Modulator Simulations with Focusing Fields

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Introduction Thermal expansion Space charge expansion

Notations

- *nz* : Longitudinal density distribution along *z* axis
- vz : Longitudinal velocity distribution along z axis
- nx : Transverse density distribution along x axis
- vx : Transverse velocity distribution along x axis
- ny : Transverse density distribution along y axis
- vy : Transverse velocity distribution along y axis

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Introduction Thermal expansion Space charge expansion

- Electron beam has normal distribution in transverse (x,y) and uniform distribution in longitudinal (z)
- Expansion in transverse because of thermal velocity and space charge

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Average Range Modulator with linear focusing Modulator with quadrupoles focusing Introduction Thermal expansion Space charge expansion

Initial nx



Average Range Modulator with linear focusing Modulator with quadrupoles focusing Introduction Thermal expansion Space charge expansion

Initial vx



Average Range Modulator with linear focusing Modulator with quadrupoles focusing Introduction Thermal expansion Space charge expansion

Focusing field

$$E(r) = \frac{m}{e} \frac{\sigma^2}{r_0^2} (r - r_0)$$

- m : rest mass of electron
- e : charge of electron
- σ : RMS of thermal velocity
- r_0 : RMS of position r

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Introduction Thermal expansion Space charge expansion

vx without focusing



Average Range Modulator with linear focusing Modulator with quadrupoles focusing Introduction Thermal expansion Space charge expansion

vx with focusing



Average Range Modulator with linear focusing Modulator with quadrupoles focusing Introduction Thermal expansion Space charge expansion

Focusing field

$$E(r) = \frac{q}{2\pi\varepsilon_0 r} \left(1 - e^{-r^2/2r_0^2}\right)$$

- q : beam's line charge density, (Total charge)/(Longitudinal length)
- r : radial distance in transverse plane
- *r*₀ : RMS of *r*

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Introduction Thermal expansion Space charge expansion

vx without focusing



Average Range Modulator with linear focusing Modulator with quadrupoles focusing Introduction Thermal expansion Space charge expansion

vx with focusing



Average Range Modulator with linear focusing Modulator with quadrupoles focusing Introduction Thermal expansion Space charge expansion

vx with focusing



Introduction Thermal expansion Space charge expansion

Beam size with focusing



Introduction Linear focusing Quadrupoles focusin

- Calculate density and velocity distribution using electrons within some transverse ranges (1σ, 2σ, 3σ, entire domain)
- In previous comparison with theory of uniform distribution case, average range 1σ was used, as theory also counts electrons within this range
- Now with normal distribution, different average ranges affect results

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Introduction Linear focusing Quadrupoles focusing

• $\sigma_x = \sigma_y$, and they don't change during simulation

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Introduction Linear focusing Quadrupoles focusing

nx at 0.6 m



Introduction Linear focusing Quadrupoles focusing

vx at 0.6 m



Introduction Linear focusing Quadrupoles focusing

nz at 0.6 m



Introduction Linear focusing Quadrupoles focusing

vz at 0.6 m



Introduction Linear focusing Quadrupoles focusing

Beam size changes



Introduction Linear focusing Quadrupoles focusing

- $\sigma_x \neq \sigma_y$, and they change during simulation
- σ_x and σ_y are calculated dynamically at each visualization step

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Introduction Linear focusing Quadrupoles focusing

nx at 0.6 m



Introduction Linear focusing Quadrupoles focusing

vx at 0.6 m



Introduction Linear focusing Quadrupoles focusing

ny at 0.6 m



Introduction Linear focusing Quadrupoles focusing

vy at 0.6 m



Introduction Linear focusing Quadrupoles focusing

nz at 0.6 m



Introduction Linear focusing Quadrupoles focusing

vz at 0.6 m



Introduction Linear focusing Quadrupoles focusing

nx at 3 m



Introduction Linear focusing Quadrupoles focusing

vx at 3 m



Introduction Linear focusing Quadrupoles focusing

ny at 3 m



Introduction Linear focusing Quadrupoles focusing

vy at 3 m



Introduction Linear focusing Quadrupoles focusing

nz at 3 m



Introduction Linear focusing Quadrupoles focusing

vz at 3 m



Introduction Linear focusing Quadrupoles focusing

• All following results use entire domain as average range

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Linear Focusing Average Range Modulator with linear focusing Modulator with quadrupoles focusing	
	VZ

