

Effect of energy jitter on CeC cooling

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Joint CeC meeting, July 23, 2021

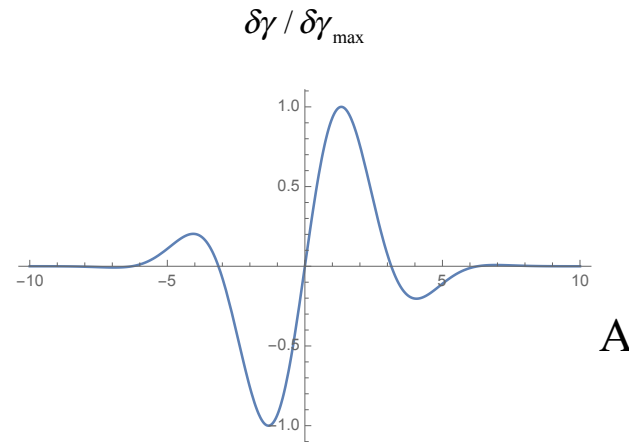
Continue with a simple 1D model

- Real 3D model is replaced by a trivial longitudinal kick with shape described by dimensionless parameter r repressing the relative bandwidth of the amplifier

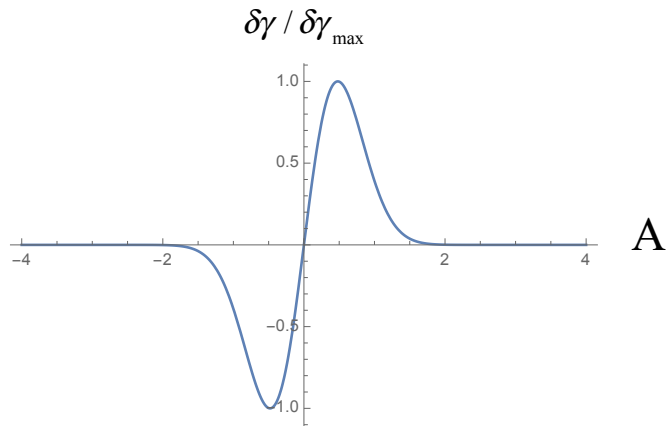
$$\delta\gamma = \Delta\gamma \cdot \sin\omega(t - \Delta t) \cdot e^{-\frac{(t-\Delta t)^2}{2\tau^2}} \equiv \Delta\gamma \cdot \sin kz \cdot e^{-\frac{\kappa^2 z^2}{2}}; z = R_{56}(\delta_e - \delta_h); \delta_{h,e} = \frac{\gamma_{h,e} - \gamma_o}{\gamma_o};$$

$$\frac{d\delta_h}{dn} = \frac{\delta\gamma_h}{\gamma_o} = \frac{\Delta\gamma}{\gamma_o} \cdot \sin kz \cdot e^{-\frac{\kappa^2 z^2}{2}} = \xi \cdot \sin kz \cdot e^{-\frac{z^2}{2\zeta^2}}; r = \frac{1}{\omega\tau} = \frac{1}{k\zeta} = \frac{\kappa}{k}$$

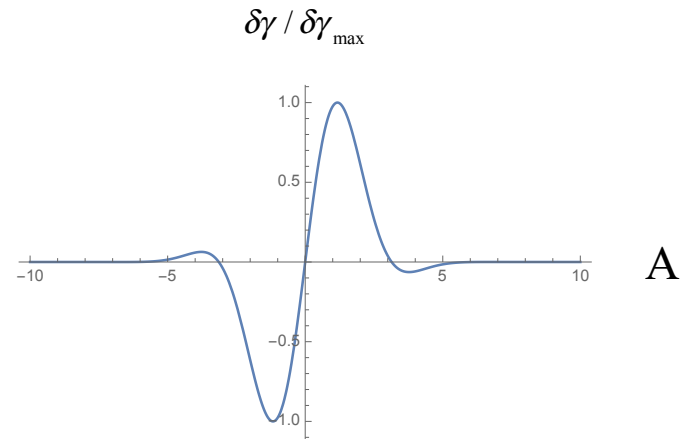
$$g(A, B) = \bar{g}(A, r) = \sin A \cdot e^{-\frac{B^2}{2}} = \sin A \cdot e^{-r^2 \frac{A^2}{2}};$$



