HW 1 (5 points): RF cavity beam loading/unloading.

A short ultra-relativistic (1-v/c <<1) bunch with charge of 5 nC is passing through a 0.3 meter long 500 MHz pillbox accelerating cavity operating at the fundamental TM_{010} with peak accelerating field of 5 MV/m.

(1) Find the change of the cavity voltage ΔV/V (accelerating field) after the beam passes through it as function of the phase of the beam passing the cavity. What are the maximum and minimum ΔV/V?

(2) How the beam loading ΔV/V depends on the accelerating field? At what level of accelerating it reaches ΔV/V 1%?

(a) Assume that beam does not change velocity in the cavity;
(b) Hint – use energy conservation law
(c) Assume that relative change of the voltage ΔV/V is small, e.g. the beam loading can be treated as a perturbation.

HW 2 (3 points): Cavities filled with ferrite material are used for RF system requiring large frequency tuning range. The frequency is controlled by applying external magnetic field, B_{ext}, to the ferrite material and by doing so to change its magnetic permeability μ(B_{ext}). A 300 m in circumference AGS synchrotron accelerates polarized protons from total energy of 2.5 GeV to 25 GeV.

(a) Calculate the range of the beam revolution frequency in AGS;
(b) Assuming 100% filling by ferrite, what should be ratio of μ_{max} to μ_{min}. Where μ should have maximum value?

Note: RF systems operate on a fixed integer harmonic of the revolution frequency.

HW 3 (2 points): In RF cavity operating at 500 MHz, amplitude of the magnetic field at the part surface is 500 Gs or 500 Oe. Find power losses per square meter of the surface for:

(a) Cu cavity*
(b) SRF cavity with surface resistance, R_s = 5 \times 10^{-9} \text{ Ohm}.

How much water you can heat from 20°C to 40°C in one hour (3,600 second) by cooling such Cu cavity?

*Hint: you may use the conductivity of Cu or scale R_s from results shown in Lecture 10. Thermal capacitance of water is 4,179 J/kg/°C.