USPAS
Hadron beam cooling
-- electron cooling

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Beam Cooling

- Beam cooling is synonymous for a reduction of beam temperature

Temperature is equivalent to terms as phase space volume, emittance and momentum spread

- Beam Cooling processes are not following Liouville's Theorem:

in a system where the particle motion is controlled by external conservative forces the phase space density is conserved´

(This neglect interactions between beam particles.)

- Beam cooling techniques are non-Liouvillean processes which violate the assumption of a conservative force.

e.g. interaction of the beam particles with other particles
(electrons, photons, matter)

Cooling Force

Generic (simplest case of a) cooling Force:

$$\boxed{F_{x,y,s} = -\alpha_{x,y,s} v_{x,y,s}}$$

$v_{x,y,s}$ velocity in the rest frame of the beam

non conservative, cannot be described by a Hamiltonian

For a 2D subspace distribution function $f(z, z', t)$

$$F_z = -\alpha_z v_z \qquad z = x, y, s \qquad v_z = v_0 \cdot z'$$

$$\frac{df(z, z', t)}{dt} = -\lambda_z f(z, z', t)$$

λ_z cooling (damping) rate

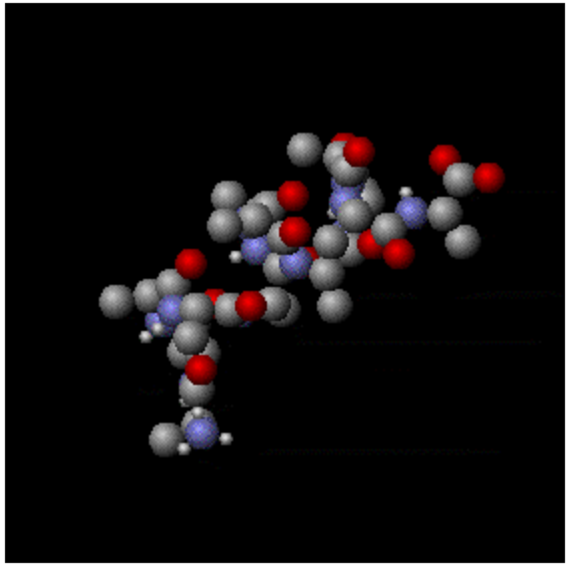
in a circular accelerator:

Transverse (emittance) cooling rate $\quad \epsilon_{x,y}(t_0 + t) = \epsilon_{x,y}(t_0)\ e^{-\lambda_{x,y} t}$

Longitudinal (momentum spread) cooling rate $\quad \frac{\delta p_\parallel}{p_0}(t_0 + t) = \frac{\delta p_\parallel}{p_0}(t_0)\ e^{-\lambda_\parallel t}$

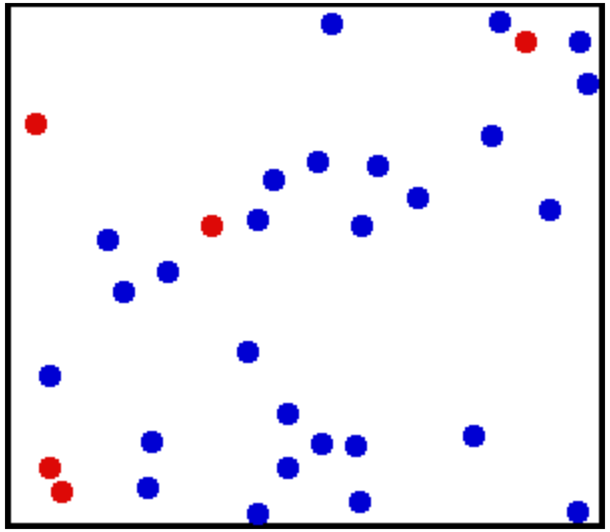
What is temperature

- What we feel as a temperature is a motion of micro-particles



Thermal vibration of a segment of protein

Motion of gas molecules is observed by us as temperature



Monoatomic molecules

temperature

mass of a particle

$$T = \frac{m(\Delta v)^2}{3k_B}$$

velocity spread of particles in the gas

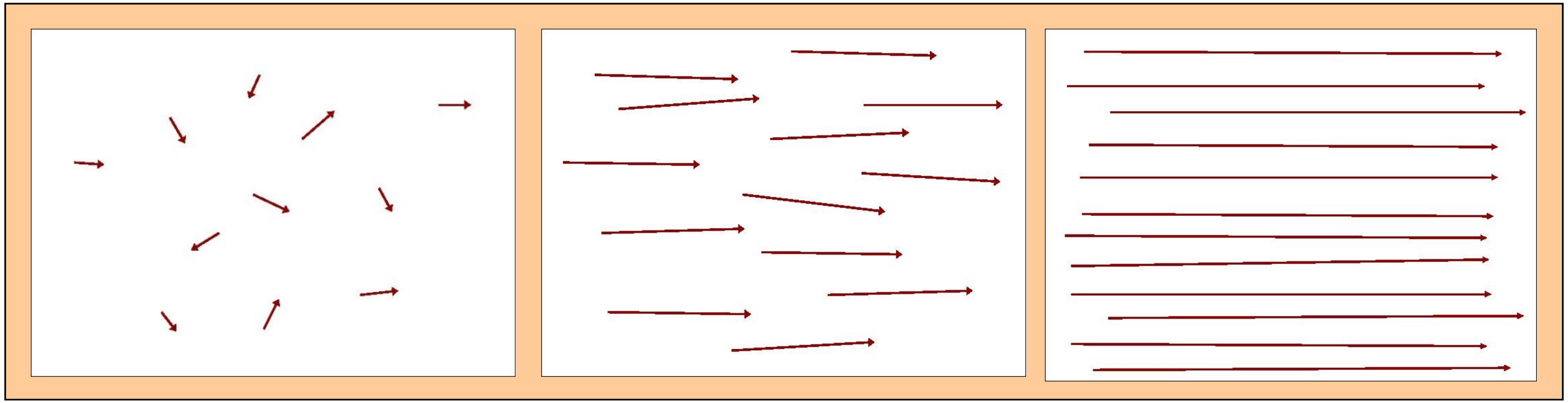
Boltzman n constant

The heavier the particles of the gas are and the faster they are moving - the higher is the gas temperature
And vice versa, the lighter are gas particles and the slower is their motion - the lower is the temperature

Beam Temperature

Where does the beam temperature originate from?

The beam particles are generated in a ‘hot’ source



at rest (source) **at low energy** **at high energy**

In a standard accelerator the beam temperature is not reduced
(thermal motion is superimposed the average motion after acceleration)

but: many processes can heat up the beam

e.g. heating by mismatch, space charge, intrabeam scattering,
internal targets, residual gas, external noise

Beam Temperature Definition

Longitudinal beam temperature

$$\boxed{\frac{1}{2}k_B T_{\|} = \frac{1}{2}mv_{\|}^2 = \frac{1}{2}mc^2\beta^2(\frac{\delta p_{\|}}{p})^2}$$

Transverse beam temperature

$$\boxed{\frac{1}{2}k_B T_{\perp} = \frac{1}{2}mv_{\perp}^2 = \frac{1}{2}mc^2\beta^2\gamma^2\theta_{\perp}^2}$$

$$\theta_{\perp} = \frac{v_{\perp}}{\beta c}, \qquad \theta_{\perp}(s) = \sqrt{\frac{\epsilon}{\beta_{\perp}(s)}}$$ uniform beam

Distribution function

$$f(v_{\perp}, v_{\|}) \propto \exp(-\frac{mv_{\perp}^2}{2k_B T_{\perp}} - \frac{mv_{\|}^2}{2k_B T_{\|}})$$

Particle beams can be anisotropic: $k_B T_{\|} \neq k_B T_{\perp}$

e.g. due to laser cooling or the distribution of the electron beam

Don‘t confuse: beam energy $\leftrightarrow$ beam temperature

(e.g. a beam of energy 100 GeV can have a temperature of 1 eV)

