3D cooling
with PCA μ-bunching CeC

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Join CeC meeting

October 15, 2021

Why am I presenting this again?

- In original "*Coherent Electron Cooling*, PRL **102**, 2008" paper with Derbenev we assumed that electron and hadron beams will be separated and coupling to transverse degrees of freedom will be done using chromatic transport either for electron or hadrong beam
- With non-zero R_{15}, R_{25} time of flight will depend on x and x' of particle

$$\Delta t = \frac{R_{25}x - R_{15}x'}{v_o}$$

which is connection with nonzero hadron beam D,D' results in re-distribution of cooling decrement. This scheme can be used for any CeC scheme, including PCA-based, when beam's separation is included

- But one of main attractions of PCA the it can be used without separating electron and hadron beams and, therefore, can save a lot of money
- Without beam's separation $R_{15}=0$; $R_{25}=0$ and I was asked how transverse cooling can be achieved in such scheme. While this was presented at number of CeC X reviews and COOL'13 and COOL'19 workshop, not everybody were there, and I decided to present it at this meeting to close this gap

Short answer

- The first step is to demonstration of longitudinal cooling
- Then use horizontal dispersion to connect to horizontal degree of freedom
- Vertical degree of freedom will be cooled by coupling to the horizontal degree of freedom
- Coupling to horizontal degree of freedom is achieved by adjusting trip time (change $\sim 10^{-4}$ in e-beam energy or using weak chicane to delay electrons) and horizontal offset between ion and electron beams in the CeC kicker
- The theory of 3D CeC cooling, including re-distribution of cooling decrements, was developed in 2007 and did not change since then (*these are formulae in next slide*)
- In 2007, we also introduced - *additional to IBS* - diffusion caused by amplified imprint of neighboring hadrons and the electron beam noise. It is used for for self-consistent Fokker-Plank equation describing evolutions of hadron beam with CeC (*not topic for today*)

Linearized case: cooling decrements

We will neglect transverse kicks, which are very weak in CeC

Sorry for repeating trivial formulae from my AP class

Only energy kick $\quad \Delta x_6 = \dfrac{\delta E_h}{E_o} = const - \sum_{i=1}^{6} \zeta_i \cdot x_i$

$$X = \frac{1}{2}\sum_{k=1}^{3}\left(a_k Y_k(s) e^{i\psi_k} + c.c.\right); \quad \mathbf{Y}_{k=1,2} = \begin{pmatrix} Y_{k\beta} \\ -Y_{k\beta}^T S D \\ 0 \end{pmatrix}; \quad \mathbf{Y}_3 \cong \frac{1}{\sqrt{\Omega}}\begin{pmatrix} D \\ -i\Omega \\ 1 \end{pmatrix}; \quad Y_{k\beta} = \begin{bmatrix} y_{k1} \\ y_{k1} \\ y_{k2} \\ y_{k4} \end{bmatrix}; \quad D = \begin{bmatrix} D_x \\ D'_x \\ D_y \\ D'_y \end{bmatrix};$$

$$\langle \Delta a_k \rangle = -\xi_k a_k \to a_k = a_{k0} e^{-\nu \xi_k} \qquad \mathrm{Re}\,\xi_{(1,2)} = -\frac{i}{2}\left(Y_{(1,2)\beta}{}^T S D\right)^* \sum_{i=1}^{4} y_{(1,2)i} \cdot \zeta_i; \quad \xi_s = \mathrm{Re}\,\xi_3 = \frac{1}{2}\left(\zeta_6 + \sum_{i=1}^{4} D_i \cdot \zeta_i\right),$$

No x-y coupling

$$Y_{1\beta} \equiv Y_x = \begin{bmatrix} w_x \\ w'_x + \dfrac{i}{w_x} \\ 0 \\ 0 \end{bmatrix}; \quad Y_{2\beta} \equiv Y_y == \begin{bmatrix} 0 \\ 0 \\ w_y \\ w'_y + \dfrac{i}{w_y} \end{bmatrix}; D = \begin{bmatrix} D \\ D' \\ 0 \\ 0 \end{bmatrix};$$

$$\beta_{x,y} = w_{x,y}^2; \quad \alpha_{x,y} = -w'_{x,y} w_{x,y}$$

$$\xi_x = \mathrm{Re}\,\xi_1 = -\left(D\zeta_1 + D'\zeta_2\right); \quad \xi_s = \xi_6 - \xi_x.$$

