Brookhaven National Laboratory

U.S. DEPARTMENT OF ENERGY

PHY564 | Advanced Accelerator Physics

Lecture #9:
Linear accelerators & RF systems for storage rings

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Outline:

- Particle acceleration: from DC to SRF accelerators
- RF fundamentals: transmission lines & cavities
- Normal & Superconducting RF: figures of merit

References:

- This lecture is an adaptation of the material taught by S. Belomestnykh, S. Verdú-Andrés, S. Posen and myself at the USPAS'22 "RF Superconductivity for Particle Accelerators" and by V. Litvinenko in PHY564'20 "Advanced Accelerator Physics"
- For more information on RF fundamentals, please refer to the following literature:

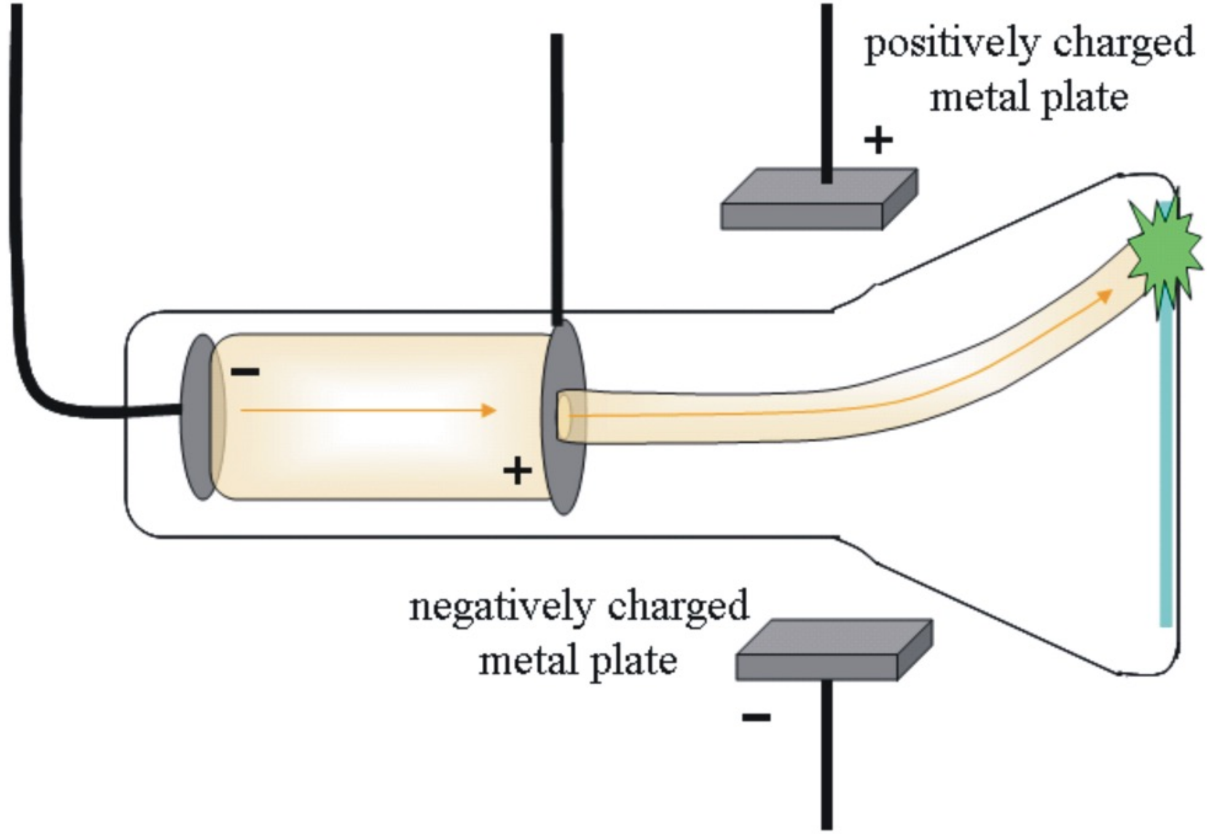
[1] D. M. Pozar, "Microwave Engineering", Wiley (2005).
[2] R. E. Collin, "Field Theory of Guided Waves", Wiley-IEEE Press (1991).
[3] S. J. Orfanidis, "Electromagnetic Waves and Antennas" (2016), http://eceweb1.rutgers.edu/~orfanidi/ewa/
[4] F. Caspers, "RF Course", Joint Universities Accelerator School (2019), https://indico.cern.ch/event/779575/contributions/3244508/attachments/1797628/2931054/JUAS2019_RF_lecture_rev1.pdf
[5] T. Wangler, "RF Linear Accelerators", Wiley (2008)
[6] H. Padamsee and M. Reiser. "RF Superconductivity." Wiley (1999).

Particle acceleration: where should we start?

A charged particle can only gain or lose energy by its interaction with electric field $\vec{E}$:

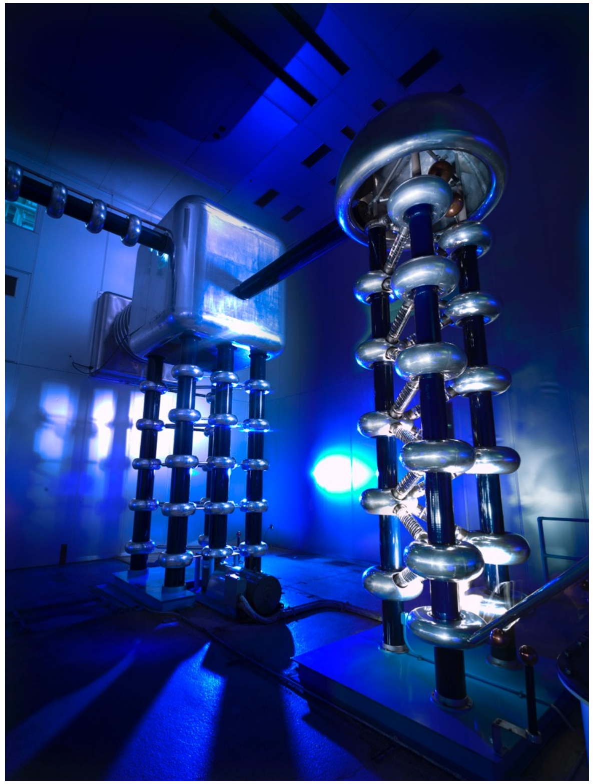
$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

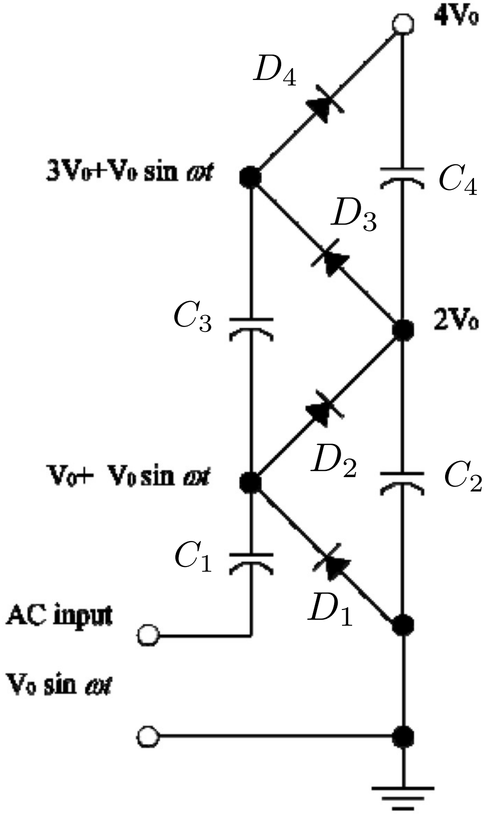
- Late 1800s: Cathode Ray Tube (CRT)



Particle acceleration: where should we start?

- 1932: Cockroft-Walton electrostatic accelerator – an electric circuit that generates a high DC voltage from a low-voltage AC or pulsing DC.





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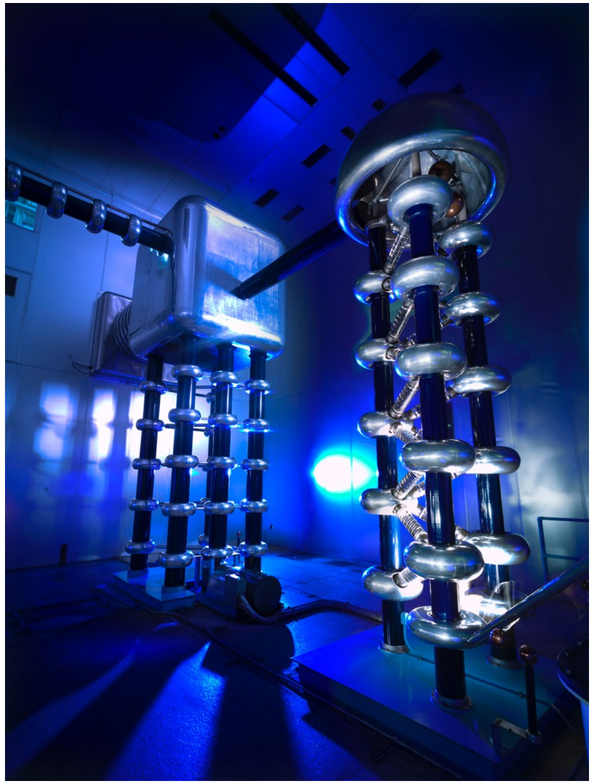

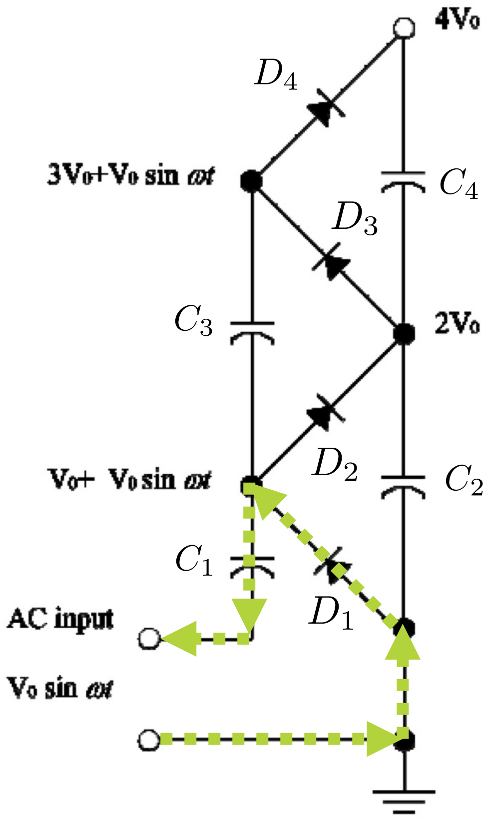

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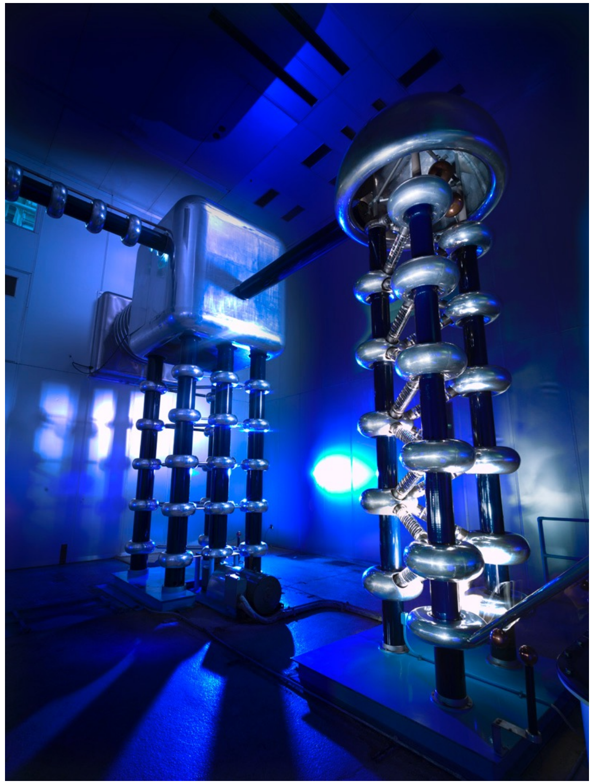

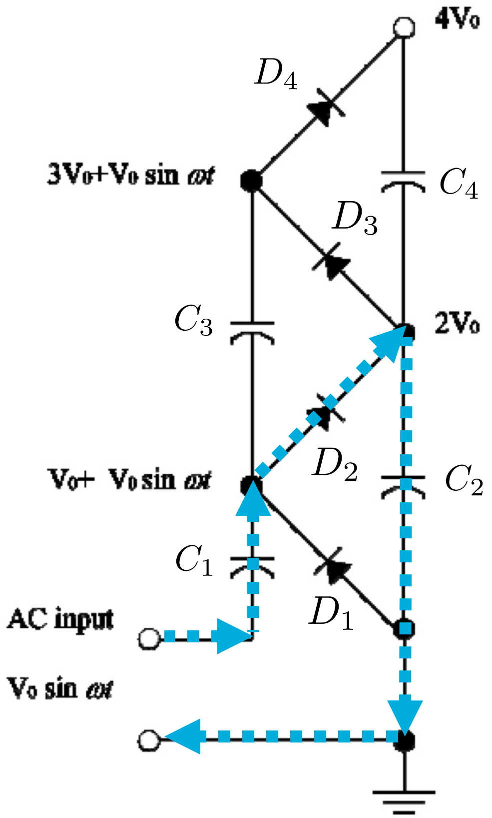

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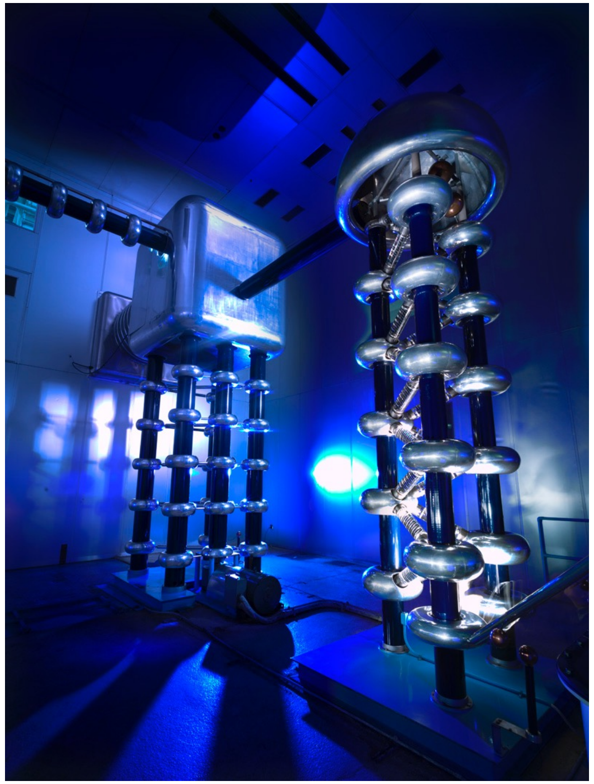
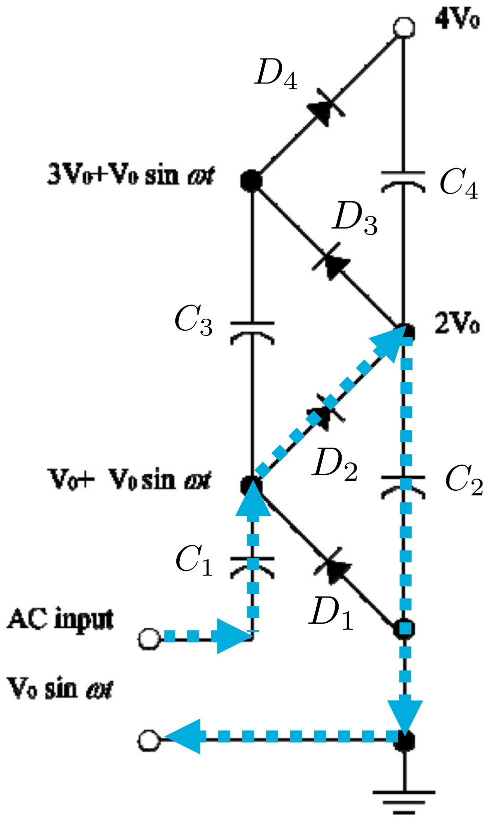
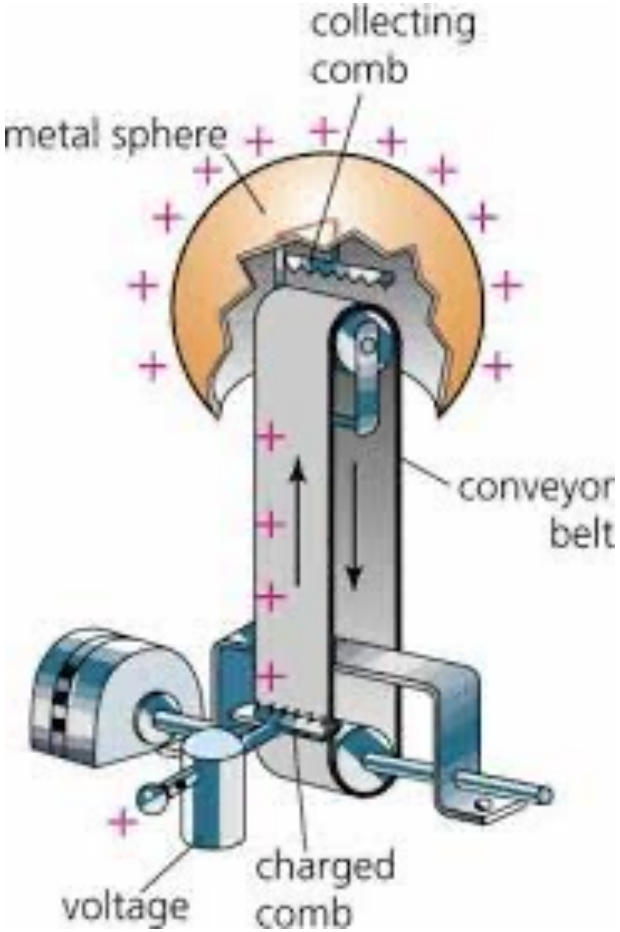




- 1931: Van de Graaff and tandem accelerators



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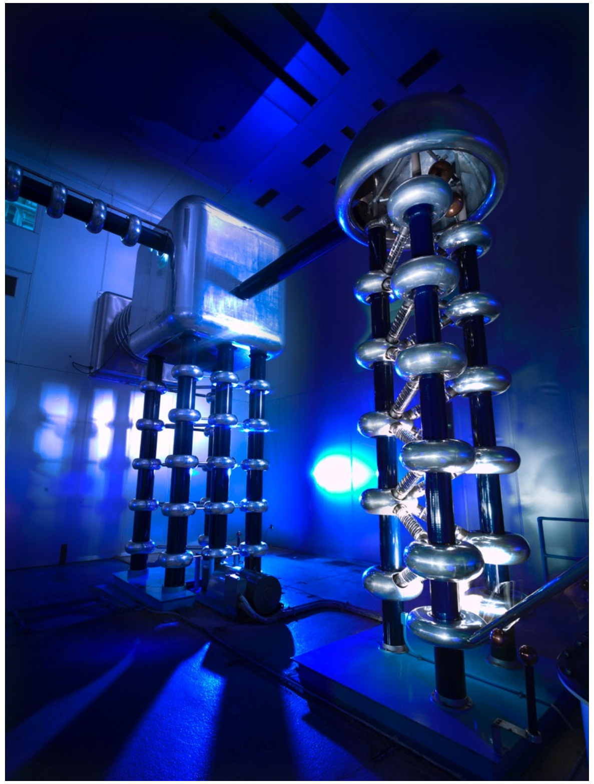

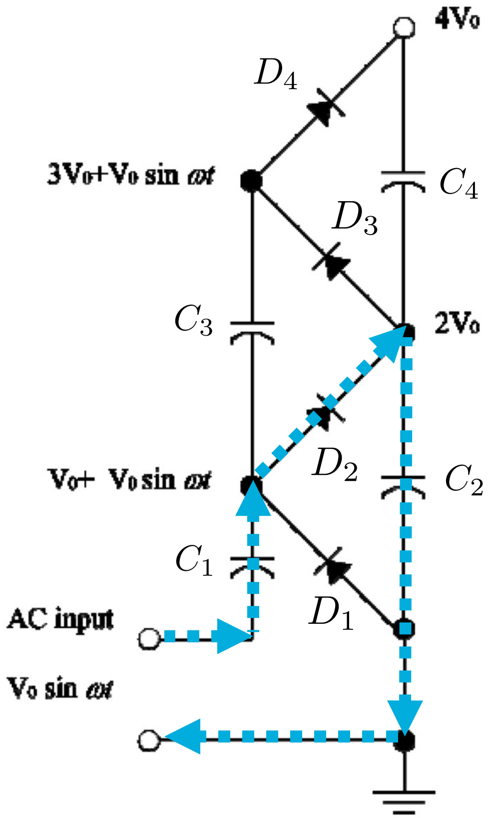

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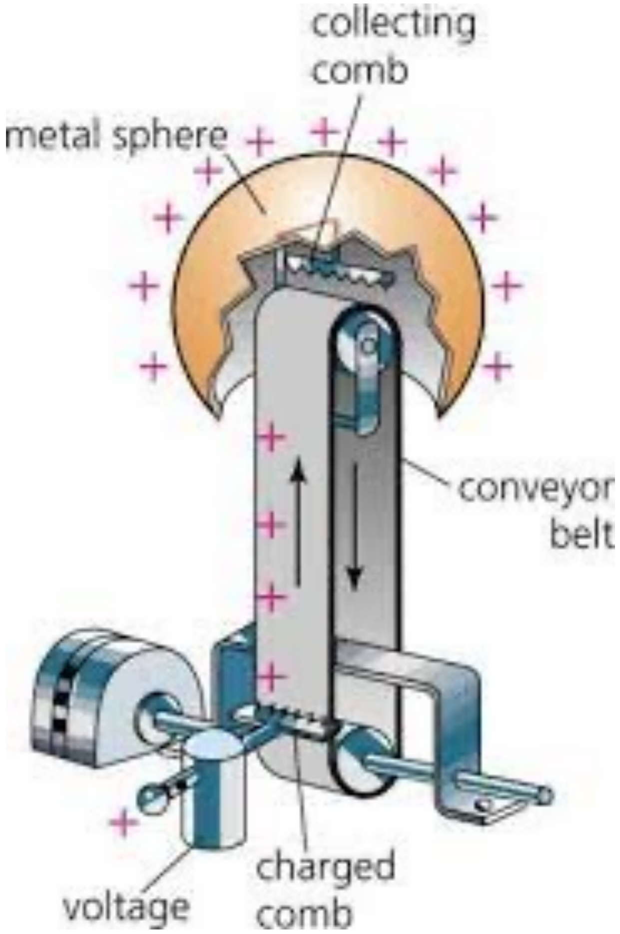
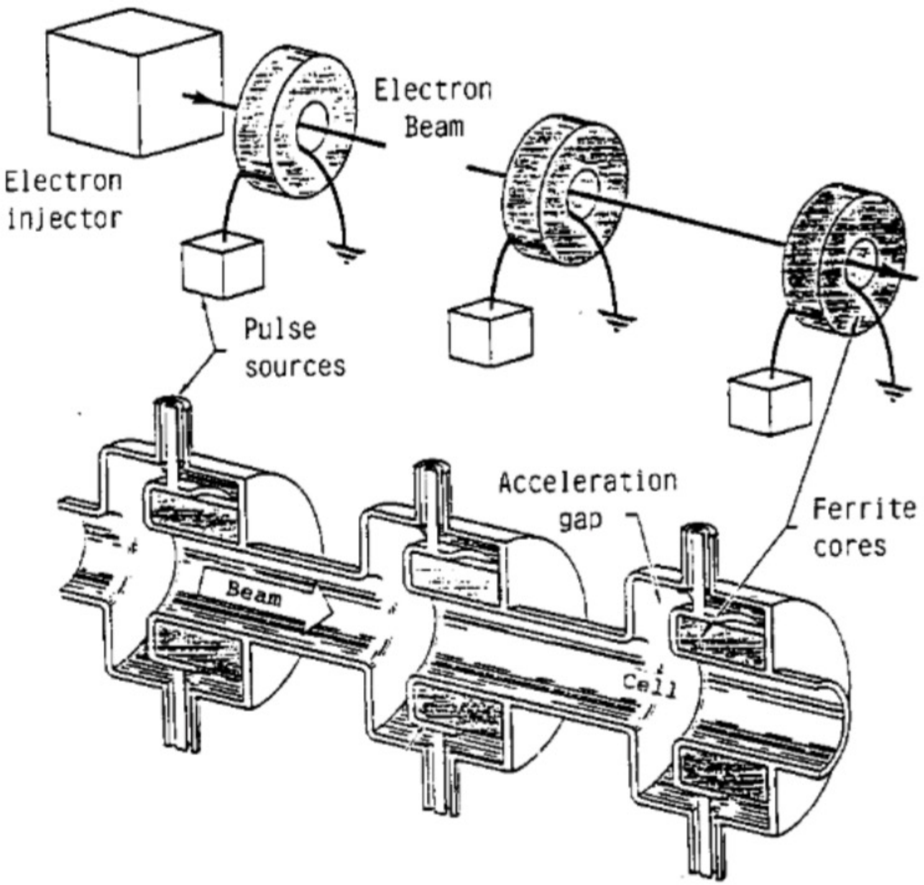


The main limitation of electrostatic accelerators is that the maximum energy obtainable cannot exceed the product of the charge and the potential difference that can be maintained, and in practice this potential difference is limited by **electric breakdown** to no more than a few tens of megavolts.

Alternative: induction accelerators

Let's use the electric fields associated with changing magnetic flux!

$$\oint \vec{E} \cdot d\vec{l} = -\frac{\partial}{\partial t}\left(\int \vec{H} \cdot d\vec{s}\right)$$



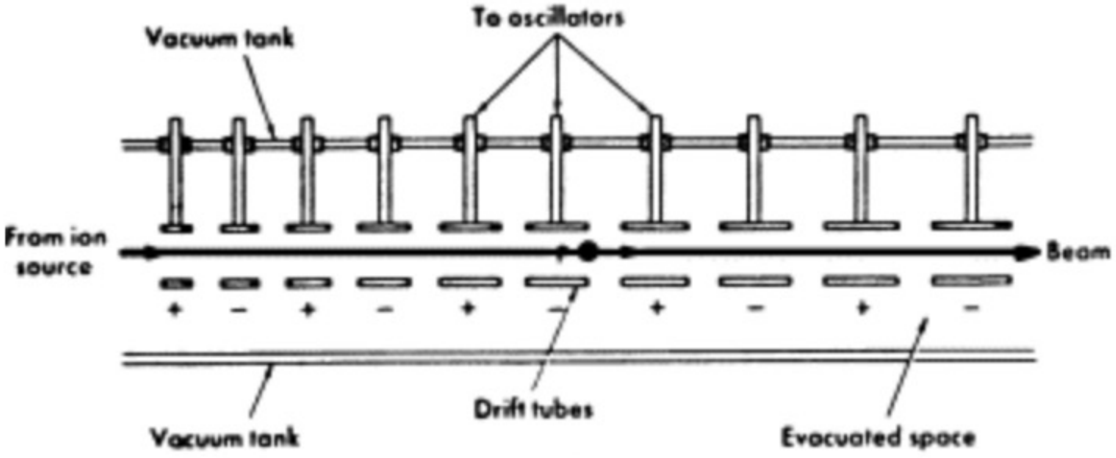
Induction accelerators can be thought of in simple terms as a **series of 1:1 transformers** where pulsed voltage sources form the transformer primary circuit and the charged particle beam pulse acts as the secondary.

- 1960s: N.C. Christofilos developed, the linear induction accelerator technology.
- Utilize ferrite-loaded, non-resonant magnetic induction cavities.
- Useful for high power and high current beams.
- Have **limited accelerating field**.
- Are pulsed by nature but have **low repetition rate** (~kHz).