Benefits of Beam Cooling

- Improved beam quality
  - Precision experiments
  - Luminosity increase

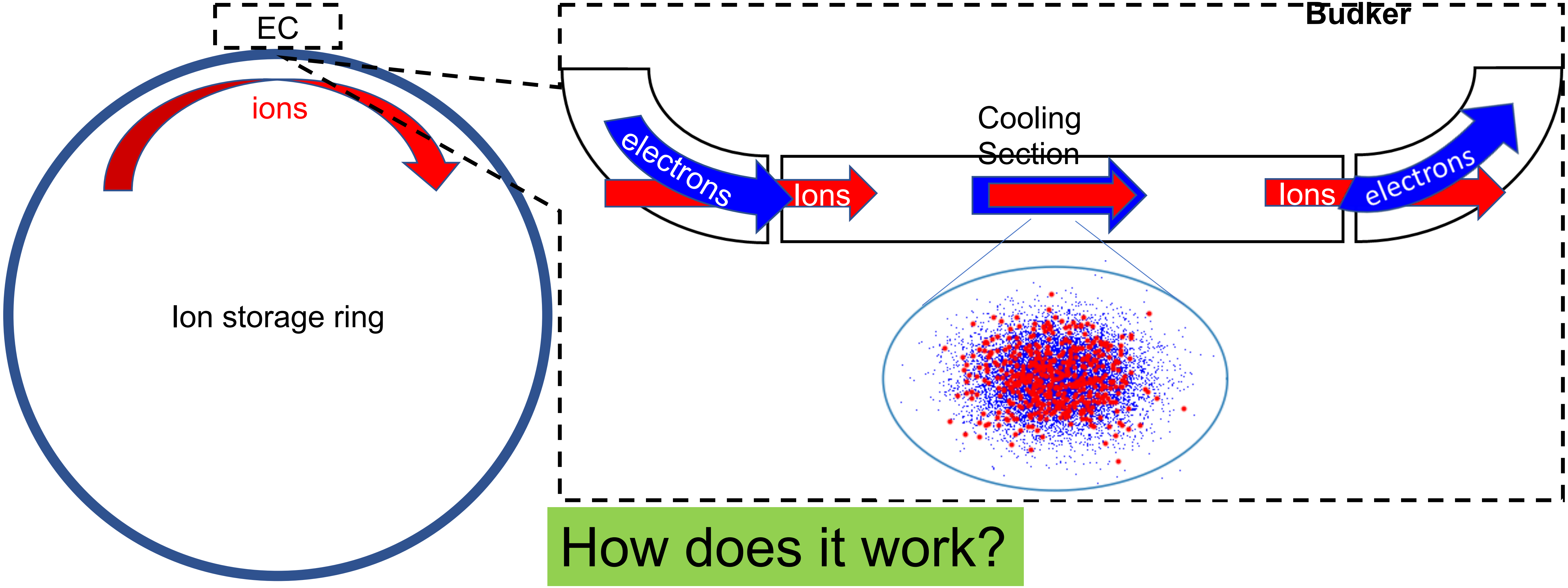
- Compensation of heating
  - Experiments with internal target
  - Colliding beams

- Intensity increase by accumulation
  - Weak beams from source can be enhanced
  - Secondary beams (antiprotons, rare isotopes)

The basics of Electron Cooling

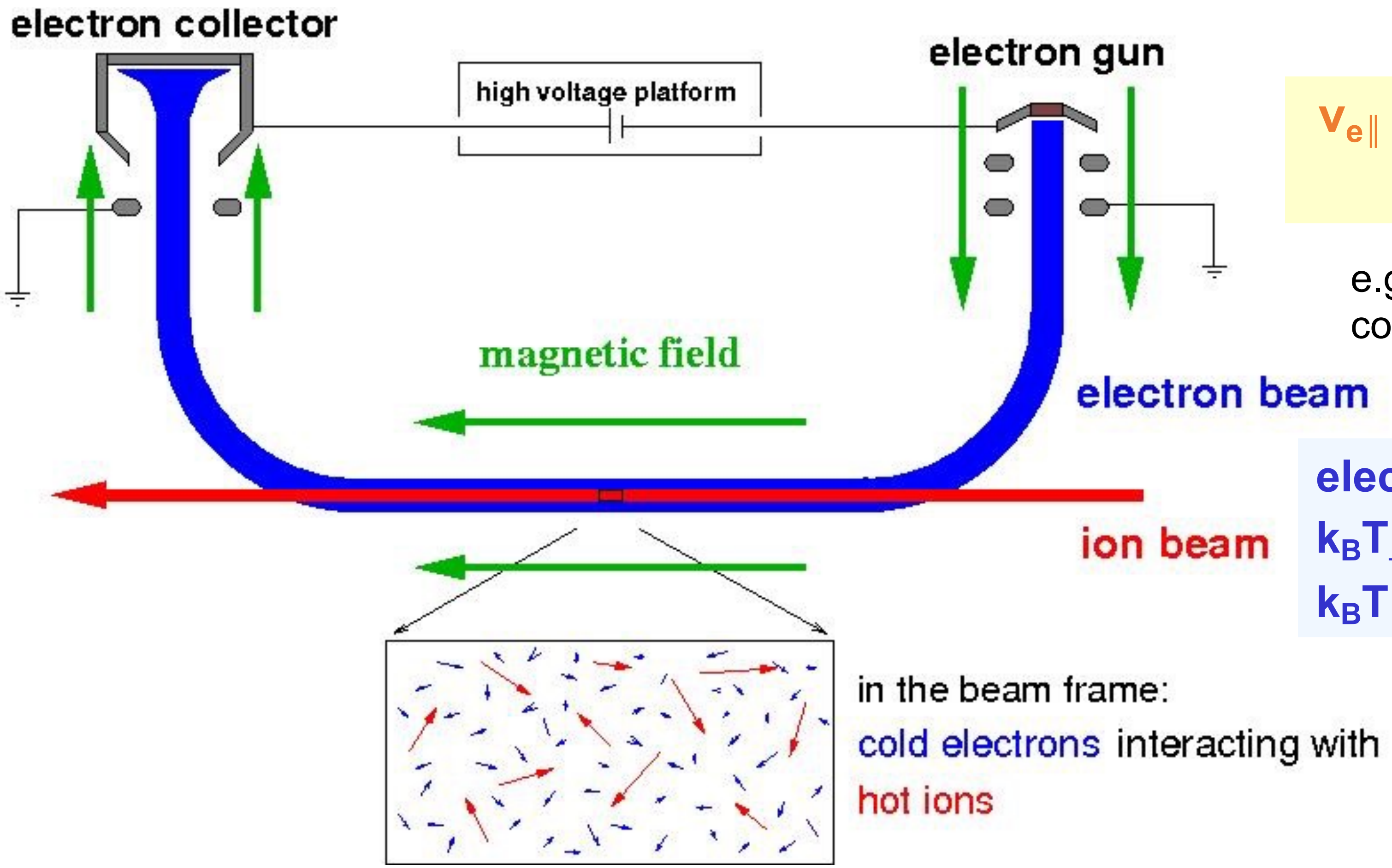
- Mix a bunch of ions with an electron bunch traveling with the same speed
- Let the two bunches travel together over some length
- The velocity spread of the ions will get reduced

Gersh Budker



How does it work?