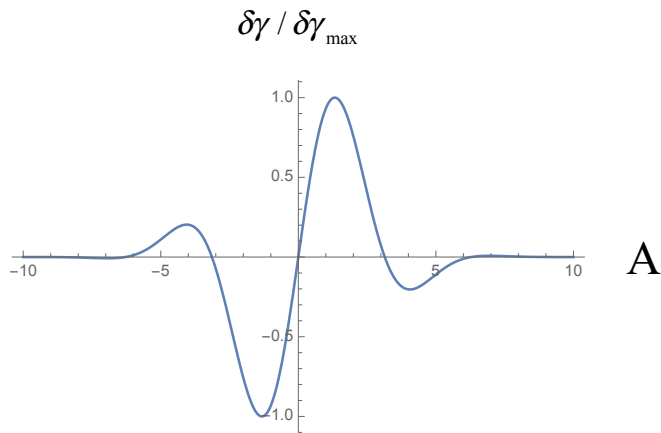
Reminder of parameter r - it is relative bandwidth



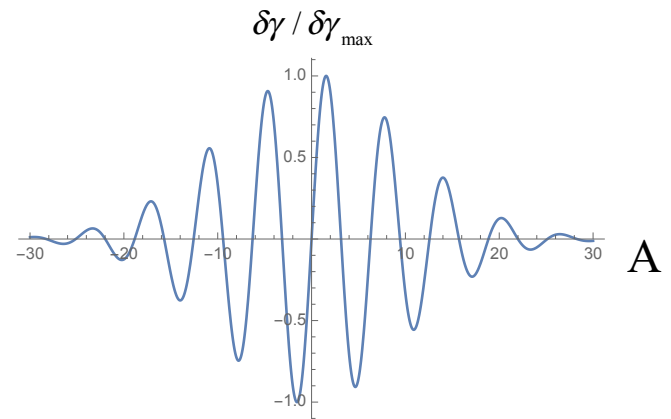
$r=2$



$r=0.6$

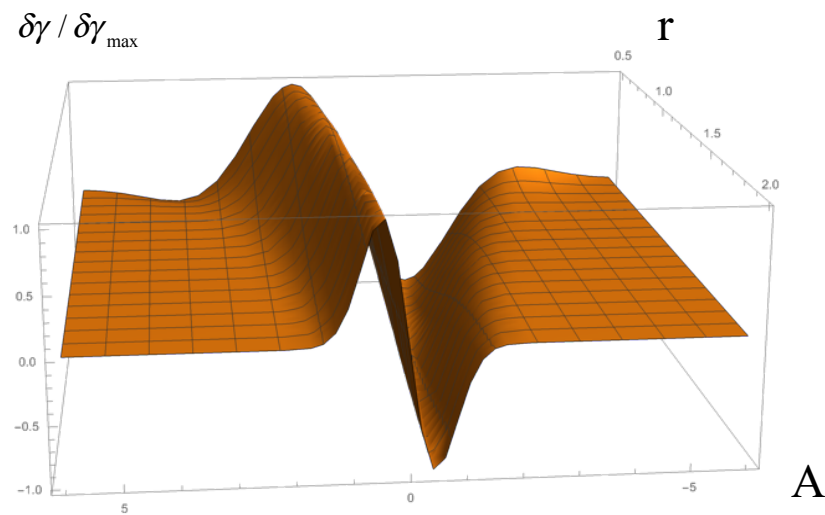
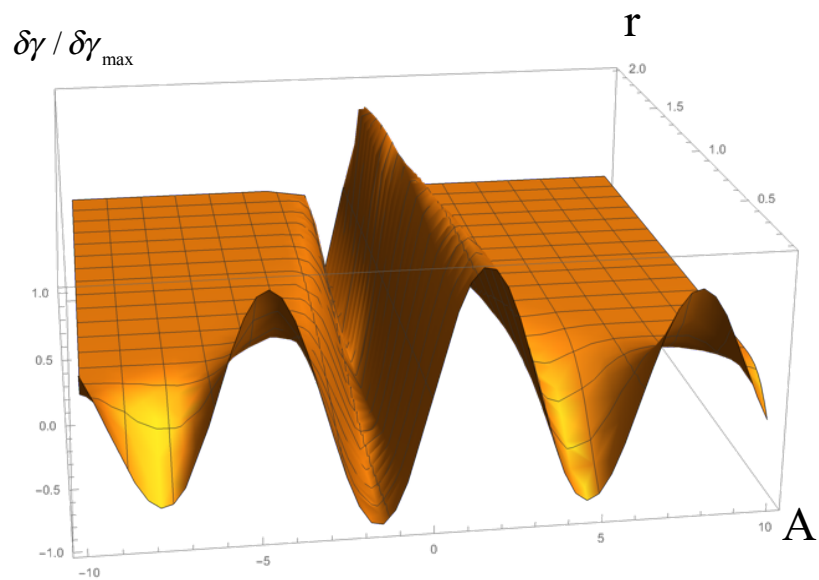


