Today schedule:

1. Questions & Answers regarding Computational simulation
2. Beam transport and RF acceleration short lecture. (30-40 minutes)
3. 10 minutes break.
4. Demonstration of beam manipulation. Go to ATF control.
5. Questions
Emittance compensation

- After initial acceleration, space-charge field is mainly transverse (beam is long in rest frame).
- Both radial and longitudinal forces scale as $\gamma^{-2}$
- Transverse force dependent almost exclusively on local value of current density $I / \sigma^2$

$$\sigma_x''(\zeta,s) + \kappa^2 \cdot \sigma_x(\zeta,s) = \frac{r_x \lambda(\zeta)}{2\gamma^3 \sigma_x(\zeta,s)} + \frac{\varepsilon_{n,x}^2}{2\gamma \sigma_x^3(\zeta,s)}$$

$$\zeta = s - v_b t$$
$$I(\zeta) = \lambda(\zeta) \cdot v_b$$

Simple model how the emittance compensation works [*]

Beam transport and acceleration

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BNL ERL layout. 
~20m circumference

SRF Gun with photocathode

Injection line

SRF 5cell linac

Returning loop

Beam dump

~7m straight section
Schematic Layout of the BNL ERL

- Source (Gun)
- Beam transport elements (magnets)
- Acceleration (RF cavities)
- Dipole magnets
- Quadrupole magnets
- SC RF Gun
- Merger system
- Cryo-module
- SRF cavity
- Beam dump
- 50 kW 703.75 MHz system
- 1 MW 703.75 MHz Klystron
- Laser
- Control room

Electrons:
- e⁻ 2.5 MeV

Electron束 (束) 2.5 MeV
Main accelerators components

- Source
- Beam transport
- Beam Acceleration
• Each particle is defined in 6-D space (coordinates and momentums)

\[ \tilde{x} = (x, p_x, y, p_y, z, p_z) \]

• In accelerators physics is more convenient to use reference particle and paraxial approximation \( p_z \gg p_x, p_y \) then:

\[ \tilde{x} = (x, x', v, v', c\Delta t, \Delta E/E) \]

\[ x' = \frac{p_x}{p_z}, \quad y' = \frac{p_y}{p_z} \]

• \( \Delta t, \Delta E \)-it's time and energy difference energy from reference particle.
Beam phase space modification drift space only

If there is no coupling between X and Y we can work with 2D phase spaces. For example $u=x$ or $y$.

Eventually beam spreads out and hits the aperture. Focusing is needed.

This beam is "divergent".

Vacuum pipe aperture radius = $a$ ($\pm a$)
Magnetic lattice

- Usually the set of different kinds of magnets is needed in order to successfully propagate charge beam through the system.
How we can say these are quadrupoles?
Why magnetic field not electric field?

Ratio of magnetic and electric forces

- For ultra-reletivistic particles $v \sim c$
  - $B=1T$ is equivalent to $E=300MV/m$!!!
- For low energy ($v=0.01c$)
  - $B=1T$ is equivalent of $E=3MV/m$

- Electrostatic accelerators existed but the use of such systems are very limited of low energy!!
The light optics similarity

The same matrix formalism can be adopted in first order and linear approximation. Vectors \((x, x')\) and \((y, y')\)
Bending magnets

• A dipole magnet with constant magnetic field
• Positive particle coming in the screen will bend to the right.
• Using combination of the dipoles one could create any kind of transport lines.

\[ \theta = \int_{S_1}^{S_2} Bdl = \frac{c}{Bp} \int_{S_1}^{S_2} Bdl \]

Where, \( p_0 \) is the momentum and \( Bp = p_0/e \) is the momentum ‘rigidity’ of the beam.
Dispersion

- Particle with different momentum will be bend on different angle
- Can cause beam quality degradation but also used for some experiments.

- Mask at ATF
Quadrupoles

- Due to special field symmetry focus beam in one direction but defocus in other.
- Particles moving at axis are not experience any force.
Quadrupoles (cont.)