Electron Cooling



$v_{e\parallel} = \beta_e c = \beta_i c = v_{i\parallel}$

$E_e = m_e / M_i \cdot E_i$

e.g.: 220 keV electrons
cool 400 MeV/u ions

electron temperature

$k_B T_\perp \approx 0.1$ eV

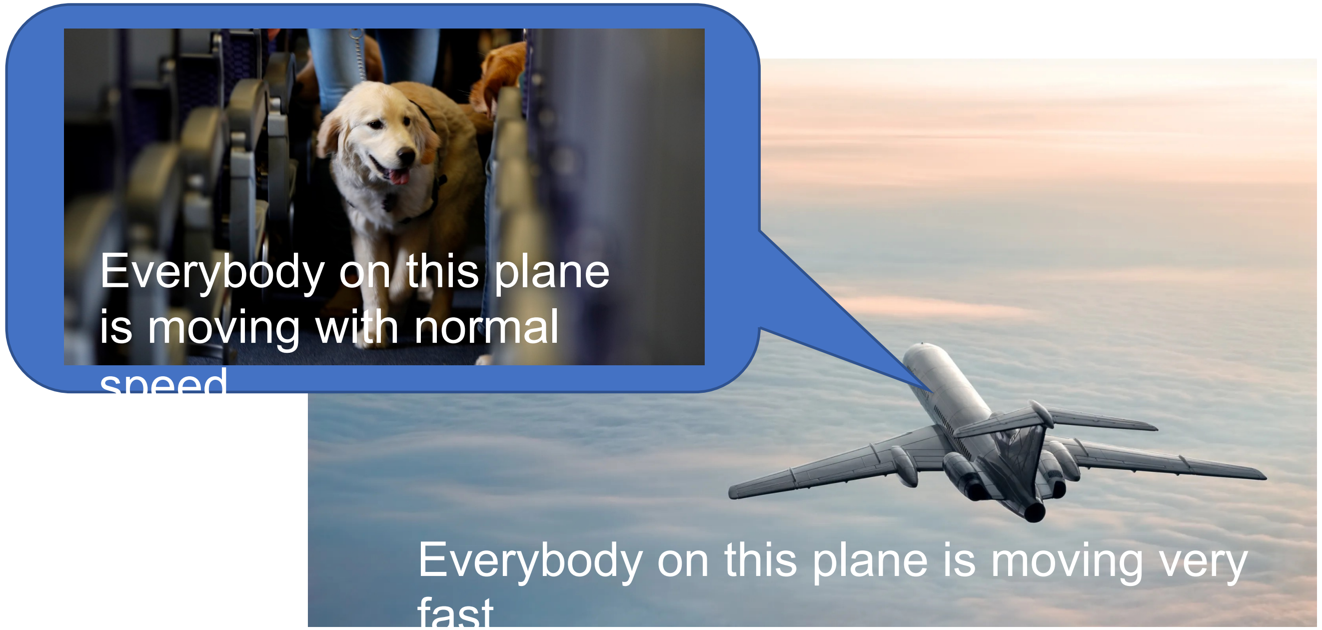
$k_B T_\parallel \approx 0.1 - 1$ meV

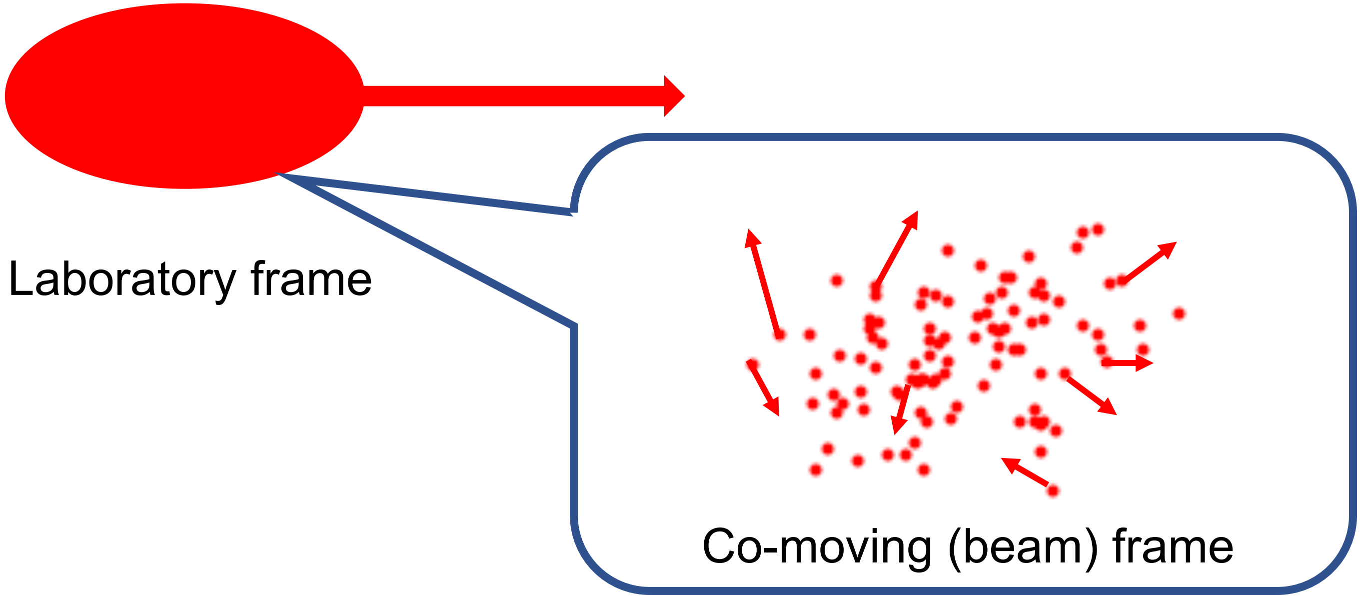
superposition of a cold intense electron beam with the same velocity

**momentum transfer by Coulomb collisions
cooling force results from energy loss
in the co-moving gas of free electrons**

Co-moving reference frame

- The motion is relative
- When we look at a bunch of accelerated particles from our reference frame, we see that all the particles move in the same direction with nearly the speed of light

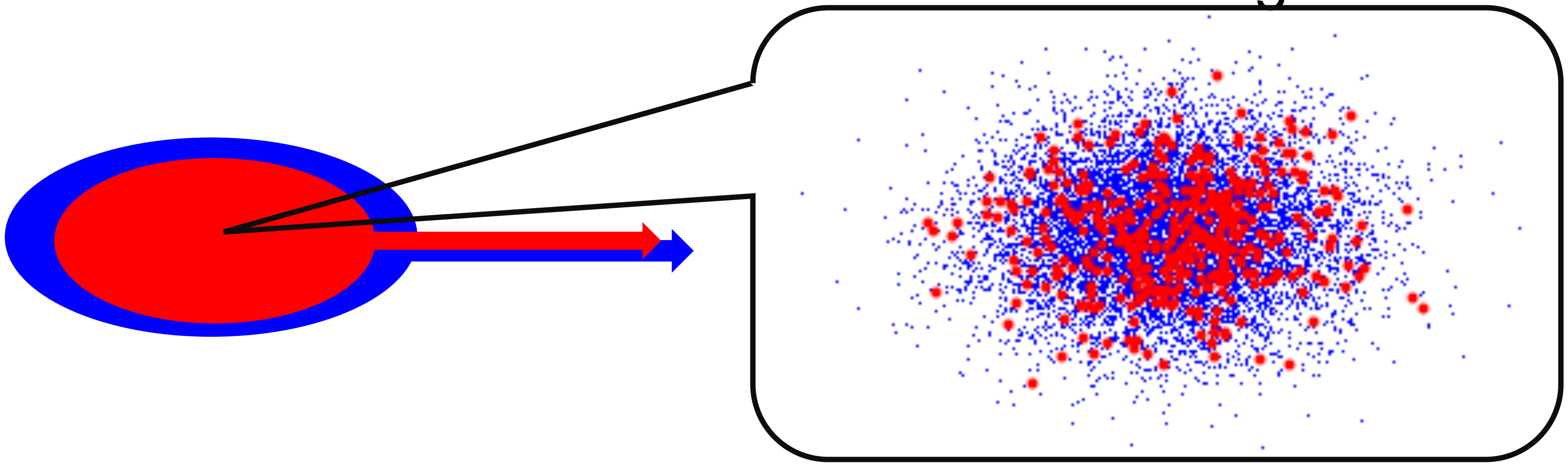




- But in the co-moving reference frame our bunch of particles looks like a gas, with particles moving randomly in all directions

Electron cooling as a heat exchange

- In the co-moving frame, the mixture of an ion bunch and an electron bunch looks like a mixture of two gases – a gas of ions and a gas of electrons
- Electrons are much lighter than the ions ($m_{Au} \approx 400000 \cdot m_e$). This means that the gas of electrons is much colder than the gas of ions.
- Each time the ion bunch passes the cooling section it transfers a small fraction of its heat to the electron bunch, and becomes a little bit colder. Hence the name – Electron Cooling.



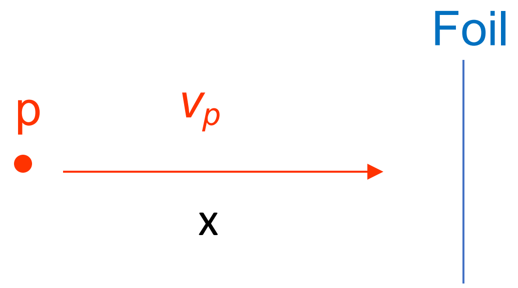
$$m_e \ll m_i$$

$$T_e \ll T_i$$

Moving foil analogy

- Consider electrons as being represented by a foil moving with the average velocity of the ion beam.
- Ions moving faster (slower) than the foil (electrons) will penetrate it and will lose energy along the direction of their momentum (dE/dx losses) during each passage until all the momentum components in the moving frame are diminished.