Alternative: RF accelerators

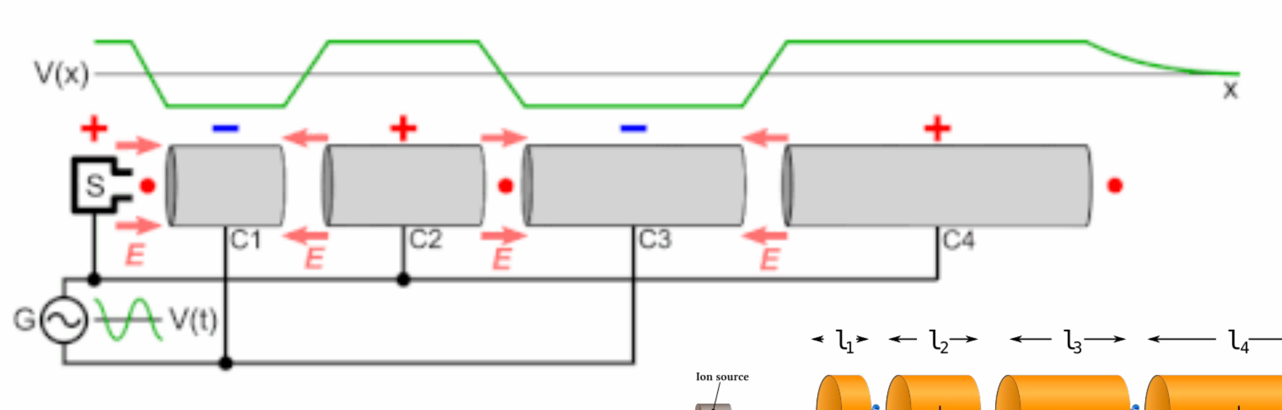
- Oscillating longitudinal electric field + Longitudinal structure (cells) which alternates the direction of the field
- When particle propagates through the RF accelerator, **the field direction in each cell is synchronized with the particle arrival** and the effect from all cells is added coherently

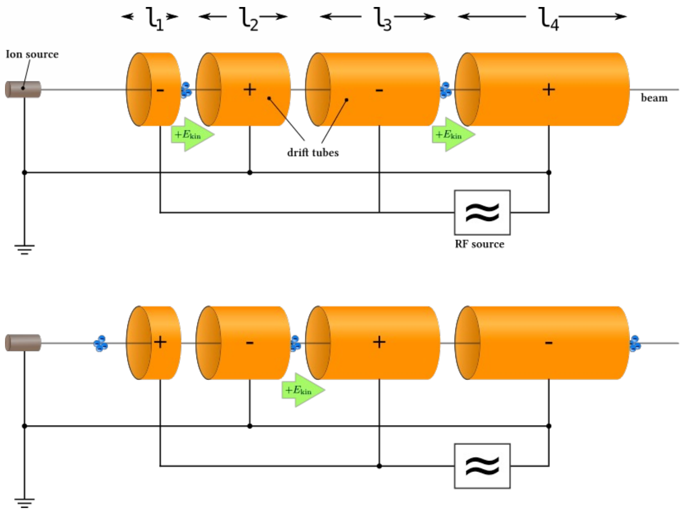
(a)

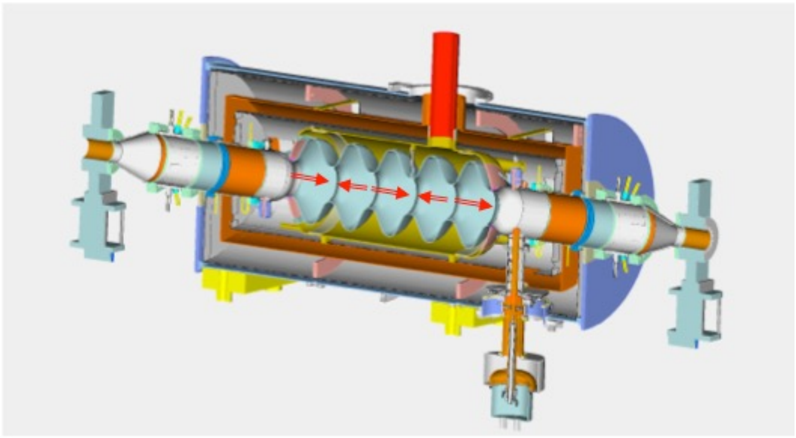


(b)

1928 Wideroe linac: tube length is adjusted for changing β

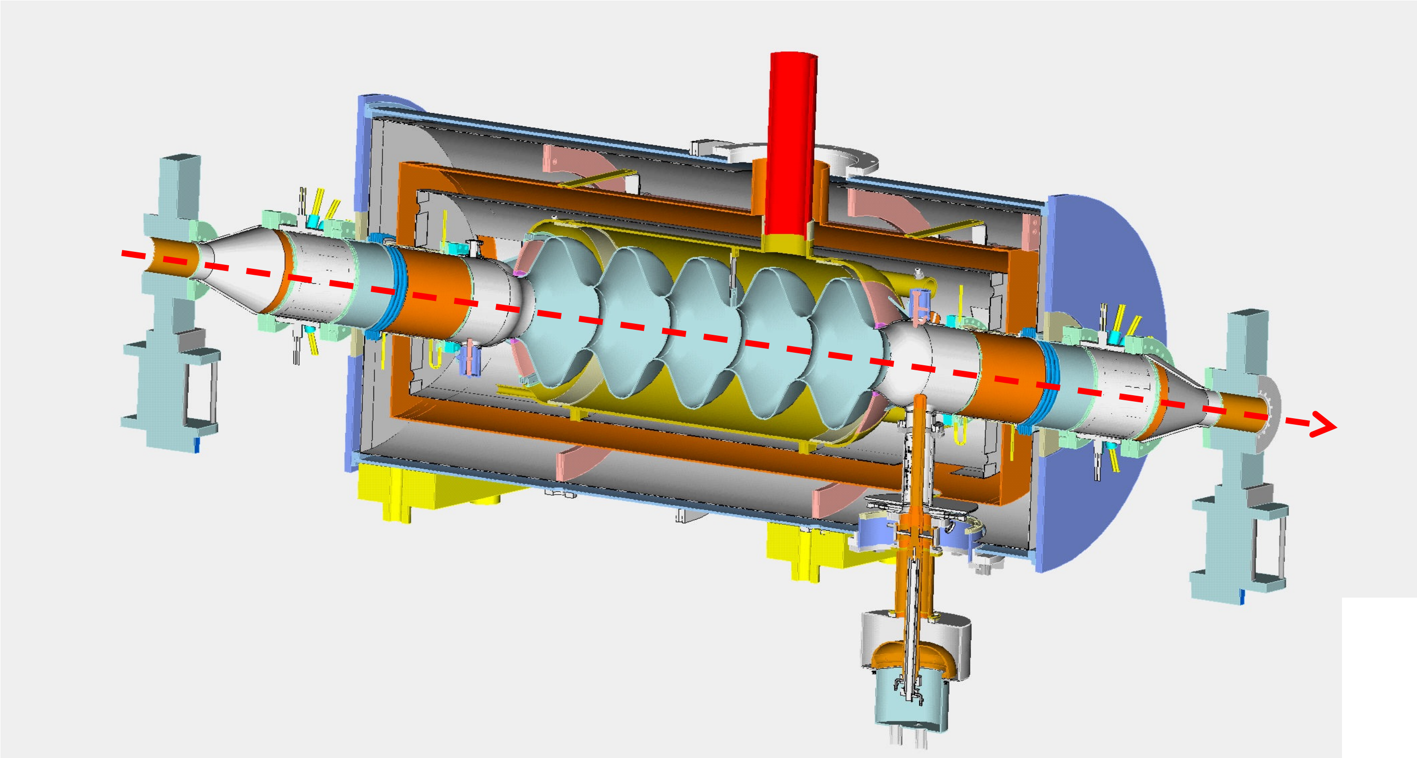


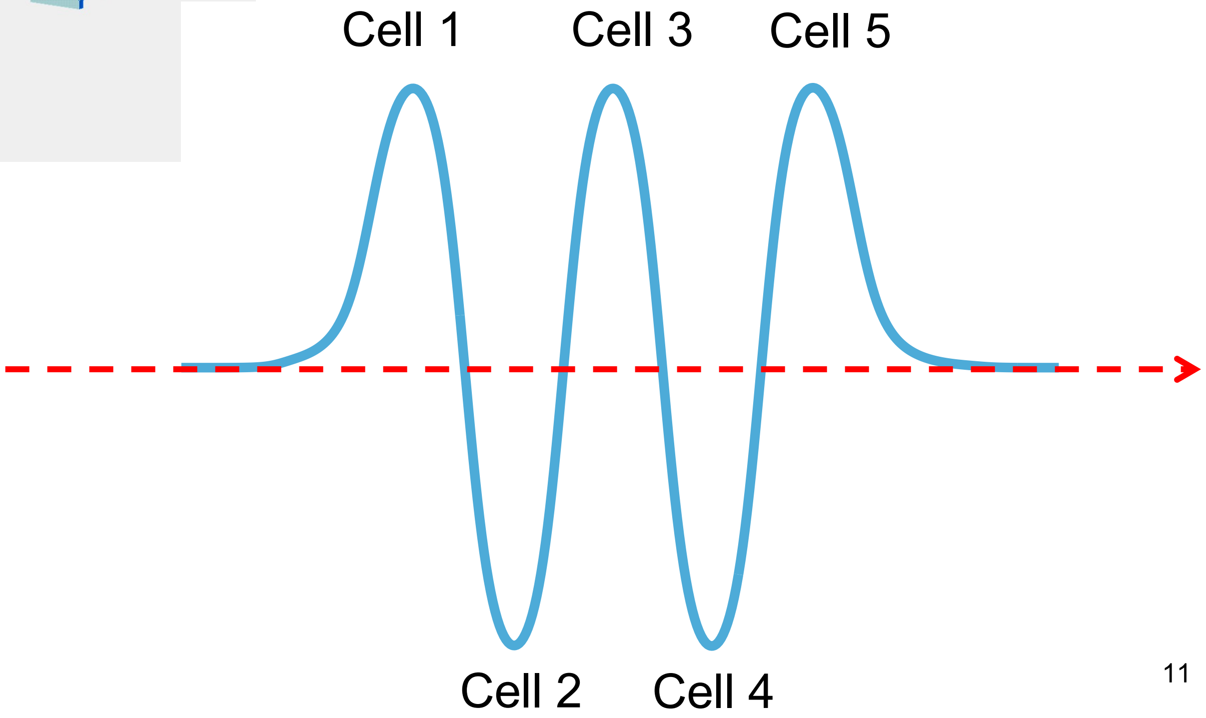




$\beta = 1$ electron linacs

Example of an RF accelerator: 5-cell cavity





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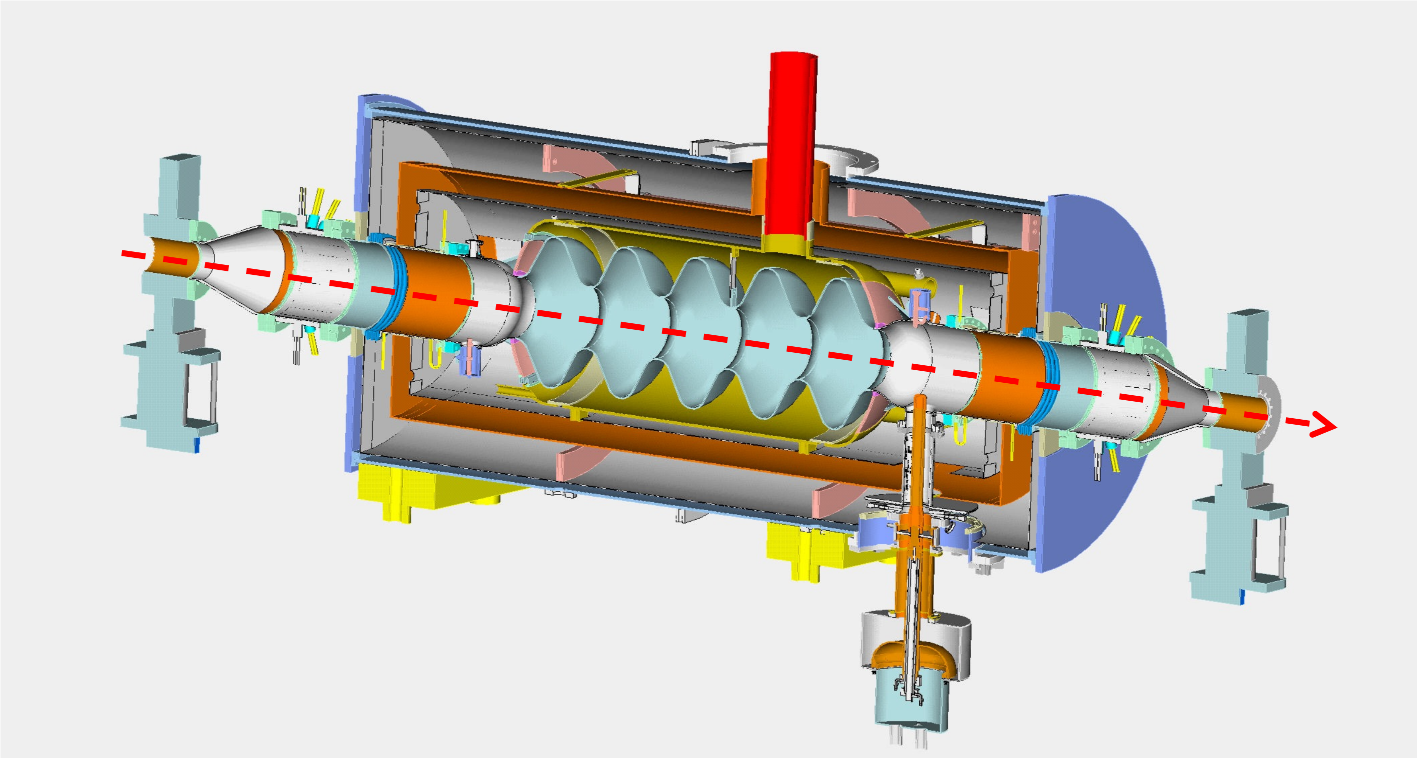

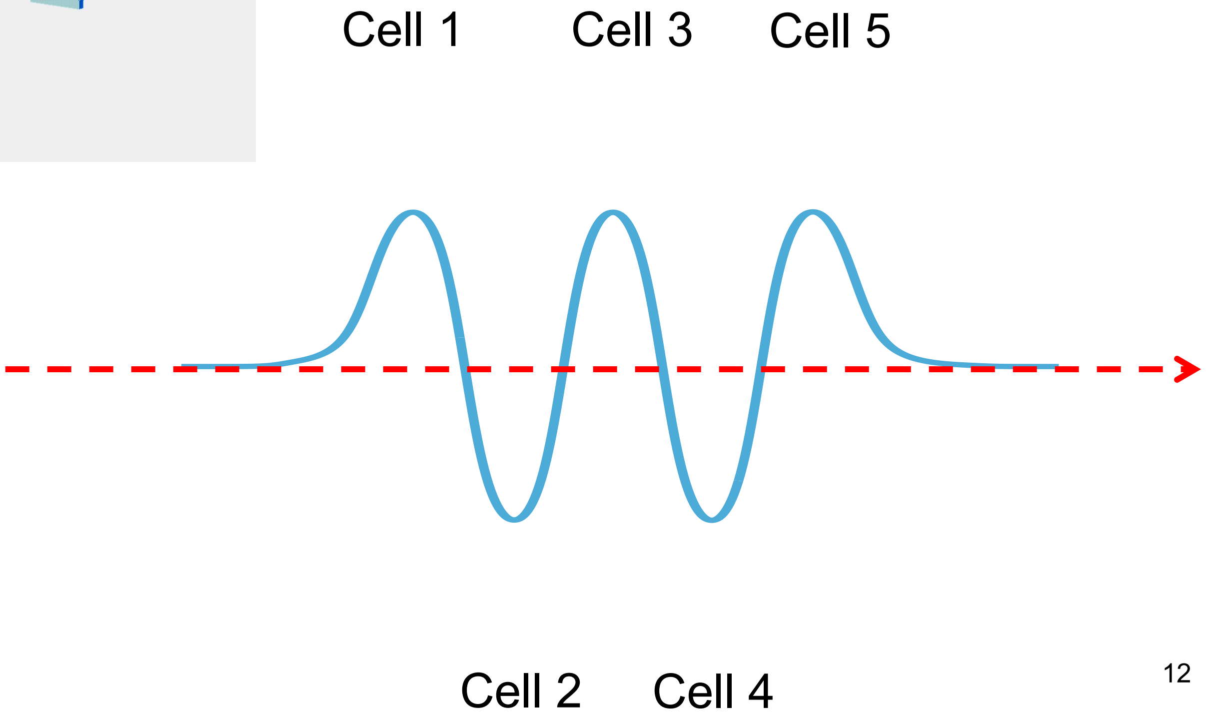

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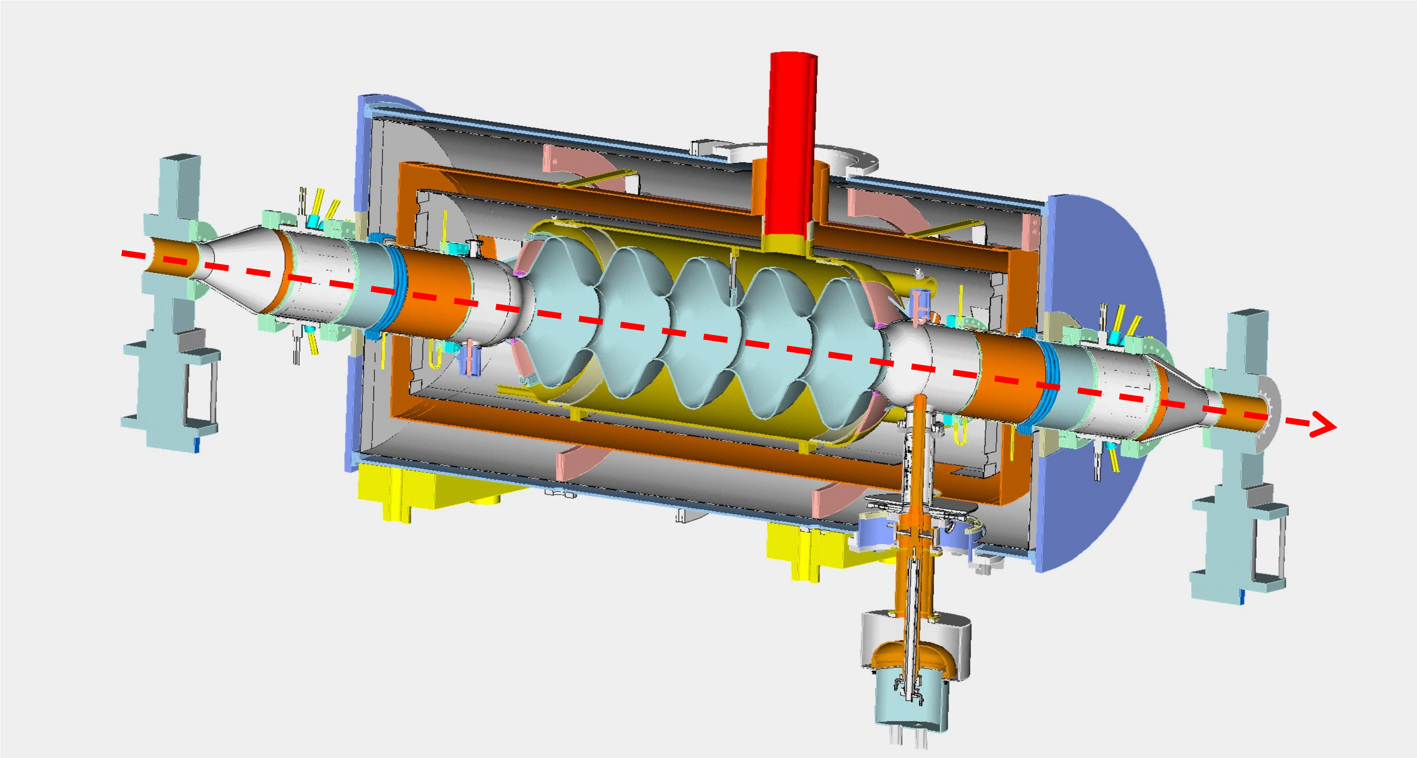

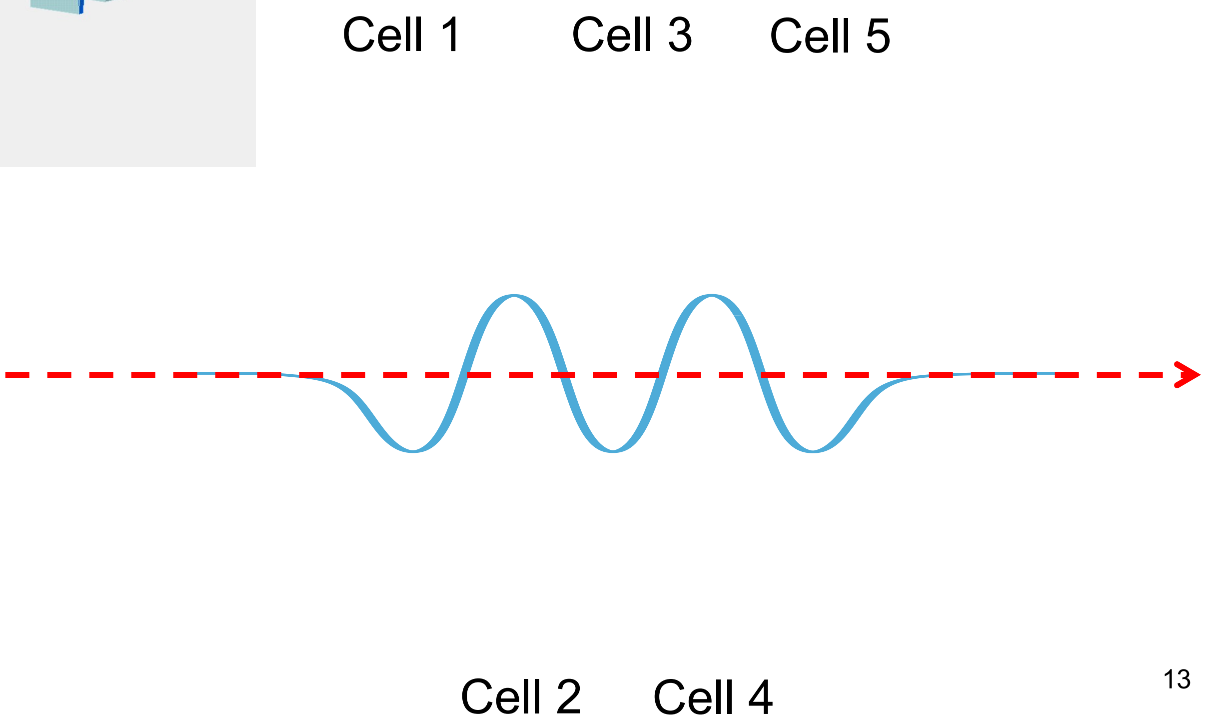

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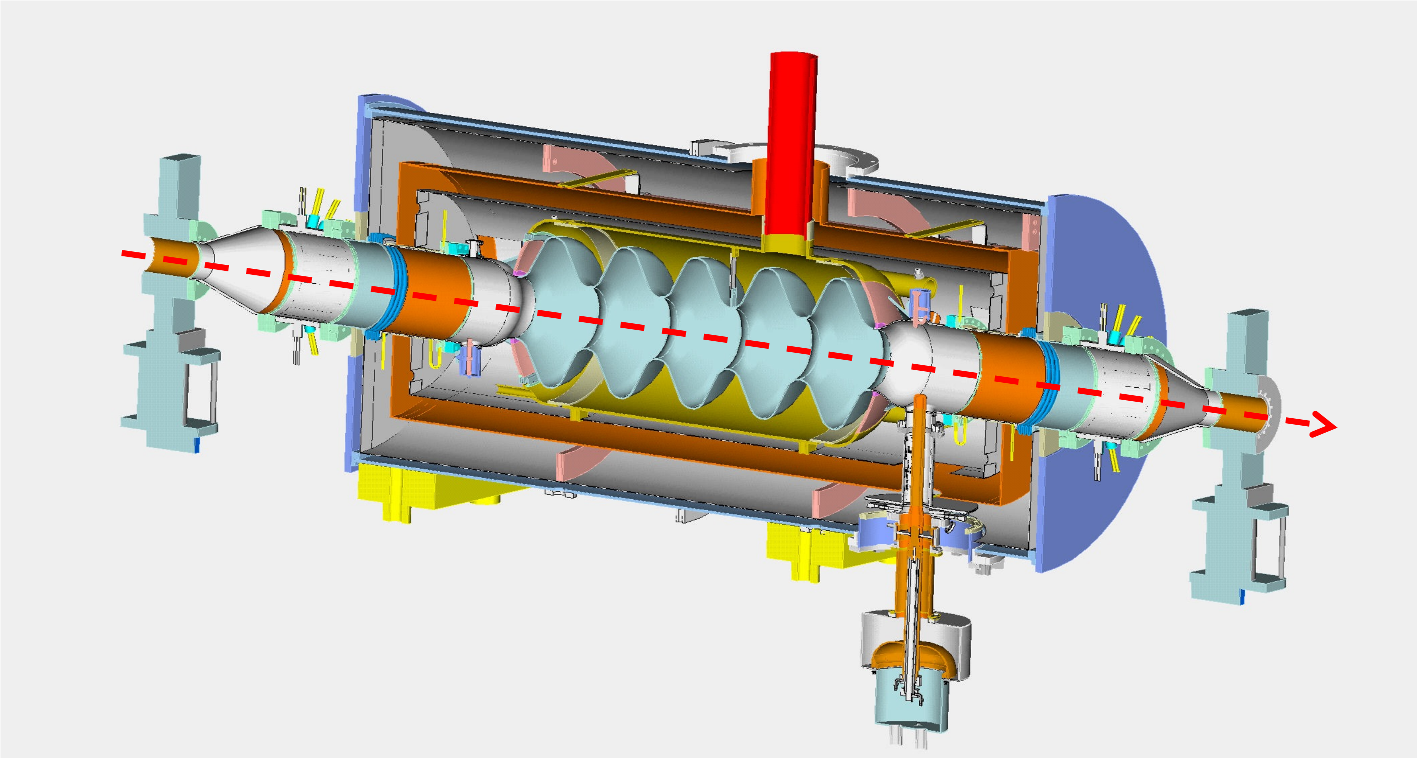

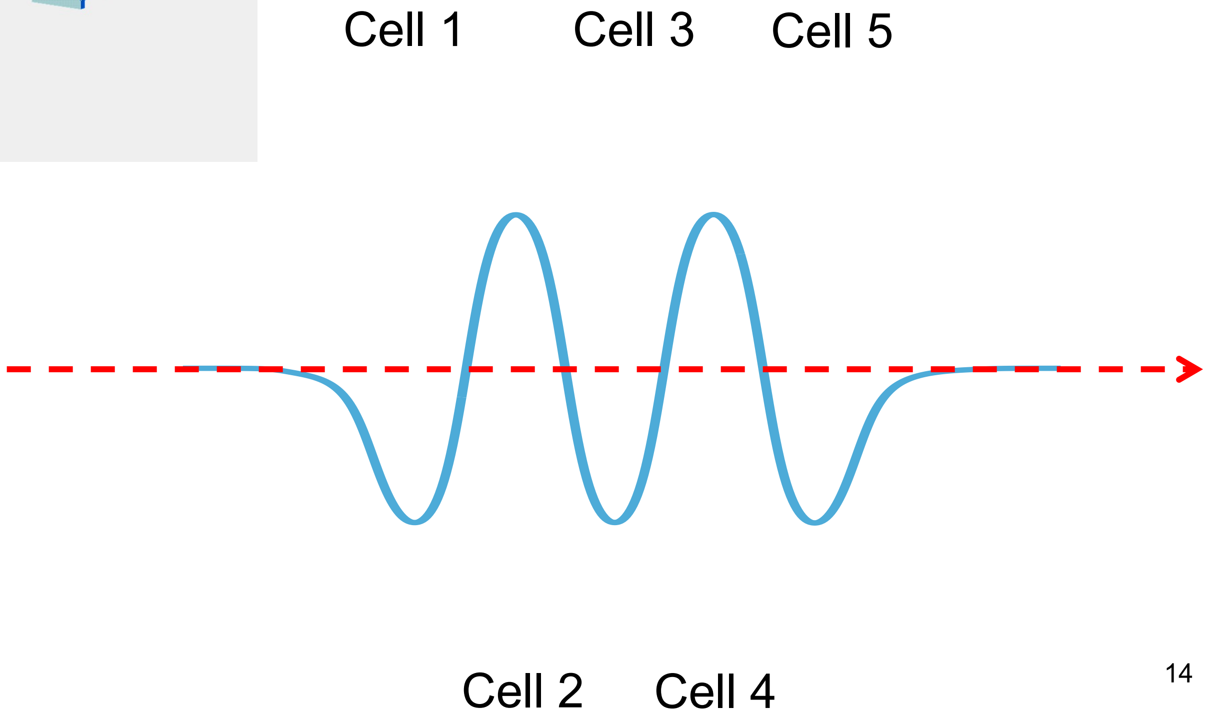

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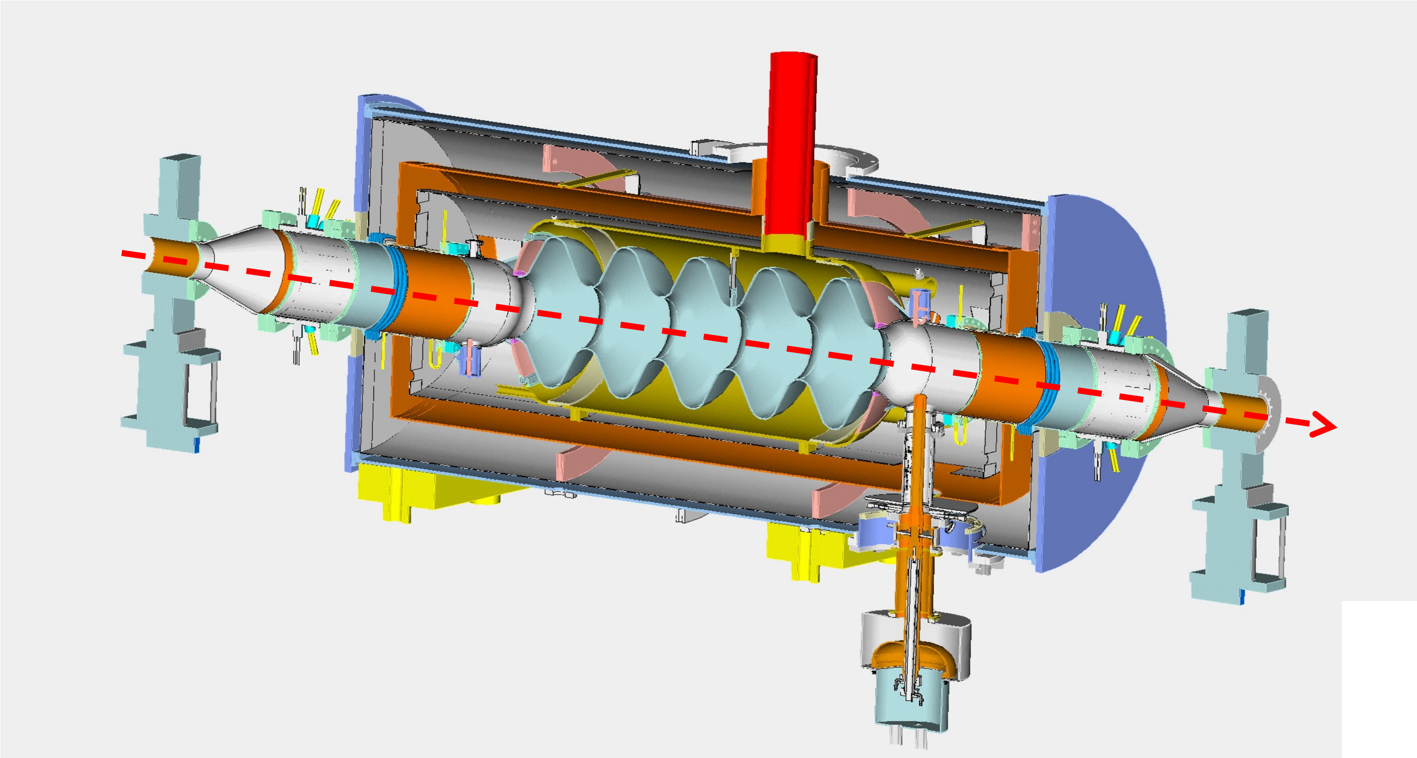

