu \\ -\frac{1+\alpha_1\alpha_2}{\sqrt{\beta_1\beta_2}}\sin\mu - \frac{\alpha_1 - \alpha_2}{\sqrt{\beta_1\beta_2}}\cos\mu & \sqrt{\frac{\beta_2}{\beta_1}}(\cos\mu - \alpha_1 \sin\mu) \end{pmatrix}$$

$$= \begin{pmatrix} \sqrt{\beta_2} & 0 \\ -\frac{\alpha_2}{\sqrt{\beta_2}} & \frac{1}{\sqrt{\beta_2}} \end{pmatrix} \begin{pmatrix} \cos\mu & \sin\mu \\ -\sin\mu & \cos\mu \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{\beta_1}} & 0 \\ -\frac{\alpha_1}{\sqrt{\beta_1}} & \sqrt{\beta_1} \end{pmatrix}$$

The values of the Courant–Snyder parameters α_2, β_2, γ_2 at s_2 are related to α_1, β_1, γ_1 at s_1 by

$$\begin{pmatrix}\beta\\ \alpha\\ \gamma\end{pmatrix}_2 = \begin{pmatrix} M_{11}^2 & -2M_{11}M_{12} & M_{12}^2 \\ -M_{11}M_{21} & M_{11}M_{22}+M_{12}M_{21} & -M_{12}M_{22} \\ M_{21}^2 & -2M_{21}M_{22} & M_{22}^2 \end{pmatrix}\begin{pmatrix}\beta\\ \alpha\\ \gamma\end{pmatrix}_1$$

The evolution of the betatron amplitude function in a drift space is

$$\beta_2 = \frac{1}{\gamma_1} + \gamma_1 (s - \frac{\alpha_1}{\gamma_1})^2 = \beta^* + \frac{(s-s^*)^2}{\beta^*},$$

$$\alpha_2 = \alpha_1 - \gamma_1 s = -\frac{(s-s^*)}{\beta^*}, \quad \gamma_2 = \gamma_1 = \frac{1}{\beta^*}$$

Passing through a thin-lens quadrupole, the evolution of betatron function is

$$\beta_2 = \beta_1, \quad \alpha_2 = \alpha_1 + \frac{\beta_1}{f}, \quad \gamma_2 = \gamma_1 + \frac{2\alpha_1}{f} + \frac{\beta_1}{f^2}$$

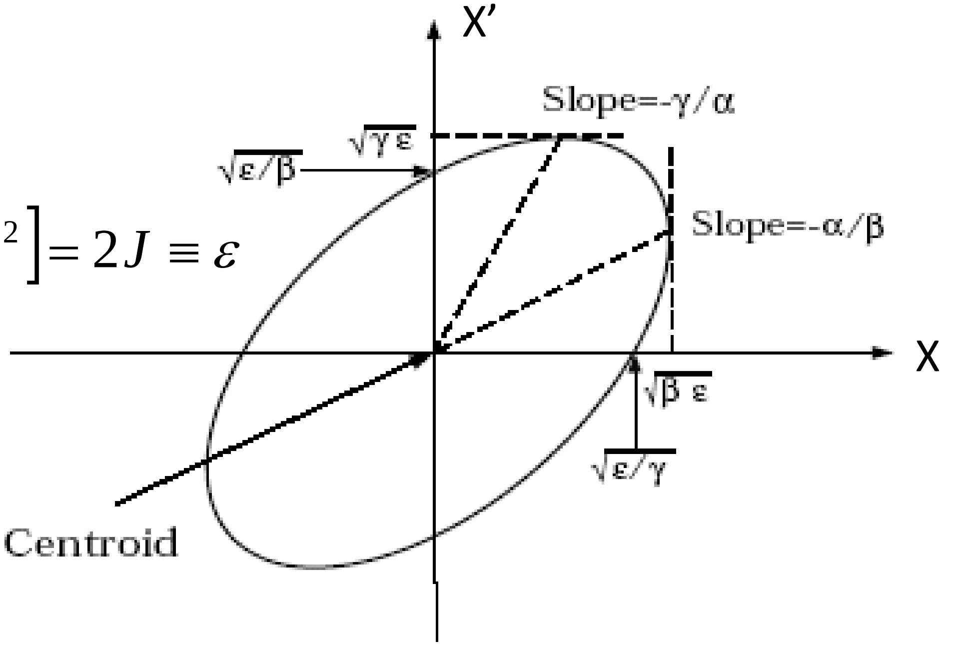
$$X = \sqrt{2\beta J}\cos\psi, \quad X' = -\sqrt{\frac{2J}{\beta}}(\sin\psi + \alpha\cos\psi)$$

$$P_X = \beta X' + \alpha X = -\sqrt{2\beta J}\sin\psi$$

(X, P_X) form a normalized phase space coordinates with $X^2+P_X^2=2\beta J$, here J is called **action**.