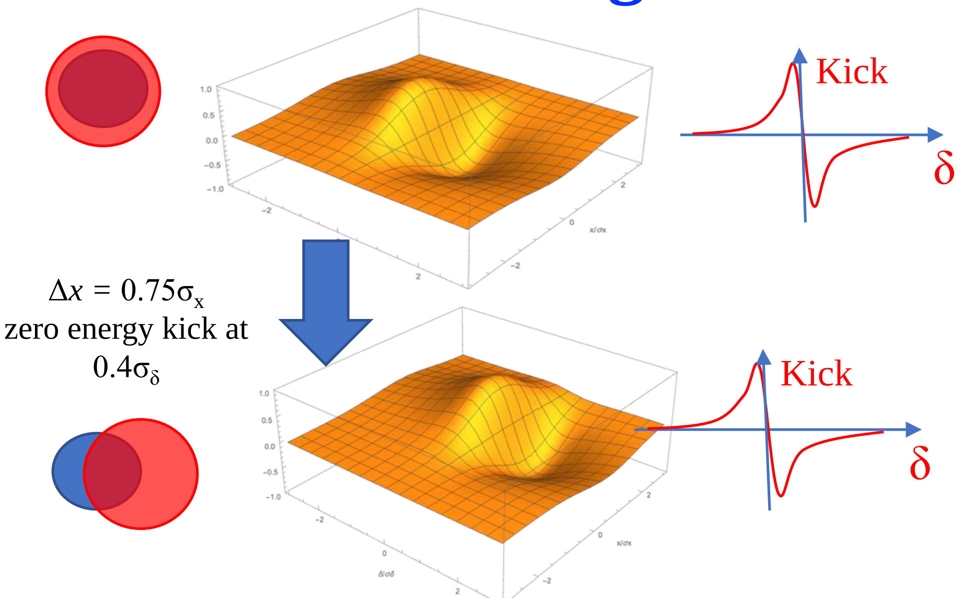
Qx-Qy resonance

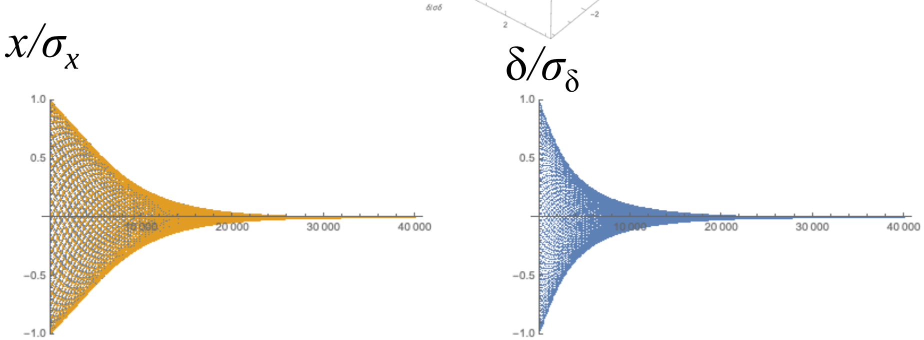
$$Y_1 = \frac{1}{\sqrt{1+|\alpha|^2}}\left(Y_x + \alpha Y_y\right); \quad Y_2 = \frac{1}{\sqrt{1+|\alpha|^2}}\left(-\alpha^* Y_x + Y_y\right)$$