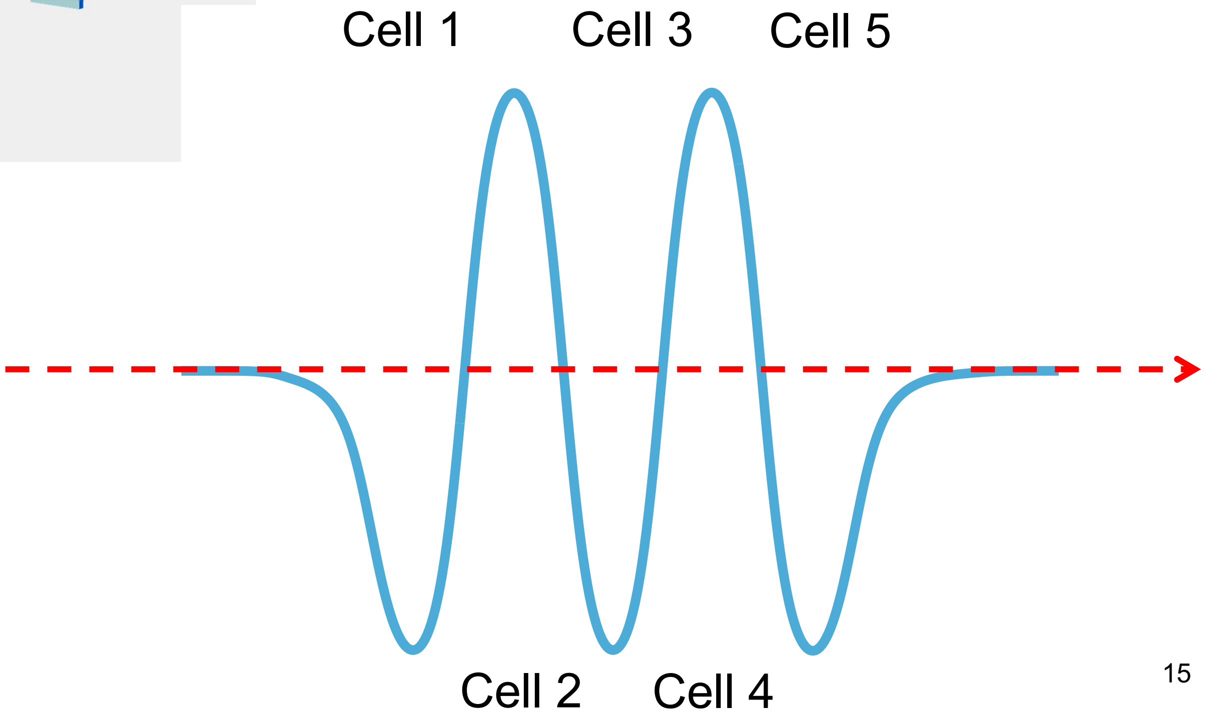

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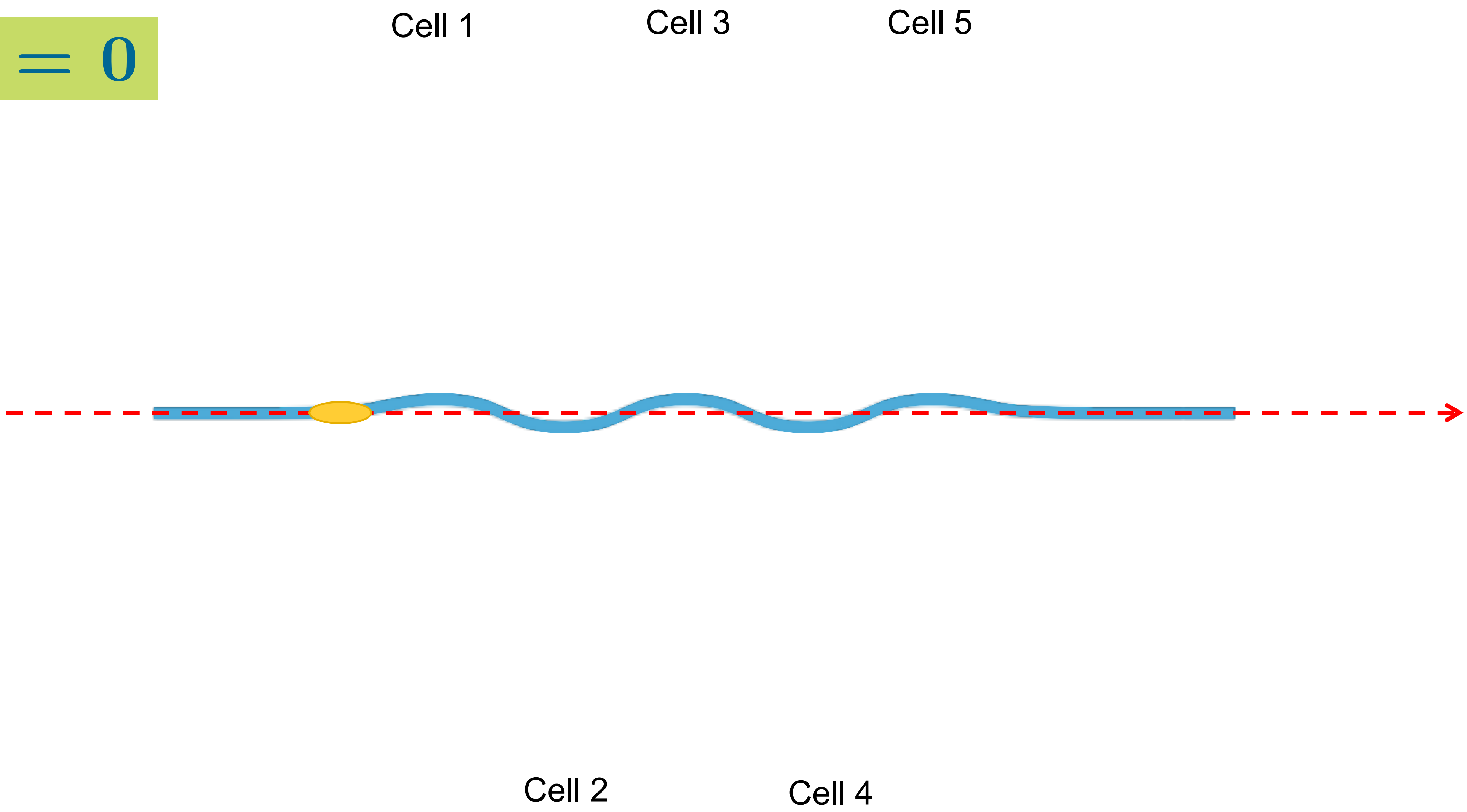
Example of an RF accelerator: 5-cell cavity

$t = 0$



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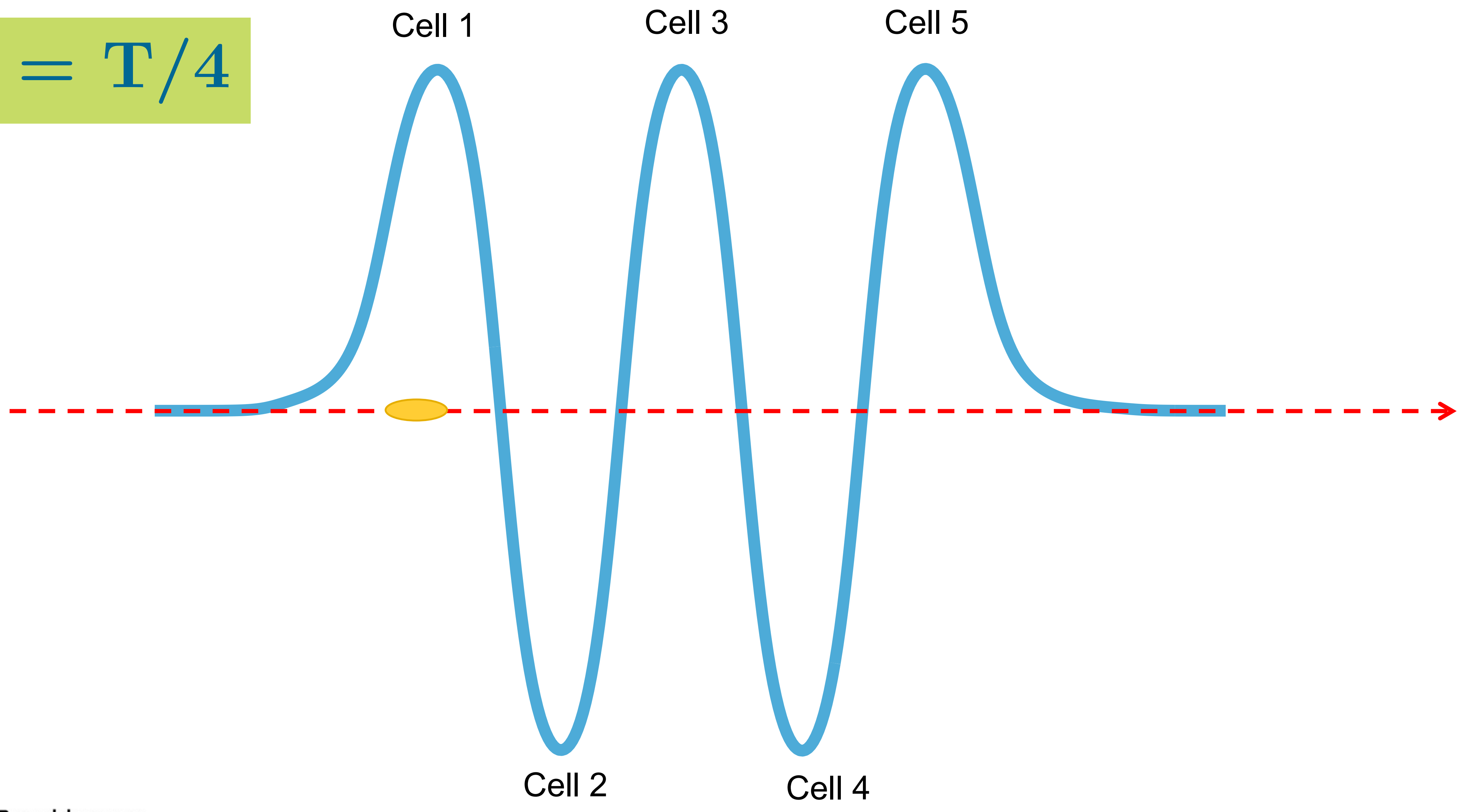
Example of an RF accelerator: 5-cell cavity

$$t = T/4$$



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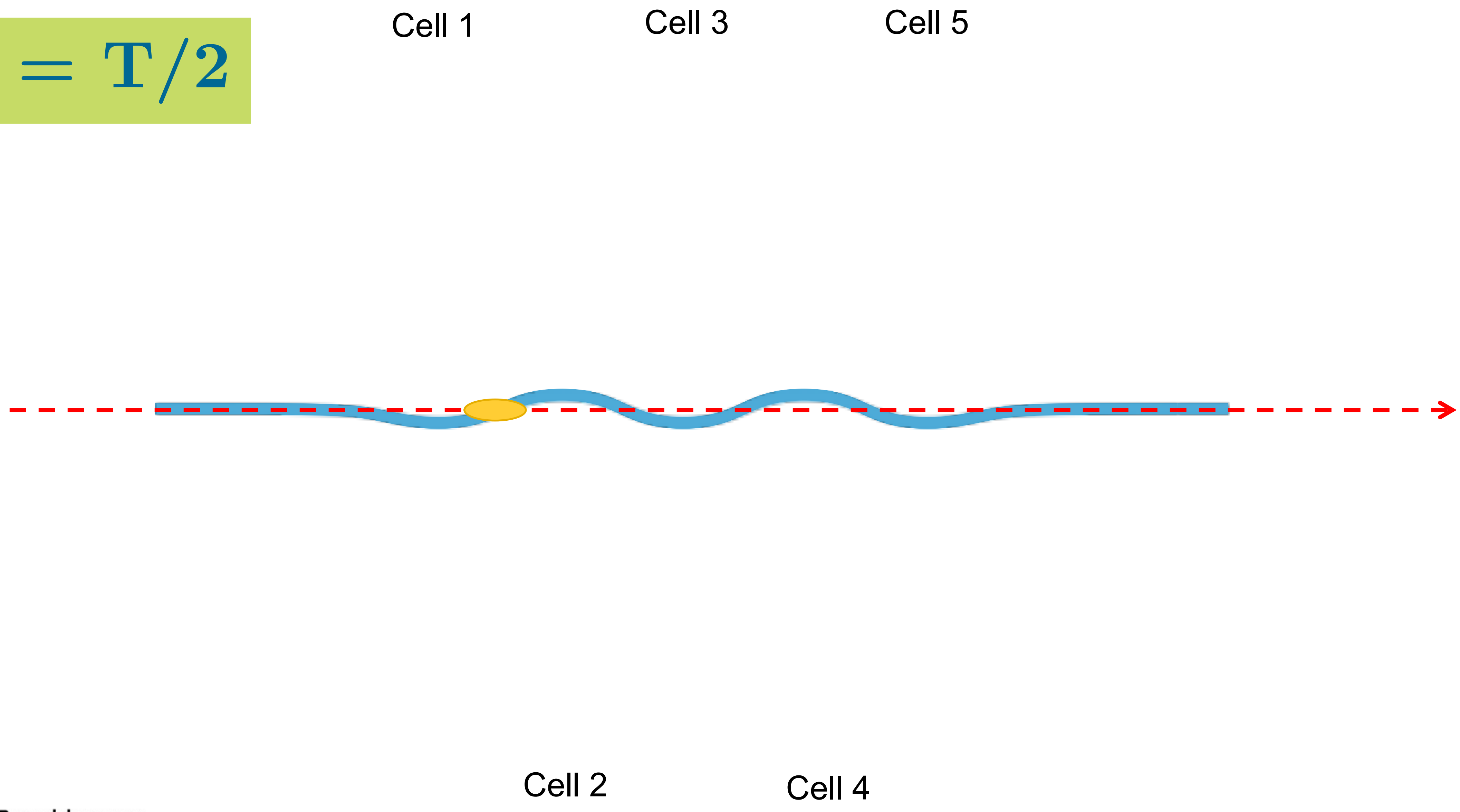
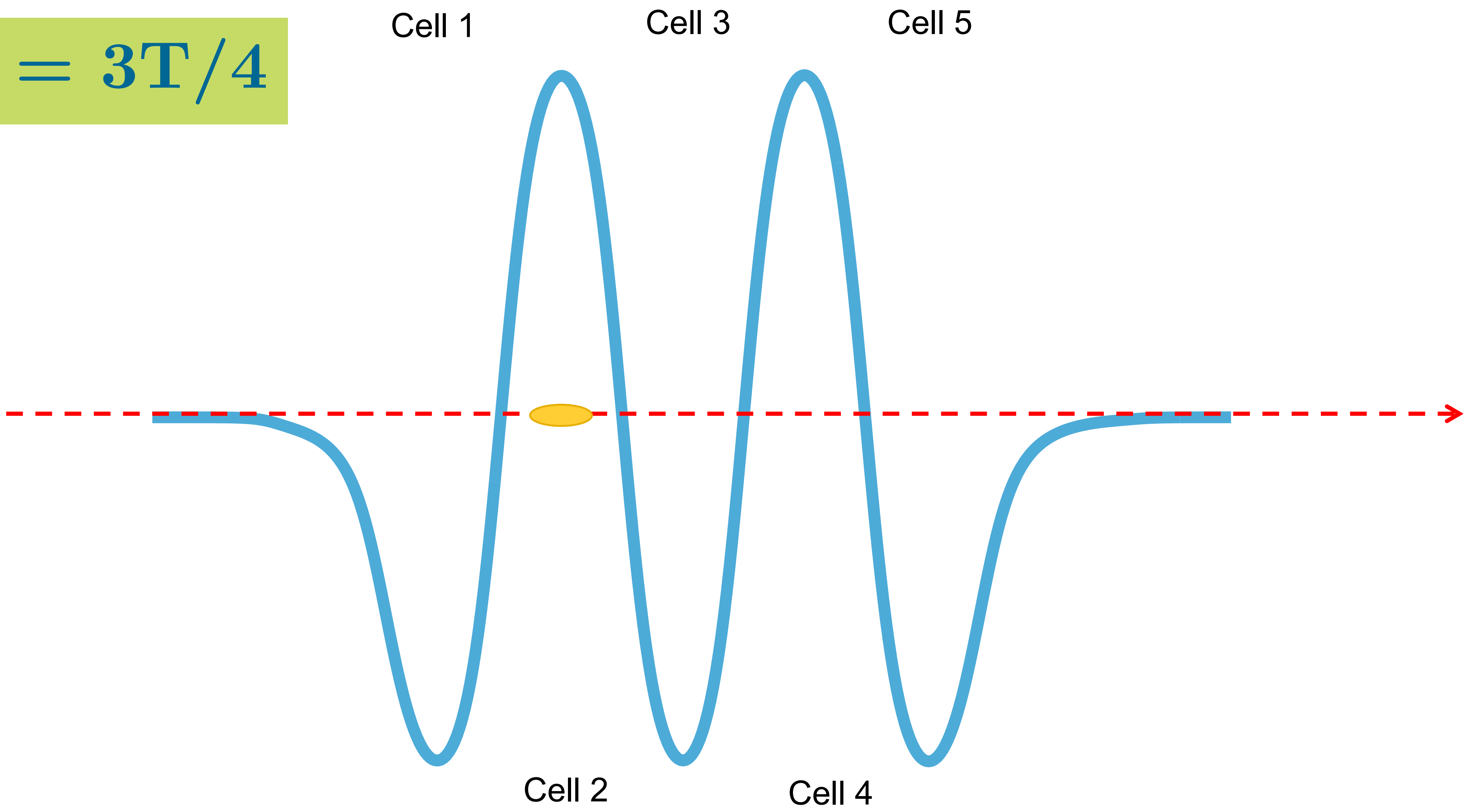
Example of an RF accelerator: 5-cell cavity

$$t = T/2$$



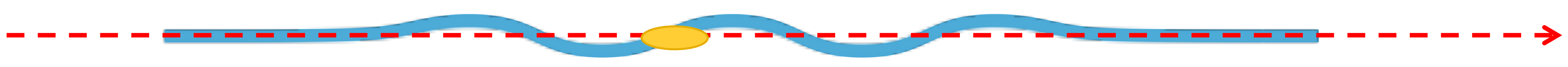
Example of an RF accelerator: 5-cell cavity

$$t = 3T/4$$



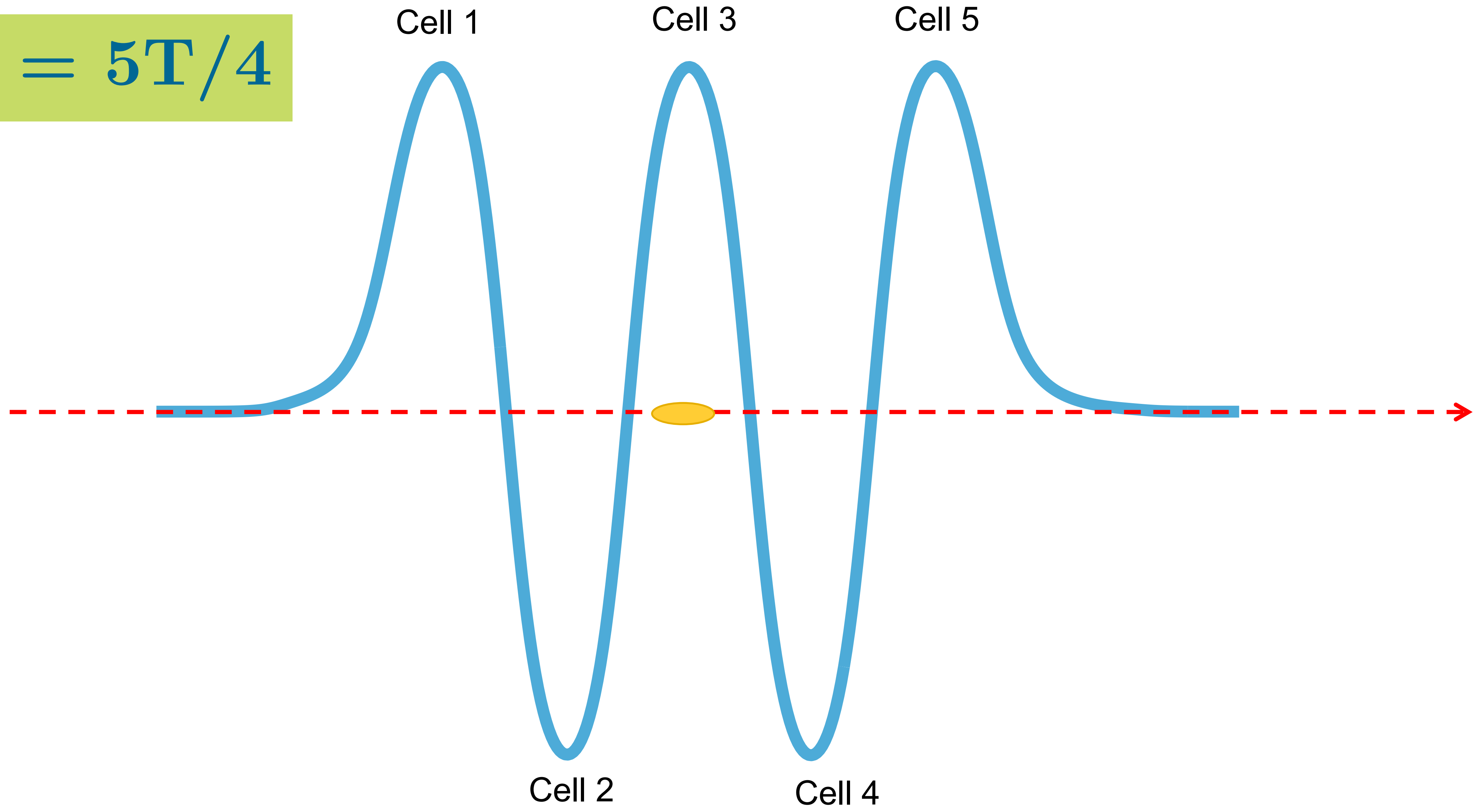
Example of an RF accelerator: 5-cell cavity

$$t = T$$



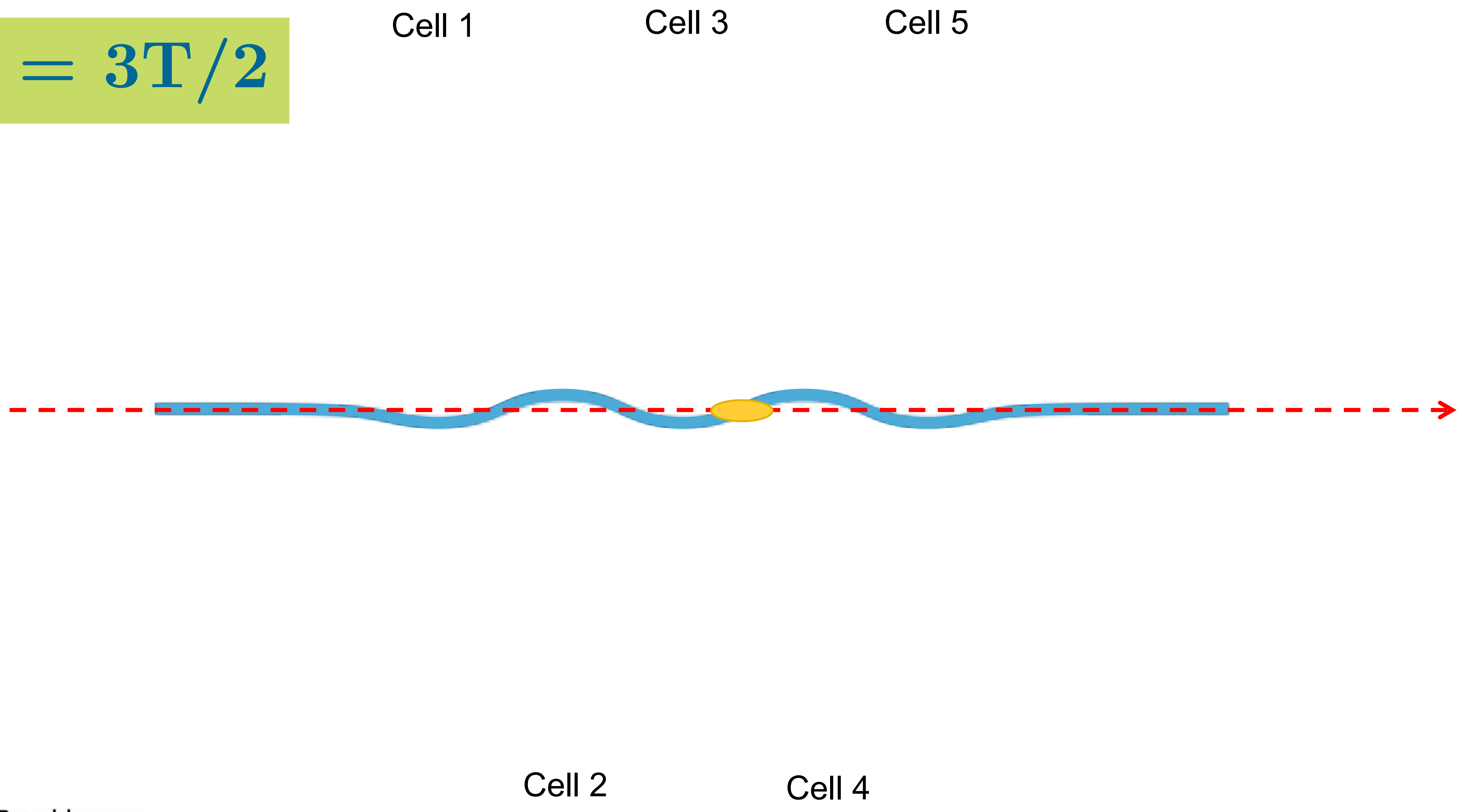
Example of an RF accelerator: 5-cell cavity