Courant-Snyder Invariant

$$\gamma X^2 + 2\alpha XX' + \beta X'^2 = \frac{1}{\beta}\left[X^2 + (\alpha X + \beta X')^2\right] = 2J \equiv \varepsilon$$



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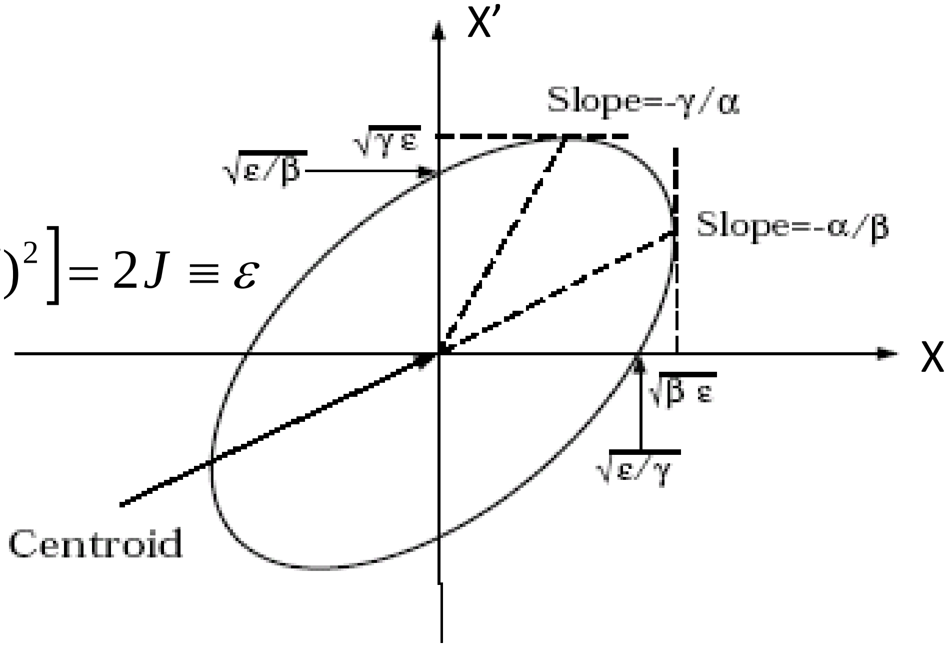


Emittance of a beam

$$\langle X \rangle = \int X \rho(X, X') dX dX', \quad \langle X' \rangle = \int X' \rho(X, X') dX dX',$$

$$\sigma_X^2 = \int (X - \langle X \rangle)^2 \rho(X, X') dX dX', \quad \sigma_{X'}^2 = \int (X' - \langle X' \rangle)^2 \rho(X, X') dX dX',$$

$$\sigma_{XX'} = \int (X - \langle X \rangle)(X' - \langle X' \rangle) \rho(X, X') dX dX' = r \sigma_X \sigma_{X'}$$

$$\varepsilon_{rms} = \sqrt{\sigma_X^2 \sigma_{X'}^2 - \sigma_{XX'}^2} = \sigma_X \sigma_{X'} \sqrt{1 - r^2}$$

The rms emittance is invariant in linear transport:

$$\frac{d\varepsilon^2}{ds} = 0$$

normalized emittance $\varepsilon_n = \varepsilon\beta\gamma$ is invariant when beam energy is changed.

Adiabatic damping – beam emittance decreases with increasing beam momentum, i.e. $\varepsilon = \varepsilon_n/\beta\gamma$, which applies to beam emittance in **linacs**.

In storage rings, the beam emittance **increases** with energy ($\sim\gamma^2$). The corresponding normalized emittance is proportional to γ^3.

The Gaussian distribution function

$$\rho(X, P_X) = \frac{1}{2\pi\sigma_X^2} e^{-(X^2 + P_X^2)/2\sigma_X^2}$$

$$\rho(\varepsilon) = \frac{1}{2\varepsilon_{rms}} e^{-\varepsilon/2\varepsilon_{rms}}$$

$\epsilon/\epsilon_{\mathrm{rms}}$	2	4	6	8
Percentage in 1D [%]	63	86	95	98
Percentage in 2D [%]	40	74	90	96

Effects of Linear Magnetic field Error

$$x'' + [K_x(s) + k(s)]x = \frac{b_0}{\rho}, \qquad y'' + [K_y(s) - k(s)]y = -\frac{a_0}{\rho}$$

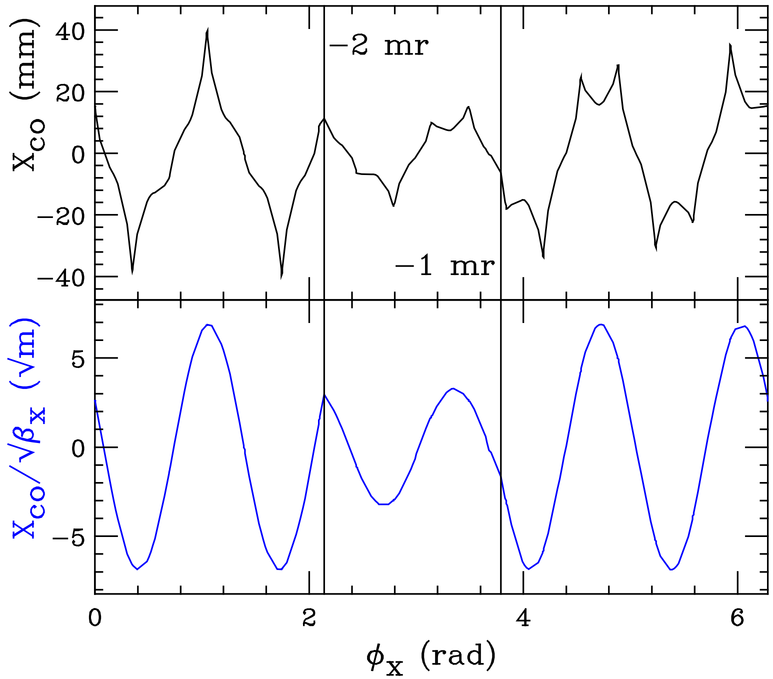
For a localized dipole field error: θ=ΔBℓ/Bρ

$$X'' + K_X(s)X = \theta\delta(s - s_0)$$

$$X_0 = \frac{\beta_0\theta}{2\sin\pi\nu}\cos\pi\nu,$$

$$X_0' = \frac{\theta}{2\sin\pi\nu}(\sin\pi\nu - \alpha_0\cos\pi\nu)$$

$$X_{co}(s) = G(s, s_0)\theta$$



$$G(s, s_0) = \frac{\sqrt{\beta(s_0)\beta(s)}}{2\sin\pi\nu}\cos[\pi\nu - |\psi(s) - \psi(s_0)|]$$