Bohr's model

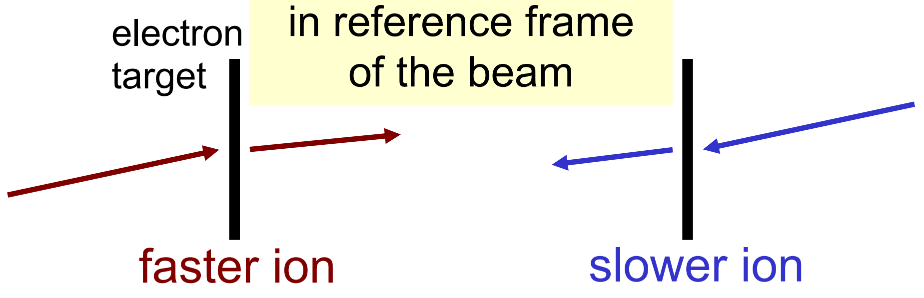


$$F = -\frac{dE}{dx} = \frac{4\pi Z^2 n_e e^4}{(4\pi\varepsilon_0)^2 m_e v_i^2}\alpha_1$$

In moving frame

Simple Derivation of the Electron Cooling Force

Analogy: energy loss in matter
(electrons in the shell)



Rutherford scattering: $2\ tan(\frac{\theta}{2}) = \frac{2Z_1Z_2e^2}{4\pi\epsilon_0 \Delta p v b} \quad Z_1 = Q\ (ion),\ Z_2 = -1\ (electron)$

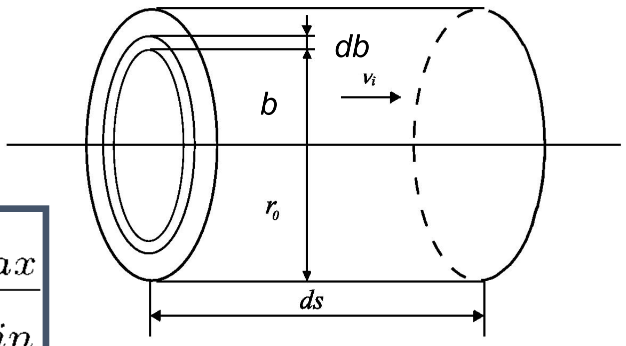
Energy transfer: $\Delta E(b) = \frac{(\Delta p)^2}{2m_e} \simeq \frac{2Q^2e^4}{(4\pi\epsilon_0)^2 m_e v^2}\frac{1}{b^2} \quad (for \quad b \gg b_{min})$

Minimum impact parameter: $b_{min} = \frac{Qe^2}{(4\pi\epsilon_0)^2 m_e v^2}$

from: $\Delta E(b_{min}) = \Delta E_{max} \simeq 2m_e v^2$

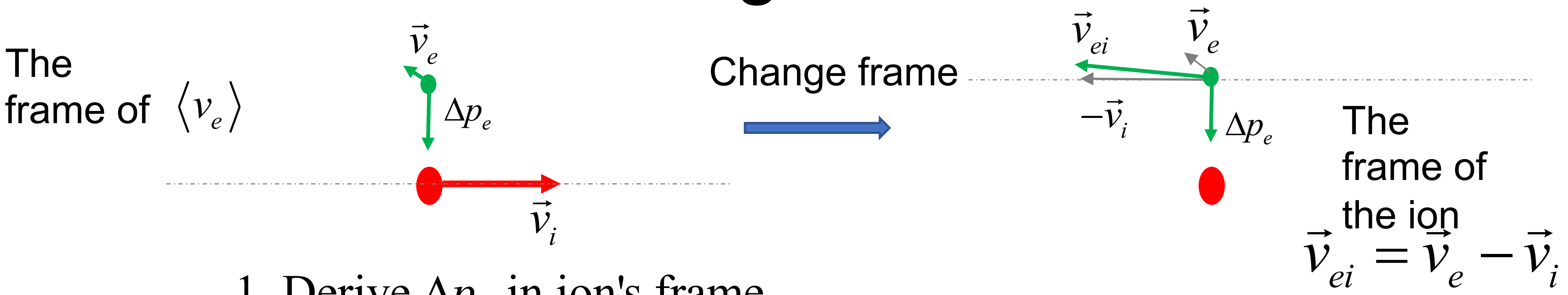
Energy loss:

$$\boxed{-\frac{dE}{dx} = 2\pi \int_{b_{min}}^{b_{max}} b n_e \Delta E\, db = \frac{4\pi Q^2 e^4}{(4\pi\epsilon_0)^2 m_e v^2} n_e \ln \frac{b_{max}}{b_{min}}}$$



Coulomb logarithm $L_C = \ln (b_{max}/b_{min}) \approx 10$ (typical value)

Derivation of non-magnetized electron cooling friction force



1. Derive Δp_e in ion's frame

2. Δp_e is also the momentum change in the frame of $\langle v_e \rangle$

3. Get the energy change of the electron in the frame of $\langle v_e \rangle$ from

$$\Delta E_e = \frac{(p_{e0} + \Delta p_e)^2 - p_{e0}^2}{2m_e} \approx \frac{\Delta p_e{}^2}{2m_e} \text{ assuming } p_{e0} << \Delta p_e$$

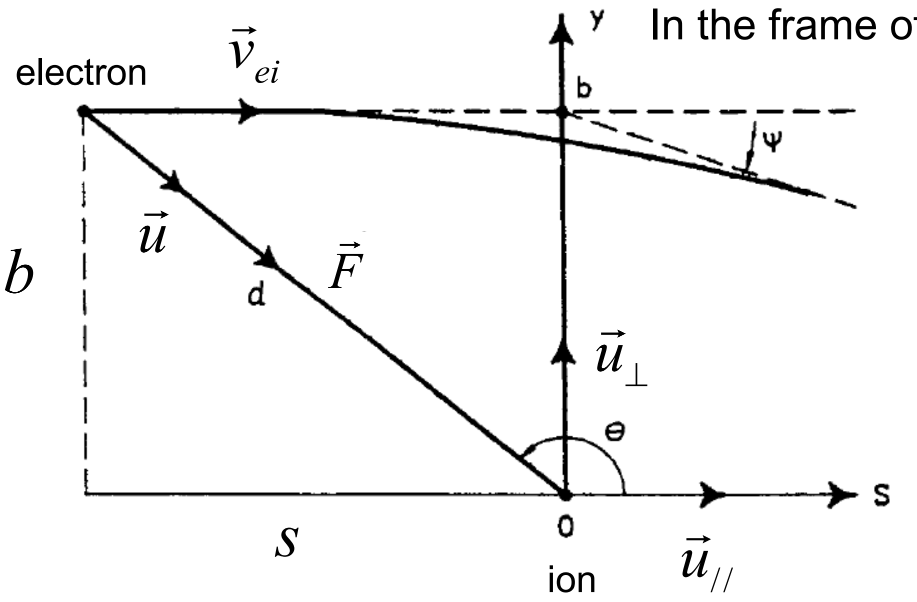
4. Get the energy loss of the ion in the frame of $\langle v_e \rangle$ from

$$\Delta E_{i,loss} = \sum_j \Delta E_{e,j}$$

since the total energy is conserved.