$r=0.435$



$r=0.1$

Kick shapes



Energy jitter

- Assume that energy of election beam is random and has Gaussian distribution

$$f(\delta_e) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{\delta_e^2}{2\sigma^2}}; z = R_{56}(\delta_e - \delta_h); x = kz = kR_{56}(\delta_e - \delta_h);$$

$$dx = kR_{56}d\delta_e; \delta_e = \frac{x}{kR_{56}} + \delta_h$$

$$\langle \delta\gamma \rangle = \frac{\Delta\gamma}{\sqrt{2\pi}\sigma} \int_{-\infty}^{\infty} e^{-\frac{\delta_e^2}{2\sigma^2}} \sin kz \cdot e^{-\frac{\kappa^2 z^2}{2}} d\delta_e = \frac{\Delta\gamma}{\sqrt{2\pi}\sigma kR_{56}} \int_{-\infty}^{\infty} e^{-\frac{\left(\frac{x}{kR_{56}\sigma} + \frac{\delta_h}{\sigma}\right)^2}{2}} \sin x \cdot e^{-\frac{\kappa^2 x^2}{2}} dx;$$

$$\langle \delta\gamma \rangle = \frac{\Delta\gamma}{\sigma kR_{56}} I; I = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{\left(\frac{x}{kR_{56}\sigma} + \frac{\delta_h}{\sigma}\right)^2}{2}} \sin x \cdot e^{-\frac{\kappa^2 x^2}{2}} dx$$

- Integration results in similar functional dependence but with reduced amplitude and broader curve

$$\langle \delta\gamma \rangle = \langle \Delta\gamma \rangle e^{-\frac{1}{2}(\kappa' R_{56} \delta_h)^2} \sin k' R_{56} \delta_h;$$

$$\langle \Delta\gamma \rangle = \frac{\Delta\gamma}{\sqrt{1 + (\sigma\kappa R_{56})^2}} e^{-\frac{1}{2} \frac{(kR_{56}\sigma)^2}{1 + (\sigma\kappa R_{56})^2}}; \kappa' = \frac{\kappa}{\sqrt{1 + (\sigma\kappa R_{56})^2}}; k' = \frac{k}{1 + (\sigma\kappa R_{56})^2}.$$

Dimensionless Z parameter

$$X = kR_{56}\delta_h; Y = \kappa R_{56}\delta_h = rX; \textcolor{red}{Z} = \textcolor{red}{\sigma}kR_{56}$$