$$t = 5T/4$$



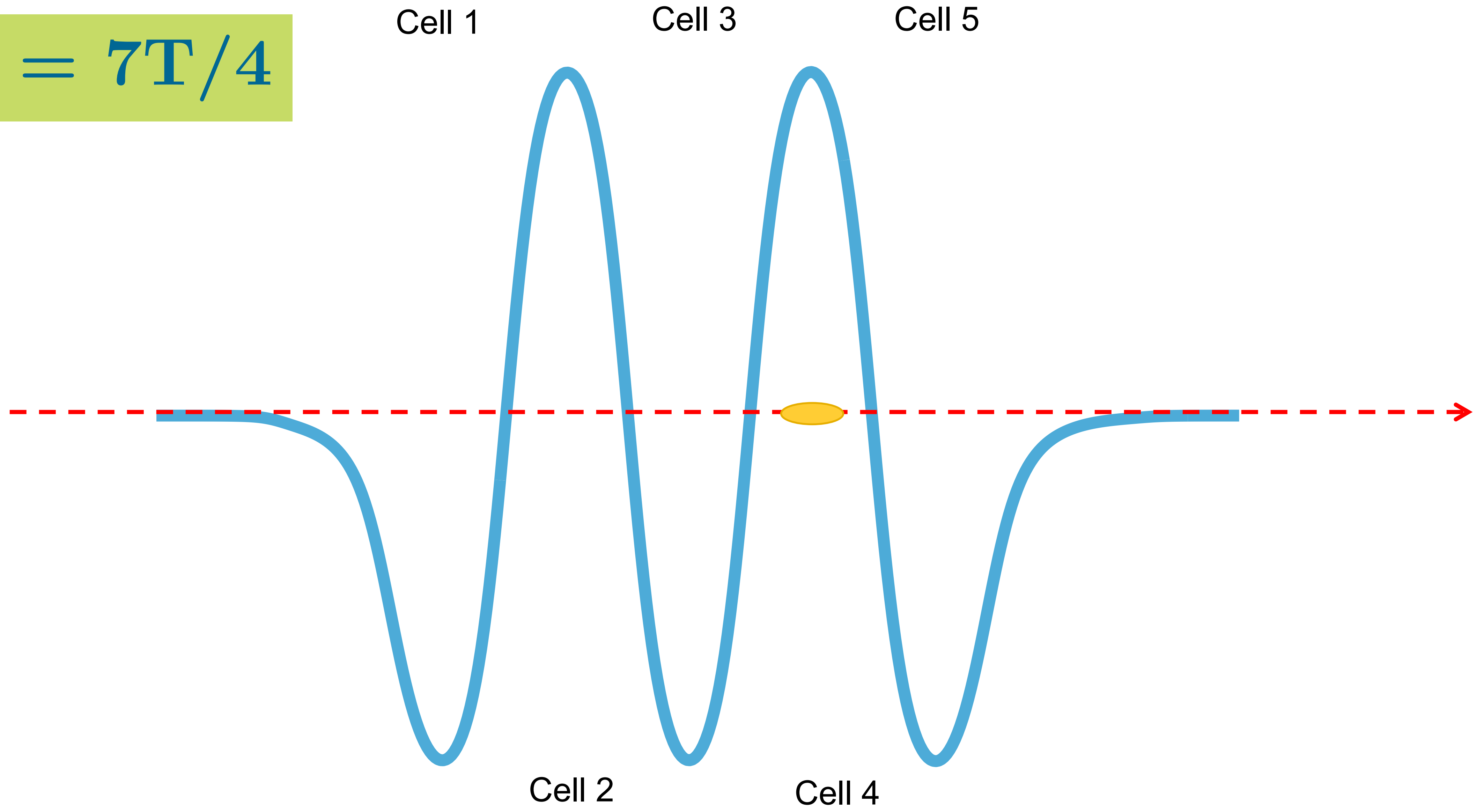
Example of an RF accelerator: 5-cell cavity

$$t = 3T/2$$



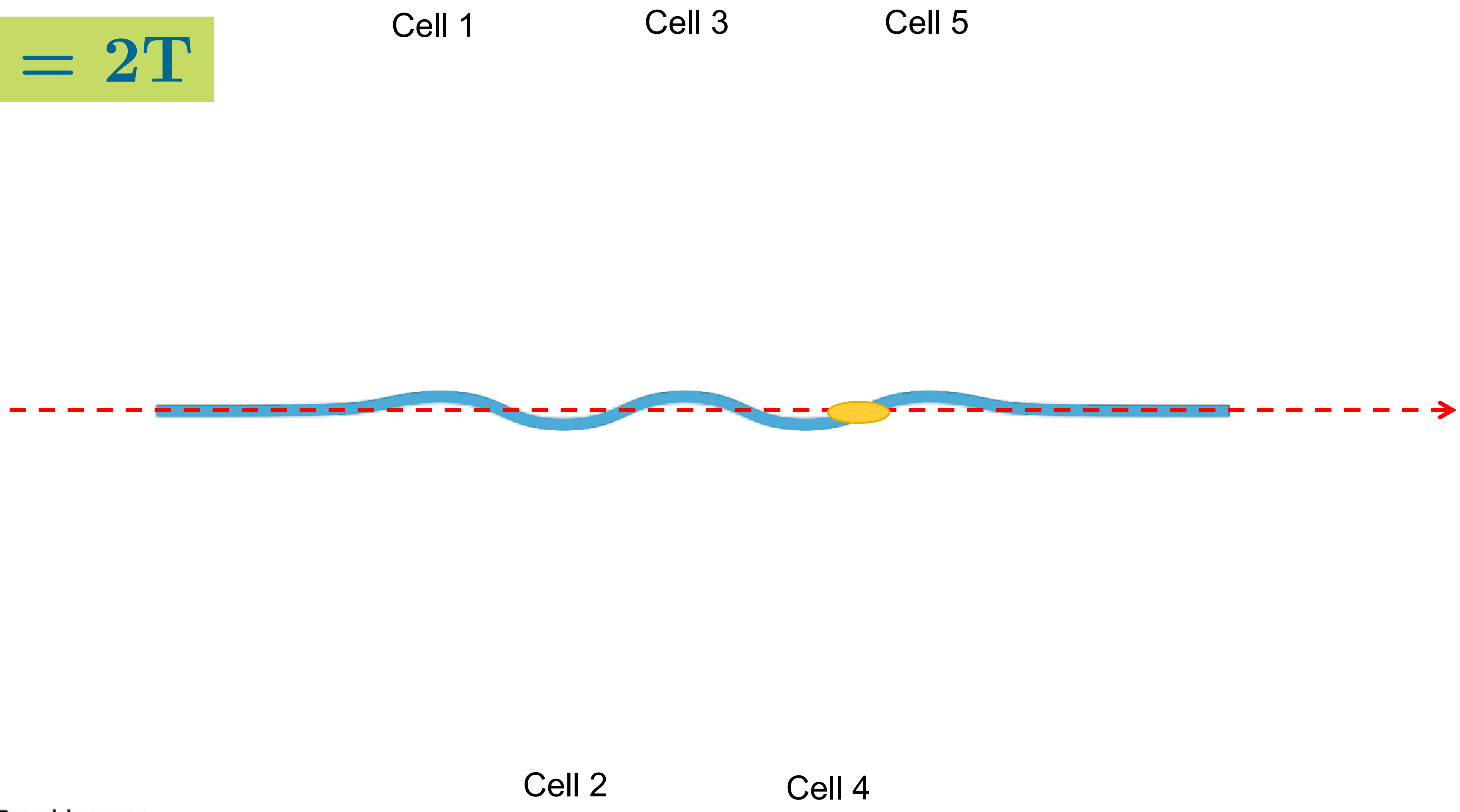
Example of an RF accelerator: 5-cell cavity

$$t = 7T/4$$



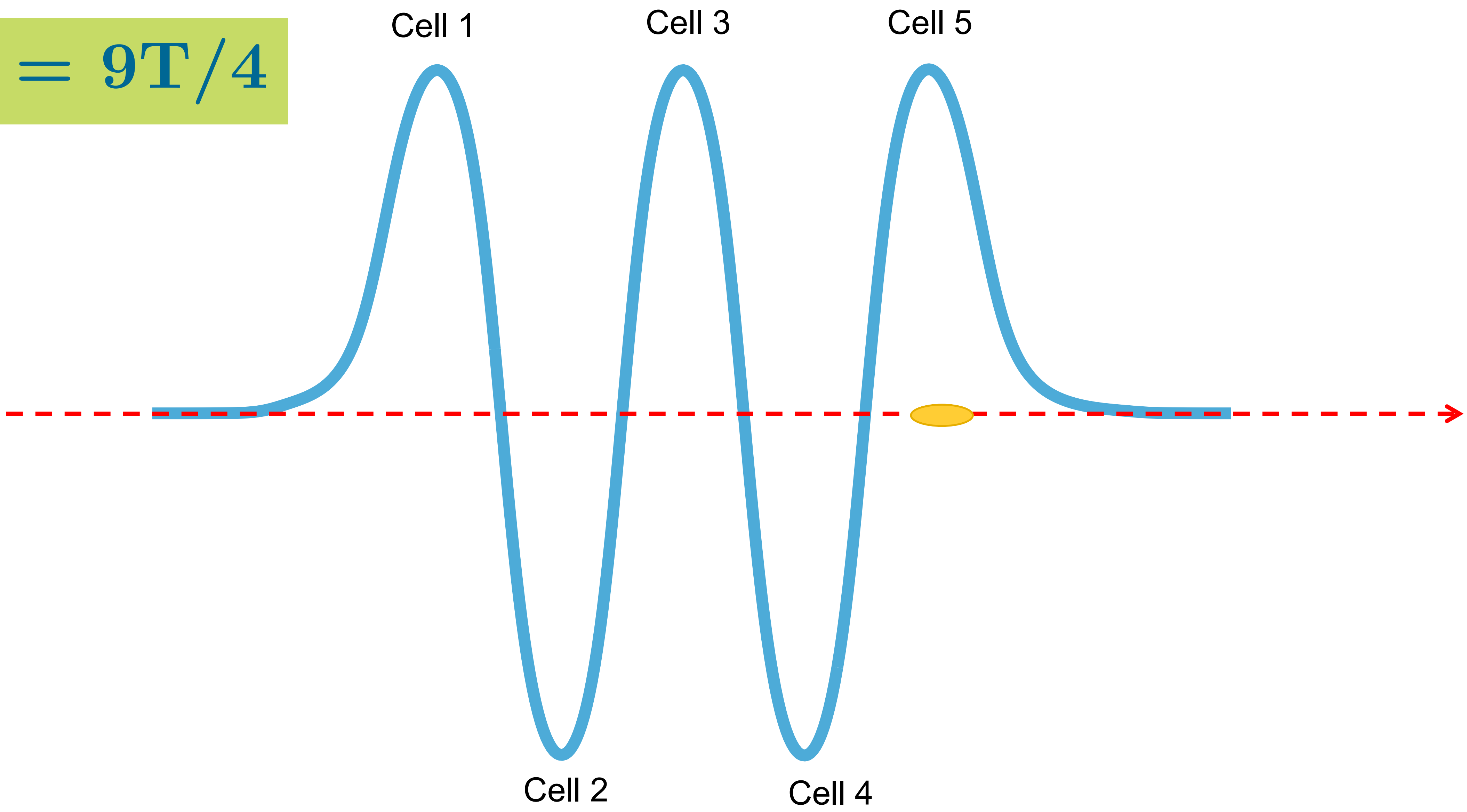
Example of an RF accelerator: 5-cell cavity

$$\mathbf{t = 2T}$$



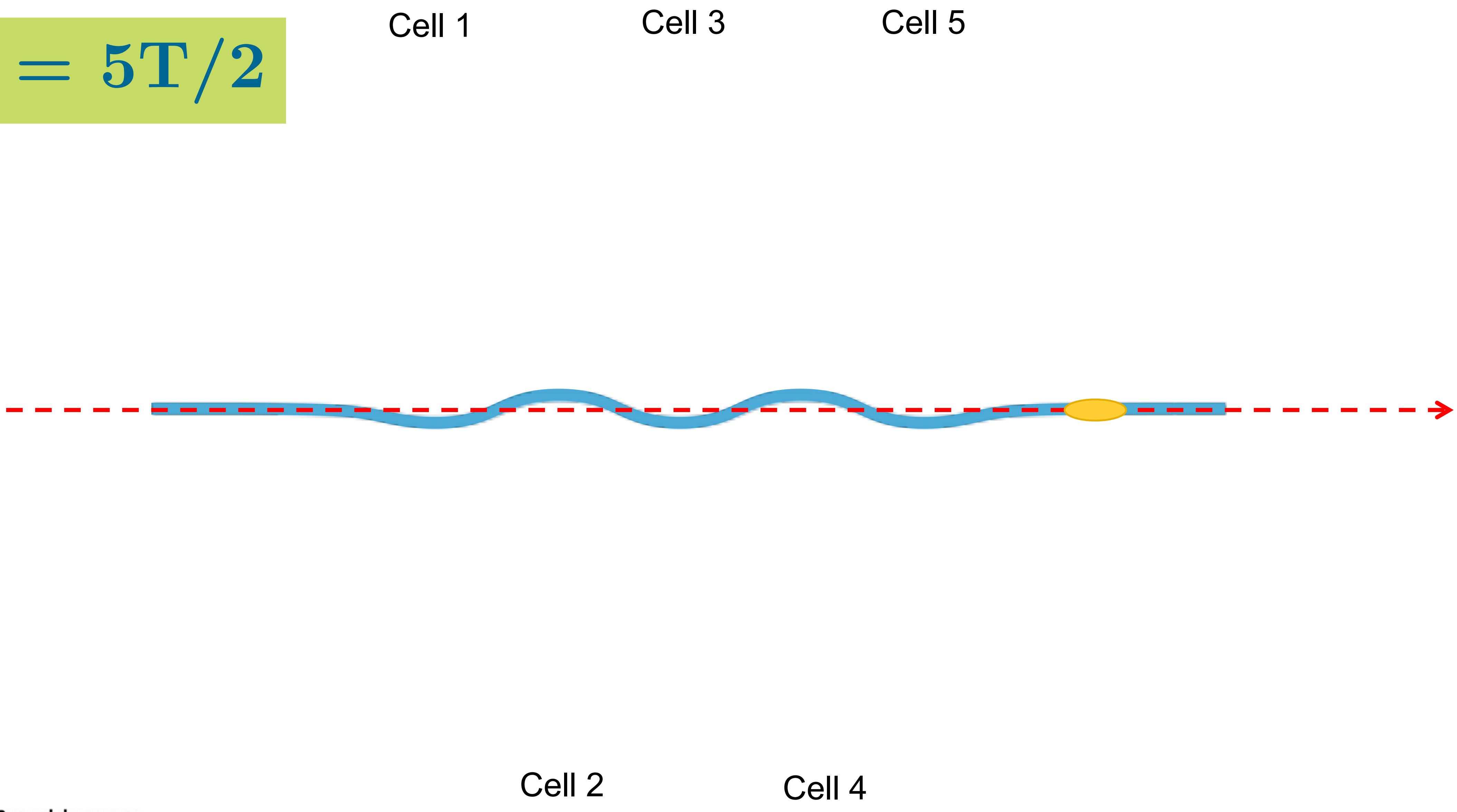
Example of an RF accelerator: 5-cell cavity

$$t = 9T/4$$



Example of an RF accelerator: 5-cell cavity

$$t = 5T/2$$



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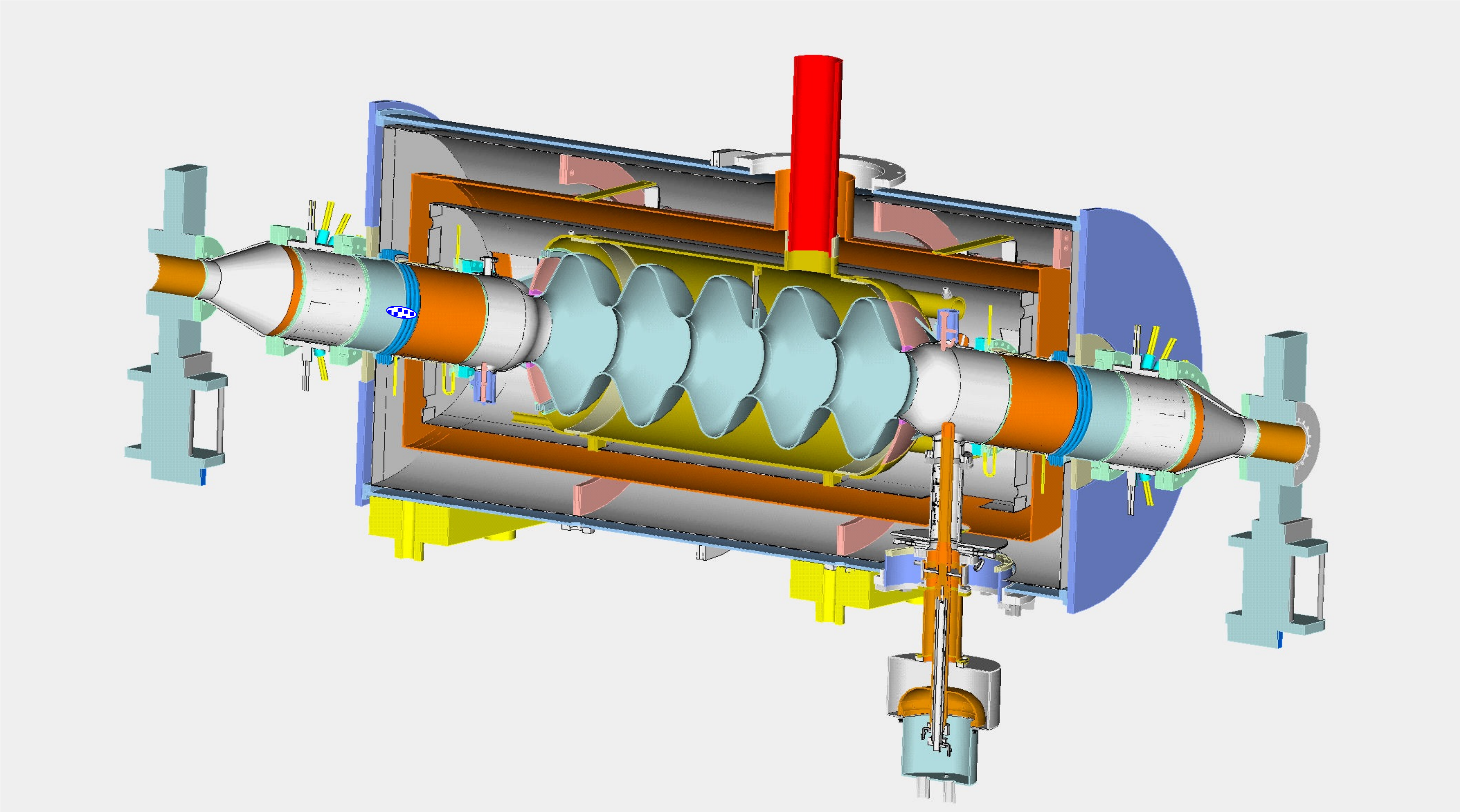
Example of an RF accelerator: 5-cell cavity



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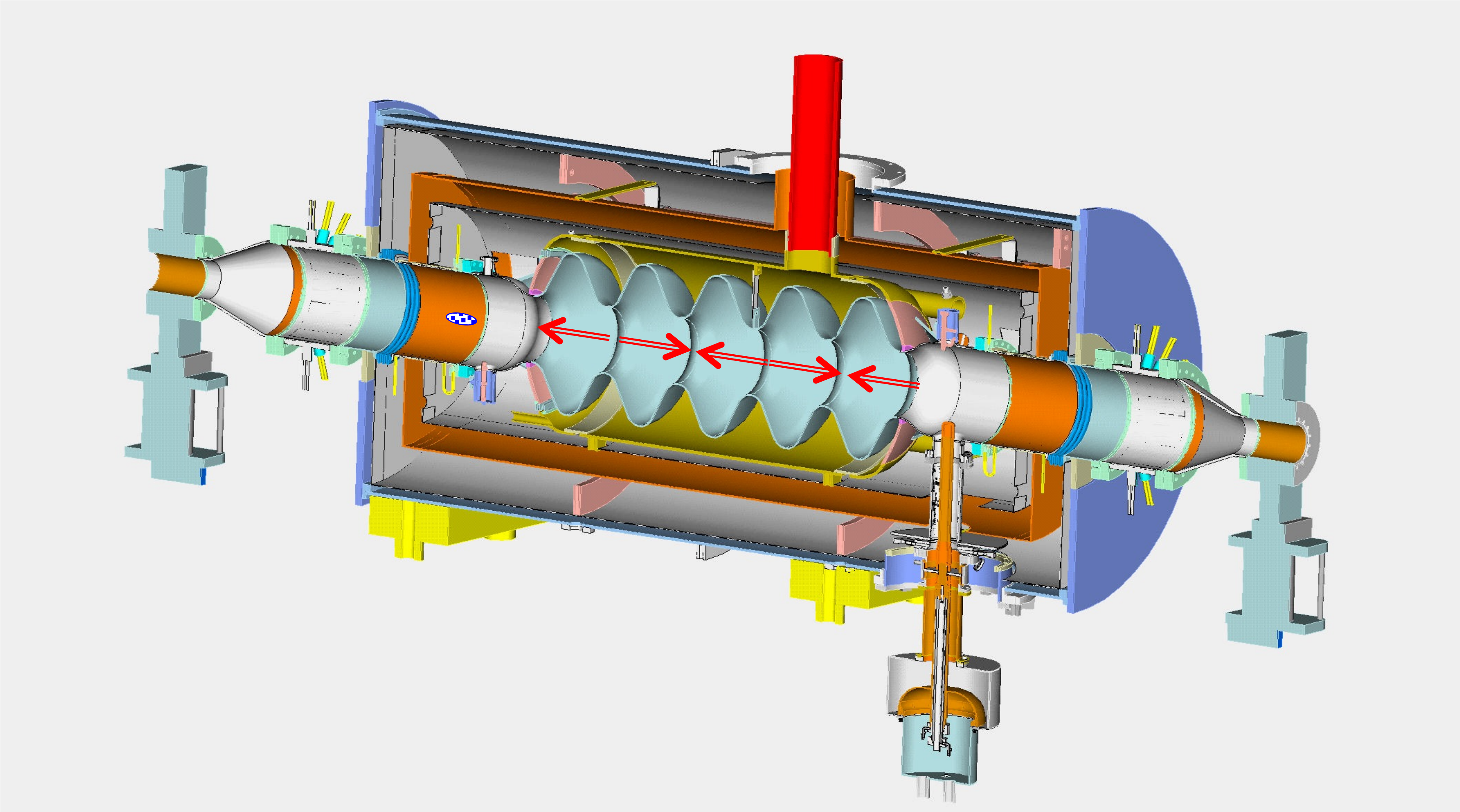
Example of an RF accelerator: 5-cell cavity



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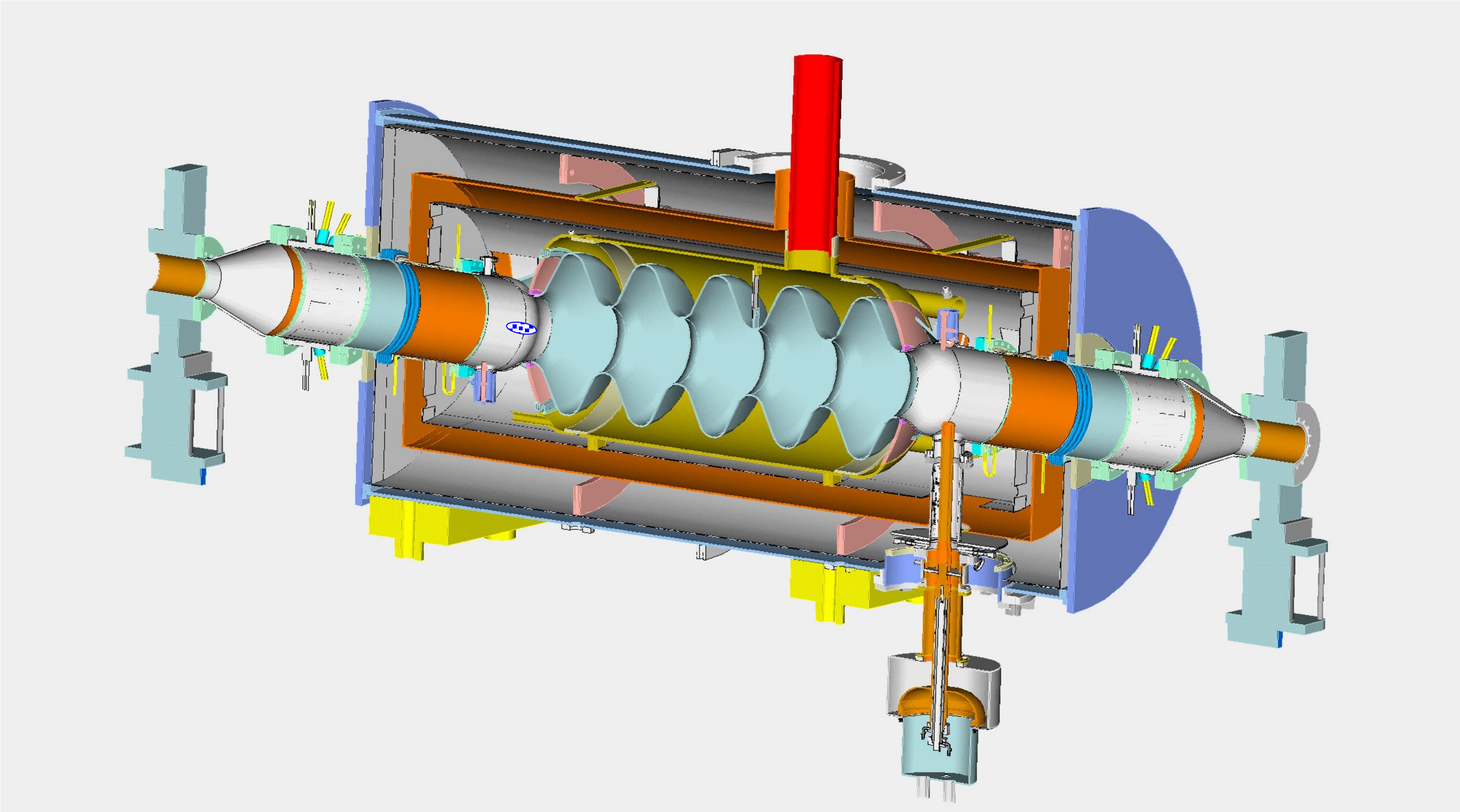

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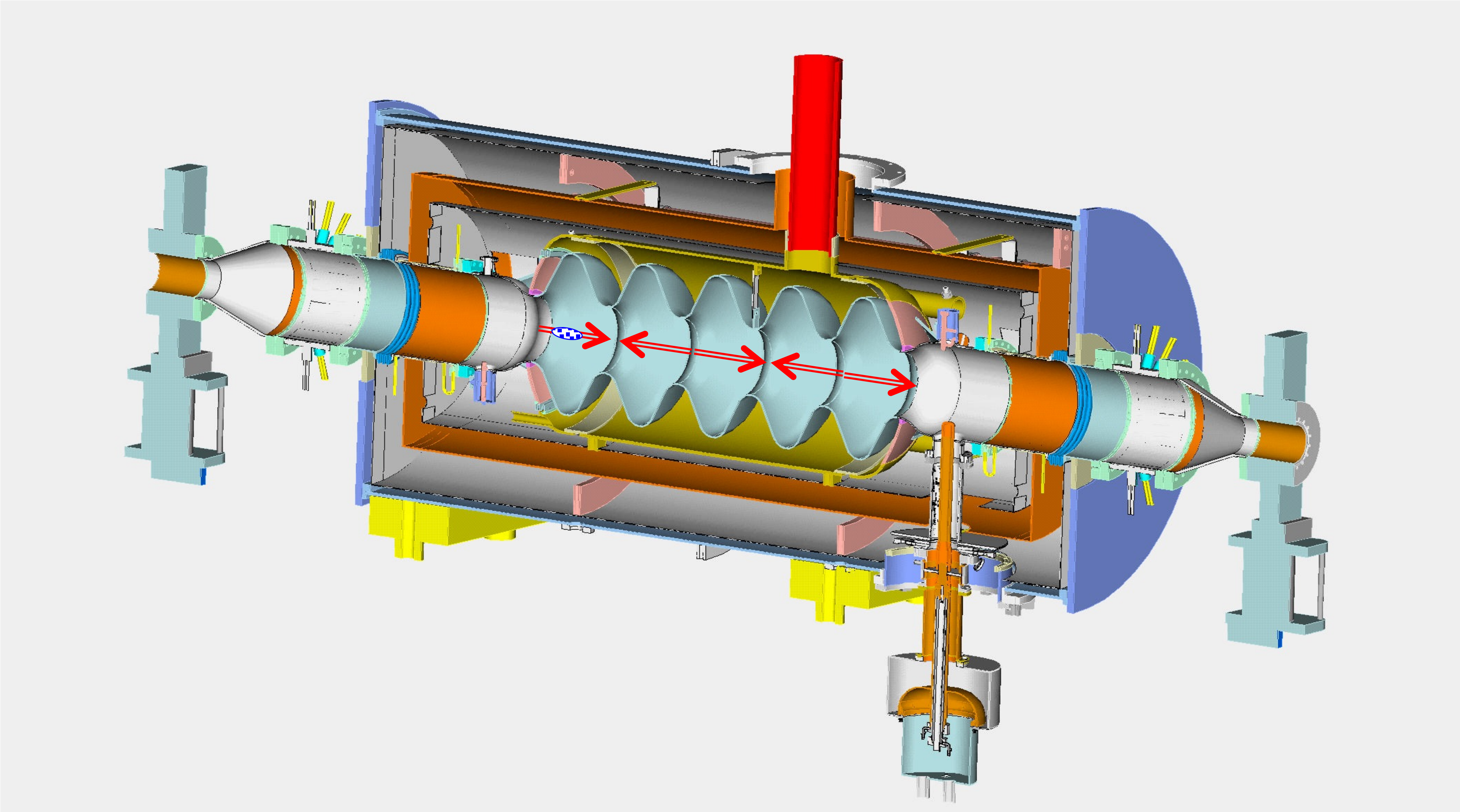