Momentum transfer

In the frame of the moving ion



The energy gained by the electron is equal to the energy loss by ion.

$$\vec{F} = \frac{Ze^2}{4\pi\varepsilon_0 d^2}\vec{u}$$

$$\vec{v}_{ei} = \vec{v}_e - \vec{v}_i$$

Assuming ψ is small

$$\Delta\vec{p} = \int_{-\infty}^{\infty} \vec{F}dt \approx \frac{1}{v_{ei}}\int_{-\infty}^{\infty} \vec{F}ds$$

$$\Delta p_{//} \approx \frac{1}{v_{ei}}\int_{-\infty}^{\infty} F_{//}ds = -\frac{1}{v_{ei}}\int_{-\infty}^{\infty} F\cos\theta ds = -\frac{1}{v_{ei}}\int_{-\infty}^{\infty} \frac{Ze^2}{4\pi\varepsilon_0\left(s^2+b^2\right)}\frac{s}{\sqrt{s^2+b^2}}ds = 0$$

$$\Delta p_{\perp} \approx -\frac{1}{v_{ei}}\int_{-\infty}^{\infty} F\sin\theta ds = -\frac{1}{v_{ei}}\int_{-\infty}^{\infty} \frac{Ze^2 b}{4\pi\varepsilon_0\left(s^2+b^2\right)^{3/2}}ds = -\frac{1}{v_{ei}}\frac{2Ze^2 b}{4\pi\varepsilon_0 b^2}\int_0^{\infty} d\frac{s}{\left(s^2+b^2\right)^{1/2}} = -\frac{1}{v_{ei}}\frac{2Ze^2}{4\pi\varepsilon_0 b}$$

Energy loss

- Back to the frame of the electrons' average velocity, the electron's energy gain is

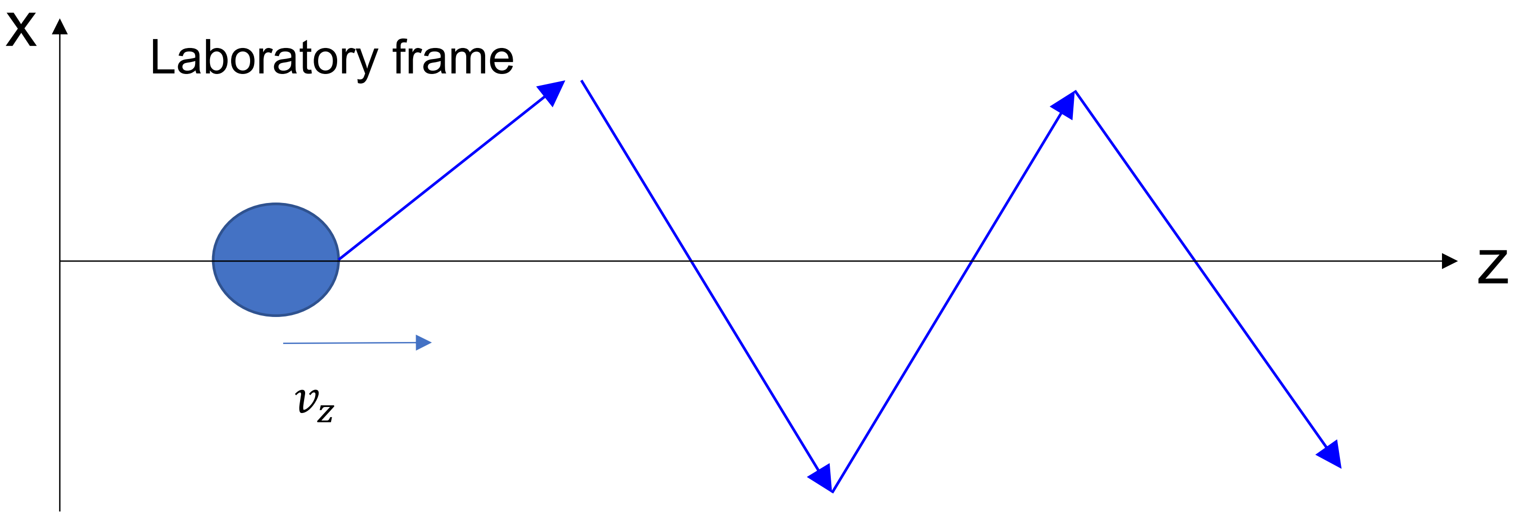
$$\Delta E(b) \approx \frac{\Delta p_\perp^2}{2m_e} = \frac{2Z^2e^4}{m_e {v_{ei}}^2 (4\pi\varepsilon_0)^2 b^2}$$

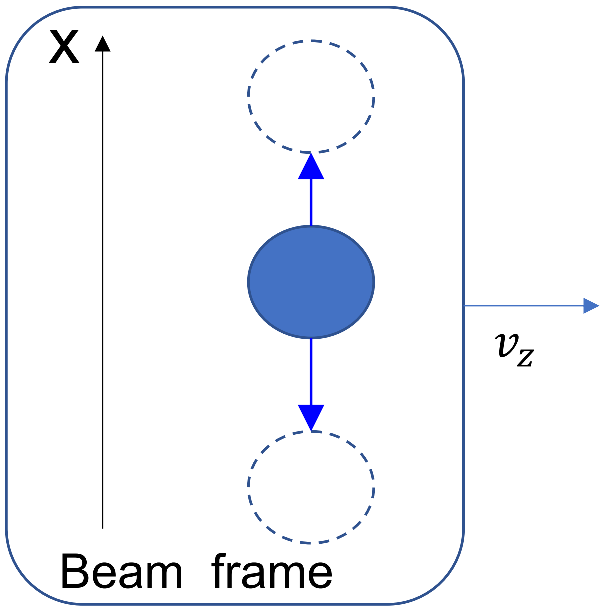
which has to be the energy loss by the ion.

$$\Delta E_{loss}(b) = \frac{2Z^2e^4}{m_e {v_{ei}}^2 (4\pi\varepsilon_0)^2 b^2}$$

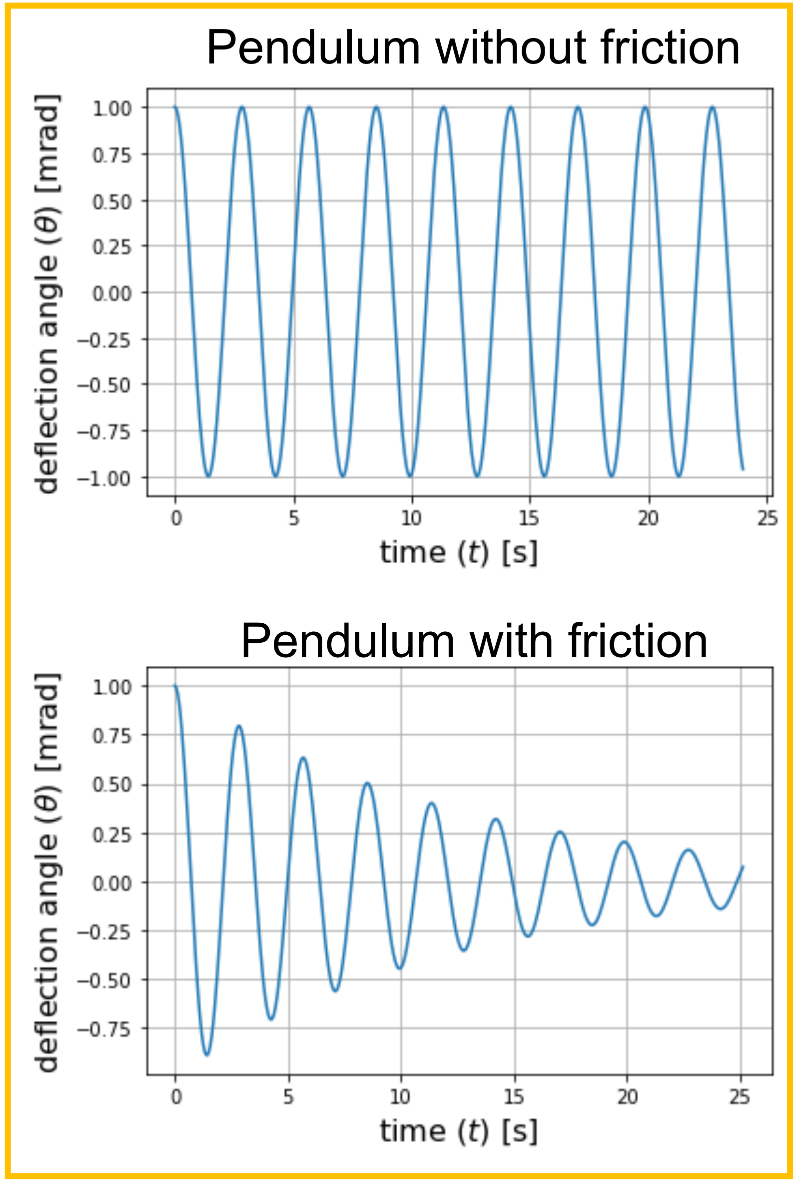
Energy loss by a moving ion due to its interaction with one electron sitting at impact parameter b.