For a distributed dipole field error:

$$X_{co}(s) = \sqrt{\beta(s)} \sum_{k=-\infty}^{\infty} \frac{\nu^2 f_k}{\nu^2 - k^2} e^{jk\phi(s)}$$

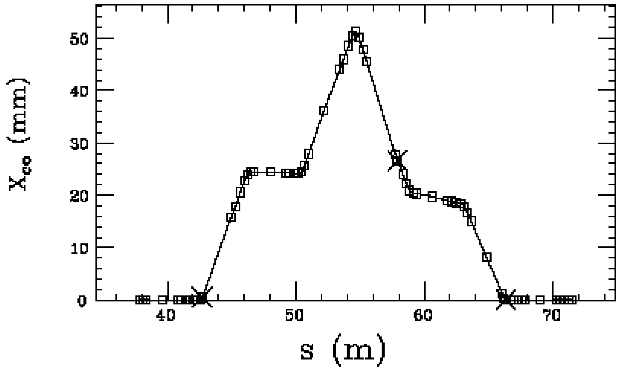
Where the field error is expanded in Fourier series $\left[\beta^{3/2}(\varphi)\frac{\Delta B(\varphi)}{B\rho}\right] = \sum_{k=-\infty}^{\infty} f_k e^{jk\varphi}$

$$f_k = \frac{1}{2\pi}\oint\left[\beta^{3/2}(\varphi)\frac{\Delta B(\varphi)}{B\rho}\right]e^{-jk\varphi}d\varphi = \frac{1}{2\pi\nu}\oint\left[\beta^{1/2}(\varphi)\frac{\Delta B(\varphi)}{B\rho}\right]e^{-jk\varphi}ds$$

$$\text{Sensitivity factor} \equiv \frac{\left\langle \left(X_{co}(s)\right)^2\right\rangle^{1/2}}{\theta_{rms}} \propto \sqrt{\beta(s)}$$

closed orbit bump: $X_{co}(s_f) = 0,\ X'_{co}(s_f) = 0$

$$\Delta x_{co}(s) = \left(\sqrt{\beta_x(s_k)\beta_x(s)}\sin(\Delta\psi_x(s))\right)\theta_k$$



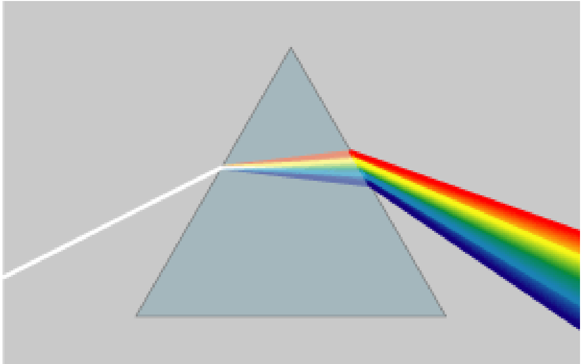
Orbit length change:

$$\Delta C = C - C_0 = \theta_0 \oint \frac{G_x(s,s_0)}{\rho} ds = D(s_0)\theta_0$$

$$\Delta C = \oint D(s_0)\frac{\Delta B_y(s_0)}{B\rho}ds_0$$

Off-momentum and dispersion

For different particle energy $\delta = \frac{p - p_0}{p_0}$



$$x = x_\beta + D\delta \qquad x' = x'_\beta + D'\delta$$

$$x''_\beta + K_x(s) x_\beta = 0, \qquad K_x(s) = \frac{1}{\rho^2} - K(s)$$

$$D'' + K_x(s) D = \frac{1}{\rho}$$

$$\begin{pmatrix} D(s_2) \\ D'(s_2) \end{pmatrix} = M(s_2|s_1) \begin{pmatrix} D(s_1) \\ D'(s_1) \end{pmatrix} + \begin{pmatrix} d \\ d' \end{pmatrix},$$

Extend the matrix representation to 3 by 3

$$\begin{pmatrix} D(s_2) \\ D'(s_2) \\ 1 \end{pmatrix} = \begin{pmatrix} M(s_2|s_1) & \bar{d} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} D(s_1) \\ D'(s_1) \\ 1 \end{pmatrix}.$$

For a pure dipole (K=0):

$$M = \begin{pmatrix} \cos\theta & \rho\sin\theta & \rho(1-\cos\theta) \\ -\frac{1}{\rho}\sin\theta & \cos\theta & \sin\theta \\ 0 & 0 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & L & \frac{1}{2}L\theta \\ 0 & 1 & \theta \\ 0 & 0 & 1 \end{pmatrix}$$

