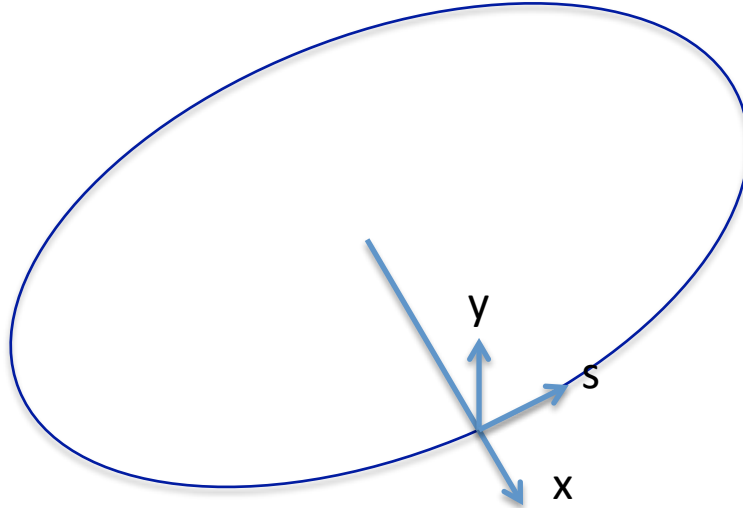


**Homework 3. PHY 564 September 9, 2015**  
 Due September 16, 2015

**Problem 1. 5 points. Plane symmetry and plane trajectories.**

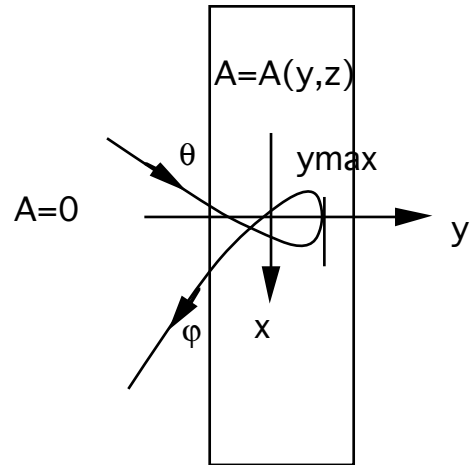


- (a) Plane reference orbit (torsion  $\kappa=0$ ) requires that total out-of plane force is equal zero. Find ratio between radial (x, horizontal) magnetic field and of-plane (vertical, y) electric field to satisfy this condition. What's happens when both of them are equal zero?
- (b) Define full set of condition on EM field providing that all in-plane trajectories (e.g. all trajectories with  $y=0$  and  $y'=0$ , but otherwise arbitrary) to stay in-plane, i.e.  $y=0$  is a solutions. Consider that particles have different energies.

*Hint: use Lorentz force*

**Problem 2. 10 points. Magnetic Mirror:** An electron propagates through a magnetic field with vector potential  $\vec{A} = \vec{A}(y,z)$ . Find an additional invariant of motion caused by independence of vector potential on x. Write explicit expression for  $p_x$  using this invariant. Consider a magnet with mid-plane symmetry ( $\vec{H} = \hat{e}_z H(y)$  at  $z=0$ ; z is perpendicular to the plane of figure) shown below with  $\vec{A} = \vec{A}(y,z)$  inside the magnet and  $\vec{A} = 0$  outside the magnet. Let's consider an electron entering the magnet in the middle plane  $z=0$  with mechanical momentum  $\vec{p} = \hat{e}_x p_x + \hat{e}_y p_y = p(\hat{e}_x \cos \theta + \hat{e}_y \sin \theta)$  ( $\vec{A} = 0$ ) laying in the x-y plane, making turn in the magnet and coming out.

1. Show that trajectory of electron remains in the plane  $z=0$ ;
2. Find angle  $\varphi$  of out-coming trajectory of the electron (reflected angle).
3. Find equation defining depth of penetration of electron inside the magnet  $y_{\max}$  using  $A(y, z=0)$ .



Clues: use Lorentz force to find (1), Use canonical momentum to connect mechanical momentum with  $\vec{A} = \vec{A}(y, z = 0)$  for (2,3)