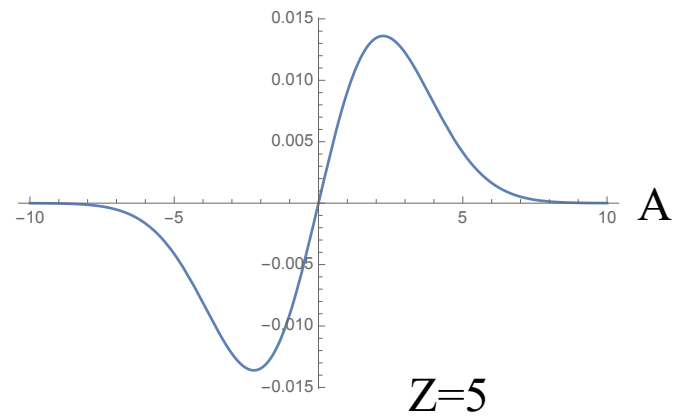
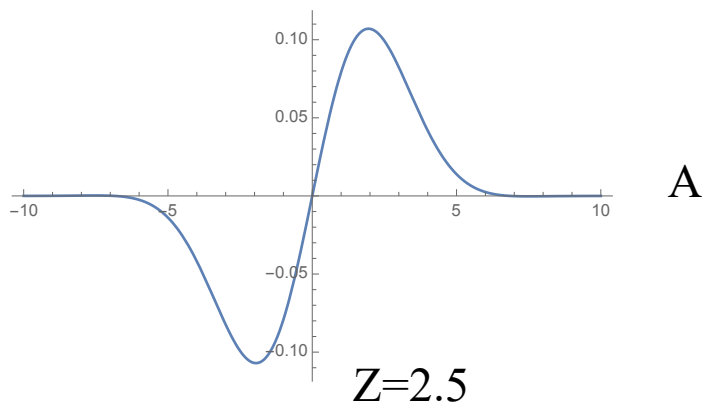
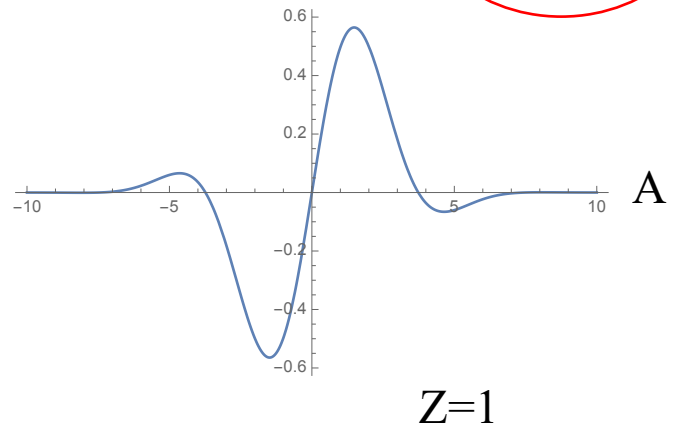
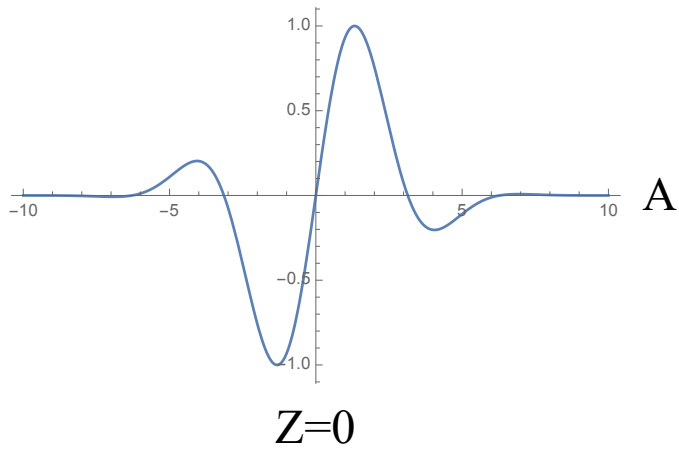
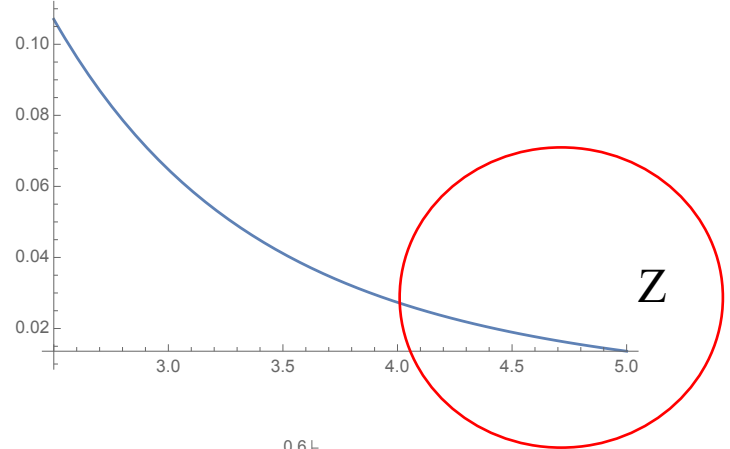
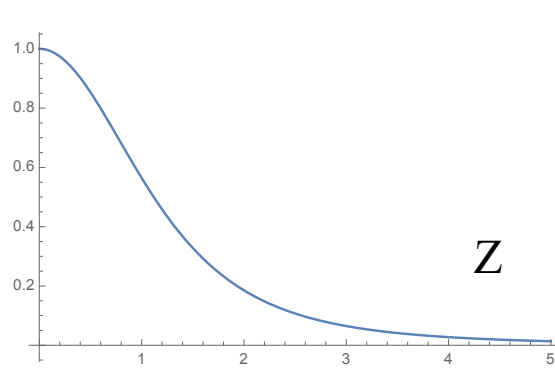
$$\langle \Delta\gamma \rangle = \frac{\Delta\gamma}{\sqrt{1 + \textcolor{red}{r}^2 \textcolor{red}{Z}^2}} e^{-\frac{1}{2} \frac{\textcolor{red}{Z}^2}{1 + \textcolor{red}{r}^2 \textcolor{red}{Z}^2}}; \kappa' = \frac{\kappa}{\sqrt{1 + \textcolor{red}{r}^2 \textcolor{red}{Z}^2}}; k' = \frac{k}{1 + \textcolor{red}{r}^2 \textcolor{red}{Z}^2}.$$

$$\langle \delta\gamma \rangle = \langle \Delta\gamma \rangle e^{-\frac{1}{2} \frac{(rX)^2}{1 + \textcolor{red}{r}^2 \textcolor{red}{Z}^2}} \sin \frac{X}{1 + \textcolor{red}{r}^2 \textcolor{red}{Z}^2};$$