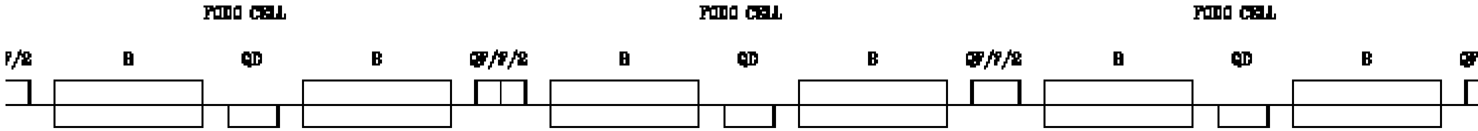
$$\theta \ll 1 \quad i.e. \quad L \ll \rho$$

For quadrupoles:

$$M(s, s_0) = \begin{pmatrix} \cos\sqrt{K}\ell & \frac{1}{\sqrt{K}}\sin\sqrt{K}\ell & 0 \\ -\sqrt{K}\sin\sqrt{K}\ell & \cos\sqrt{K}\ell & 0 \\ 0 & 0 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 0 \\ -1/f & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Defocusing
change K -> -K

FODO cell



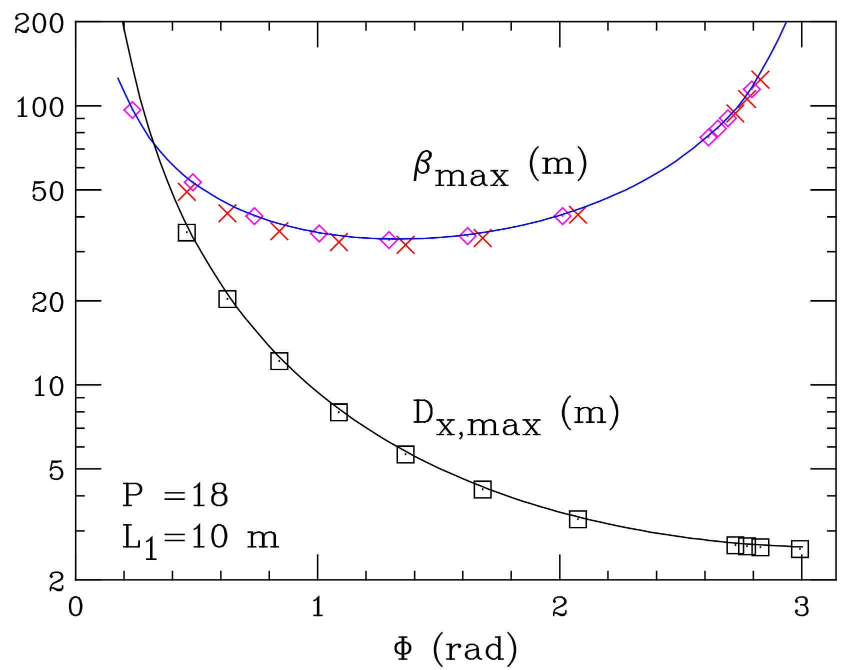
$$\mathbf{M} = \begin{pmatrix} 1 & 0 & 0 \\ -\frac{1}{2f} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & L & \frac{1}{2}L\theta \\ 0 & 1 & \theta \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ \frac{1}{f} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & L & \frac{1}{2}L\theta \\ 0 & 1 & \theta \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ -\frac{1}{2f} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Closed orbit condition:

$$\begin{pmatrix} D_F \\ D'_F \\ 1 \end{pmatrix} = \begin{pmatrix} 1-\frac{L^2}{2f^2} & 2L(1+\frac{L}{2f}) & 2L\theta(1+\frac{L}{4f}) \\ -\frac{L}{2f^2}+\frac{L^2}{4f^3} & 1-\frac{L^2}{2f^2} & 2\theta(1-\frac{L}{4f}-\frac{L^2}{8f^2}) \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} D_F \\ D'_F \\ 1 \end{pmatrix}$$

$$D_F = \frac{L\theta(1+\frac{1}{2}\sin\frac{\Phi}{2})}{\sin^2\frac{\Phi}{2}}, \quad D'_F = 0$$

$$\beta_{\max} = \frac{2L_1(1+\frac{L_1}{2f})}{\sin\Phi} = \frac{2L_1(1+\sin\frac{\Phi}{2})}{\sin\Phi}$$



Tune shift, or tune spread, due to chromatic aberration:

$$\Delta \nu_x = \left[-\tfrac{1}{4\pi} \oint \beta_x(s) K_x(s) ds \right] \delta \equiv C_x \delta, \quad C_x = d\nu_x / d\delta$$

$$\Delta \nu_y = \left[-\tfrac{1}{4\pi} \oint \beta_y(s) K_y(s) ds \right] \delta \equiv C_y \delta, \quad C_y = d\nu_y / d\delta$$

The chromaticity induced by quadrupole field error is called natural chromaticity. For a simple FODO cell, we find

$$\Delta \nu_x = \left[-\frac{1}{4\pi} \oint \beta_x(s) K_x(s) ds \right] \delta \approx -\frac{1}{4\pi} \sum \frac{\beta_{xi}}{f_i} \delta$$

$$C_{X,\mathrm{nat}}^{\mathrm{FODO}} = -\frac{1}{4\pi} N \left(\frac{\beta_{\max}}{f} - \frac{\beta_{\min}}{f} \right) = -\frac{\tan(\Phi/2)}{\Phi/2} \nu_X \approx -\nu_X$$