Particle motion in co-moving frame



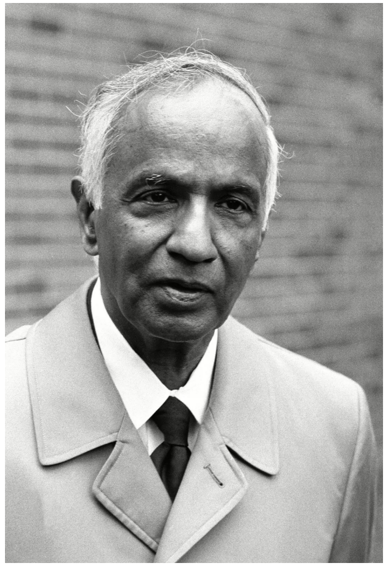


- An individual particle's motion in the beam frame looks exactly like a motion of a frictionless oscillator
- As a matter of fact, in first approximation, it is fair to say that in the beam frame our bunch looks like a billion oscillators swinging back and forth
- In our mechanical model of a pendulum, we had to introduce friction to slow down the motion of an oscillator.
- **Maybe, adding friction to the system is what the Electron Cooling does?**

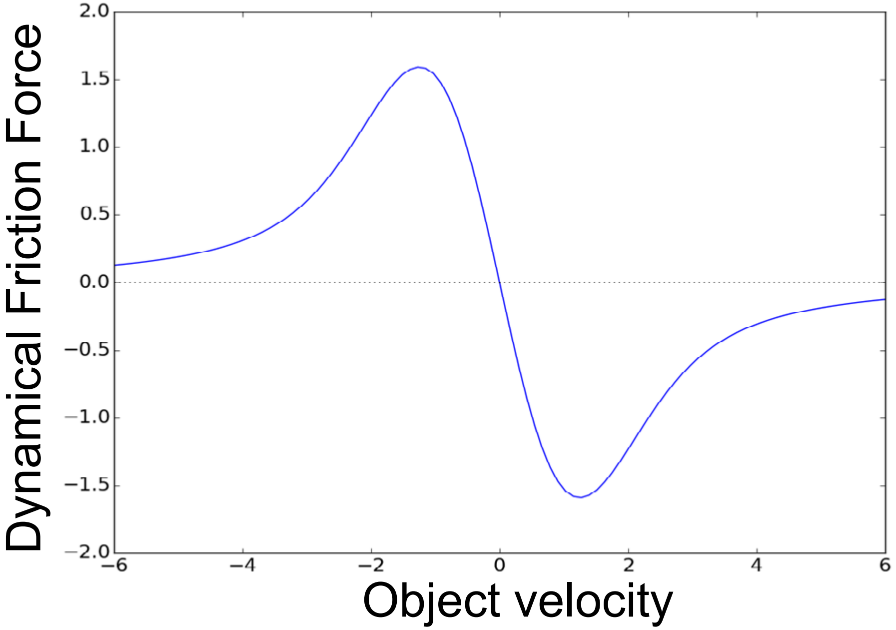


Dynamical Friction

As a massive celestial object (a star, for example) is moving through a cloud of smaller lighter bodies (for instance, space dust) the gravitational pull from the "space dust cloud" slows down the star. This effect is called **Dynamical Friction**.



Subrahmanyan Chandrasekhar



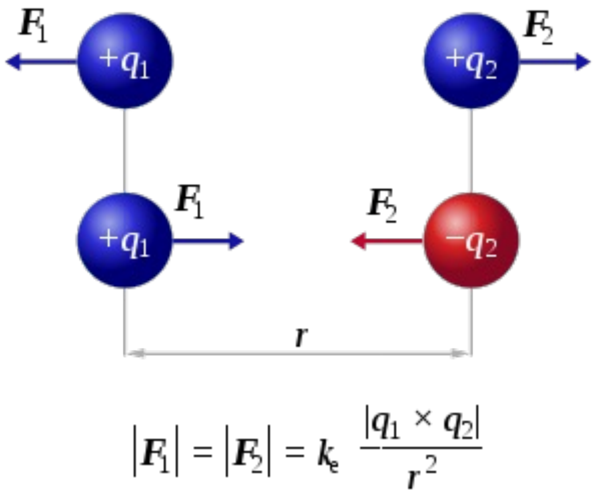
Conceptually, this effect is similar to the friction that is experienced by a massive body moving through water, or a dense gas

The dynamical friction force has an interesting dependence on velocity of the heavy object

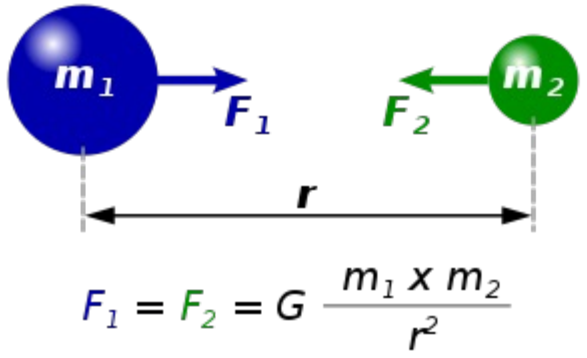
Dynamical friction and Electron Cooling (I)

- In the co-moving frame, electron bunch looks like a “gas of electrons”
- An ion is much heavier than the electrons
- So, in the co-moving frame we have a massive object (an ion) moving through the cloud of much lighter particles
- The electric force and the gravitational force look very similar:

Coulomb’s law:



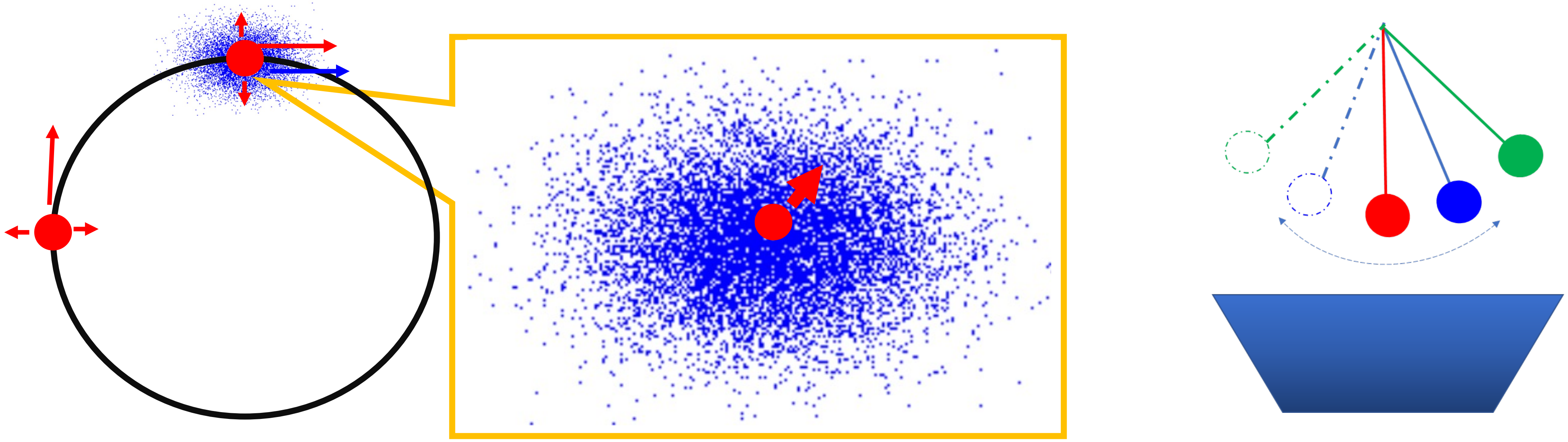
Newton’s law:



- It is reasonable to expect that in Electron cooling we will see the same effect of dynamical friction, which we see in astrophysics.