Example

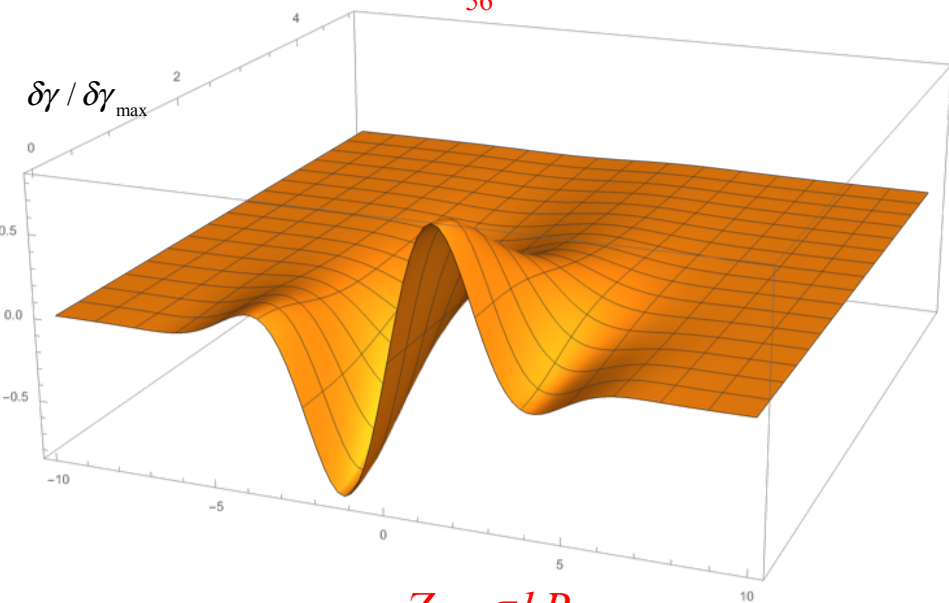
$$r=0.435$$

$$Z = \sigma k R_{56}$$

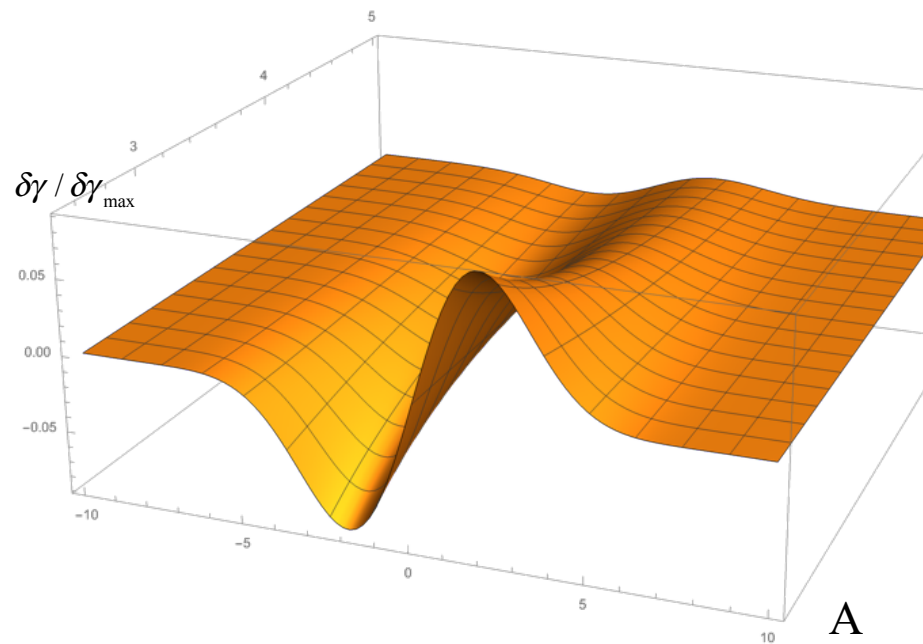


3D pics

$$Z = \sigma k R_{56}$$

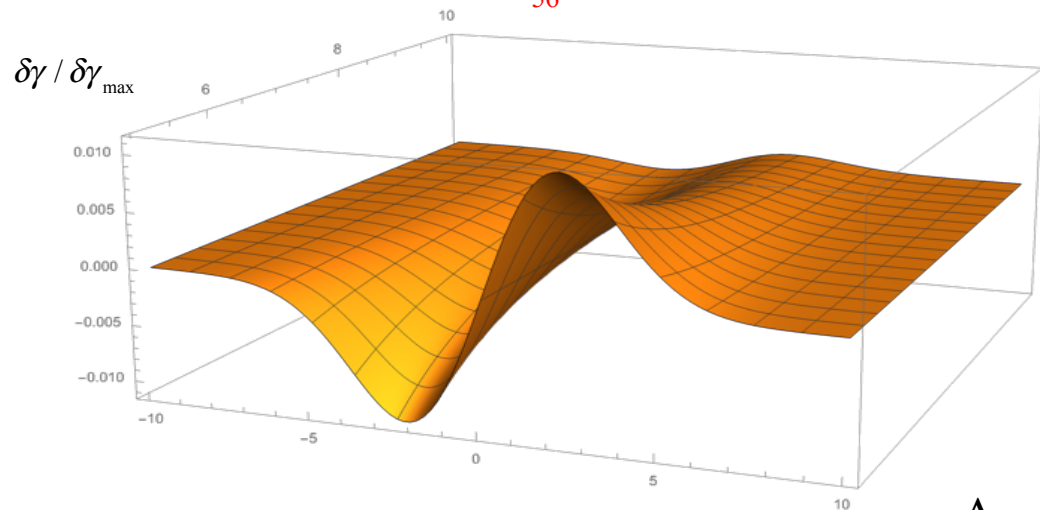


$$Z = \sigma k R_{56}$$



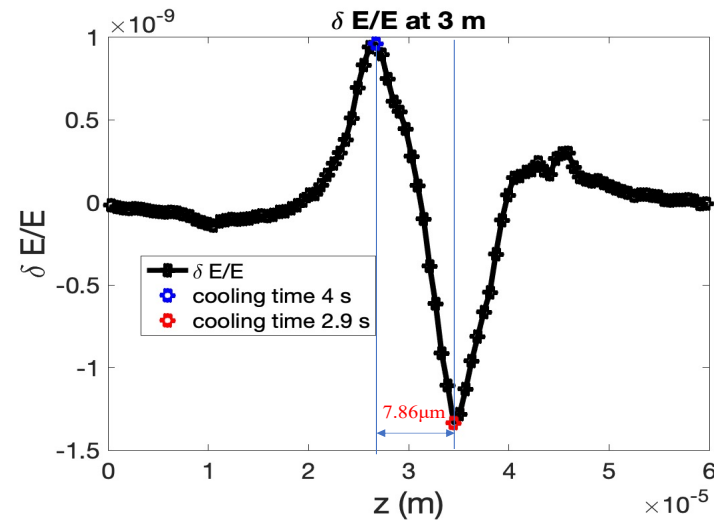
A

$$Z = \sigma k R_{56}$$



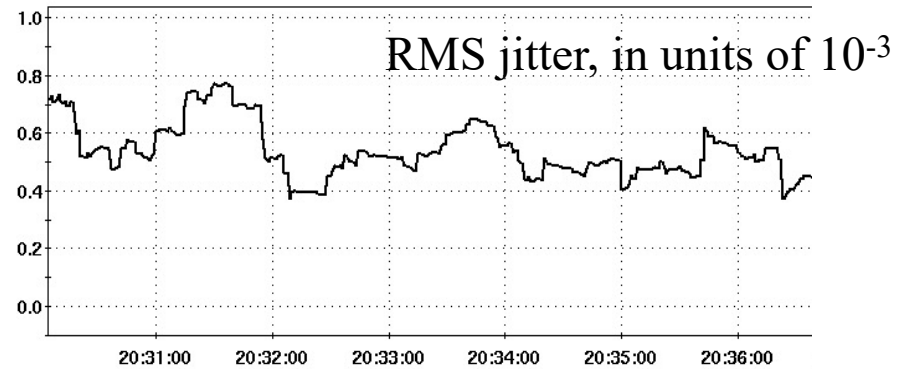
A

Some numbers

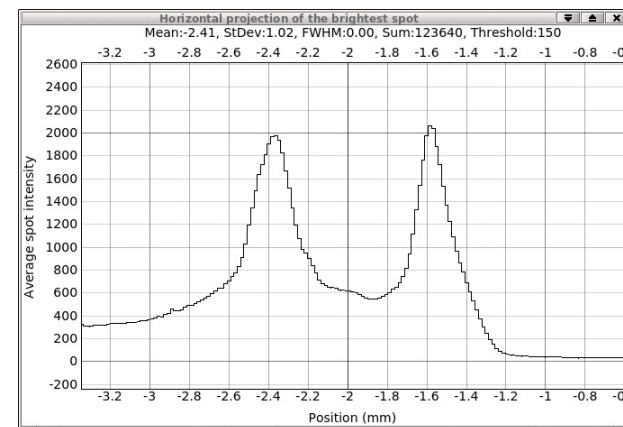
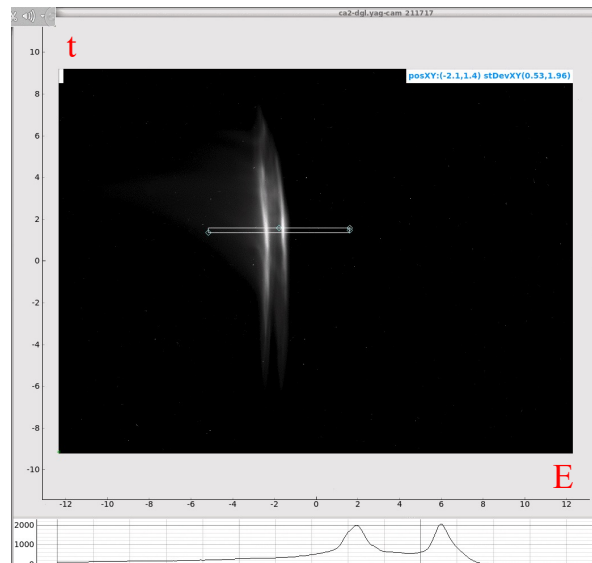


- σ is RMS values of relative energy jitter
- For $r=0.435$ and $Z=0$ maximum is located at $X=1.32512$, hence $k=3.372 \times 10^5, m^{-1}$ and $Z=1$ for $\sigma=2 \times 10^{-4}$.
- It means for $\sigma=0.1\%$ we would have $Z=5$
- The kick is not only weaker by also 70% broader:
 - Maximum[A, $r=0.435$, $Z=0$] \rightarrow 1 at $A=1.325$
 - Maximum[A, $r=0.435$, $Z=5$] \rightarrow 0.0136 at $A=2.239$
- It means that with $\sigma=0.1\%$ cooling for the central particles (with energy spread up to 0.06%) is 0.8% of that for $\sigma=0$.
- It may explain very slow cooling or the fact that we did not detect it yet

Supporting data



- We scanned the laser phase to see how it affects the beam position on dogleg YAG: 0.36% energy change per 100 psec jump.
- On June 10, 2021 peak to peak jumps in energy were 0.35%. It can be result of 100 psec timing jitter in the laser pulses timing
- We indeed observed pulse-to-pulse energy jumps at this level