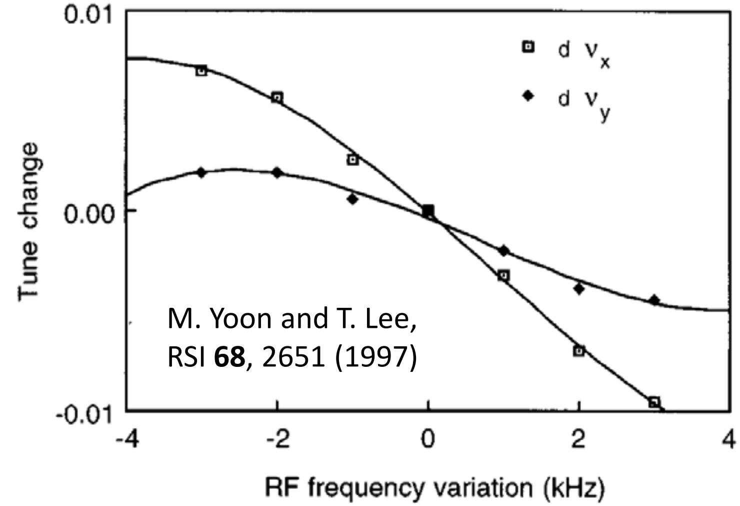
We define the specific chromaticity as $\xi_x = C_x / \nu_x, \quad \xi_y = C_y / \nu_y$

The **specific chromaticity is about −1 for FODO cells**, and can be as high as -4 for high luminosity colliders and high brightness electron storage rings.

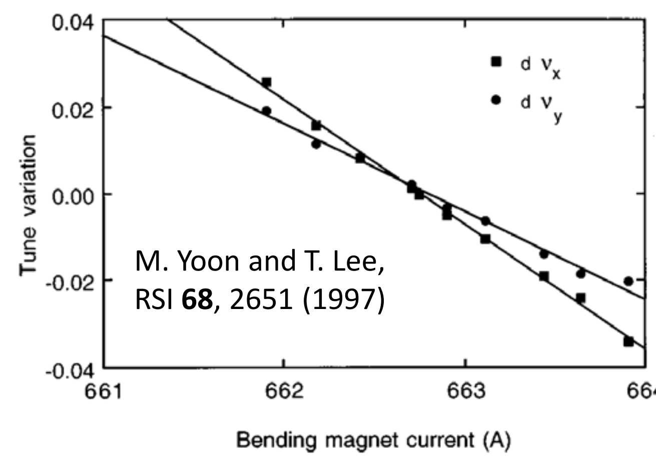
$$\sin \frac{\Phi}{2} = \frac{L_1}{2f} \qquad \beta_{\max} = \frac{2L_1(1+\sin(\Phi/2))}{\sin\Phi}, \quad \beta_{\min} = \frac{2L_1(1-\sin(\Phi/2))}{\sin\Phi}$$

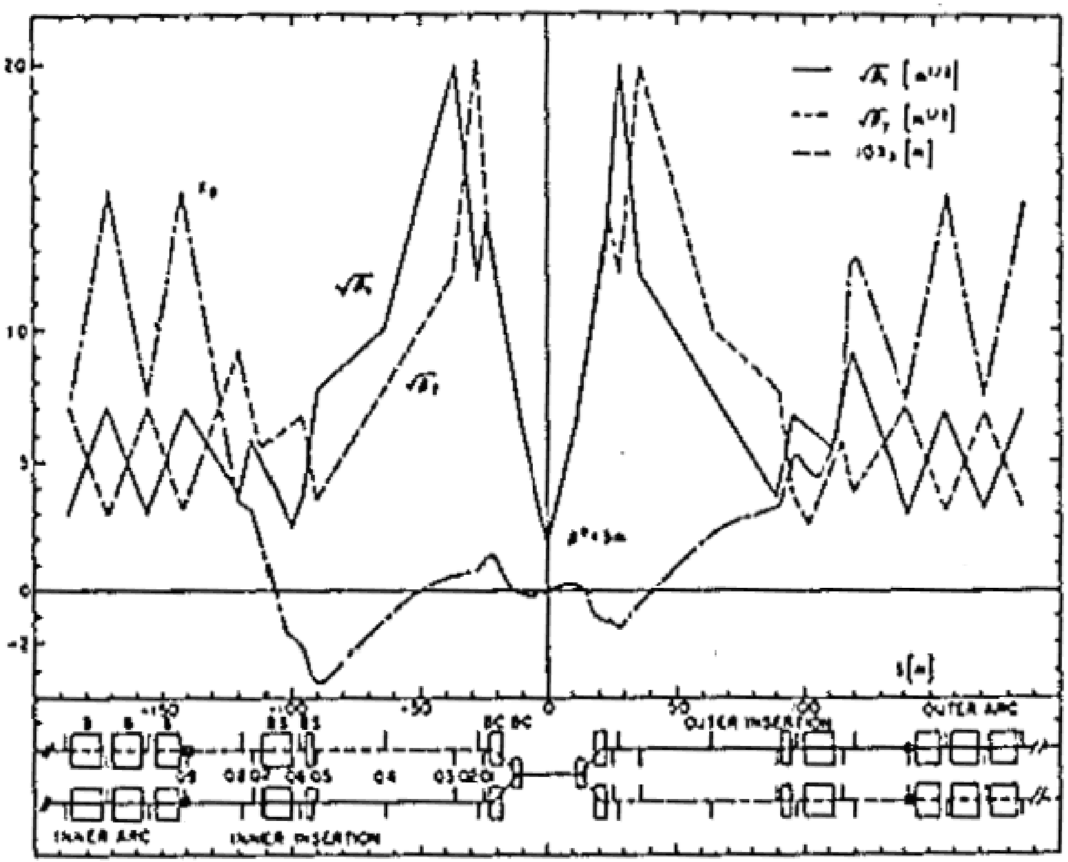
Chromaticity measurement:

The chromaticity can be measured by measuring the betatron tunes vs the rf frequency



The **chromaticity** can be obtained by measuring the tune variation vs the bending-magnet current at a **constant rf frequency**. May not apply for combined function magnets





Contribution of low β triplets in an IR to the natural chromaticity is

$$C_{total} = N_{IR} C_{IR} + C_{ARCs}$$

$$C_{IR} = -\frac{2\Delta s}{4\pi\beta^*} \approx -\frac{1}{2\pi}\sqrt{\frac{\beta_{\max}}{\beta^*}}$$

$$x''_\beta + (K_x(s) + K_2 D\delta)x_\beta = 0, \quad y''_\beta + (K_y(s) - K_2 D\delta)y_\beta = 0$$

$$x = x_\beta + D\delta$$

$$\Delta K_x(s) = K_2(s)D(s)\delta, \quad \Delta K_y(s) = -K_2(s)D(s)\delta$$

$$C_x = -\tfrac{1}{4\pi}\oint \beta_x(s)[K_x(s) - K_2(s)D(s)]ds$$
$$C_y = -\tfrac{1}{4\pi}\oint \beta_y(s)[K_y(s) + K_2(s)D(s)]ds$$

- In order to minimize their strength, the chromatic sextupoles should be located near quadrupoles, where $\beta_x D_x$ and $\beta_y D_x$ are maximum.
- A large ratio of β_x/β_y for the focusing sextupole and a large ratio of β_y/β_x for the defocussing sextupole are needed for optimal independent chromaticity control.
- The families of sextupoles should be arranged to minimize the systematic half-integer stopbands and the third-order betatron resonance strengths.

Resonances

- Parametric Resonances: $m\nu_{x,y}=\ell$, ℓ=integer.
- Coupling resonances:
  - ✓ Linear: $\nu_x-\nu_y = \ell$ – skew quadrupoles; solenoids; vertical closed orbit in sextupoles
  - ✓ Sum resonances: $m\nu_x+n\nu_y=\ell$: Order of resonance = m + n
  - ✓ Difference resonances: $m\nu_x-n\nu_y=\ell$

Nonlinear resonances: sextupole field

Hill's equations $$x'' + K_x(s)x = \frac{\Delta B_y}{B\rho}, \quad y'' + K_y(s)y = -\frac{\Delta B_x}{B\rho}$$

$$\Delta B_y + j\Delta B_x = B_0 \sum_n (b_n + ja_n)(x + jy)^n,$$

~~$B_y = B_0 b_0, \quad B_x = B_0 a_0,$ Dipole field error~~

$B_y = B_0 b_1 x, \quad B_x = B_0 b_1 y,$ Quadrupole field error

~~$B_y = -B_0 a_1 y, \quad B_x = B_0 a_1 x,$ Skew Quadrupole field error~~

$B_y = B_0 b_2 (x^2 - y^2), \quad B_x = 2B_0 b_2 xy,$ Sextupole field

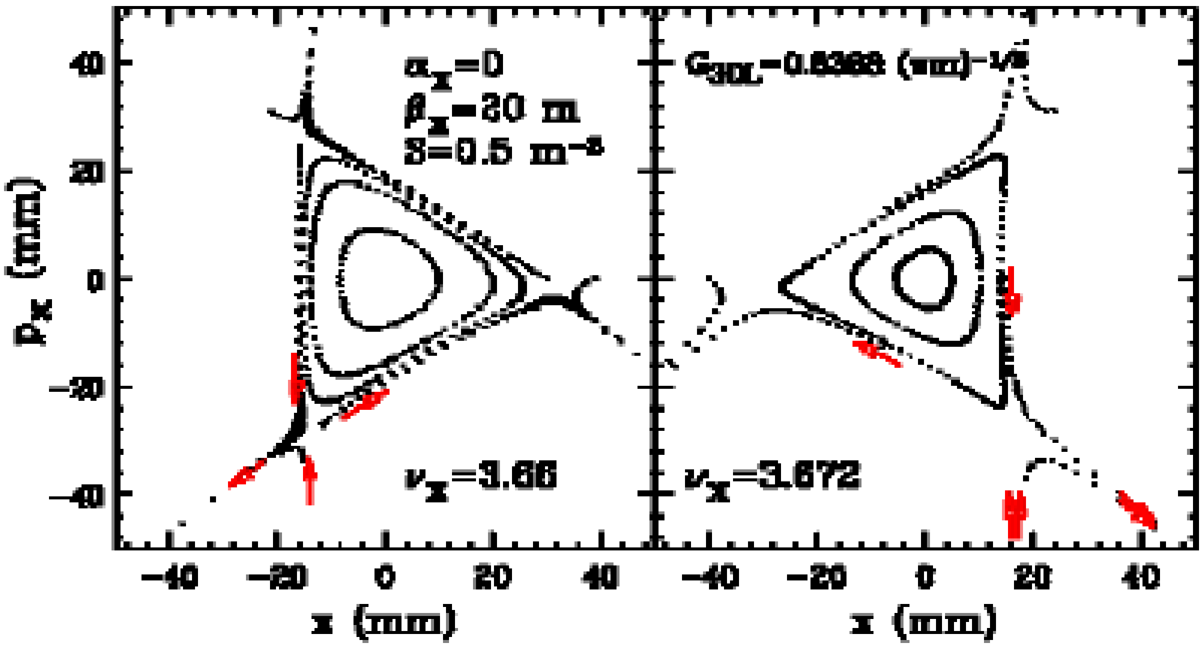
$B_y = -2B_0 a_2 xy, \quad B_x = B_0 a_2 (x^2 - y^2),$ Skew Sextupole field