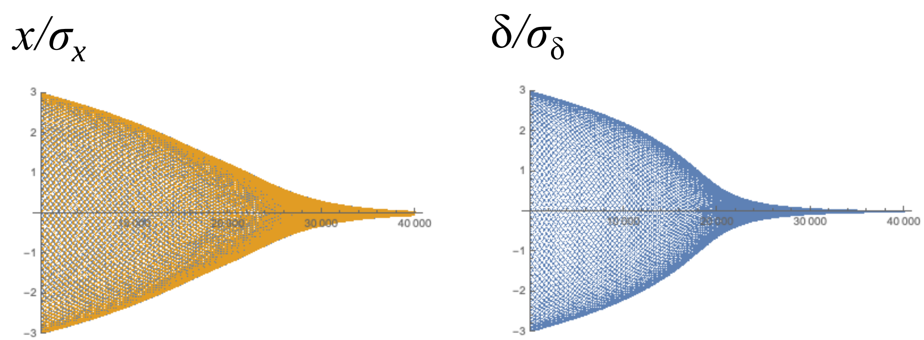
$$\mathrm{Re}\,\xi_1 = -\frac{D\zeta_1 + D'\zeta_2}{1+|\alpha|^2}; \quad \mathrm{Re}\,\xi_2 = -|\alpha|^2 \frac{D\zeta_1 + D'\zeta_2}{1+|\alpha|^2}.$$

Can use a non-achromatic transport (time of flight dependence) or transverse beam separation to couple longitudinal and transverse cooling

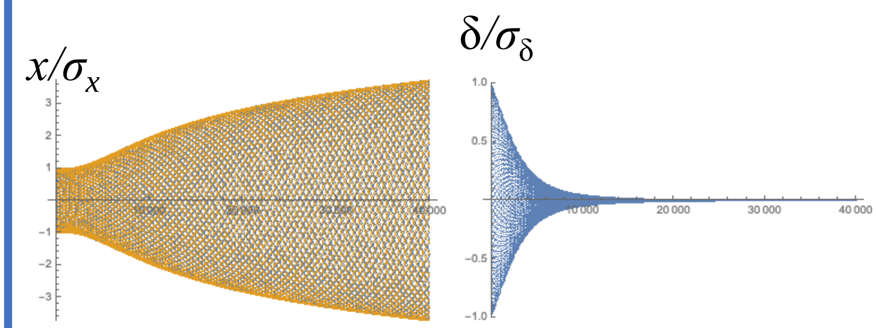
Distribution of cooling between longitudinal and transverse degrees of freedom – real kick



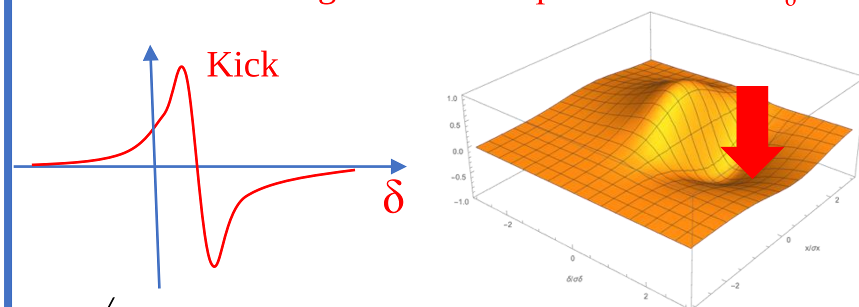


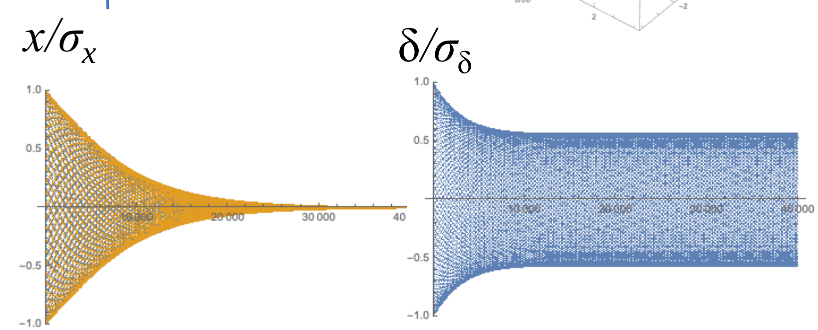


Wrong sign of displacement
$\Delta x = -0.75\sigma_x$



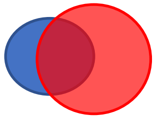
Excessive shifting of zero-kick point to $\delta = 0.6\sigma_\delta$

