PHY 554. Homework 2.

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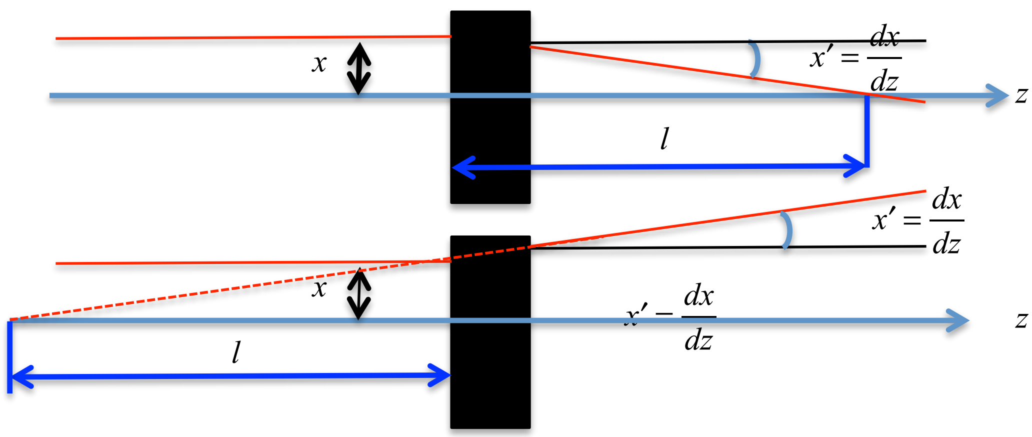
HW 1 (5 point): Let's first determine an effective focal length, *F*, of the of a paraxial (e.g. small angles!) focusing object (a black-box) as ratio between a parallel displacement of trajectory at its entrance to corresponding change of the angle at its exit (see figure below):

$$F = -\frac{x}{x'}; x' \equiv \frac{dx}{dz}$$

see figure below for



For completeness, the distance from the entrance to the object to the trajectory crossing the axis, *l*, in general is not equal to the focal length. In beam optics this is frequently, but not correctly, referred as astigmatism – in contrast, the astigmatism is defined as dependence of the focal strengths on the direction of propagation of the ray (particle).
Let consider a doublet of two thin lenses: a focusing (*F*) and defocusing (*D*) lenses with equal but opposite in sign focal length F with center separated by distance L as in Fig. 1.