$$x'' + K_x(s)x = \frac{1}{2}S(s)(x^2 - y^2), \qquad y'' + K_y(s)y = -S(s)xy \qquad S(s) = \frac{B_2}{B\rho}$$

$$x'' + K_x(s)x = \frac{1}{2}S(s)(x^2 - y^2), \qquad y'' + K_y(s)y = -S(s)xy$$

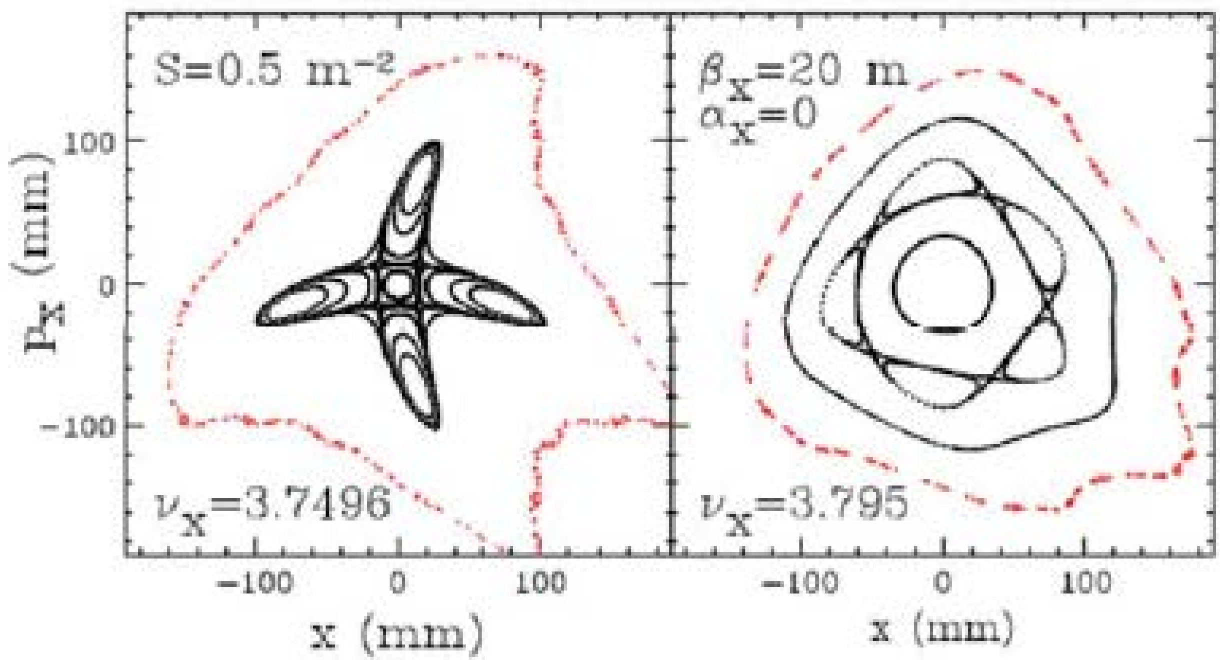
$$\Delta x' = \frac{1}{2}\int S(s)(x^2 - y^2)ds = \frac{1}{2}\overline{S}(x^2 - y^2), \qquad \Delta y' = -\int S(s)xyds = -\overline{S}xy$$

Thus particle motion in existence of sextupole fields can be tracked thru a combination of linear transfer map $M(s_1,s_2)$ and a local kick in the x' which is proportional to the integrated sextupole field strength.

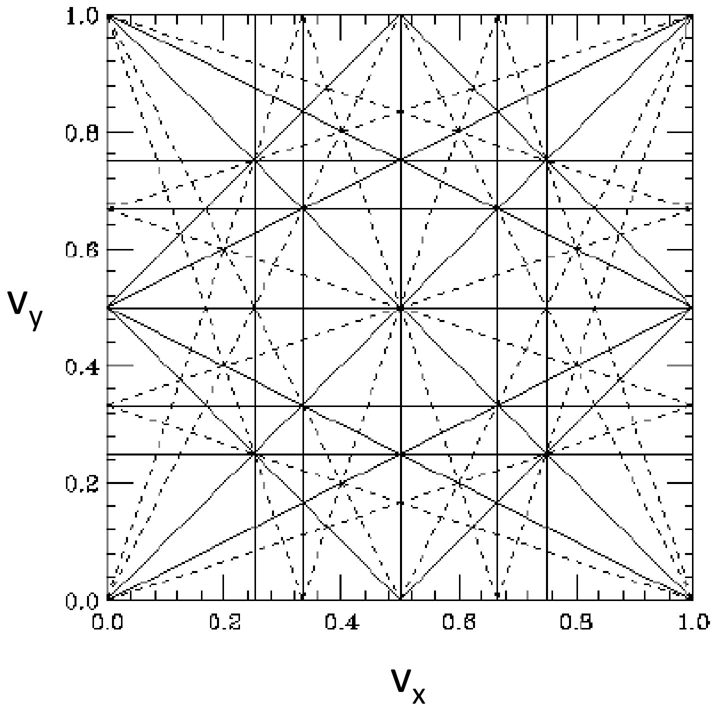


Normalized phase space plots at a tune below (left) and above (right) a third order resonance driven by a single sextupole magnet. Four particles with various initial actions were used in the tracking. The integrated sextupole strength is S = 0.5 m^{-2} with lattice parameters β_x = 20 m and α_x = 0.