• Particle displaced by \((x,z)\) from the center

\[
B = B_1(z\hat{x} + x\hat{z})
\]

\[
\vec{F} = evB_1\hat{s} \times (z\hat{x} + x\hat{z}) = -evB_1\hat{z} + evB_1\hat{x}
\]

the equations of motion become:

\[
\frac{1}{v^2} \frac{d^2x}{dt^2} = \frac{eB_1}{\gamma mv} x, \quad \frac{1}{v^2} \frac{d^2z}{dt^2} = -\frac{eB_1}{\gamma mv} z
\]

or

\[
\frac{d^2x}{ds^2} = x'' = \kappa x, \quad \frac{d^2z}{ds^2} = -\kappa z \quad \text{where} \quad \kappa = \frac{eB_1}{\gamma mv}
\]

When matrix transformation from entrance to exit of quadrupole:

\[
\begin{pmatrix}
  x' \\
  x''
\end{pmatrix} = \begin{pmatrix}
  \cos\sqrt{\kappa}L & \frac{1}{\sqrt{\kappa}} \sin\sqrt{\kappa}L \\
  -\sqrt{\kappa}\sin\sqrt{\kappa}L & \cos\sqrt{\kappa}L
\end{pmatrix} \begin{pmatrix}
  x_0 \\
  x_0'
\end{pmatrix}
\]

\[
\begin{pmatrix}
  z' \\
  z''
\end{pmatrix} = \begin{pmatrix}
  \cosh\sqrt{\kappa}L & \frac{1}{\sqrt{\kappa}} \sinh\sqrt{\kappa}L \\
  \sqrt{\kappa}\sinh\sqrt{\kappa}L & \cosh\sqrt{\kappa}L
\end{pmatrix} \begin{pmatrix}
  z_0 \\
  z_0'
\end{pmatrix}
\]
Thin lens approximation

- For thin lens when \( K << 1/L^2 \)

\[
\begin{pmatrix}
\cos(\sqrt{KL}) & \frac{1}{\sqrt{K}} \sin(\sqrt{KL}) \\
-\sqrt{K} \sin(\sqrt{KL}) & \cos(\sqrt{KL})
\end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 \\ -KL & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{F} & 1 \end{pmatrix}
\]

- If the quadrupole is thin enough, the particles coordinate doesn’t change while momentum change. The quad works almost as a optical lens...

\[\Delta x' = \frac{x}{f}\]

- With only one difference:

Focus in one plane and defocus in other plane
Focus the beam in both directions.

• Using doublets
• Using optical analogy one can calculate

\[ \frac{1}{f_{combined}} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} \]

What if \( f_1 = -f_2 \)?

\[ f_{combined} = \frac{f_1^2}{d} \]

• A quadrupole doublet is focusing in both planes.
• Strong focusing by sets of quadrupole doublets with alternative gradient. Could keep beam inside vacuum chamber.
Solenoid

- A solenoid is a set of helical coils.
- Typically solenoid radius is smaller than the length.
- Magnetic field is generated along the axis line.
- Solenoid couples X and Y motion.
- Solenoid produced focusing in both direction

\[ 1/f = e \int Bz^2 \, dz / (2pc)^2 \]

- Solenoids are preferable at low energy
Back up
Correction Coils – axial view

- skew quad. (sQ);
- normal dipole (nD);
- skew dipole (sD);
- (solenoid winding starts R=9.3 cm)
- normal quad. (nQ);
Linear transformation preserves emittance.

Dipole magnet:
\[ x = x_0 \cos(\phi) + x_0' R \sin(\phi) \]
\[ x' = -\frac{x_0}{R} \sin(\phi) + x_0' \cos(\phi) \]

Thin focusing lens with f- focal length:
\[ x = x_0 ; x' = x_0' - \frac{x_0}{f} \]

Drift space L - length:
\[ x = x_0 + L x_0 ; x' = x_0' \]

Nonlinear transformation increases emittance:

Thin Sextupole:
\[ x = x_0 ; x' = x_0' - S x_0^2 \]
Injection combine function magnets

Due to very small real estate and large beam size: each magnet includes 4 sets of coils: 1) vertical bend, 2) quadrupole focusing, 3) sextupole correction and 4) horizontal steering.

The quadrupole coil is used to split focusing equally between the planes. The amount of the sextupole component is controlled by the gap between the yoke and the main dipole coil. A small additional coil in the corners is a sextupole trim coil, intended for use if sextupole component needs to be adjusted to reduce emittance growth.

Window-frame dipole for Z-bend

Analysis predicts that the influence of various field components on the emittance growth are complicated by the fact that the beam trajectory bends significantly in the fringe fields. Hence, direct tracking in the calculated fields extracted from Opera3d was used of test beam to evaluate and to minimize influence of magnetic field on the beam emittance.

initial emittances 0
After tracking emittances:
\( \varepsilon_{xn} = 0.6 \text{ mm-mrad} \)
\( \varepsilon_{yn} = 0.6 \text{ mm-mrad} \)
Main accelerators components

- Source
- Beam transport
- Beam Acceleration
Acceleration is needed!!