- $\sigma_x = \sigma_y = \sigma$, and they don't change during simulation
- ion's position
 - x : 0.0/0.5/1.0/1.5/2.0 σ off center
 - y : center
 - z : center

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	nx	
Linear Focusing	VX	
Average Range	ny	
Modulator with linear focusing	vy	
Modulator with quadrupoles focusing	nz	
	VZ	

0.0σ off center


	nx	
Linear Focusing		
Average Range		
Modulator with linear focusing		
Modulator with quadrupoles focusing		



	nx	
Linear Focusing	VX	
Average Range	ny	
Modulator with linear focusing	vy	
Modulator with quadrupoles focusing	nz	
	VZ	



	nx	
Linear Focusing		
Average Range		
Modulator with linear focusing		
Modulator with quadrupoles focusing		



	nx	
Linear Focusing		
Average Range		
Modulator with linear focusing		
Modulator with quadrupoles focusing		



Linear Focusing	
Average Range	
Modulator with linear focusing	
Modulator with quadrupoles focusing	



Linear Focusing	
Average Range	
Modulator with linear focusing	
Modulator with quadrupoles focusing	



Linear Focusing	
Average Range	
Modulator with linear focusing	
Modulator with quadrupoles focusing	



Linear Focusing	
Average Range	
Modulator with linear focusing	
Modulator with quadrupoles focusing	



Linear Focusing	
Average Range	
Modulator with linear focusing	
Modulator with quadrupoles focusing	



Linear Focusing		
Average Range	ny	
Modulator with linear focusing		
Modulator with quadrupoles focusing		



	nx	
Linear Focusing	VX	
Average Range	ny	
Modulator with linear focusing	vy	
Modulator with quadrupoles focusing	nz	
	vz	



	nx	
Linear Focusing	VX	
Average Range	ny	
Modulator with linear focusing	vy	
Modulator with quadrupoles focusing	nz	
	vz	



Linear Focusing		
Average Range	ny	
Modulator with linear focusing		
Modulator with quadrupoles focusing		



	nx	
Linear Focusing	VX	
Average Range	ny	
Modulator with linear focusing	vy	
Modulator with quadrupoles focusing	nz	
	VZ	



	nx
Linear Focusing	vx
Average Range	ny
Modulator with linear focusing	vy
Modulator with quadrupoles focusing	nz
	VZ



	nx
Linear Focusing	vx
Average Range	ny
Modulator with linear focusing	vy
Modulator with quadrupoles focusing	nz
	VZ



	<u>n</u>
Linear Focusing	V.
Average Range	n
Modulator with linear focusing	v
Modulator with quadrupoles focusing	n
	V



Linear Focusing	
Average Range	
Modulator with linear focusing	vy
Modulator with quadrupoles focusing	



Linear Focusing	
Average Range	
Modulator with linear focusing	1
Modulator with quadrupoles focusing	

vx ny vy nz



Linear Focusing		
Average Range		
Modulator with linear focusing		
Modulator with quadrupoles focusing	nz	



Linear Focusing		
Average Range		
Modulator with linear focusing		
Modulator with quadrupoles focusing	nz	



Linear Focusing		
Average Range		
Modulator with linear focusing		
Modulator with quadrupoles focusing	nz	



Linear Focusing		
Average Range		
Modulator with linear focusing		
Modulator with quadrupoles focusing	nz	



Linear Focusing		
Average Range		
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Modulator with quadrupoles focusing	nz	



Linear Focusing		
Average Range		
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Modulator with quadrupoles focusing		
	VZ	



Linear Focusing		
Average Range		
Modulator with linear focusing		
Modulator with quadrupoles focusing		
	VZ	



Linear Focusing		
Average Range		
Modulator with linear focusing		
Modulator with quadrupoles focusing		
	VZ	



	nx	
Linear Focusing	VX	
Average Range	ny	
Modulator with linear focusing	vy	
Modulator with quadrupoles focusing	nz	
	VZ	