It appears that sextupoles will not produce resonances higher than the third order. However, strong sextupoles are usually needed to correct chromatic aberration. Concatenation of strong sextupoles can generate high-order resonances such as $4\nu_x$, $2\nu_x \pm 2\nu_y$, $4\nu_y$, $5\nu_x$,…etc. The figure below shows the phase space plots of the single sextupole model at ν_x = 3.7496 and ν_x = 3.795, i.e. a single sextupole can also drive the 4th and 5th order resonances. The largest phase space map marks the boundary of stable motion.

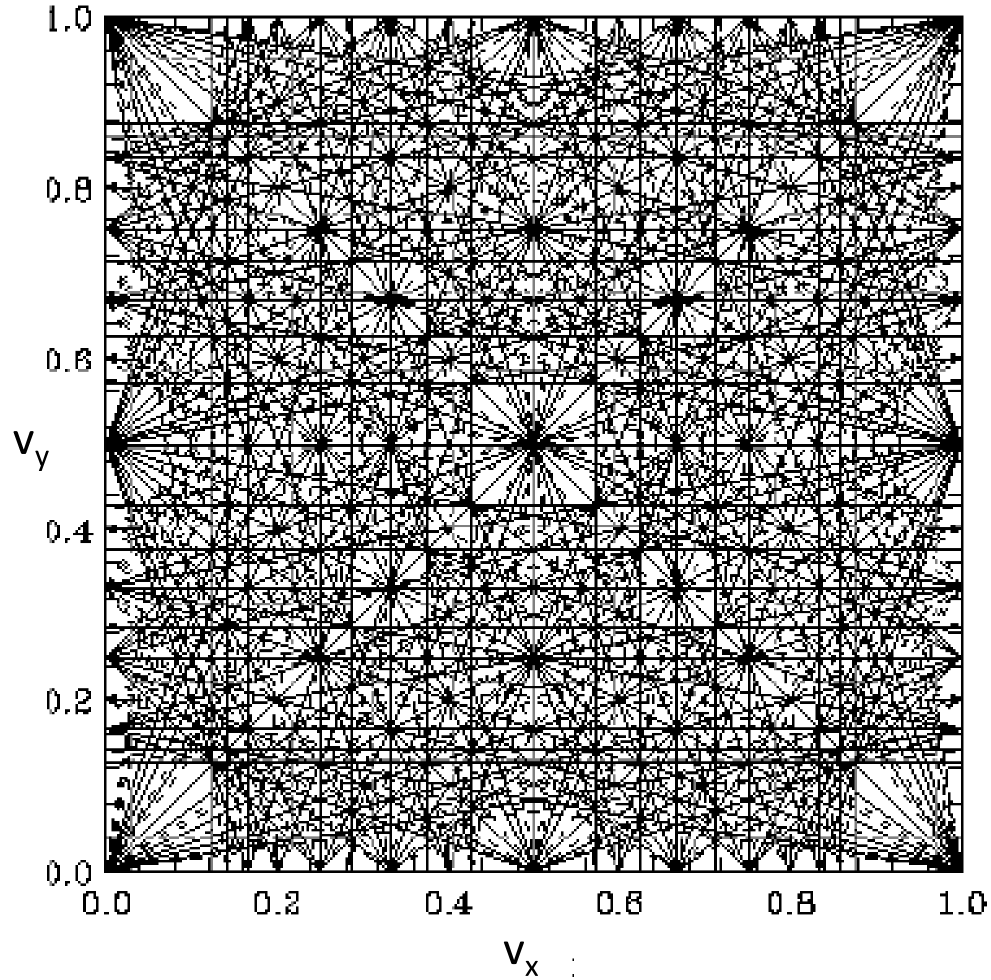


Resonance lines in tune space



Up to 4th order

Up to 8th order



Lattice Design Strategy

Based on our study of linear betatron motion, the lattice design of accelerator can be summarized as follows. The lattice is generally classified into three categories: low energy booster, collider lattice, and low-emittance lattice storage rings.

- The betatron tunes should be chosen to avoid systematic integer and half-integer stopbands and systematic low-order nonlinear resonances; otherwise, the stopband width should be corrected.
- The betatron amplitude function and the betatron phase advance between the kicker and the septum should be optimized to minimize the kicker angle and maximize the injection or extraction efficiency.
- Local orbit bumps can be used to alleviate the demand for a large kicker angle. Furthermore, the injection line and the synchrotron optics should be properly “matched” or “mismatched” to optimize the emittance control.
- To improve the slow extraction efficiency, the β value at the (wire) septum location should be optimized. The local vacuum pressure at the high-β value locations should be minimized to minimize the effect of beam gas scattering.

- The chromatic sextupoles should be located at high dispersion function locations. The focusing and defocusing sextupole families should be located in regions where $\beta x \gg \beta y$, and $\beta x \ll \beta y$ respectively in order to gain independent control of the chromaticities.
- It is advisable to avoid the transition energy for low to medium energy synchrotrons in order to minimize the beam dynamics problems during acceleration.

Besides these design issues, problems regarding the dynamical aperture, nonlinear betatron detuning, collective beam instabilities, rf system, vacuum requirement, beam lifetime, etc., should be addressed.