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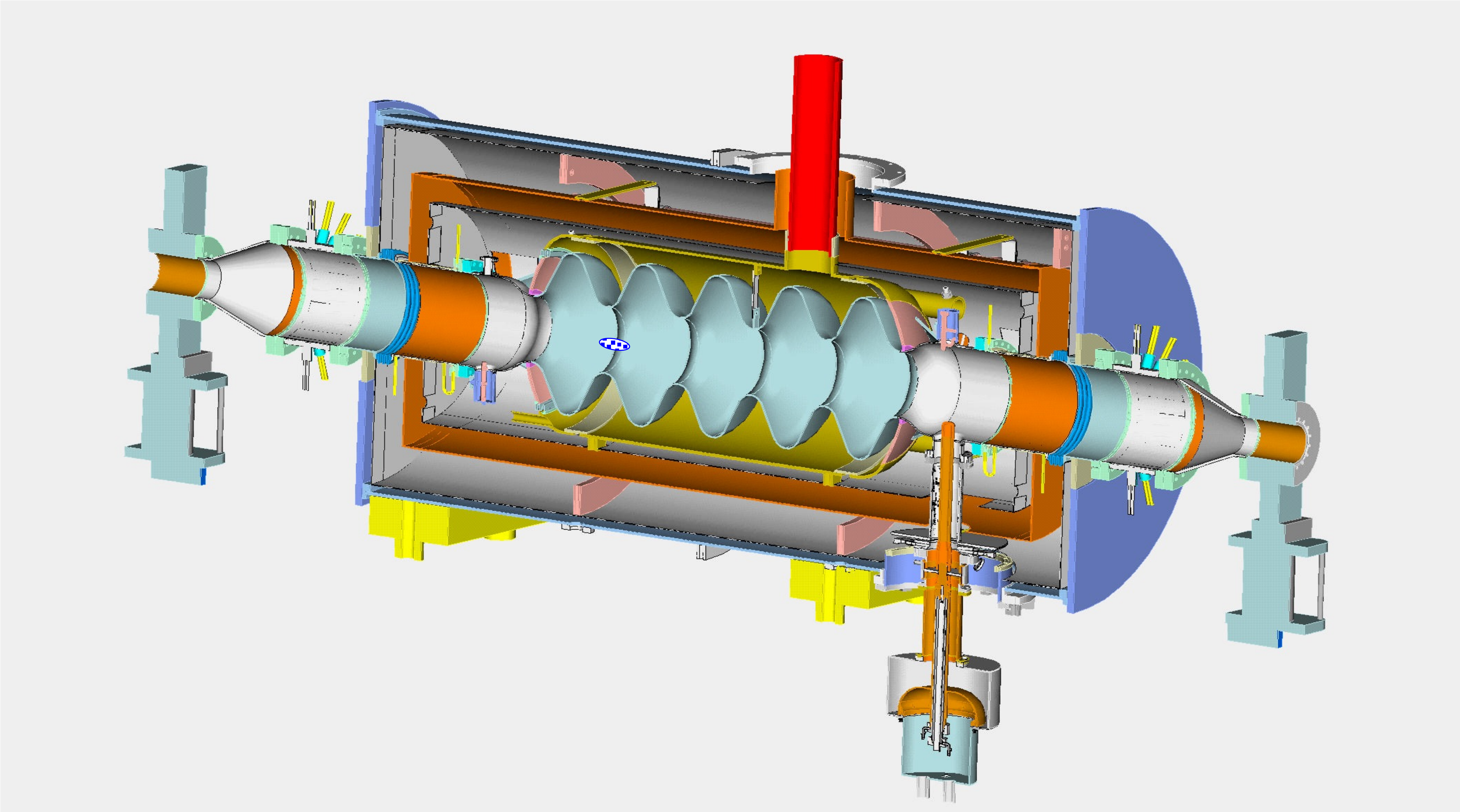

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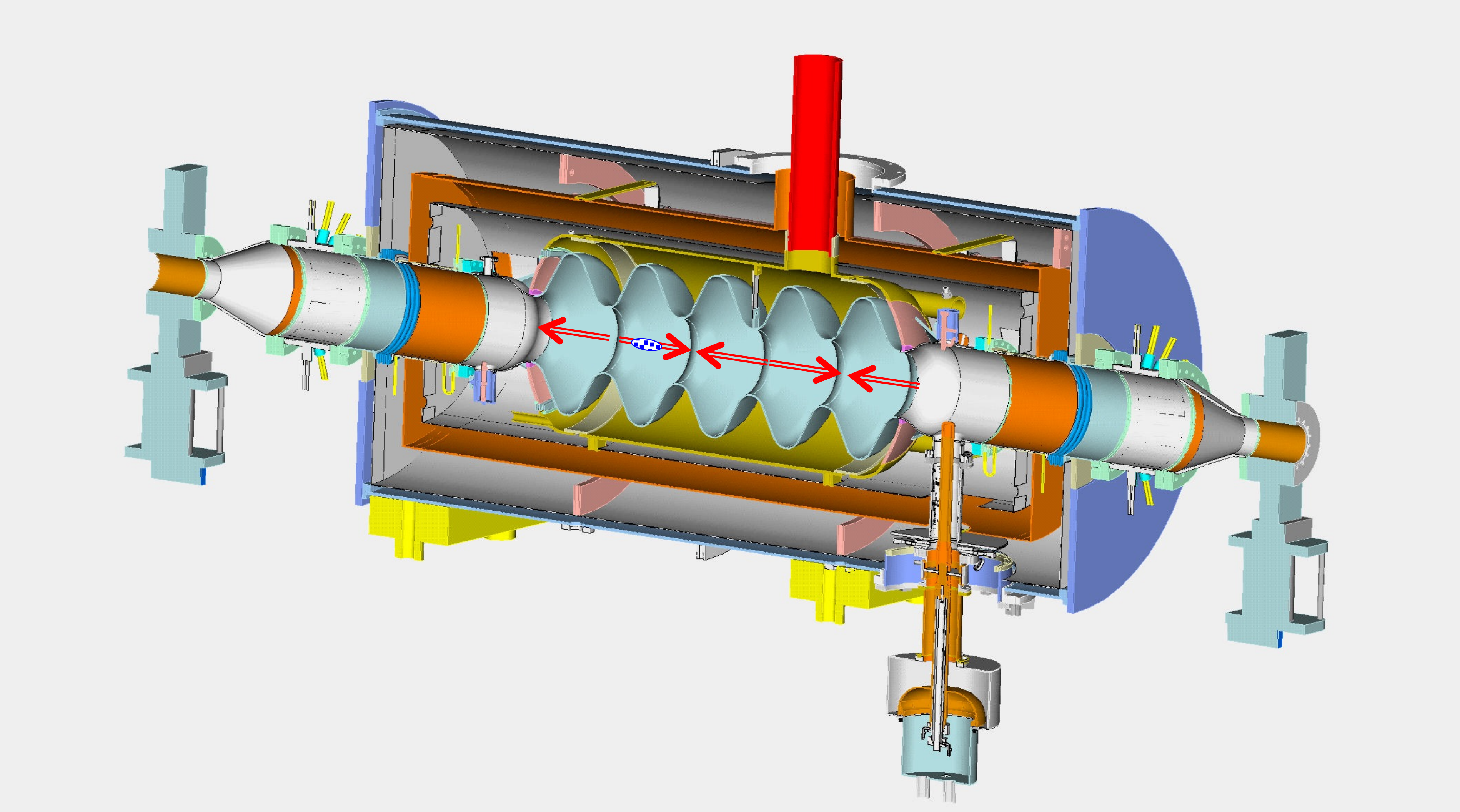
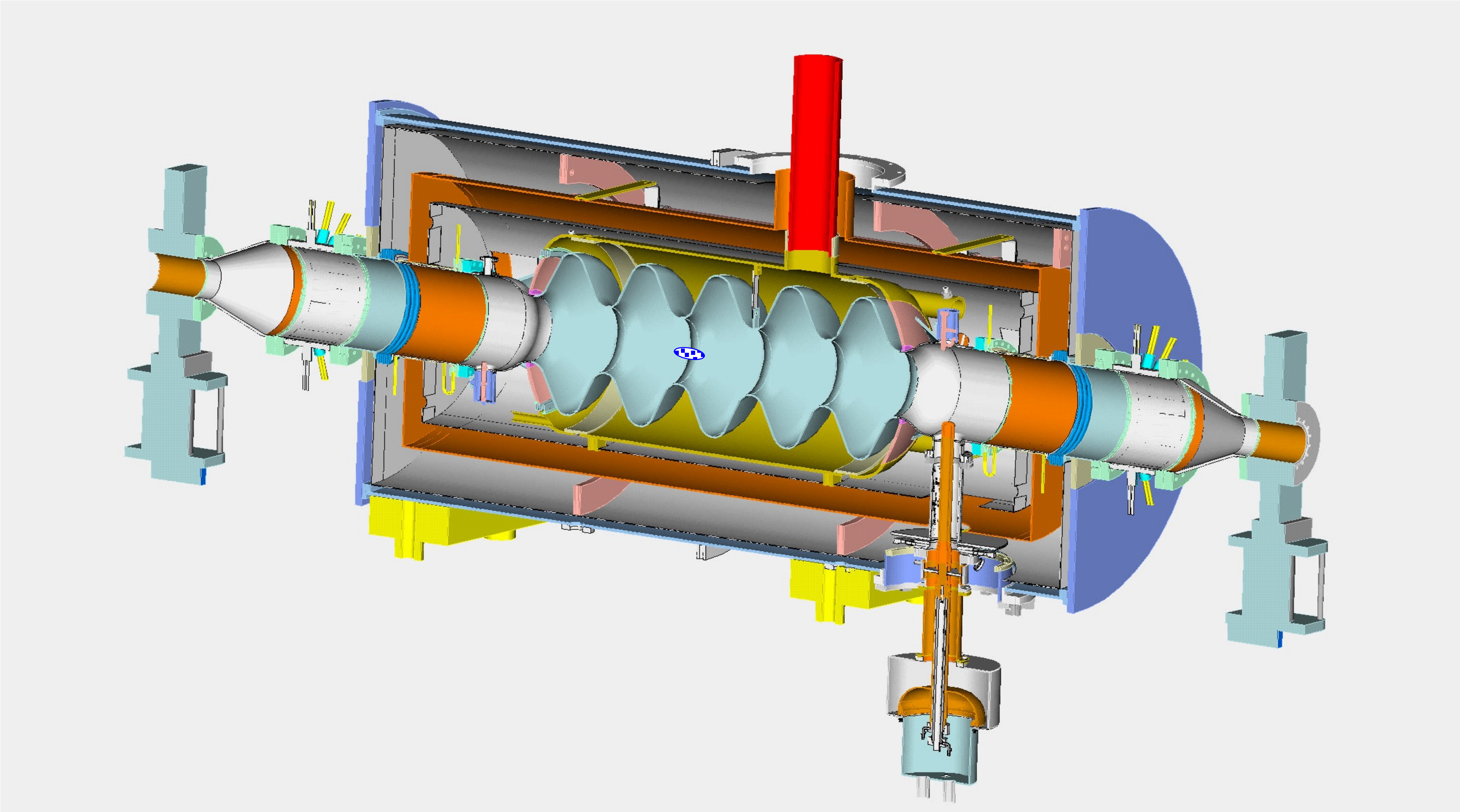
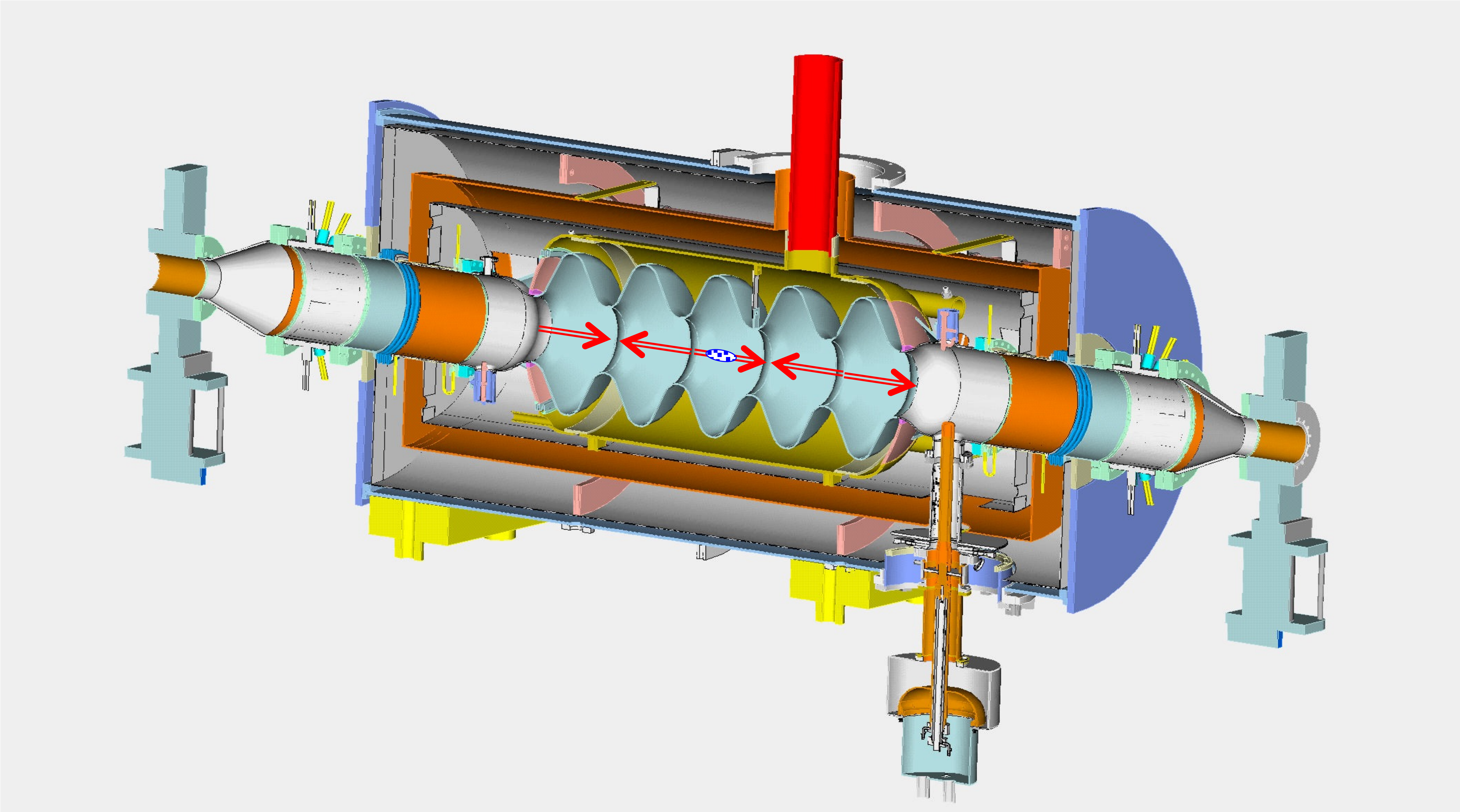


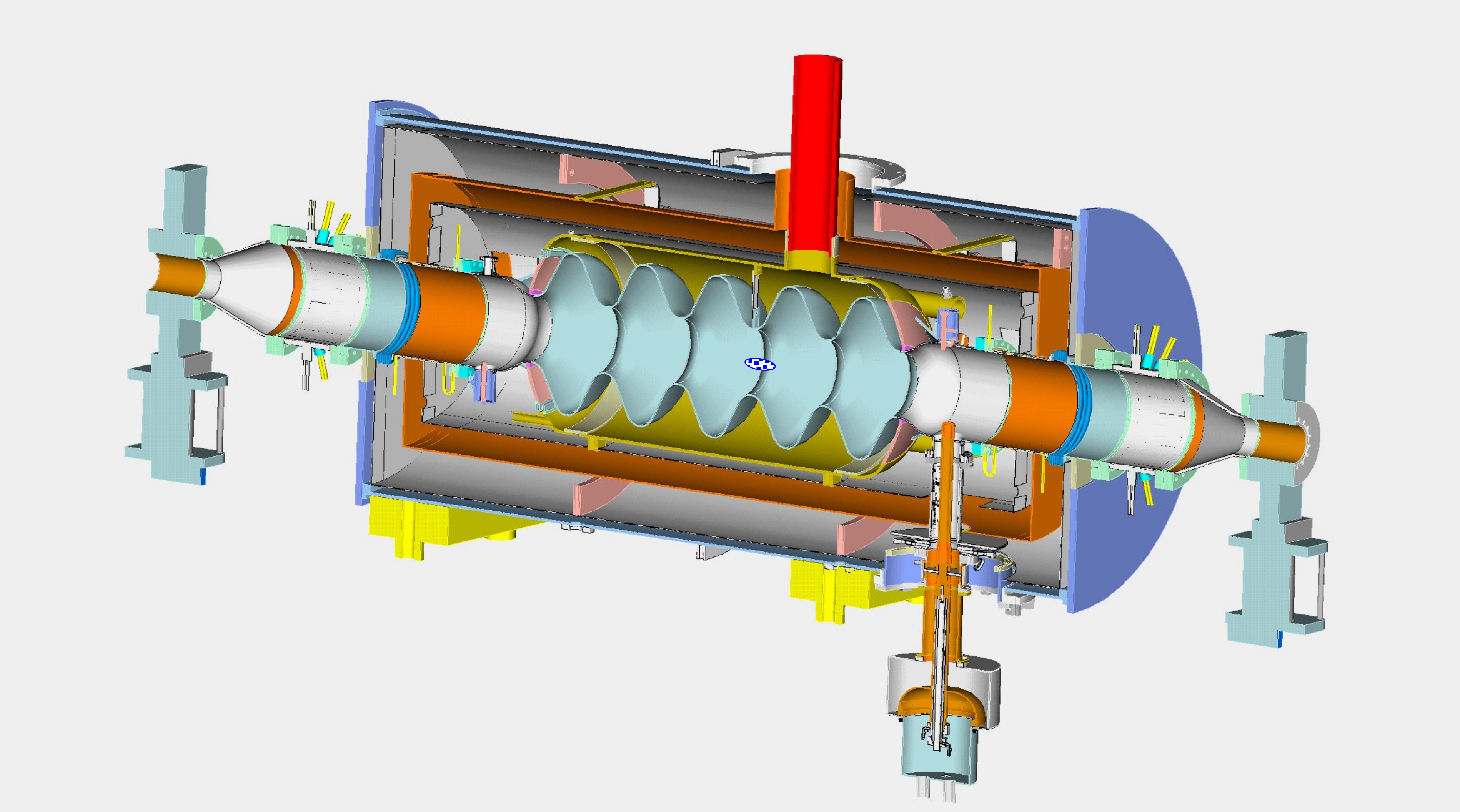
Example of an RF accelerator: 5-cell cavity



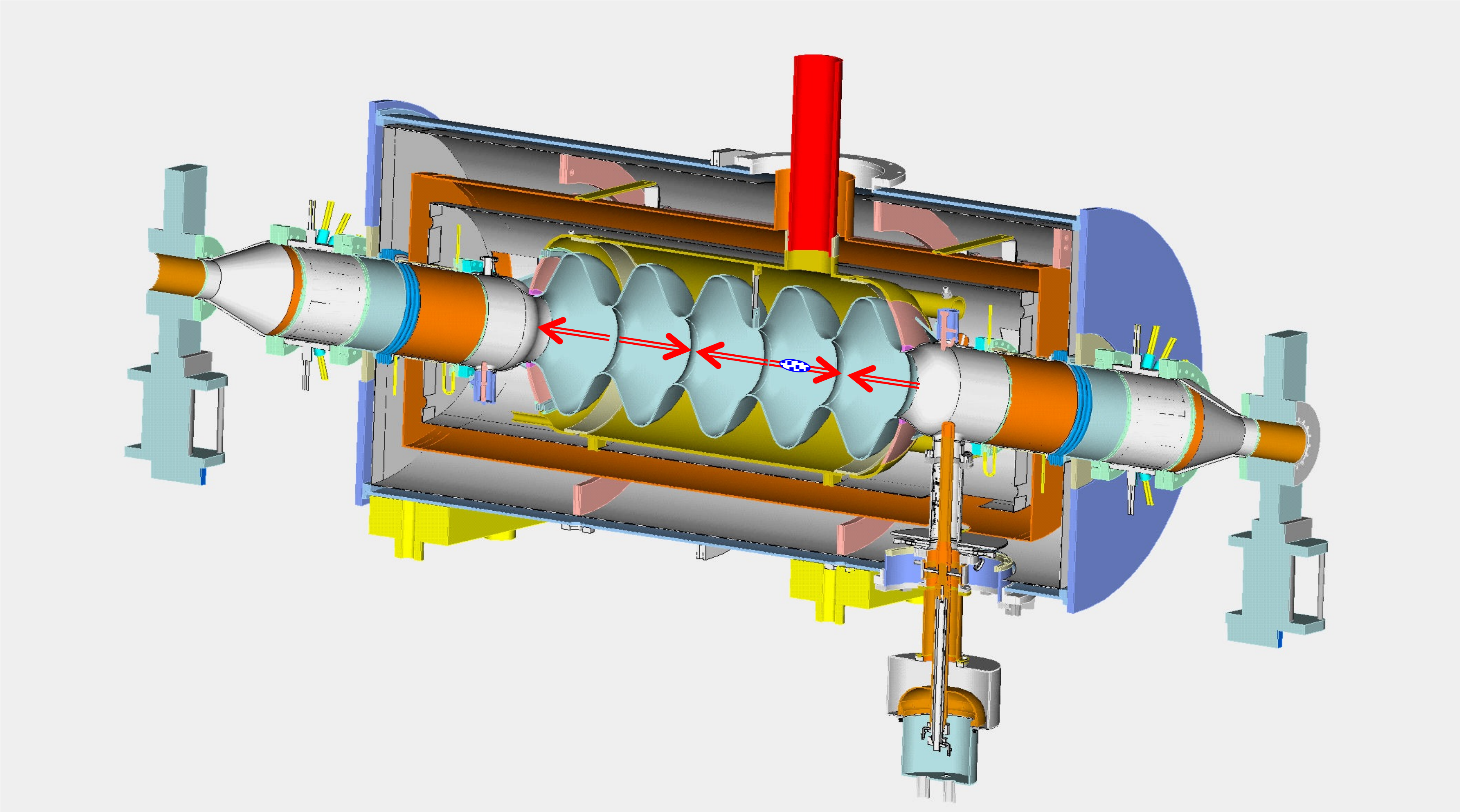
Example of an RF accelerator: 5-cell cavity



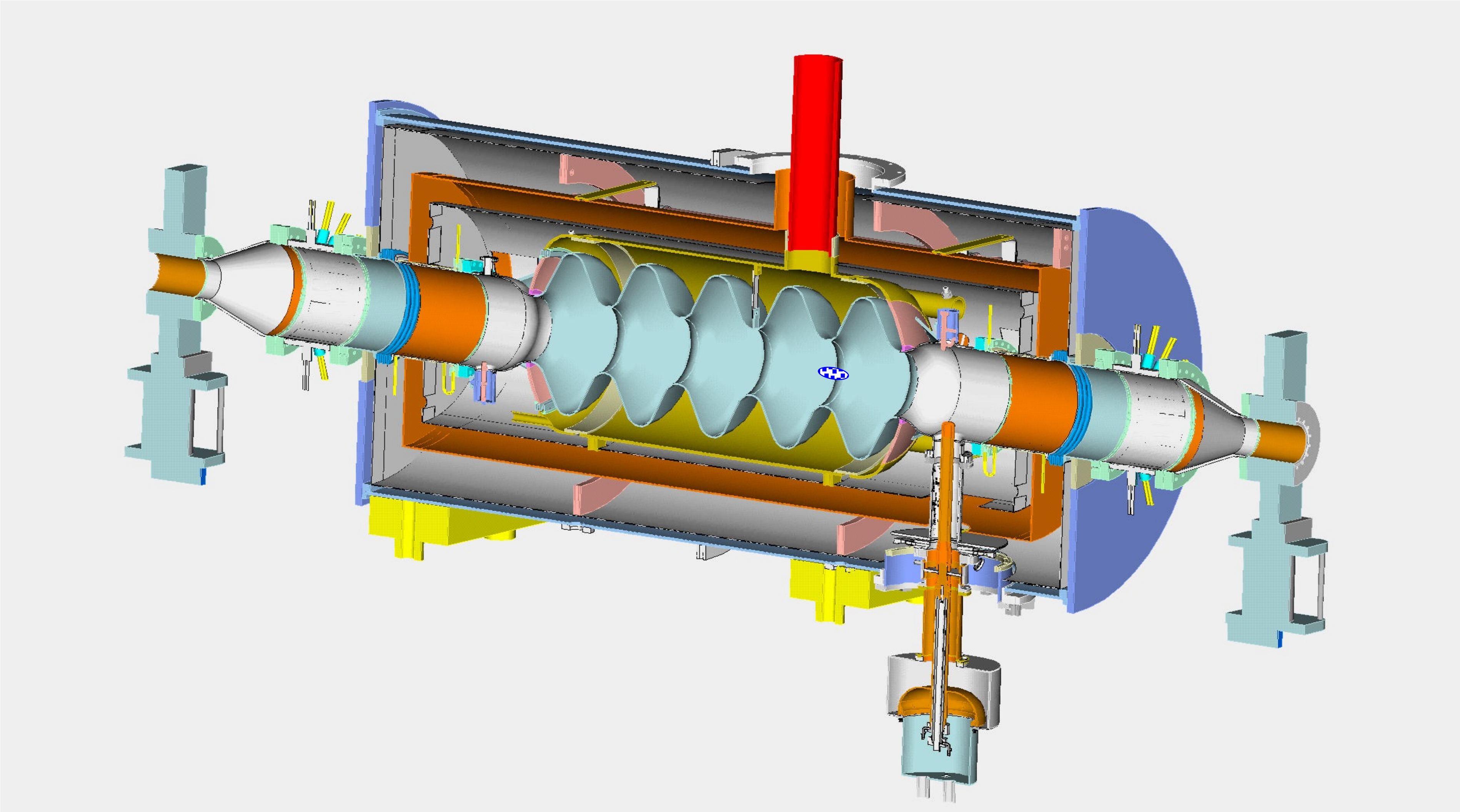
Example of an RF accelerator: 5-cell cavity



