Modulator Simulations and Prelim Dipole Simulations

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• Ion moves with 1 or 2 $\beta_z$ along longitudinal direction
• In the code, ion stays and electron bunch moves instead
$v_{ion} = 1\beta_z$, longitudinal number distribution
$v_{ion} = 1\beta_z$, longitudinal velocity distribution
$v_{ion} = 2\beta_z$, longitudinal number distribution
$v_{ion} = 2\beta_z$, longitudinal velocity distribution
Magnetic field of quadrupole

\[
\begin{pmatrix}
B_x \\
B_y \\
B_z
\end{pmatrix} = -\frac{G}{b_1} \cdot \begin{pmatrix}
B_{\text{fringe},x}(b_1 x, b_1 y, b_1 (z - \frac{1}{2} L)) + B_{\text{fringe},x}(b_1 x, b_1 y, b_1 (-z - \frac{1}{2} L)) \\
B_{\text{fringe},y}(b_1 x, b_1 y, b_1 (z - \frac{1}{2} L)) + B_{\text{fringe},y}(b_1 x, b_1 y, b_1 (-z - \frac{1}{2} L)) \\
B_{\text{fringe},z}(b_1 x, b_1 y, b_1 (z - \frac{1}{2} L)) - B_{\text{fringe},z}(b_1 x, b_1 y, b_1 (-z - \frac{1}{2} L))
\end{pmatrix}
\]
Magnetic field of quadrupole (continue)

\[
B_{\text{fringe}}(x, y, z) = \frac{1}{4} \left[ -y - 2 \arctan \left( -\frac{\sin(y)}{e^{-z} + \cos(y)} \right) + \frac{y \sinh(z)}{\cos(x) + \cosh(z)} \right]
\]

\[
- x - 2 \arctan \left( -\frac{\sin(x)}{e^{-z} + \cos(x)} \right) + \frac{x \sinh(z)}{\cos(y) + \cosh(z)}
\]

\[
\frac{y \sin(x)}{\cos(x) + \cosh(z)} + \frac{x \sin(y)}{\cos(y) + \cosh(z)}
\]
Magnetic field of quadrupole (continue)

- $G = K = K_1 \cdot B_\rho$
- $b_1 = \pi/r_{bore}$
- $r_{bore} = 3\text{cm \ or \ 12\text{cm}}$
$r_{bore} = 3\, cm$, longitudinal number distribution
Moving Ion Modulator with Quadrupoles Dipoles

$r_{bore} = 3cm$, longitudinal velocity distribution

At 2.7 m

Graph showing the average velocity in m/s as a function of $1/$Debye length.
$r_{bore} = 12\text{cm}$, longitudinal number distribution

![Graph showing distribution of electrons per Debye length](image-url)
$r_{bore} = 12\text{cm}$, longitudinal velocity distribution
Magnetic field of dipoles

- \( B(s) = B_0 \sin(k_w \cdot s) \)
- \( k_w = \frac{2\pi}{\lambda_w} \)
- \( \lambda_w = 4 \text{ cm} \)
- \( B_0 = 0.2 \text{ T} \)
Motion of electron bunch

- Length of dipoles is 7 m.
- Dipoles’ magnetic field makes electron bunch move along negative z direction, which requires even larger computational domain than 3-pole wiggler simulations.
- Use coarse mesh for prelim dipoles simulations
Dipoles effect at 1 m

![Graph showing the effect of dipoles at 1 m](image_url)