Linear Focusing		
Average Range		
Modulator with linear focusing		
Modulator with quadrupoles focusing		
	VZ	



Linear Focusing	
Average Range	
Modulator with linear focusing	
Modulator with quadrupoles focusing	

Beam size changes





- $\sigma_x \neq \sigma_y$, and they change during simulation
- ion's position
 - x : 0.0/0.5/1.0/1.5/2.0 $\sigma(\text{initial})$ off center
 - y : center
 - z : center
- Quadrupoles change beam's transverse velocity, and longitudinal velocity
- Electron beam has the same longitudinal velocity as ion initially, but has some longitudinal motion with respect to ion during simulation because of quadrupoles, so the vz plot is not symmetric

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Linear Focusing	
Average Range	
wodulator with linear focusing	
Modulator with quadrupoles focusing	

vx ny vy nz vz



Linear Focusing	
Average Range	
wodulator with linear focusing	
Modulator with quadrupoles focusing	

vx ny vy nz vz



Linear Focusing	
Average Range	
Modulator with linear focusing	
Modulator with quadrupoles focusing	



Linear Focusing	
Average Range	
Modulator with linear focusing	
Modulator with quadrupoles focusing	

vy nz



Linear Focusing	
Average Range	
Modulator with linear focusing	
Modulator with quadrupoles focusing	


Linear Focusing	١
Average Range	
Modulator with linear focusing	
Modulator with quadrupoles focusing	



	n
Linear Focusing	V.
Average Range	n
Modulator with linear focusing	v
Modulator with quadrupoles focusing	n
	V



Linear Focusing	vx
Average Range	
Modulator with linear focusing	
Modulator with quadrupoles focusing	



	n.
Linear Focusing	V.
Average Range	n
Modulator with linear focusing	V
Modulator with quadrupoles focusing	n.
	V



	<i>n</i> 2
Linear Focusing	V
Average Range	ny
Modulator with linear focusing	vj
Modulator with quadrupoles focusing	n
	V2



Linear Focusing	1
Average Range	1
Modulator with linear focusing	١
Modulator with quadrupoles focusing	1



	n
Linear Focusing	V.
Average Range	n
Modulator with linear focusing	v
Modulator with quadrupoles focusing	n
	V



Linear Focusing	
Average Range	
Modulator with linear focusing	
Modulator with quadrupoles focusing	



Linear Focusing	1
Average Range	1
Modulator with linear focusing	١
Modulator with quadrupoles focusing	1



Linear Focusing	
Average Range	
Modulator with linear focusing	
Modulator with quadrupoles focusing	



Linear Focusing	
Average Range	
Modulator with linear focusing	
Modulator with quadrupoles focusing	

vx ny vy nz vz



Linear Focusing	
Average Range	
Modulator with linear focusing	
Modulator with quadrupoles focusing	

vx ny **vy** nz vz



Linear Focusing	
Average Range	
Modulator with linear focusing	
Modulator with quadrupoles focusing	



Linear Focusing	
Average Range	
Modulator with linear focusing	
Modulator with quadrupoles focusing	

vx ny **vy** nz vz



Average Range	
Modulator with linear focusing	
Modulator with quadrupoles focusing	

vx ny **vy** nz vz



Linear Focusing	
Average Range	
Modulator with linear focusing	
Modulator with quadrupoles focusing	



Linear Focusing	
Average Range	
Modulator with linear focusing	
Modulator with quadrupoles focusing	



Linear Focusing	
Average Range	
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Linear Focusing	
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Modulator with quadrupoles focusing	



Linear Focusing	
Average Range	
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Modulator with quadrupoles focusing	



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Linear Focusing	١
Average Range	1
Modulator with linear focusing	١
Modulator with quadrupoles focusing	1



Linear Focusing	
Average Range	
Modulator with linear focusing	
Modulator with quadrupoles focusing	



Linear Focusing	
Average Range	
Modulator with linear focusing	
Modulator with quadrupoles focusing	



Linear Focusing	
Average Range	
Modulator with linear focusing	
Modulator with quadrupoles focusing	



Linear Focusing	
Average Range	
Modulator with linear focusing	
Modulator with quadrupoles focusing	