Example of an RF accelerator: 5-cell cavity



Example of an RF accelerator: 5-cell cavity



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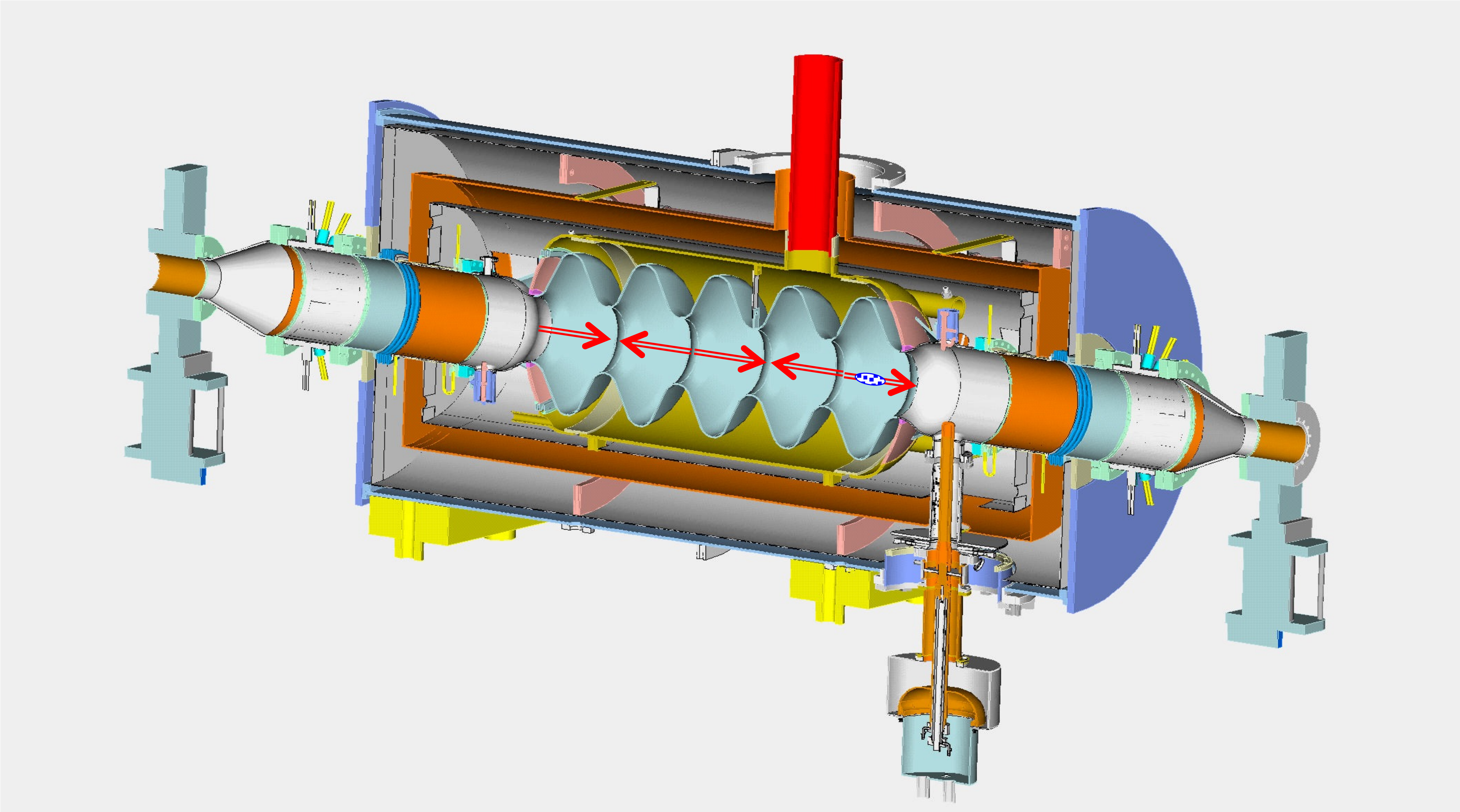
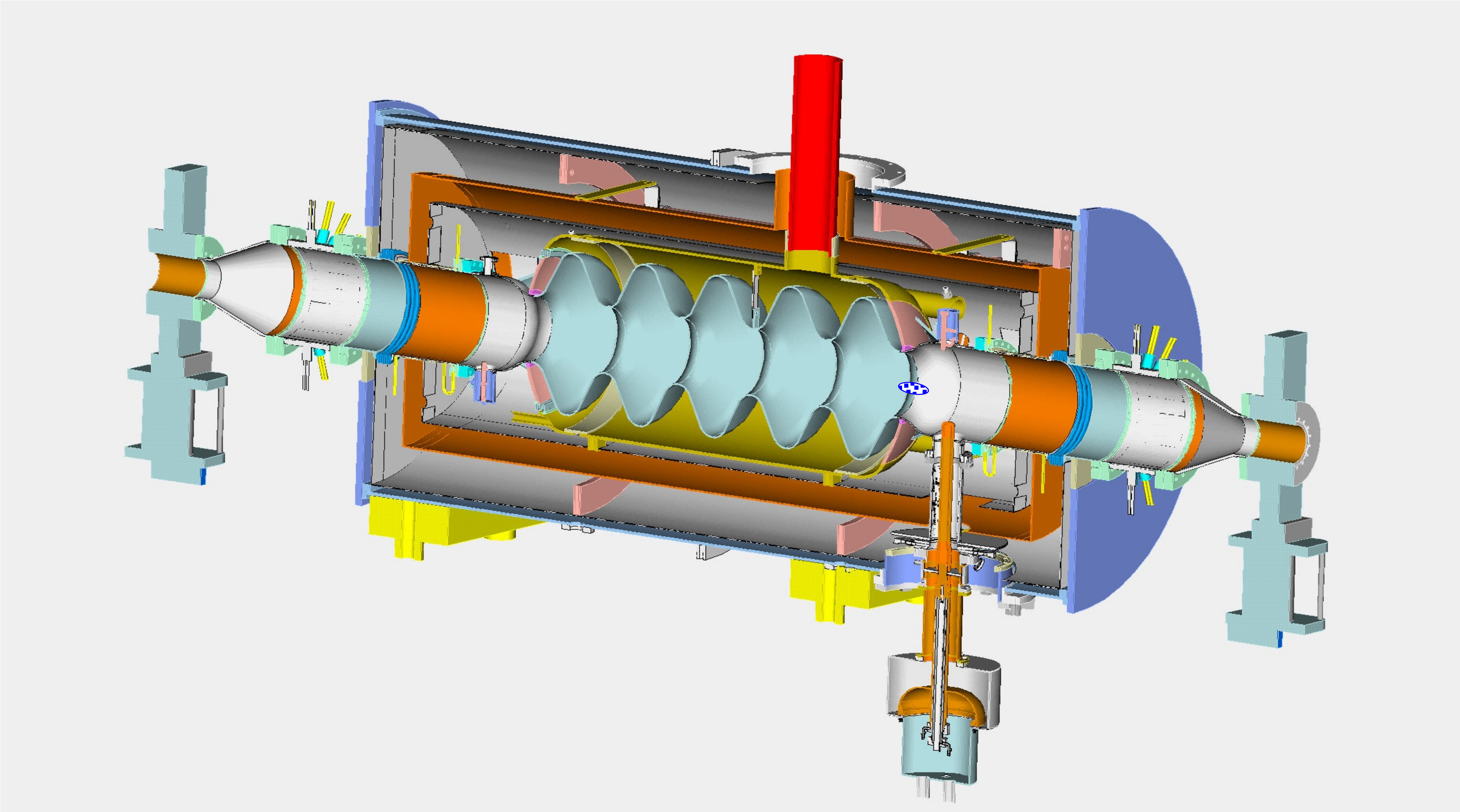
Example of an RF accelerator: 5-cell cavity



Example of an RF accelerator: 5-cell cavity



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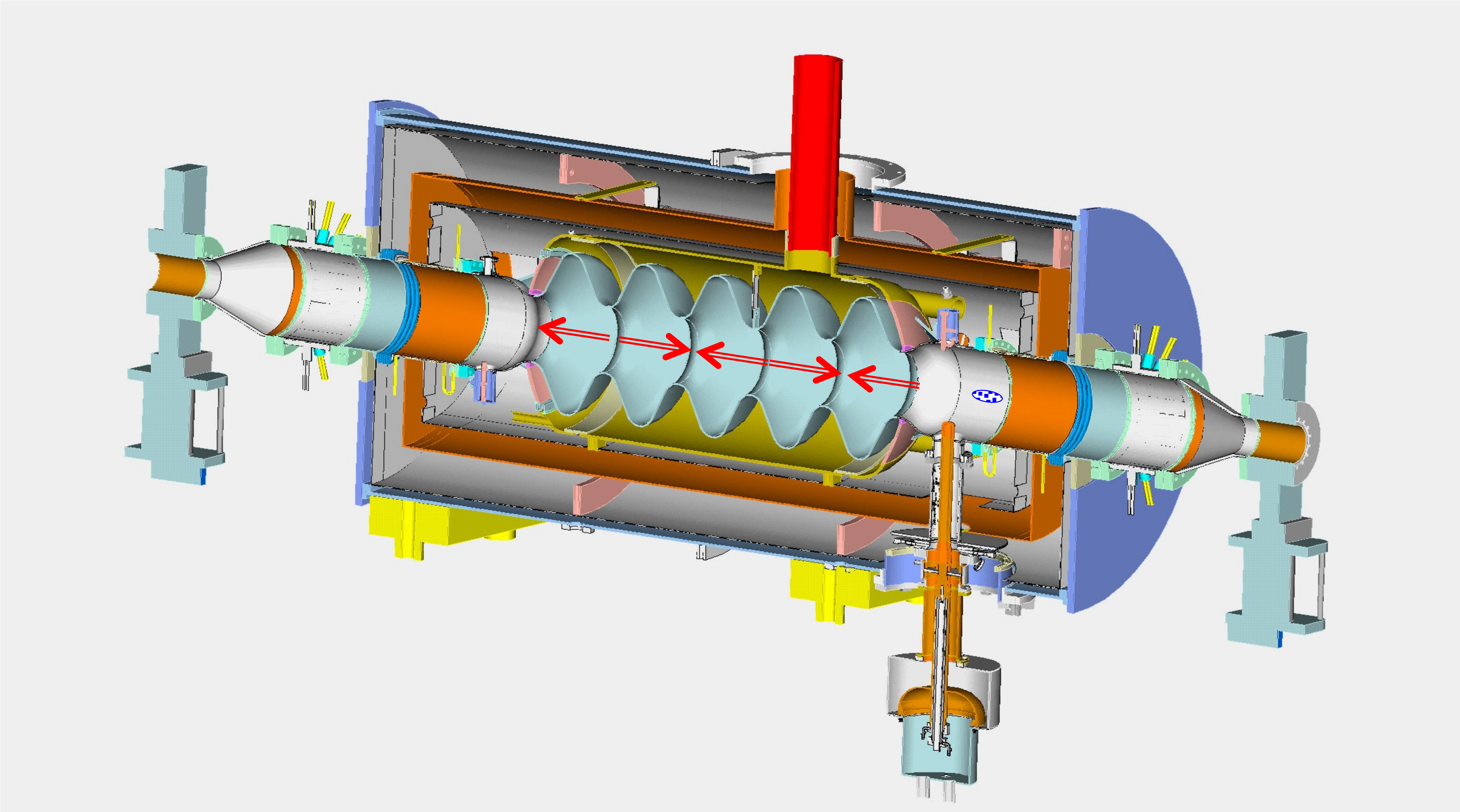

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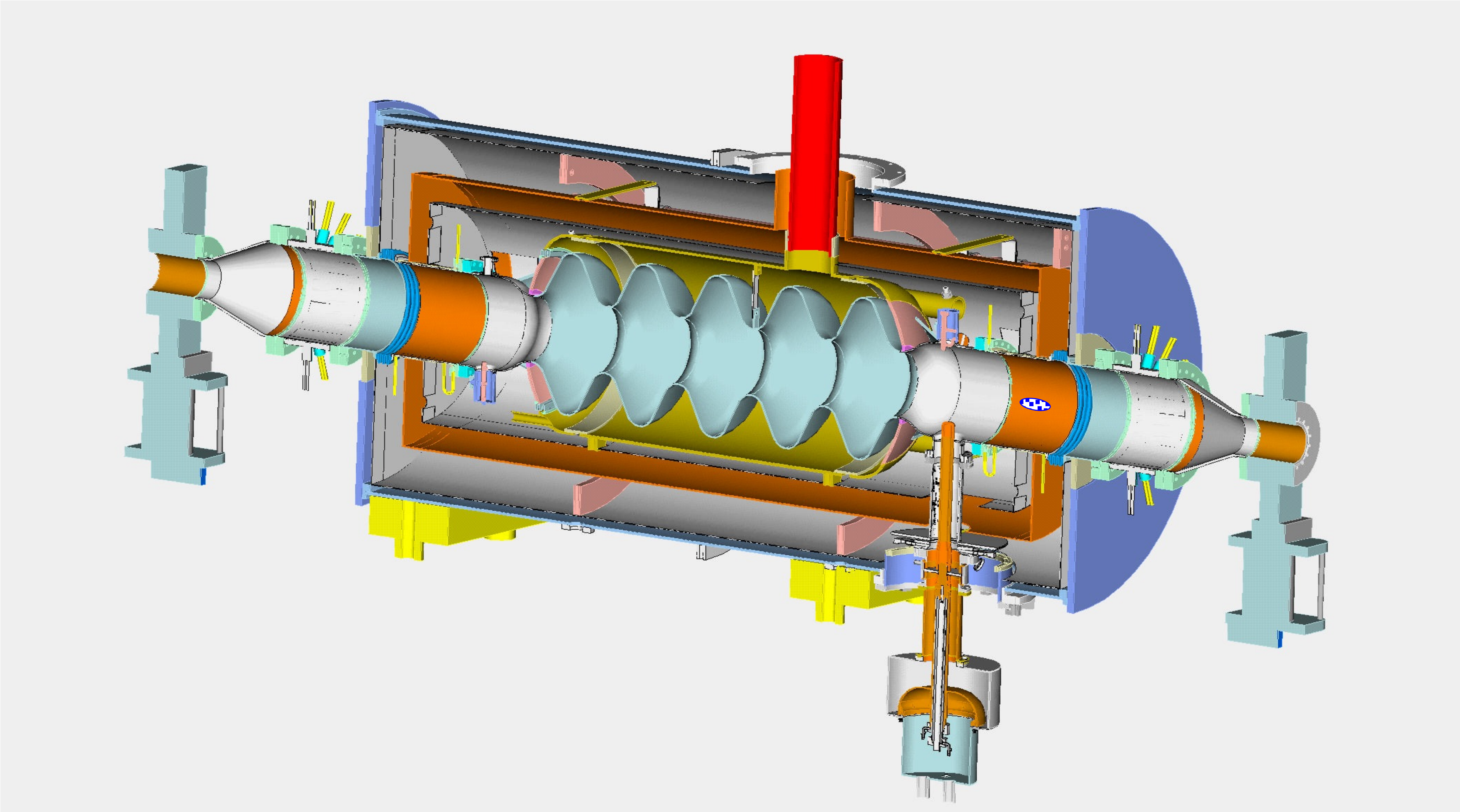


Takeaways # 1:

- Maximum energy obtainable in a **DC accelerator** cannot exceed the product of the charge and voltage drop in the gap. **Electric breakdown** limits achievable acceleration to ~ few tens of MV.

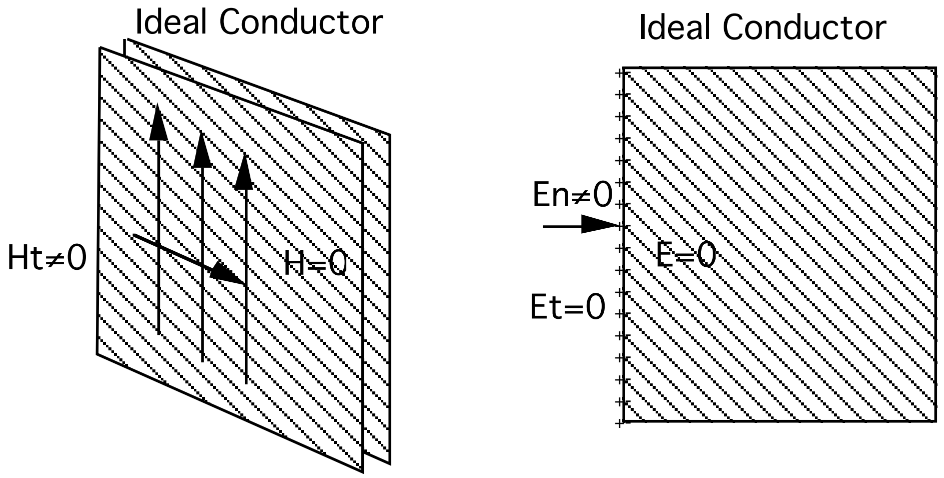
- **RF linear accelerators help to avoid the limitation**.

- In RF linacs, the **coherent addition/subtraction of the energy gain from cell to cell** happens by design: period of the electric field oscillation is matched to the travel time of electron between the cells.

- **Accurate synchronization of RF** linac is important task for any linear accelerator.

RF acceleration: how do we deliver RF to the beam?

- We are considering **oscillating EM fields in RF structures**.
- RF structures are built from **highly conducting material**, both to **contain EM fields** inside and to provide **low losses**.
- In first approximation we can **consider an ideal boundary conditions** and take finite conductivity as a perturbation later.



No perfect conductors exist, but certain metals are very good conductors. Copper, with a room-temperature resistivity of $\rho = {}^{1}/_{\sigma} = 1.7\times10^{-8}\Omega\mathrm{m}$, is the most commonly used metal for accelerator applications.

In a real conductor, the electric and magnetic fields, and the current decay exponentially with distance from the surface of the conductor, a phenomenon known as the **skin effect**. The **skin depth** is given by:

$$\delta = \sqrt{\frac{2}{\sigma\mu_0\omega}}$$

Because of the skin effect, the AC and DC resistances are not equal. It is convenient to define the AC or RF surface resistance. **The AC surface resistance is proportional to the square root of the frequency**.

$$R_s = \frac{1}{\sigma\delta} = \sqrt{\mu_0\omega/2\sigma}$$

What is a transmission line?

- In general, a transmission line is a **medium or structure that forms a path to direct energy from one place to another**.

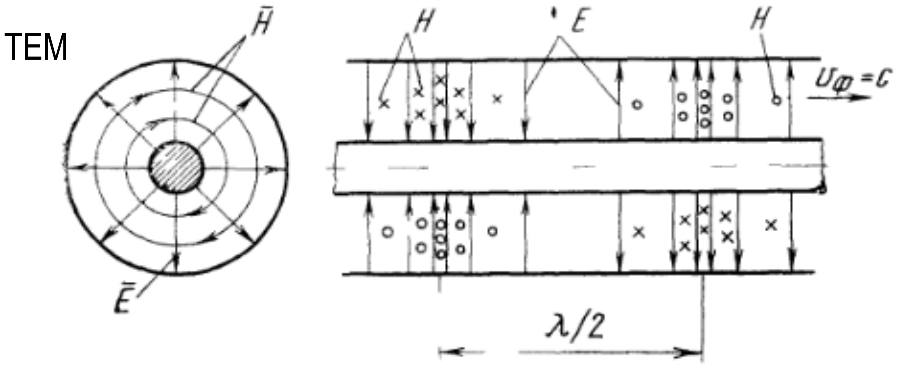
- In radio frequency (RF) and microwave engineering, transmission lines are **specialized cables and waveguides designed to carry an alternating current (AC) of radio frequency** (~20 kHz to 300 GHz).

Coaxial cable

- Electric **currents oscillating at radiofrequencies** present specific properties as a **manifest** of their **wave nature**. (Hereafter referred to as "RF waves".)
- The **RF transmission lines** "enclose" electromagnetic waves, and thus **prevent radiation out of the line**, which would cause **power loss**.

Transmission line: coaxial line

Two types of transmission lines are **commonly used in accelerator RF systems**: **coaxial lines** and **rectangular waveguides**.



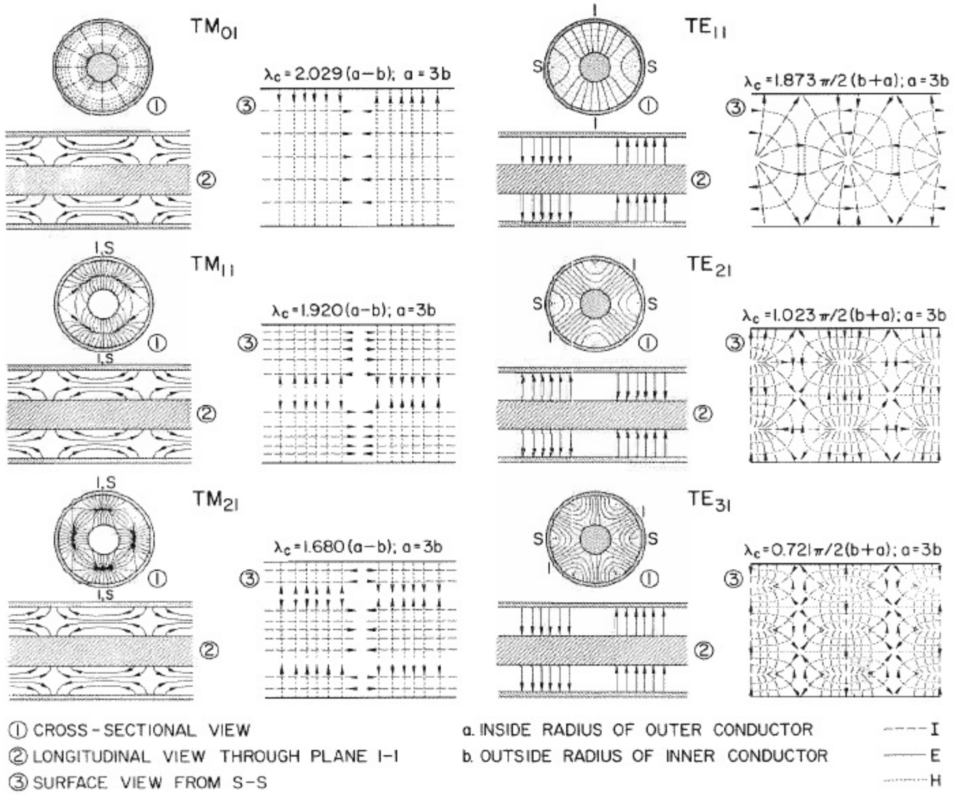
a = radius of outer conductor; b = radius of inner conductor

- The **coaxial line** has two conductors and therefore **can support TEM mode (as well as TE and TM modes).**

- The **bandwidth** of a coaxial line is **theoretically infinite**; in practice, the maximum frequency is **limited to the cutoff of the lowest waveguide mode** (TE11):

$$f_c[TE_{11}] \simeq \frac{c}{\pi(a+b)}$$

Consequently, the line dimensions become smaller at high frequencies and ultimately limits power handling capacity of the coaxial line.



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Transmission line: rectangular waveguide

A **rectangular waveguide** is the **most common transmission line type to power RF cavities in particle accelerators**.

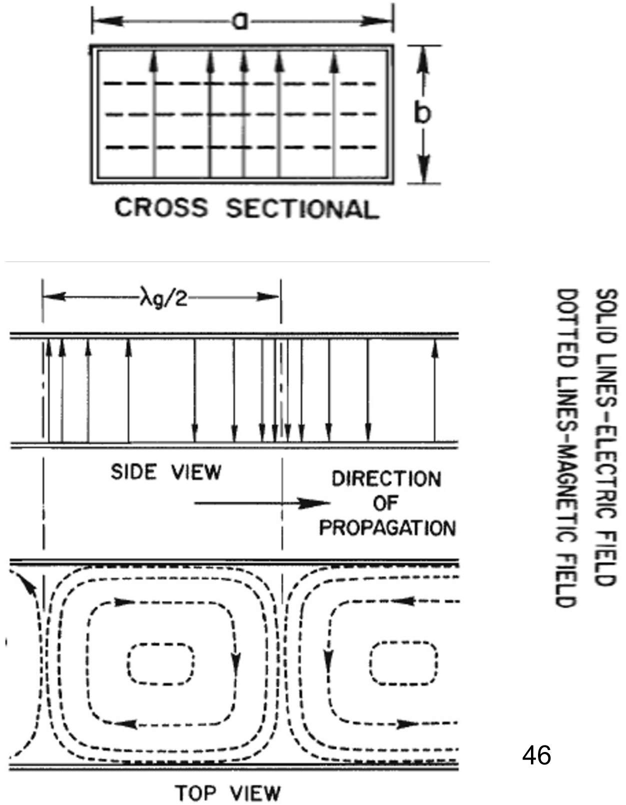
Source: Global Invacom

- Waveguides can **support only TE and TM modes**. Usually the lowest mode, (TE_{10}) is used. The bandwidth is limited by the cutoff frequencies of this and the next lowest modes:

$$\omega_{\mathrm{c,mn}} = \frac{c}{\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

- Waveguides are usually less lossy than coaxial lines due to bigger dimensions and absence of inner conductor. The **attenuation due to conductor losses** for the TE10 mode is:

$$\alpha_{\mathrm{cond}} = \frac{R_s}{b\sqrt{\mu/\varepsilon}\sqrt{1-(\lambda/2a)^2}}\left[1 + \frac{2b}{a}\left(\frac{\lambda}{2a}\right)^2\right]$$



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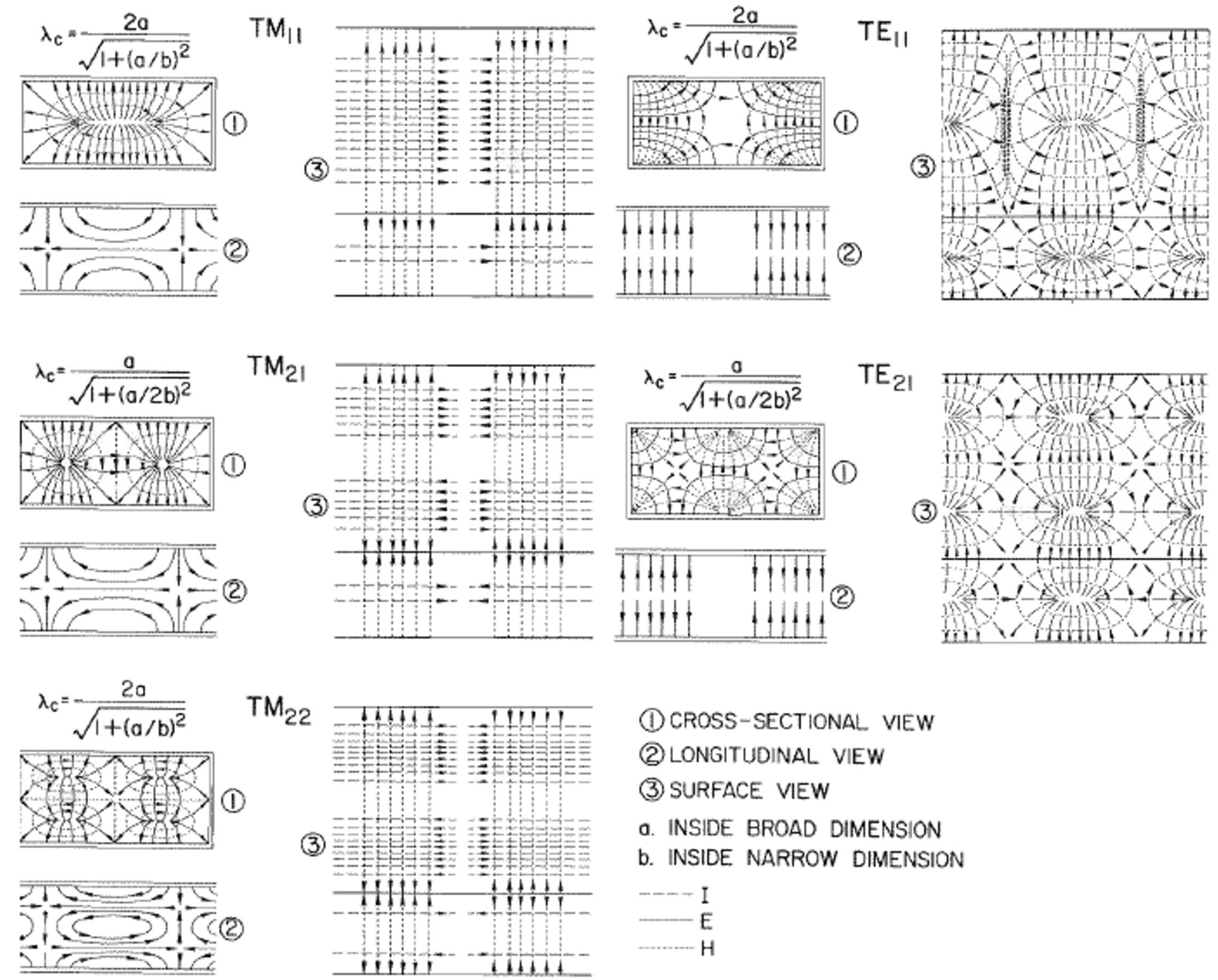
Transmission line: rectangular waveguide

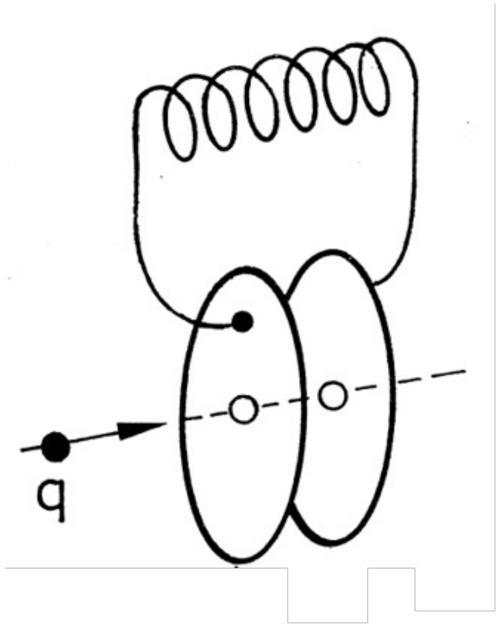
Some modes of the rectangular waveguide and their cutoff frequencies:



RF resonators

An $(R)LC$ circuit is the the simplest form of an RF resonator.

To use such a circuit **for particle acceleration**, it must have **opening for beam passage in the area of high electric field** (capacitor).



As particles are accelerated in vacuum, the structure must **provide vacuum space**.

- A ceramic vacuum break (between the two electrodes of the capacitor) can be used to separate the beam line vacuum from the rest of the resonator.

- Alternatively, the resonant structure can be enclosed in a vacuum vessel.

