

Solution of the CeC problem in USPAS final exam

- a) The resonant condition for a FEL with helical wiggler is

$$\lambda_0 = \frac{\lambda_w (1+K^2)}{2\gamma^2} \quad (1)$$

with

$$K = \frac{eB_w \lambda_w}{2\pi mc}. \quad (2)$$

Inserting the relation

$$\gamma(z) = \gamma_0 [1 - \delta_{loss}(z)],$$

into eq. (1) leads to

$$K = \sqrt{\frac{2\gamma_0^2 \lambda_0 [1 - \delta_{loss}(z)]^2}{\lambda_w} - 1} \approx K_0 \sqrt{1 - 4 \frac{\gamma_0^2 \lambda_0}{\lambda_w K_0^2} \delta_{loss}(z)} \approx K_0 - 2 \frac{\gamma_0^2 \lambda_0}{\lambda_w K_0} \delta_{loss}(z), \quad (3)$$

with

$$K_0 = \sqrt{\frac{2\gamma_0^2 \lambda_0}{\lambda_w} - 1}. \quad (4)$$

Applying eq. (2) to eq. (3), we obtain

$$B_w(z) \approx B_0 - B_0 \frac{1+K_0^2}{K_0^2} \delta_{loss}(z). \quad (5)$$

c). From the form of the cooling energy kick

$$\Delta\delta\gamma_c(\Delta z) = -\Delta\gamma_0 \sin(k_0 \Delta z) \exp\left(-\frac{\Delta z^2}{2\sigma^2}\right), \quad (6)$$

we can derive the linear part of the cooling kick as a function of the ion's energy deviation:

$$\Delta\delta\gamma_c(\Delta z)\Big|_{\Delta z \ll \sigma, 1/k_0} = -\Delta\gamma_0 k_0 R_{56} \frac{\delta\gamma}{\gamma} = -\zeta_0 T_{rev} \delta\gamma,$$

with the local cooling rate given by

$$\zeta_0 = \frac{k_0 R_{56} \Delta\gamma_0}{\gamma T_{rev}}. \quad (7)$$

Inserting eq. (7) and $\Delta z = R_{56} \delta\gamma / \gamma$ into eq. (6) yields

$$\Delta\delta\gamma_c = -\zeta_0 T_{rev} \frac{\gamma}{k_0 R_{56}} \sin\left(\frac{k_0 R_{56}}{\gamma} \delta\gamma\right) \exp\left(-\frac{R_{56}^2}{2\sigma^2 \gamma^2} \delta\gamma^2\right). \quad (8)$$

The reduction of canonical momentum is

$$\begin{aligned}\Delta P_c &= -\zeta_0 T_{rev} \frac{h|\eta|}{k_0 R_{56} v_s} \sin\left(\frac{k_0 R_{56} v_s}{h|\eta|} P\right) \exp\left(-\frac{R_{56}^2}{2\sigma^2} \frac{v_s^2}{h^2 \eta^2} P^2\right) \\ &= -\zeta_0 T_{rev} \frac{1}{k_p} \sin(k_p P) \exp\left(-\frac{P^2}{2\sigma_p^2}\right)\end{aligned}$$

with

$$P \equiv -h \frac{|\eta|}{v_s} \frac{\Delta p}{p} = \sqrt{2I} \sin \theta , \quad (9)$$

$$k_p = \frac{k_0 R_{56} v_s}{h |\eta|} , \quad (10)$$

$$\sigma_p = \frac{\sigma h |\eta|}{|R_{56}| v_s} . \quad (11)$$

Thus the reduction of the longitudinal action is given by

$$\Delta I_c = P \Delta P_c = -\zeta_0 T_{rev} \frac{1}{k_p} P \sin(k_p P) \exp\left(-\frac{P^2}{2\sigma_p^2}\right), \quad (12)$$

and the cooling rate is given by averaging eq. (12) over the synchrotron oscillation phase θ , i.e.

$$\zeta(I) = -\frac{1}{I} \left\langle \frac{\Delta I_c}{T_{rev}} \right\rangle_{T_s} = 2\zeta_0 \bar{\zeta}(I),$$

with

$$\bar{\zeta}(I) = \frac{1}{k_p \sqrt{2I}} \frac{1}{2\pi} \int_0^{2\pi} \sin \theta \sin(k_p \sqrt{2I} \sin \theta) \exp\left(-\frac{I}{\sigma_p^2} \sin^2 \theta\right) d\theta. \quad (13)$$