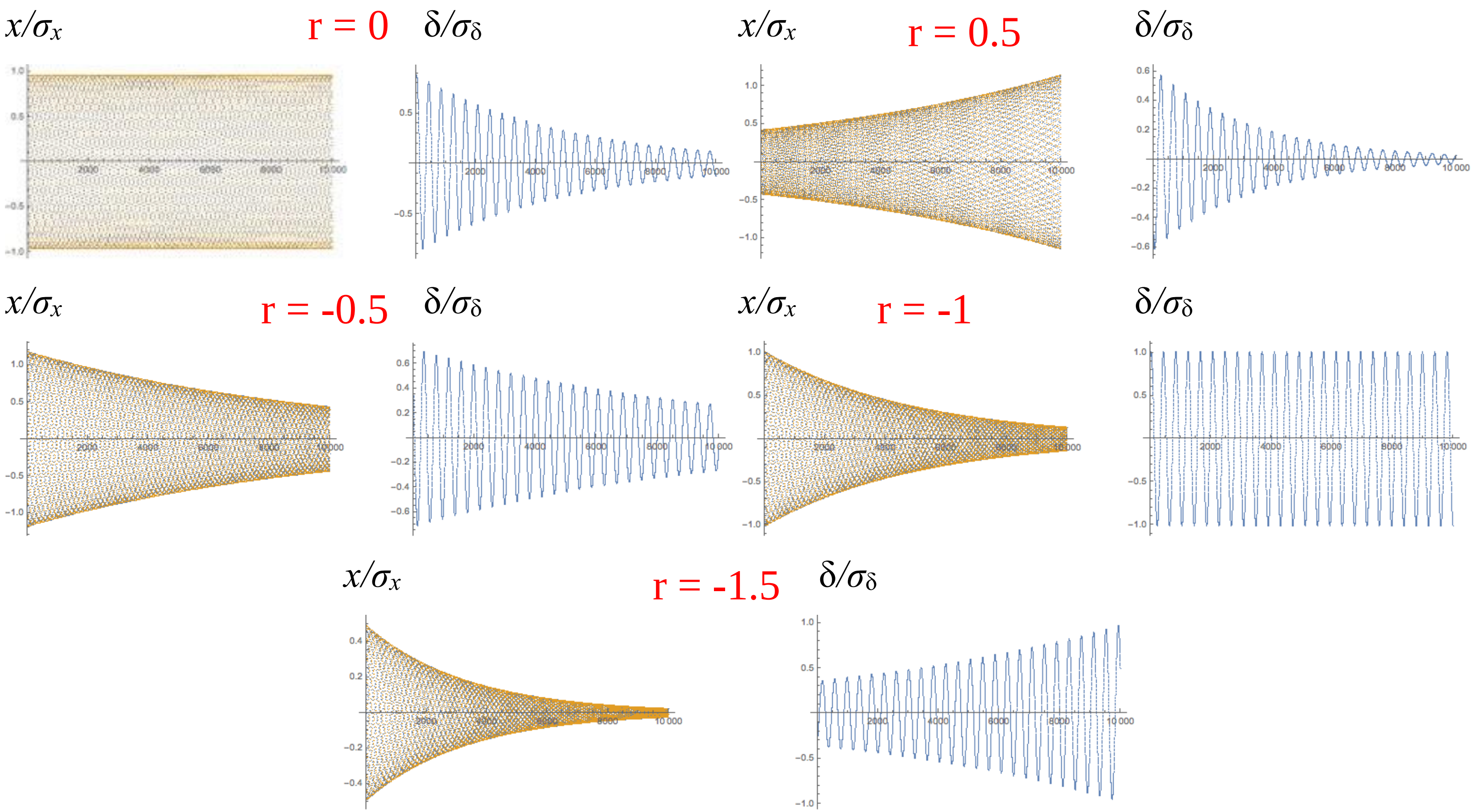




Distribution of cooling between longitudinal and transverse degrees of freedom – linearized kick

$$\frac{\delta E_h}{E_o} = const - \zeta_1 x - \zeta_6 \frac{E_h - E_o}{E_o}; \quad r = \Delta\zeta_1/\zeta_6$$





Summary

1. Conceptually there is no challenges in redistributing cooling decrements between longitudinal and horizontal degrees of freedom;
2. Cooling of vertical oscillations via coupling with horizontal oscillations was demonstrated in RHIC – both directions were cooled by a horizontal stochastic cooler. Hence, we need to repeat the fit