Dynamical friction and Electron Cooling (II)

- Indeed, the experiments confirm that the process of cooling is well described by a dynamical friction force gradually reducing the velocity spread of an ion bunch.
- A good mechanical analogy to the electron cooling process is a pendulum that, with some periodicity, is getting inserted into a bucket of water



- Only the friction that the ion experiences has an unusual dependence on the ion velocity

How to calculate friction force

density of electron bunch

electron charge

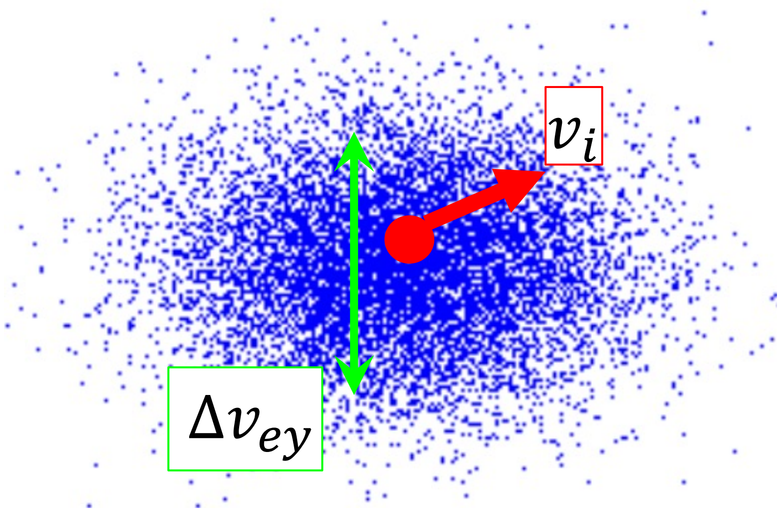
ion charge

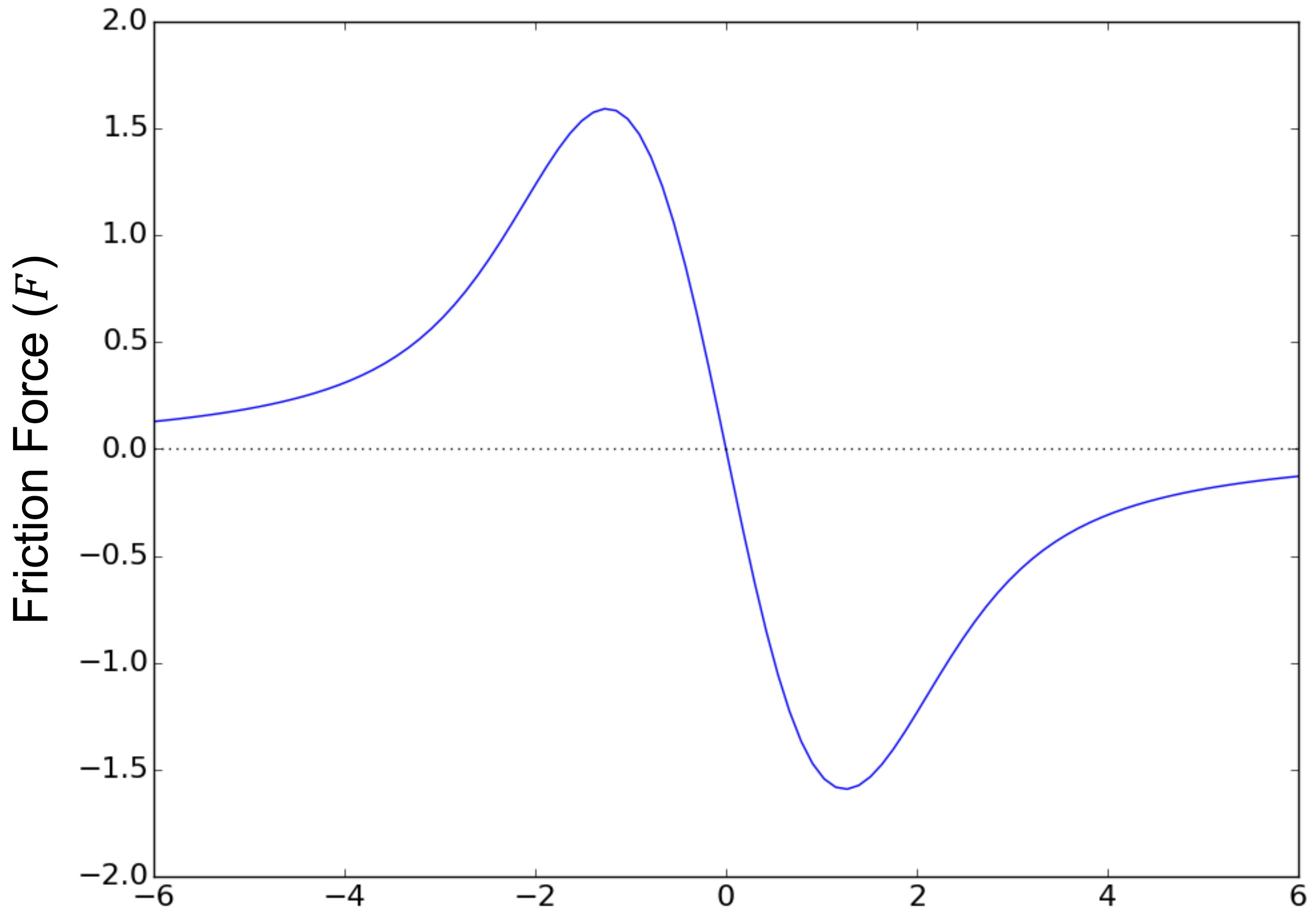
$$F = C \cdot \frac{n_e q_e q_i}{\Delta v_{ex} \Delta v_{ey} \Delta v_{ez}} \cdot f(v_i)$$

velocity spreads of electrons in three directions

C - is a constant

$f(v_i)$ - is a function of ion velocity

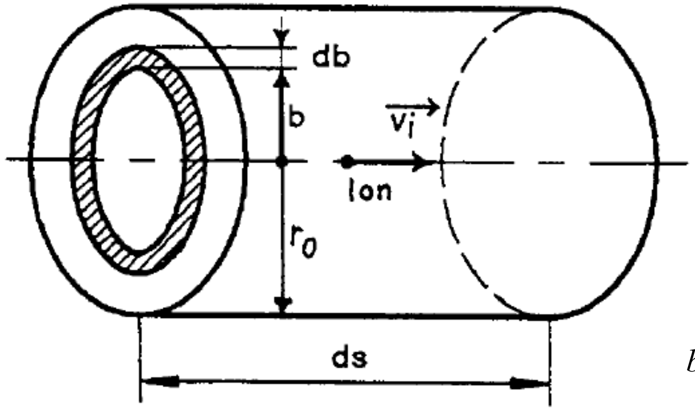




ion velocity relative to electrons' velocity spread ($v_i/\Delta v_e$)

Friction force

Energy loss by a moving ion due to its passing through a slice of electrons with velocity $\vec{v}_e$. (assuming electron spatial density is a constant)



$$b_{\min} = \frac{Ze^2}{4\pi\varepsilon_0 m_e v_i^2}$$

$b_{\max} \sim$ Debye length

$$L_c \equiv \ln\left(\frac{b_{\max}}{b_{\min}}\right) \approx 10$$

$$\Delta E_{slice} = 2\pi \Delta s n_e \int_{b_{\min}}^{b_{\max}} b \Delta E_{loss}(b)\, db$$

Coulomb Logarithm:

Ion energy loss rate: $$\frac{dE}{ds} = 2\pi n_e \int_{b_{\min}}^{b_{\max}} b \Delta E_{loss}(b)\, db = \frac{4\pi n_e Z^2 e^4}{m_e v_{ei}{}^2 (4\pi\varepsilon_0)^2} \overbrace{\ln\left(\frac{b_{\max}}{b_{\min}}\right)}^{L_c}$$

Since the energy loss slows down the ion in the frame of the electron's initial velocity, it is equivalent to a friction force in the direction of $\vec{v}_e - \vec{v}_i$ and can be defined as

$$\vec{F} = \frac{dE}{ds}\frac{\vec{v}_e - \vec{v}_i}{|\vec{v}_e - \vec{v}_i|} = \frac{4\pi n_e Z^2 e^4 L_c}{m_e (4\pi\varepsilon_0)^2}\frac{\vec{v}_e - \vec{v}_i}{|\vec{v}_e - \vec{v}_i|^3}$$

Friction force and its expression for isotropic electron velocity distribution

- The friction force due to electrons with velocity distribution $f(\vec{v}_e)$ is