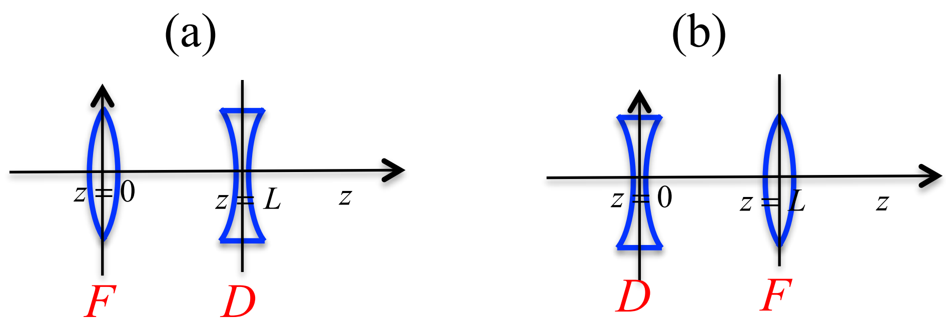


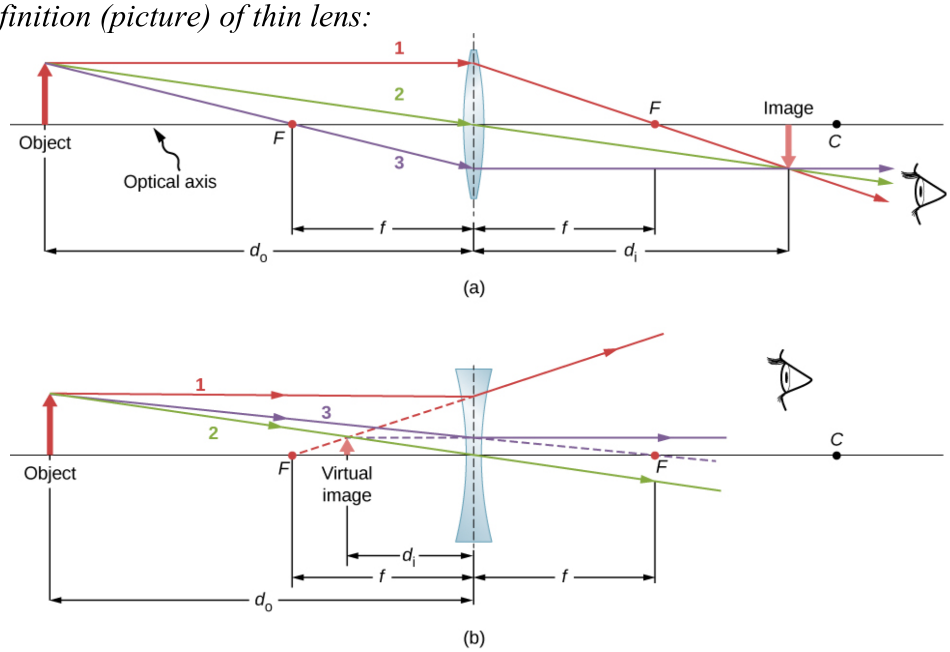
Fig.1. Two combinations of a doublet: *FD* and *DF*.

1. (3 points) Show through a calculation of the ray trajectory that the focal lengths of *FD* and *DF* doublets are equal and given by following expression:

$$F_{eff} = \frac{F^2}{L}$$

2. (2 points) Determine location of the ray crossing the axis and find their difference between *FD* and *DF* doublets – this indeed would be an astigmatism of doublet built from two quadruples.

P.S. Definition (picture) of thin lens:



HW 2 (2 points): Spectral brightness (sometimes called brilliance) of a light source is defined as

$$B = \frac{dN_{ph}}{dtd\Omega dA\left(d\lambda / \lambda\right)} = \frac{dN_{ph}}{dtd\Omega dA\left(d\omega / \omega\right)};$$

where $\frac{dN_{ph}}{dt}$ is the number of photons per second with the spectral bandwidth $d\omega / \omega$ radiated from an area dA into the solid angle $d\Omega$. The units used for brightness are expressed in photons per second

$$\left[B\right] = \frac{photons}{\sec \cdot mm^2 \cdot mrad^2 \left(10^{-3} d\lambda / \lambda\right)}$$

As an exercise, calculate spectral brightness of NdYAG laser with average power of 10 W, wavelength of λ=1.064 μm, Bandwidth of $\Delta\omega = 700\ GHz$ and with diffraction limited spot size and angular spread:

$$\Delta x \cdot \Delta \theta_x = \frac{\lambda}{4\pi}; \Delta y \cdot \Delta \theta_y = \frac{\lambda}{4\pi}.$$

HW 3 (3 points): In a fixed Cartesian coordinates for a trajectory with $\frac{dz}{dt} \neq 0$ of a particle moving in magnetic field $\vec{B} = \hat{x}B_x + \hat{y}B_y + \hat{z}B_z$ equation for its trajectory can be written in terms of z as independent variable:

$$\frac{d^2x}{dz^2} = \frac{e}{p}\sqrt{1 + x'^2 + y'^2}\left(y'B_z - (1 + x'^2)B_y + x'y'B_x\right);$$

$$\frac{d^2y}{dz^2} = -\frac{e}{p}\sqrt{1 + x'^2 + y'^2}\left(x'B_z - (1 + y'^2)B_x + x'y'B_y\right);$$

$$x' \equiv \frac{dx}{dz}; y' \equiv \frac{dy}{dz};$$

where e is the particle's charge and $p = \gamma m\mathrm{v}$ is its relativistic momentum.

Hint: consider constants of motion in a magnetic field.