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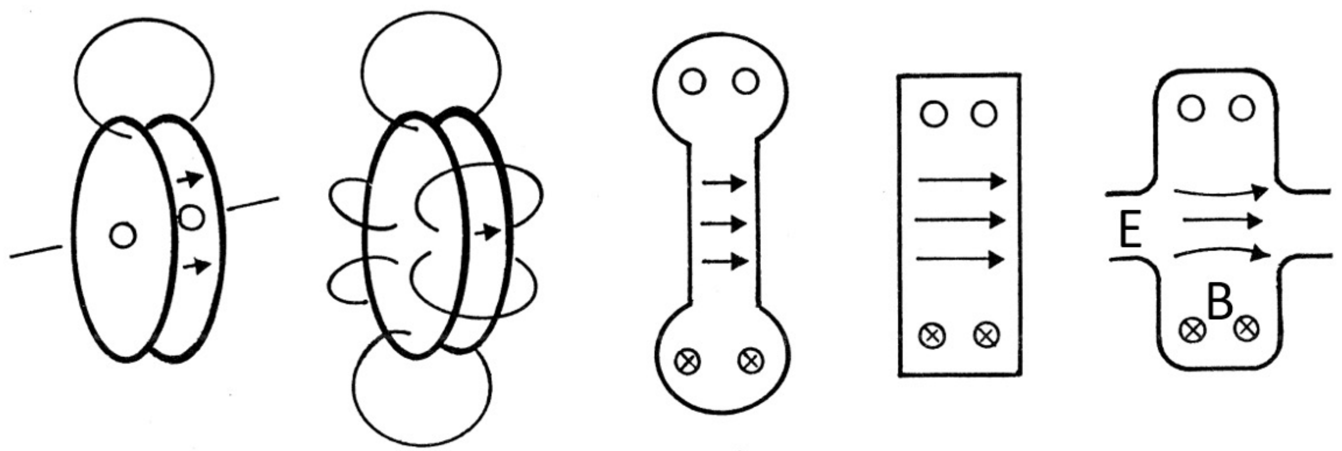
RF cavities

Or we can use "**cavity resonators**".

Metamorphosis of the LC circuit into an accelerating cavity:

1) Increase resonant frequency by lowering L, eventually have a solid wall.
2) Further frequency increase by lowering C → arriving at cylindrical, or "pillbox" cavity geometry, which can be solved analytically.
3) Add beam tubes to let particle pass through.



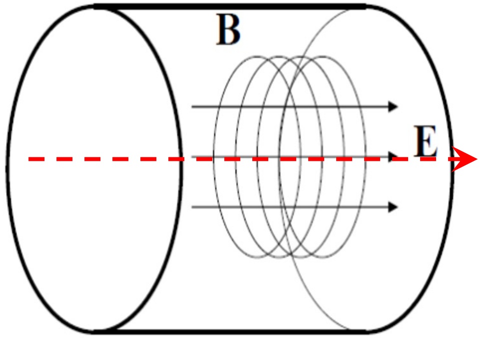
The LC circuit and a resonant cavity share common aspects:

- **Energy is stored** in the electric and magnetic fields
- **Energy is periodically exchanged** between electric and magnetic field
- Without any external input, the **stored power will eventually all turn into heat**

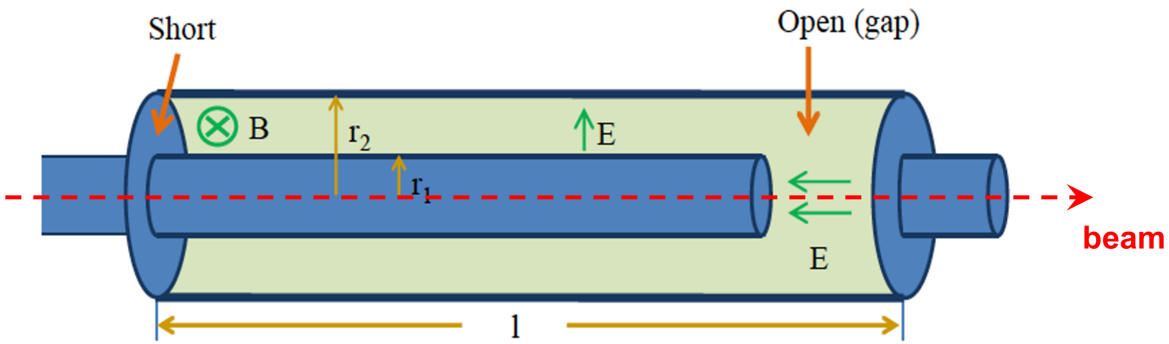
RF cavities

A cavity is a **closed metal structure that confines electromagnetic fields** in the RF or microwave region of the spectrum.

- Such cavities act as **resonant circuits with extremely low losses**.
  The Q factor for cavities made of copper is typically of the order of ten thousands compared to a few hundreds for resonant circuits made with inductors and capacitors at the same frequency.
- Resonant cavities can be made from **closed (or short-circuited) sections of a waveguide or coaxial line**.
- Electromagnetic energy is stored in the cavity; the only **losses are due to finite conductivity of cavity walls and dielectric/ferromagnetic losses of material filling the cavity**.
- The cavity wall structure can be made **stiff to allow its evacuation**.



Circular (pillbox) cavity



Coaxial cavity

Pillbox cavity

Fields in the cavity are solutions of the equation:

$$\left(\nabla^2 + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\begin{Bmatrix}\vec{E}\\ \vec{H}\end{Bmatrix} = 0$$

subject to the boundary conditions:

$$\hat{n} \times \vec{E} = 0,\ \hat{n} \cdot \vec{H} = 0$$

An infinite number of solutions (eigenmodes) belong to **two families of modes** with different field structure and eigenfrequencies:

- **TE modes have only transverse electric fields ($E_z = 0$)**;
- **TM modes have only transverse magnetic fields ($H_z = 0$).**

The modes are classified as TM_{mnp} (TE_{mnp}), where integer indices $m, n,$ and p correspond to the number of sign variations that E_z (H_z) has along $\varphi, r,$ and z directions respectively.

➢ As a longitudinal electric field (along z axis) is needed **for acceleration, only TM modes can be used**. Typically, the lowest frequency TM_{010} mode of a pillbox cavity is used.

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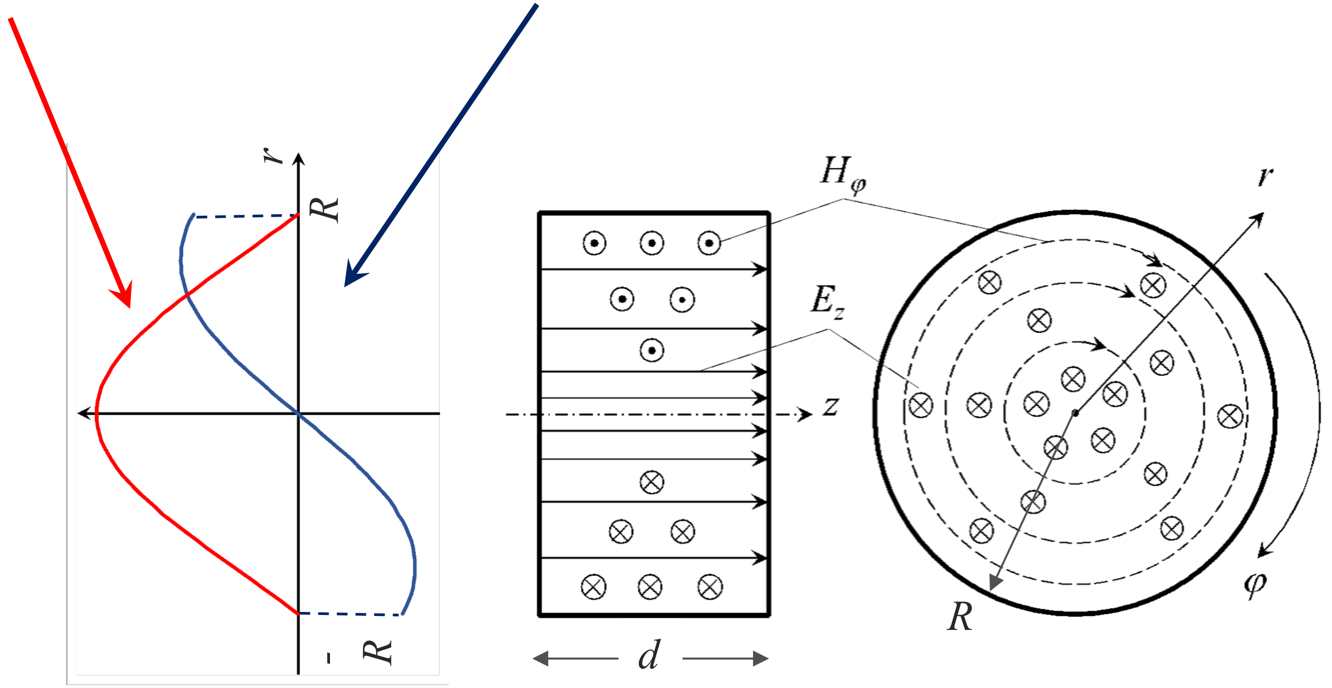
Pillbox cavity: TM_{010}

For a pillbox cavity w/o beam tubes the solution of Maxwell equations gives us:

$$E_z = E_0 J_0\left(\frac{2.405r}{R}\right)e^{i\omega t} \qquad H_\varphi = -i\frac{E_0}{\sqrt{\mu/\varepsilon}}J_1\left(\frac{2.405r}{R}\right)e^{i\omega t} \qquad \omega_{010} = \frac{2.405r}{R}$$

The electric field, concentrated near the axis, is used for acceleration.

The magnetic field is concentrated near the cylindrical wall and contributes to the RF losses.



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Pillbox cavity: Higher Order Modes (HOMs)

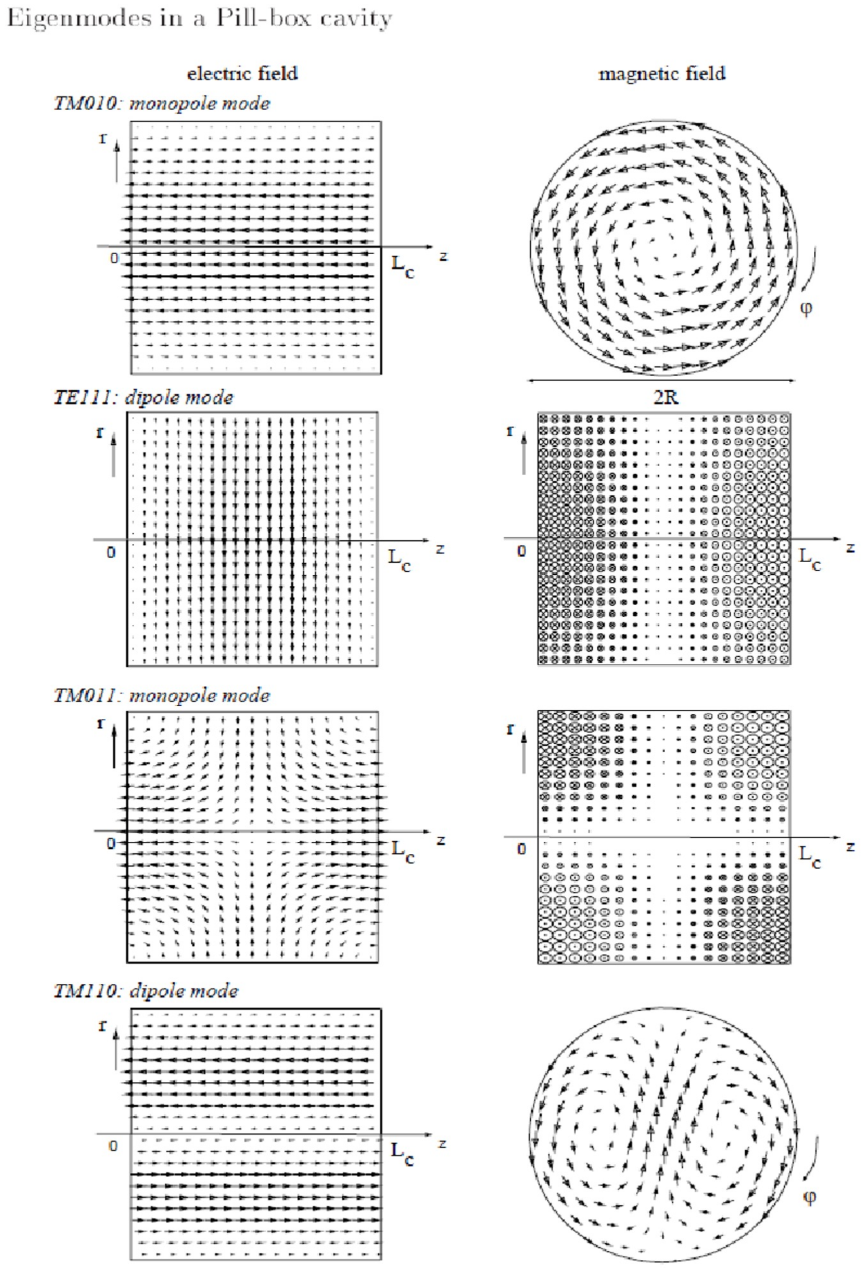
While TM_{010} mode is used for acceleration and usually is the lowest frequency mode, **all other modes are "parasitic" as they may cause various unwanted effects**. Those modes are referred to as **Higher-Order Modes** (HOMs).

The resonant wavelengths of the pillbox cavity HOMs are given by:

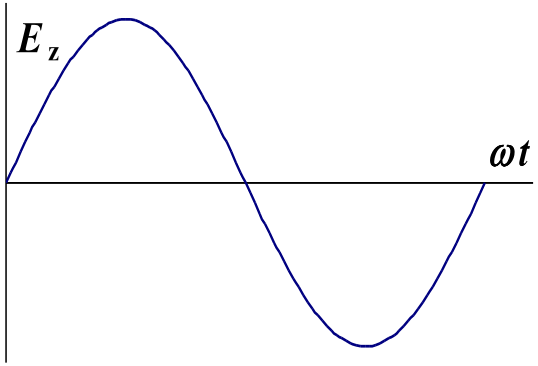
$$\omega_{c,TM_{mnp}} = \frac{2\pi c}{\sqrt{\left(\frac{\nu_{mn}}{2\pi R}\right)^2 + \left(\frac{p}{2d}\right)^2}}$$

$$\omega_{c,TE_{mnp}} = \frac{2\pi c}{\sqrt{\left(\frac{\mu_{mn}}{2\pi R}\right)^2 + \left(\frac{p}{2d}\right)^2}}$$

n	ν_{m1}	ν_{m2}	ν_{m3}	μ_{m1}	μ_{m2}	μ_{m3}
0	2.405	5.520	8.654	3.832	7.016	10.174
1	3.832	7.016	10.174	1.841	5.331	8.536
2	5.135	8.417	11.620	3.054	6.706	9.970



Acceleration inside a pillbox cavity



- Let's consider a **charged particle** passing on the axis of the cavity **with constant velocity** (either ultra relativistic or velocity change is negligible).

- On-axis electric **field depends on longitudinal position z and time:**

$$E_z(z,t) = E_0(z)\cos(\omega t + \varphi)$$

- Specific form of **$E_0(z)$ depends on the cavity design**.

- Energy change ΔE of the particle with charge q:

$$\Delta E = q\int_{-\infty}^{\infty} E_0(z)\cos(\omega t + \varphi)dz$$

$$t = \frac{z}{v} \rightarrow \Delta E = q\int_{-\infty}^{\infty} E_0(z)\cos\left(\omega\frac{z}{v} + \varphi\right)dz$$

Next, use $\cos\left(\omega\frac{z}{v} + \varphi\right) = \cos(\varphi)\cos\left(\omega\frac{z}{v}\right) - \sin(\varphi)\sin\left(\omega\frac{z}{v}\right)$

$$\Delta E = q\cos(\varphi)\underbrace{\int_{-\infty}^{\infty} E_0(z)\cos\left(\omega\frac{z}{v}\right)dz}_{V_c} - q\sin(\varphi)\underbrace{\int_{-\infty}^{\infty} E_0(z)\sin\left(\omega\frac{z}{v}\right)dz}_{V_s}$$

Define $V_{\mathrm{RF}} = \sqrt{V_c^2 + V_s^2},\ \tan(\varphi_0) = \frac{V_c}{V_s}$

$$\boxed{\Delta E = qV_{\mathrm{RF}}\cos(\varphi + \varphi_0)}$$

For a particle moving with constant velocity all cavities are described by nothing else but **accelerating voltage and phase**!

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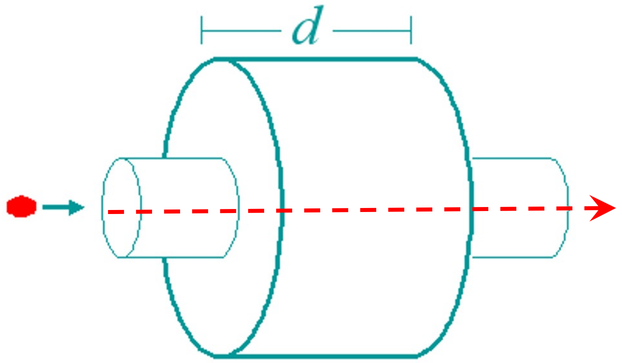
Accelerating voltage and transit time factor

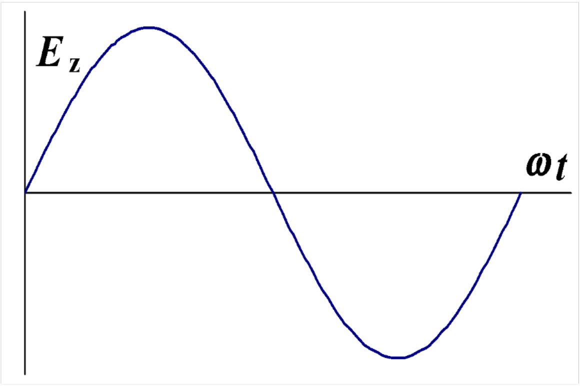


Assuming charged particles moving along the cavity axis, the accelerating voltage is given by:

$$V_c = \left| \int_{-\infty}^{\infty} E_0(\rho = 0, z) e^{i\omega z/\beta c} dz \right|$$

For the pillbox cavity we can integrate this analytically:

$$V_c = E_0 \left| \int_0^d e^{i\omega z/\beta c} dz \right| = E_0 d \frac{\sin\left(\frac{\omega d}{2\beta c}\right)}{\frac{\omega d}{2\beta c}} = E_0 d T$$



where T is the **transit time factor**. For maximum acceleration, the particle travels through the cavity in half period:

$$T_{\text{transit}} = t_{\text{exit}} - t_{\text{enter}} = \frac{\pi}{\omega} = \frac{T_0}{2} \rightarrow d = \beta\lambda/2 \rightarrow V_c = \frac{2}{\pi} E_0 d$$

Thus, for the pillbox cavity: $T = 2/\pi$.

i.e. Select $\omega_0 d$ of cavity to maximize acceleration for a particle with speed $\beta_r c_0$.

The **accelerating field** E_{acc} is defined as $E_{acc} = V_c/\mathrm{d}$.

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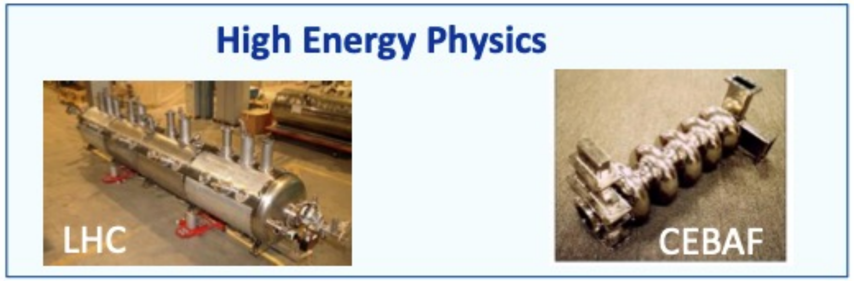
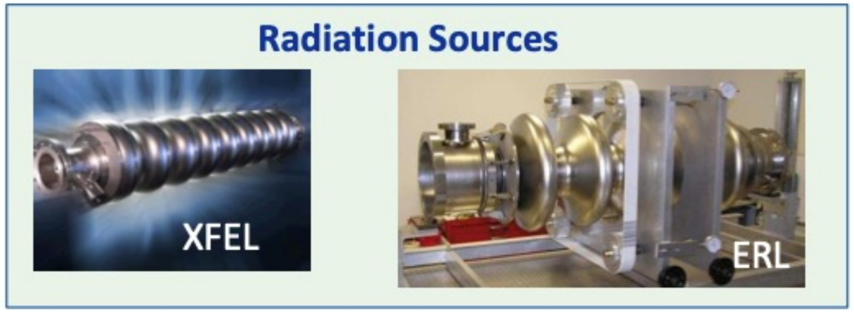
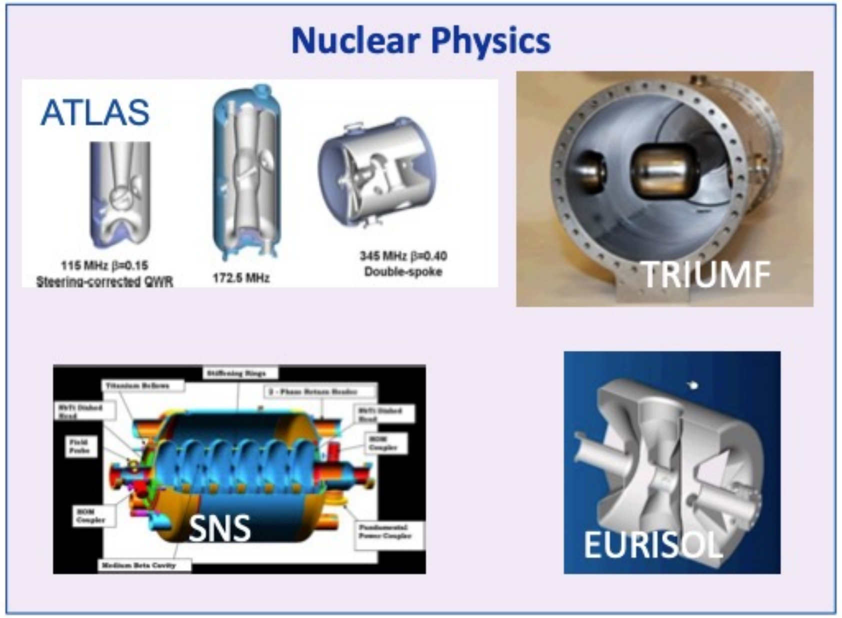
The cavity length is not as easy to specify for shapes other than pillbox so usually it is assumed to be d = βλ/2. This works OK for multi-cell cavities, but poorly for single-cell cavities or low β cavities.

The Zoo of RF cavities

The specific application, facility requirements and mode of operation will define the choice of the cavity type and operating frequency.







Low-beta & β=1 cavities

- For heavy particles like protons, it takes a lot of RF cavities to accelerate to velocity comparable to speed of the light.
- Hence, there are so called **low-β cavities designed for slow particles** (β=0.1, β=0.5…).
- Cavities used for particles traveling with $v \approx c$ are called β=1 cavities.

Cavities with β≠const

- Typically, we can use approximation that velocity of accelerating beam is nearly constant when it passing the cavity gap.
- This assumption is good for ultra-relativistic electrons/positions.
- This assumption is also a good approximation for heavy particles (ions or protons) when the energy gain per one accelerator cell is a small portion of the particle's rest mass energy.
- This assumption is violated and can not be used **for electron guns**, where electrons can accelerate from zero velocity to nearly speed of light. **Equations of motion are both time-dependent and non-linear.** You can estimate the result, nowadays it is normal to use numerical codes to get all beam dynamics correctly.

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(Intrinsic) Quality factor

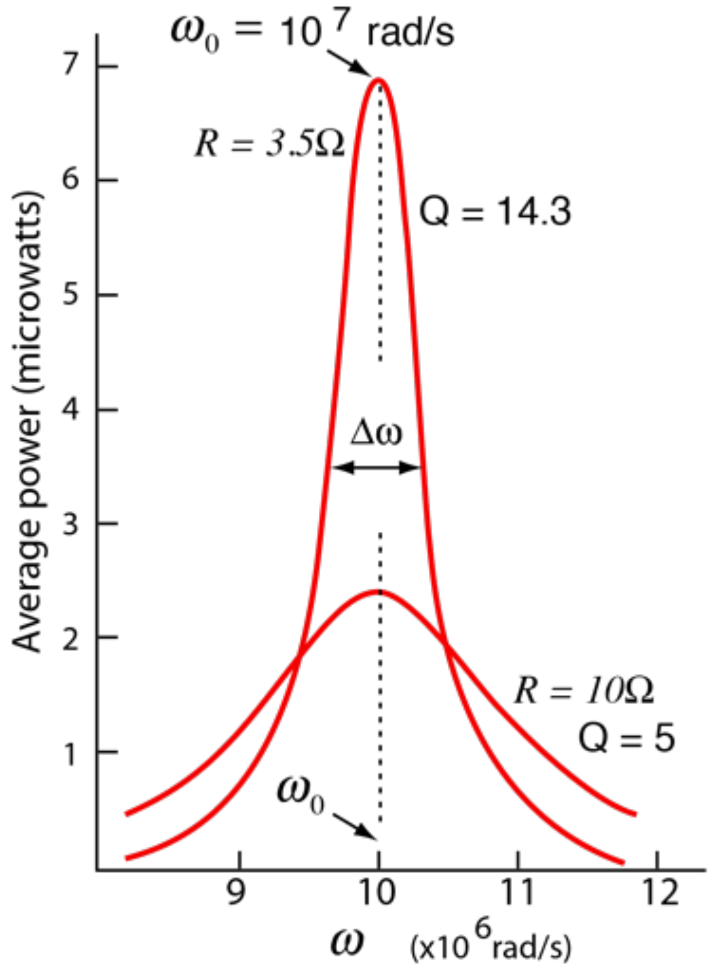
The total energy stored in the EM field of a cavity is the sum of energy stored in the electric and magnetic fields. Given the sinusoidal time dependence and the 90° phase shift between the electric and magnetic fields, the **energy oscillates back and forth between the electric and magnetic field**. Then the stored energy in a cavity is given by:

$$U = U_{\mathrm{E}} + U_{\mathrm{H}} = \frac{1}{2}\varepsilon \int_{\mathrm{V}} \left|\vec{E}\right|^2 dv = \frac{1}{2}\mu \int_{\mathrm{V}} \left|\vec{H}\right|^2 dv$$

An important figure of merit is the **(intrinsic) quality factor**, which for any resonant system is:

$$Q_0 = \omega_0 \frac{\text{stored energy}}{\text{average power loss}} = \frac{\omega_0 U}{P_0} = \frac{2\pi}{T_0}\frac{U}{P_0} = \frac{\omega_0}{\Delta\omega_0}$$

roughly 2π times the number of RF cycles it takes to dissipate the energy stored in the cavity. Q_0 is **determined by** both **material properties and cavity geometry**.



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- For NC cavities, $Q_0 \sim 10^4$; and for SC cavities at 2K, $Q_0 \sim 10^{10}$.

Geometry factor

If the surface resistance does not vary over the cavity surface:

$$Q_0 = \omega_0 \frac{\mu \int_V \left|\vec{H}\right|^2 dv}{R_s \int_S \left|\vec{H}\right|^2 ds}$$

The **ratio of the two integrals** in the last equation is **determined only by the cavity geometry**. Thus, we can re-write it as

$$Q_0 = \frac{G}{R_s}$$

with the parameter G known as the **geometry factor** or geometry constant:

$$G = \omega_0 \frac{\mu \int_V \left|\vec{H}\right|^2 dv}{\int_S \left|\vec{H}\right|^2 ds}$$

- The geometry factor depends only on the cavity shape and electromagnetic mode, but not its size: Scaling the cavity size x-fold, increases volume as x^3, reduces frequency as 1/x and increases surface as x^2. Hence, G does not change.
- The geometry factor **depends only on the cavity shape and the field configuration of an electromagnetic mode, but not on the cavity size**. Hence it is very useful for comparing different cavity shapes.

  - For the pillbox accelerating cavity, G = 257 Ohm.
  - For copper ($\sigma_c = 5.8{\times}10^7$S/m) cavity operating at 1.5 GHz: δ = 1.7 µm, $R_s =$ 10 mOhm, and $Q_0 = G\,/R_s =$ 25,700.

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Shunt impedance and R/Q

The **shunt impedance characterizes the losses in a cavity**: $R_{\rm sh} = \frac{V_{\rm c}^2}{P_0}$

Often the shunt impedance is defined following the circuit theory convention: $R_{\rm sh} = \frac{V_{\rm c}^2}{2P_0}$

and, to add to the confusion, a common definition in linacs is: $r_{\rm sh} = \frac{E_{\rm acc}^2}{P_0'}$

where $P_0{}'$ is the power dissipation per unit length and hence, the linac shunt impedance is in Ohms per meter.