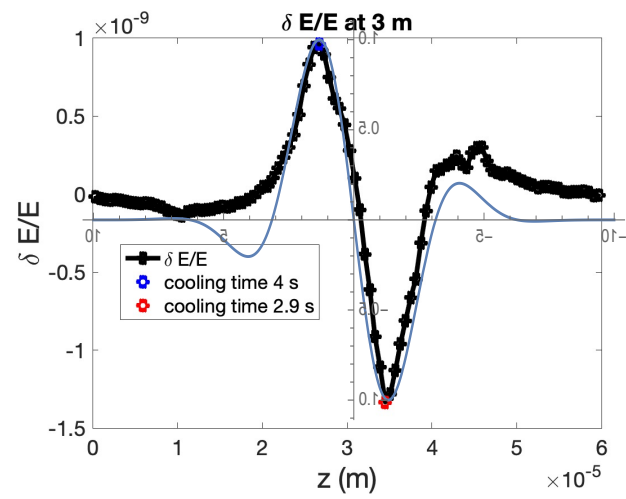


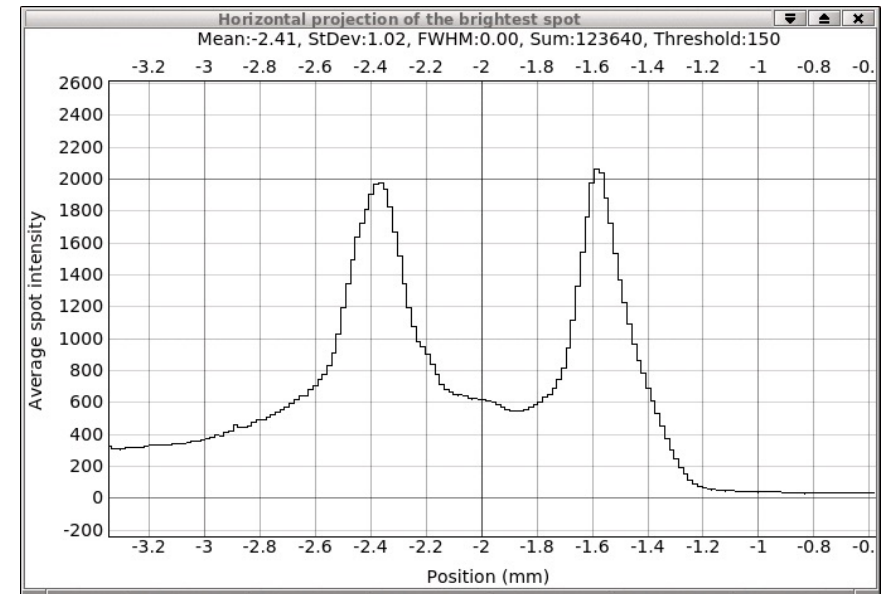
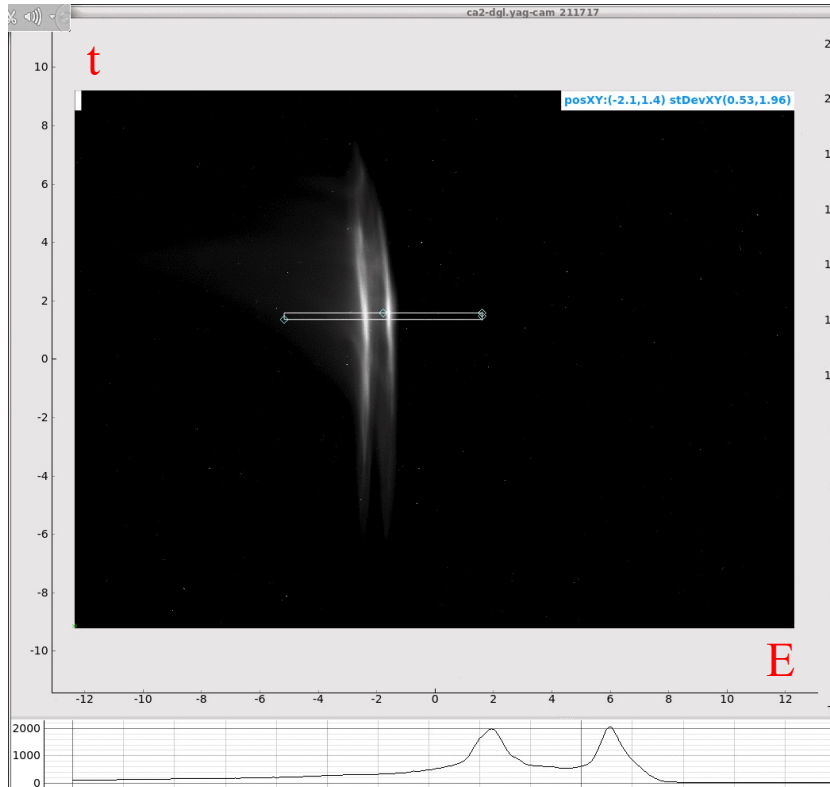
0.8 mm separation, which is 0.3% energy difference between consecutive bunches

Summary

- Pulse to pulse e-beam energy jitter – currently attributed to timing and power jitter of the drive laser – is at 0.35%-0.4% level. Estimated RMS value is in 0.05% - 0.1% range. There is no remedy to this problem except replacing the seed laser and also trimming down laser power jitter
- In addition there are slow drifts at 0.1% level, which in principle can be removed by slow feedback – still need to be verified that it is
- All these numbers significantly larger than the slice energy spread of the electron beam core ($<0.02\%$ RMS) as well as requirements for the energy stability of the electron beam for CeC to operate - $\sigma \leq 0.02\%$
- It is most likely explanation why we did not observe either cooling or anti-cooling associated with CeC process during the energy scan
- We need to fix this problem before start of the next run

Back ups





0.8 mm separation, which is 0.3% energy difference between consecutive bunches