- In colliders: The minimum energy required to create a particle (or group of particle) with total mass $M$ is: 
  \[ E_{\text{min}} = Mc^2 \]
  - High energy colliding particles $\Rightarrow$ high energy center mass $\Rightarrow$ massive particles production (cross section $\sigma$)
  - Luminosity:

\[ L = f_c \frac{N_1 N_2}{A} \approx f_c \frac{N_1 N_2}{2 \pi \sqrt{\beta_{x1} \epsilon_{x1} + \beta_{x2} \epsilon_{x2} \sqrt{\beta_{y1} \epsilon_{y1} + \beta_{y2} \epsilon_{y2}}} \]

Numbers of events

\[ N_{A \rightarrow B}^l = \sigma_{A \rightarrow B} \cdot L \]

- Normalized emittance $\sim$ preserved during acceleration, geometrical emittance reduced $\sim 1/\gamma$.

- In light source: Brightness $B \sim 1/\gamma^2$.  

The peak normalized rms brightness is given by

\[ B_n = \frac{2I}{\epsilon_{n,x} \epsilon_{n,y}} \]
Geometrical emittance transformation

Thin gap acceleration (dpz)

particle angle $x'$

particle pos

$x' = px' / pz$

$x' = px' / (pz_0 + \delta pz)$

$x' \ll x_0'$

$x = x_0$
Acceleration

\[ \frac{d\vec{p}}{dt} = q \left( \vec{E} + \frac{\vec{v}}{c} \times \vec{B} \right); \quad \frac{dE}{dt} = q \left( \vec{E} \cdot \vec{v} \right); \]

- Single pass acceleration
- Limited by maximum voltage per until discharge. \( \sim 1.5 \) MV in air

\[ \Delta E = e \oint \vec{E} \cdot d\vec{l} = -\frac{e}{c} \frac{\partial}{\partial t} \left( \int \vec{H} \cdot d\vec{s} \right) \]

*Maxwell equation prohibits multiple acceleration is DC electric field:*

- In RF cavities energy gain depends on the phase.
- The main purpose of using RF cavities in accelerators is to add (remove) energy to charged-particle beams at a fast acceleration rate
RF Field acceleration:

The RF field must be synchronous (correct phase relation) with the beam for a sustained energy transfer.

\[ E_z(z, t) = E(z) \cos \left( \omega t - \int_0^z k(z') dz' + \phi \right), \]

For efficient particle acceleration, the phase velocity of the wave must closely match the beam velocity. If we consider a particle of charge \( q \) moving along +z direction with a velocity at each instant of time equal the phase velocity of the traveling wave, then the electric force on the particle is given by

\[ F_z = q \ E(z) \ \cos \phi \]

Energy gain

\[ \Delta E = q \int_{-L/2}^{L/2} E(0, z) \cos \left[ \omega t(z) + \phi \right] \, dz \]
RF Cavity connected to RF power source
RF cavities

Typical Single cell

Quarter-wave 112MHz resonator

BNL ERL: 5Cell cavity 704MHz
ATF accelerator system

- Gun
- Accelerator section 1
  - Gun klystron
  - Gun low RF, Power and Phase control
- Linac klystron (XK-5)
- Linac low RF, Power and Phase control
- Accelerator section 2
  - Phase shifter
  - 25 MW

- Master oscillator
- Master phase shifter
- Nd:YAG laser

- 5 MeV
- 35 MeV
- 35 MeV

- 1 kW

[MeV] \[E = 10\sqrt{2P}\]

- [MW]
Multi linacs acceleration

$E = E_{inj} + E_{linac1} + E_{linac2}$

$E_{linac1} = eU_1 \cos(\phi_1)$

$E_{linac2} = eU_2 \cos(\phi_2)$

If there is enough voltage provided by one linac. The final energy can be reached by combination different phases.

For ATF:

$U_1 = U_2 = 36 \text{MV}$, $E_{inj} = 5 \text{MeV}$

$E_{final} = 35 \text{MeV}$

Why one operation could be better then others?
Few things to remember

• Space charge force depends on energy
  – Higher energy => less space charge effects

\[ \sigma_x''(\zeta,s) + \kappa'_B \cdot \sigma_x(\zeta,s) = \frac{r_e \lambda(\zeta)}{2\gamma^3 \sigma_x(\zeta,s)} + \frac{\varepsilon_{n,x}^2}{2\gamma \sigma_x^3(\zeta,s)} \]

• Focusing due to entrance and exit of RF field
  – More energy gain => stronger focusing

Entrance kick is larger than exit kick

\[ \Delta p_r = \frac{e}{c} \int E_r \, dz \]

\[ r'_{in} = \frac{\Delta p_{in,r}}{p_{in,z}} \]

\[ r'_{out} = \frac{\Delta p_{out,r}}{p_{out,z}} \]

\[ \Delta p_{in,r} \sim -\Delta p_{out,r} \]

Eout = Ein + ΔE