A related quantity is the **geometric shunt impedance** (R_{sh}/Q_0), which is **independent of the surface resistivity and the cavity size**:

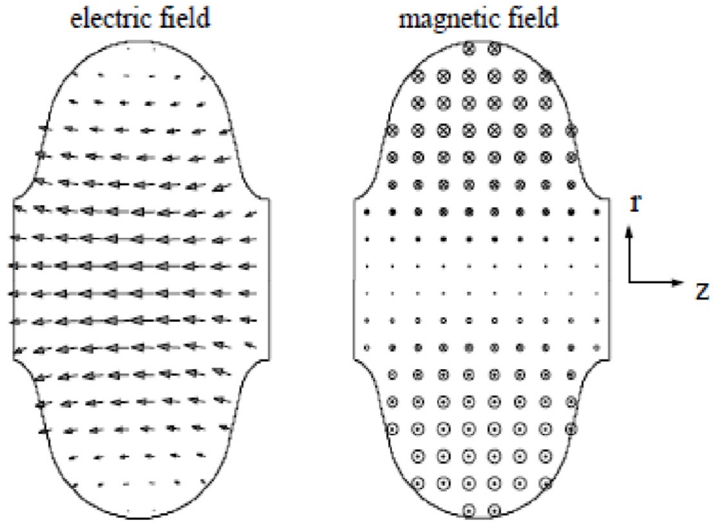
$$\frac{R_{\rm sh}}{Q_0} = \frac{V_{\rm c}^2}{\omega_0 U}$$

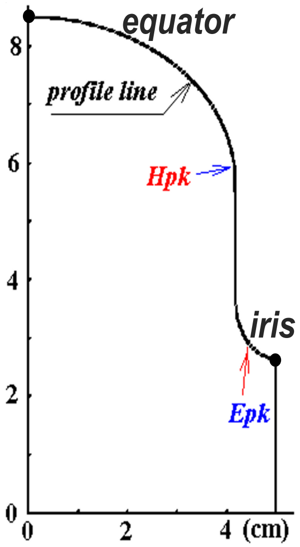
- For the pillbox cavity $\mathrm{R_{sh}/Q_0}$ = 196 Ohm.

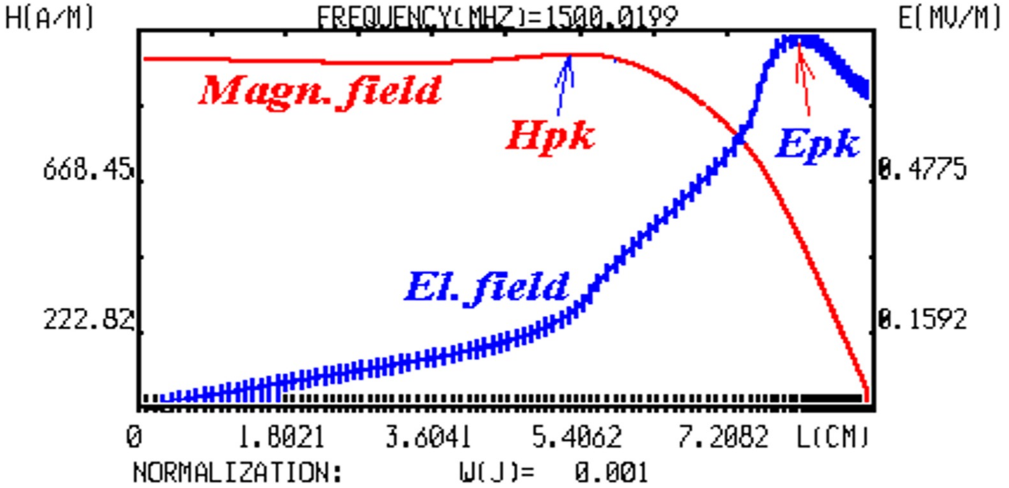
➢ The (R_{sh}/Q_0) is frequently used as a figure of merit and is **useful in determining the level of mode excitation by bunches of charged particles passing through the cavity**.

Peak surface fields

The **ratios of the max. peak surface (electric and magnetic) fields to the accelerating field**, resp. E_{pk}/E_{acc} and H_{pk}/E_{acc} are an important **figure of merit** to assess **performance limitations**. These should be made as small as possible.







Fields along the profile line

- E_{pk} **is responsible for field emission**
  - Typically for real elliptical cavities $\mathrm{E_{pk}/E_{acc}} = 2$ ... 2.6, as compared to 1.6 for a pillbox cavity.
- H_{pk} **has fundamental limit** (**critical field of SC state**) and the surface magnetic field is also responsible for **wall current losses**; rounding the equatorial edge suppresses **mutipactor** in this region.
  - Typically for real cavities $\mathrm{H_{pk}/E_{acc}} = 40$ ... 50 Oe/ MV/m, as compared to 30.5 for pillbox.

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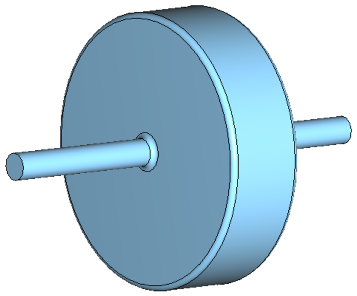
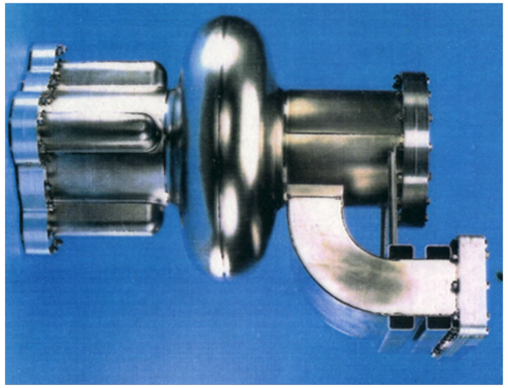
Pillbox vs “real life” SC cavity

	Pillbox	Cornell SC 500 MHz
		
G	257 Ω	270 Ω
R_{sh}/Q_0	196 Ω / cell	88 Ω / cell
E_{pk}/E_{acc}	1.6	2.5
H_{pk}/E_{acc}	30.5 Oe / MV/m	52 Oe/ MV/m

- In a high-current storage ring, it is necessary to damp higher-order modes (HOMs) to avoid beam instabilities.
- The beam pipes are made large to allow HOMs propagation toward microwave absorbers
- This enhances H_{pk} and E_{pk} and reduces R_{sh}/Q_0.

Why do we use multicell cavities?

- Single cell RF cavity has limited accelerating voltage: $V_{\mathrm{RF}}^{\mathrm{max}} = \frac{E_0 \lambda_{\mathrm{RF}}}{\pi}$

- To gain more energy we can **either use a large number of single cells or use multi-cell cavities**.

- The first path, while feasible, is **expensive** (each cavity would need individual transmitter, waveguide, controls, etc.) **and less effective** – the average accelerating gradient (energy gain per meter of real estate) would be low.

- Thus, where the acceleration gradient is important, the accelerator community uses multi-cell cavities

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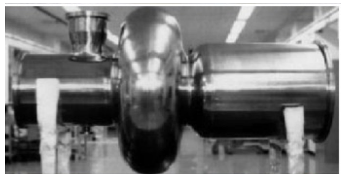
Why do we use multicell cavities?

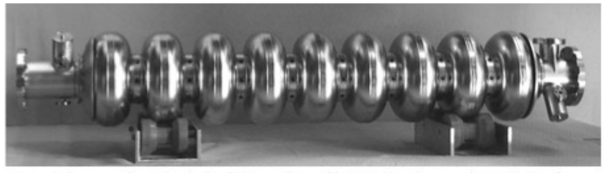
Single-cell:

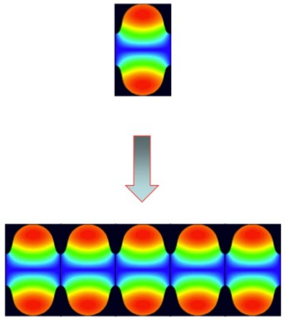
+ Easier to manage HOM damping
+ No field flatness problem
+ Input coupler transfers less power
+ Easy to clean and assemble

- Expensive to base even a small accelerator using 1-cell cavities



VS

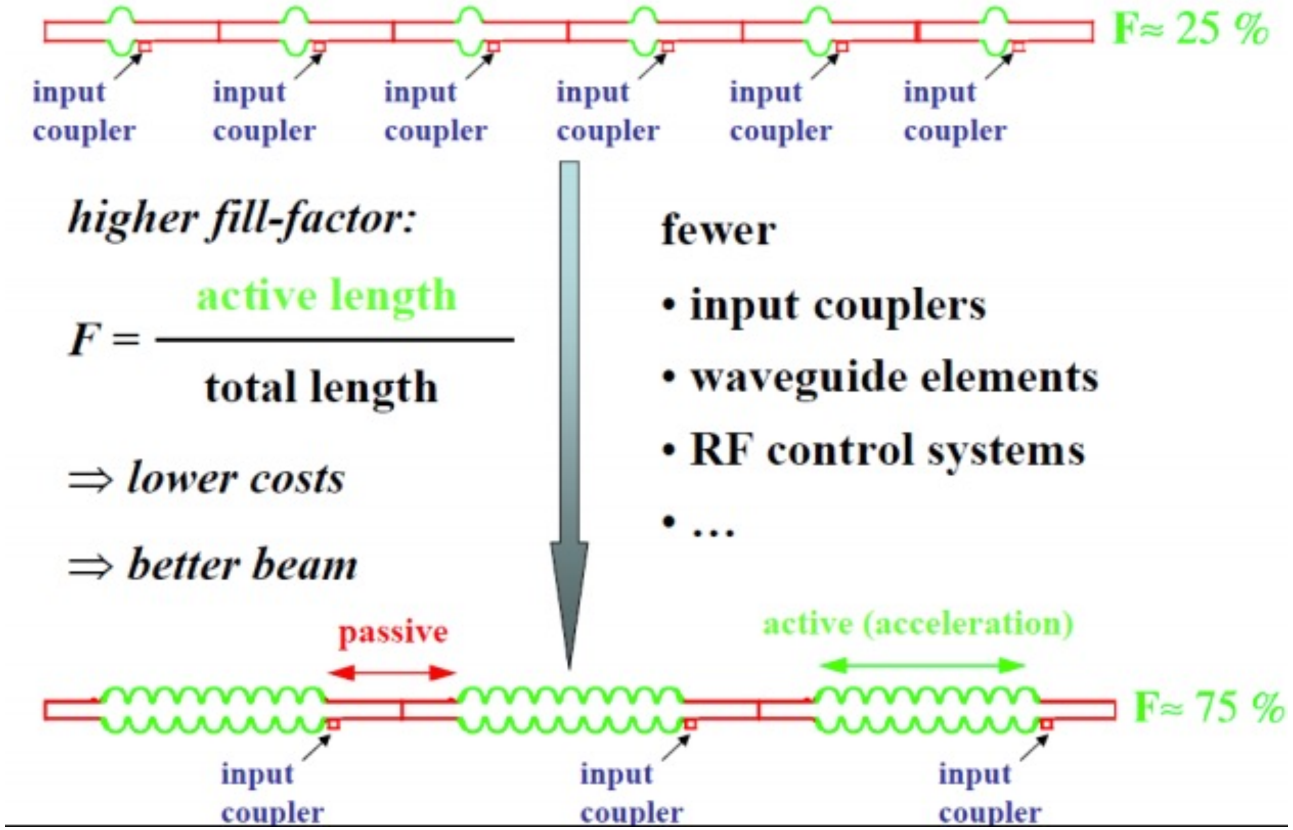




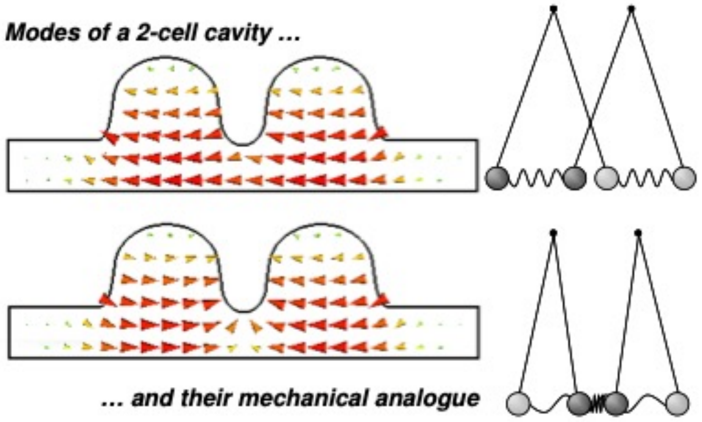
Multi-cell:

+ Less expensive and offers higher real-estate gradient

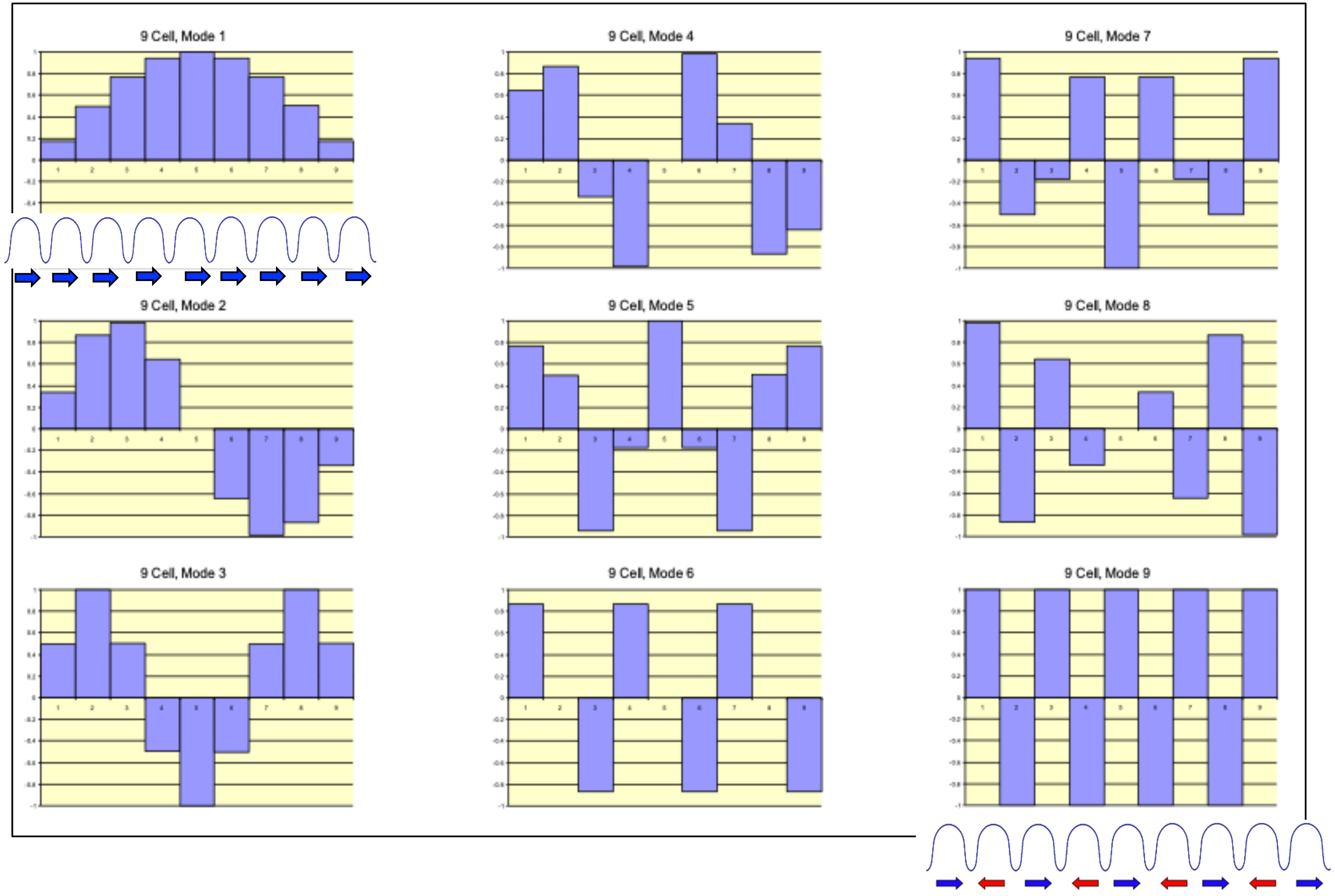
- Field flatness becomes sensitive to frequency errors of individual cells
- HOM trapping



Multicell cavities: modes

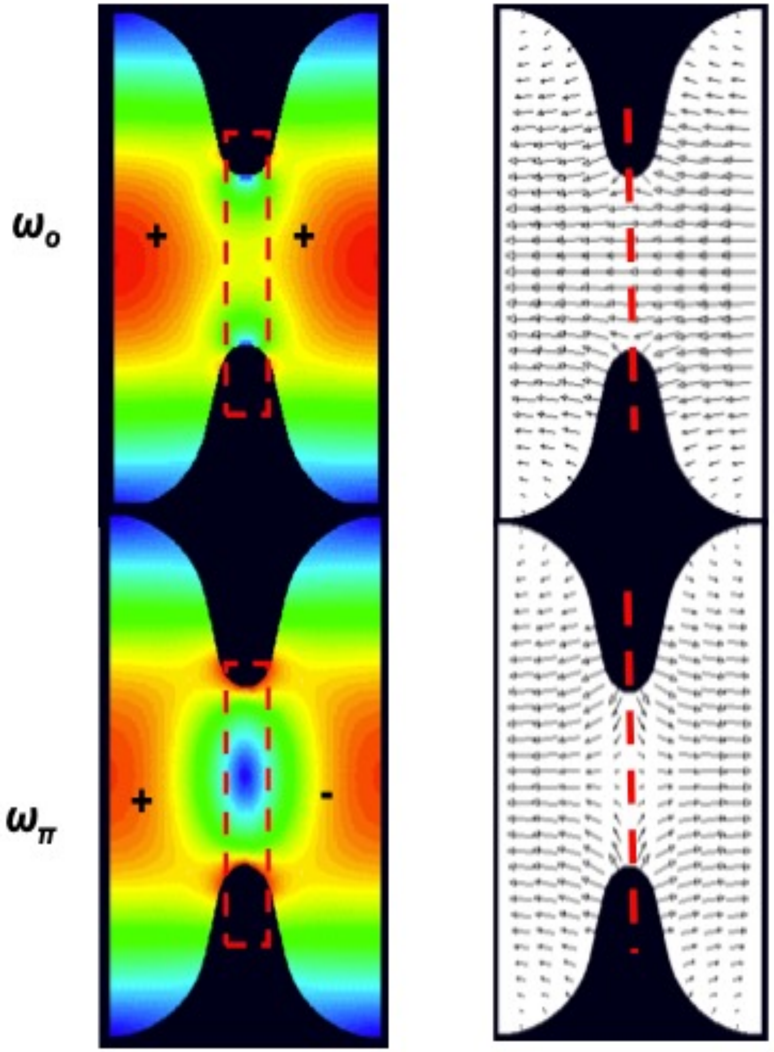


0-mode



π-mode

Coupling between cells and field flatness

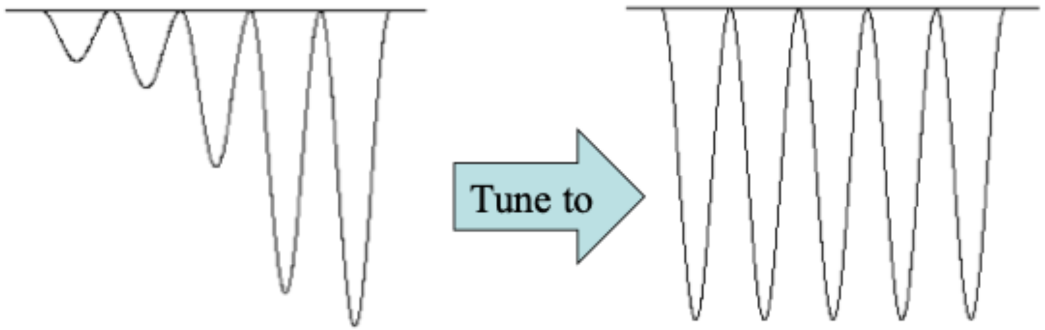


The **normalized difference between these frequencies** is a **measure of the energy flow** via the coupling region

$$k_{cc} = \frac{2(\omega_\pi - \omega_0)}{\omega_\pi + \omega_0}$$

Field flatness factor for elliptical cavities:

$$a = \frac{N^2}{k_{cc}\beta}$$



Equivalent circuit of an RF cavity

A resonant cavity can be modeled as a series of parallel RLC circuits representing the cavity eigenmodes.

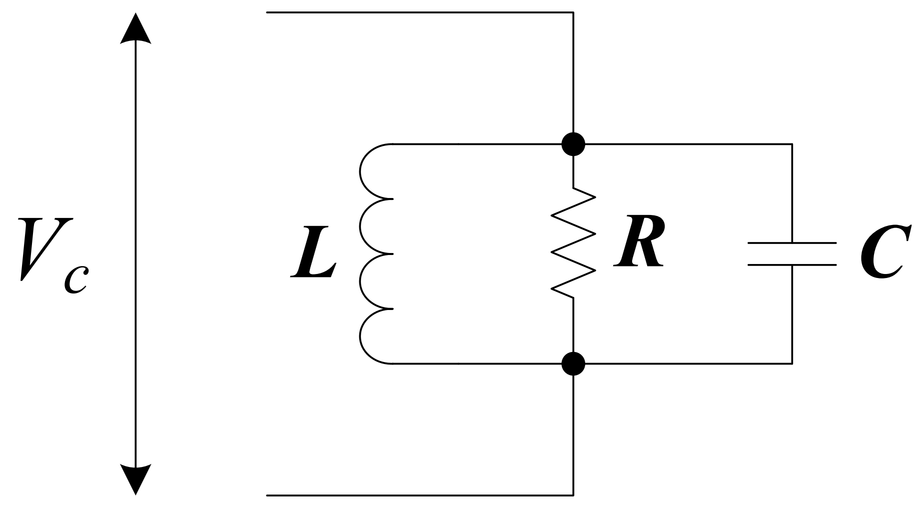
For each mode:

dissipated power $$P_{\text{loss}} = \frac{V_c^2}{2R_{\text{sh}}}$$

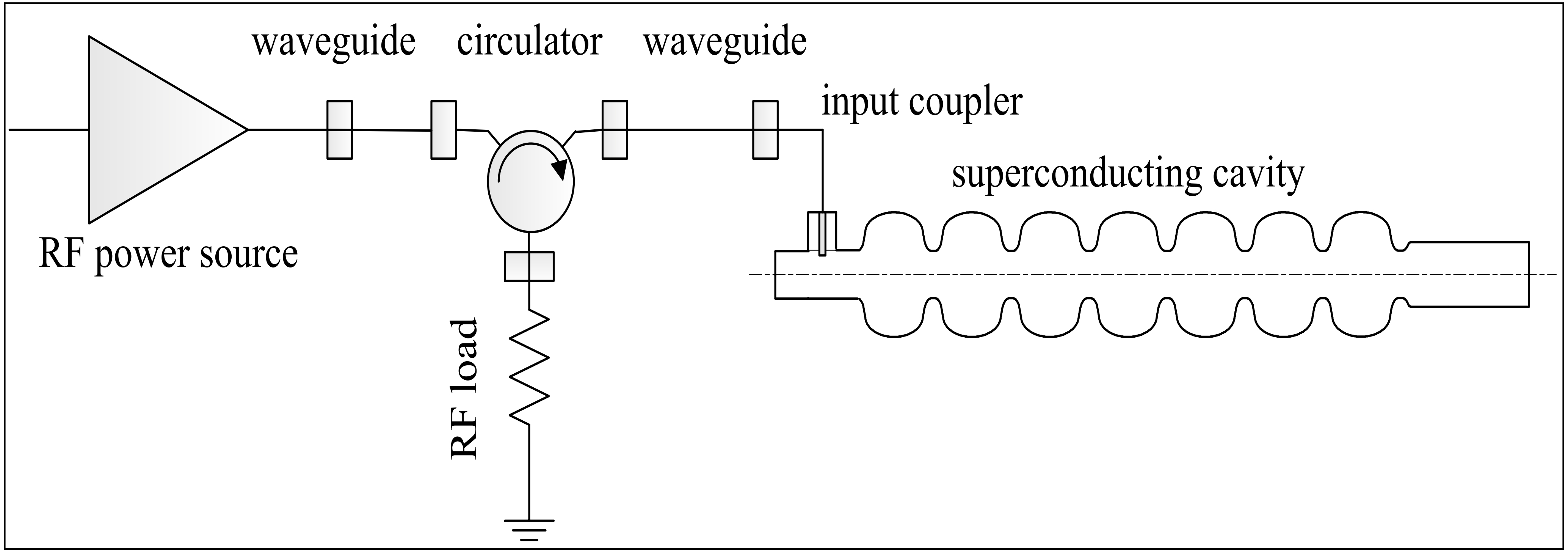
shunt impedance $$R_{\text{sh}} = 2R$$

quality factor $$Q_0 = \omega_0 C R = \frac{R}{\omega_0 L} = R\sqrt{\frac{C}{L}}$$

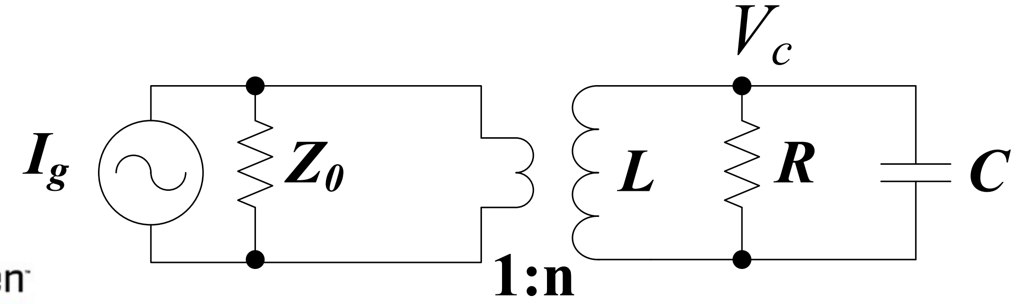
impedance $$Z = \frac{R}{1 + iQ\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)}$$

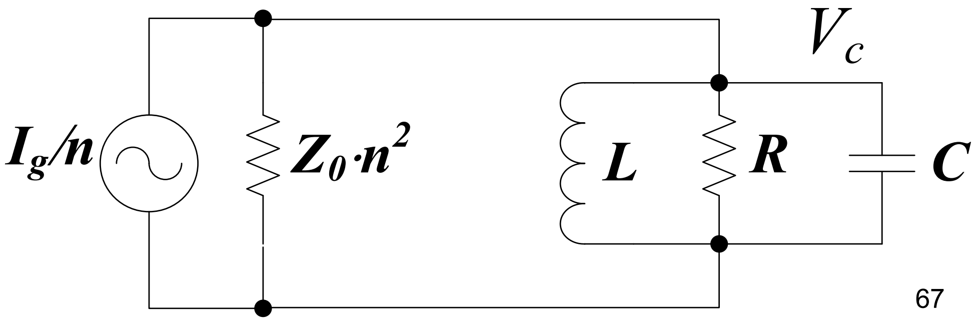


Connecting to a power source



- The input coupler can be modeled as an ideal transformer:





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Connecting to a power source

If RF is turned off, stored energy will be dissipated now not only in R, but also in Z_0/n^2, thus $P_{\mathrm{tot}} = P_0 + P_{\mathrm{ext}}$

Then analogously to the intrinsic quality factor Q_0, we can define an **external quality factor** Q_{ext}:

$$Q_{\mathrm{ext}} = \frac{\omega U}{P_{\mathrm{ext}}}$$

Such Q factors can be identified with any external ports on the cavity: input coupler, RF probe, HOM couplers, beam pipes, etc.

Then the **total power loss** can be associated with the **loaded Q-factor**: $\frac{1}{Q_{\mathrm{L}}} = \frac{1}{Q_0} + \frac{1}{Q_{\mathrm{ext1}}} + \frac{1}{Q_{\mathrm{ext2}}} + \ldots$

Coupling parameter β defines how strongly the couplers interact with the cavity: $\beta = \frac{Q_0}{Q_{\mathrm{ext}}}$

Large β implies that the power taken out of the coupler is large compared to the power dissipated in the cavity walls:

$$P_{\mathrm{ext}} = \frac{V_{\mathrm{c}}^2}{R/Q \cdot Q_{\mathrm{ext}}} = \frac{V_{\mathrm{c}}^2}{R/Q \cdot Q_0}\beta = \beta P_0$$

The total power needed from an RF power source is expressed as $P_{\mathrm{forward}} = (\beta + 1) P_0$

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Takeaways # 2:

- Two types of transmission lines are mostly used in accelerators to power RF cavities: coaxial lines and waveguides.
- Resonant modes in a cavity resonator belong to two families: TE and TM.
- There is an infinite number of resonant modes.
- The lowest frequency TM mode is usually used for acceleration.
- All other modes (HOMs) are considered parasitic as they can harm the beam.
- Several figures of merit are used to characterize RF cavities: R_s, Q_0, R/Q, G, R_{sh}…
- Superconducting RF cavities can have quality factor a million times higher than that of best Cu cavities.

The End