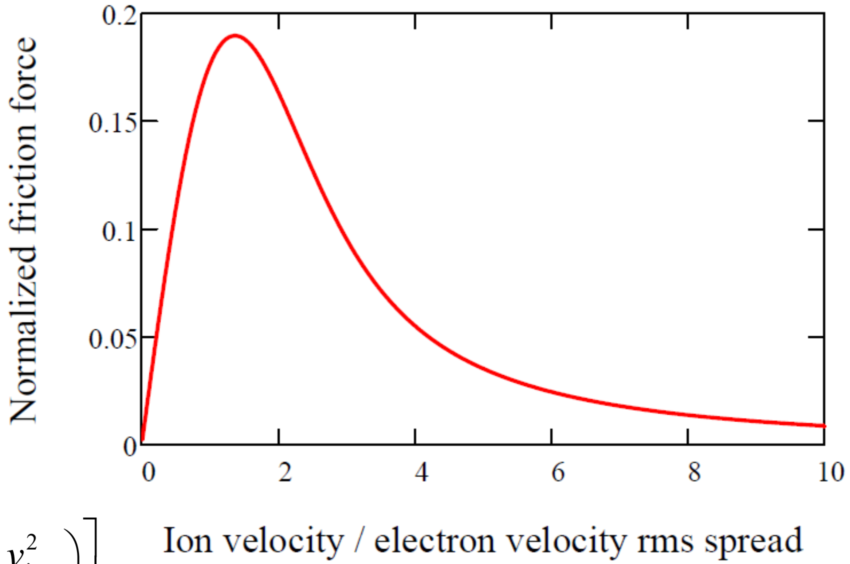
$$\vec{F} = \frac{4\pi n_e Z^2 e^4 L_c}{m_e (4\pi\varepsilon_0)^2} \int_{-\infty}^{\infty} \frac{\vec{v}_e - \vec{v}_i}{|\vec{v}_e - \vec{v}_i|^3} f(\vec{v}_e) d^3 v_e$$

Similar to Coulomb force but in velocity space.

For isotropic Gaussian electron velocity distribution,

$$f(\vec{v}_e) = \exp\left(-\frac{v_e^2}{2\sigma_{ve}^2}\right)$$

this integral can be carried out



$$\vec{F} = -\left(\frac{\vec{v}_i}{v_i}\right) \frac{n_e Z^2 e^4 L_c}{2 m_e \pi^{3/2} \varepsilon_0^{\,2} \sigma_{ve}^2} \cdot \frac{\sigma_{ve}^2}{v_i^2} \left[\frac{\sqrt{\pi}}{2} erf\left(\sqrt{\frac{v_i^2}{2\sigma_{ve}^2}}\right) - \sqrt{\frac{v_i^2}{2\sigma_{ve}^2}} \exp\left(-\frac{v_i^2}{2\sigma_{ve}^2}\right) \right]$$

Asymptotic behavior of the friction force for anisotropic velocity distribution of the electrons

For anisotropic Gaussian distribution of electrons' velocity, numerical integration is required. However, the asymptotic solution has been deriv

$$f(v_e) = \frac{e^{-\left[\frac{v_{e\parallel}^2}{2\Delta_{e\parallel}^2} + \frac{v_{e\perp}^2}{2\Delta_{e\perp}^2}\right]}}{(2\pi)^{3/2}\,\Delta_{e\perp}^2\,\Delta_{e\parallel}}$$

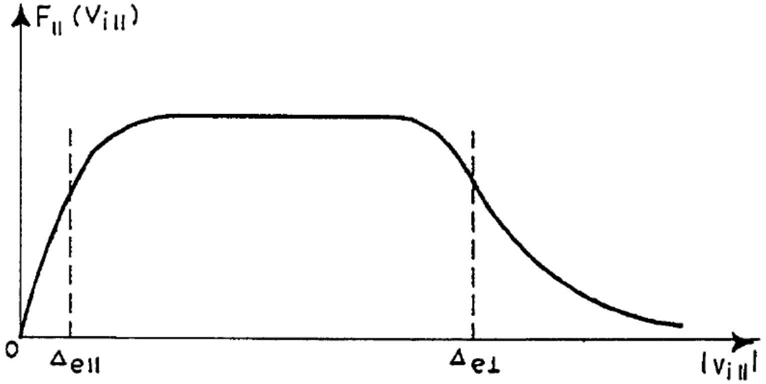


Fig. 5.6 Shape of the "non-magnetised" longitudinal cooling force

In the longitudinal direction ($v_{i\perp} = 0$) (Fig. 5.6)

$$F_\parallel(v_{i\parallel}) = -\frac{4\pi Z^2 \cdot e^4}{m_e} n_e \cdot L_c \begin{cases} \dfrac{1}{v_{i\parallel}^2} & ; \quad |v_{i\parallel}| >> \Delta_{e\perp} \\ \dfrac{1}{\Delta_{e\perp}^2} & ; \quad \Delta_{e\perp} >> |v_{i\parallel}| >> \Delta_{e\parallel} \\ \dfrac{v_{i\parallel}}{(2\pi)^{3/2}\,\Delta_{e\perp}^2\,\Delta_{e\parallel}} & ; \quad |v_{i\parallel}| << \Delta_{e\parallel} \end{cases}$$

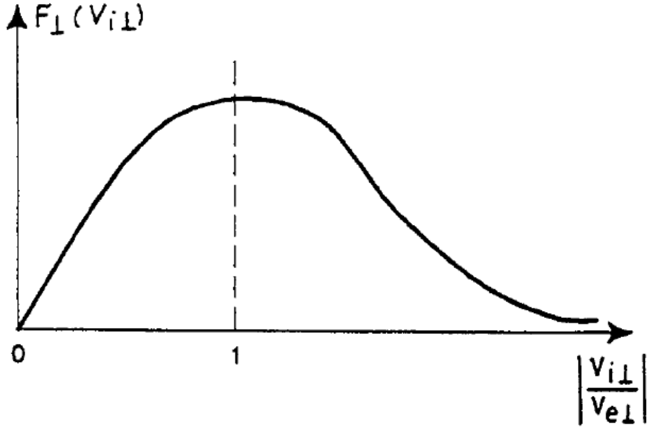


Fig. 5.7 Shape of the "non-magnetised" transverse cooling force

In the transverse direction ($v_{i\parallel} = 0$) (Fig. 5.7)

$$F(v_{i\perp}) = -\frac{4\pi Z^2 .\, e^4}{m_e} n_e \cdot L_c \begin{cases} \dfrac{1}{v_{i\perp}^2} & ; \quad |v_{i\perp}| >> \Delta_{e\perp} \\ \dfrac{\sqrt{\pi}}{8}\dfrac{v_{i\perp}}{\Delta_{e\perp}^3} & ; \quad |v_{i\perp}| << \Delta_{e\perp} \end{cases}$$

We observe that:

- the forces are not independent of the ion relative velocities
- for large ion velocities the forces scale as $1/(v_i^{\,2})$, suggesting that a beam with a relatively large emittance will have a large cooling time
- for small velocities the forces are proportional to v_i.

Binney's formula

If we assume that the electrons' velocity distribution is Gaussian, i.e.

$$f_e(\vec{v}) = \frac{1}{\left(\sqrt{2\pi}\right)^3 \sigma_\perp^2 \sigma_{v_z}} \exp\left(-\frac{v_\perp^2}{2\sigma_\perp^2} - \frac{v_z^2}{2\sigma_{//}^2}\right)$$

it is possible to reduce the 3-D integral in the electrons' velocity space

$$\vec{F} = \frac{4\pi n_e e^4 Z_i^2 L_c(\vec{v}_i)}{m_e (4\pi\varepsilon_0)^2} \int dv_x dv_y dv_z \, \frac{\vec{v} - \vec{v}_i}{\left|\vec{v} - \vec{v}_i\right|^3} f_e(\vec{v})$$

to 1-D integral and obtain the Binney's formula

$$\vec{F}_\perp = -\frac{2\sqrt{2\pi} n_e e^4 Z_i^2 L_c}{m_e (4\pi\varepsilon_0)^2} \frac{\vec{v}_{i,\perp}}{\sigma_\perp^3} B_\perp \qquad B_\perp \equiv \int_0^\infty \frac{\exp\left(-\frac{v_{i,\perp}^2}{2\sigma_\perp^2}\frac{1}{1+\tilde{q}} - \frac{v_{i,//}^2}{2\sigma_\perp^2}\frac{1}{\tilde{q} + \sigma_{//}^2/\sigma_\perp^2}\right)}{(1+\tilde{q})^2 \sqrt{\tilde{q} + \frac{\sigma_{//}^2}{\sigma_\perp^2}}} d\tilde{q}$$

$$\vec{F}_{//} = -\frac{2\sqrt{2\pi} n_e e^4 Z_i^2 L_c}{m_e (4\pi\varepsilon_0)^2} \frac{\vec{v}_{i,//}}{\sigma_\perp^3} B_{//} \qquad B_{//} \equiv \int_0^\infty \frac{\exp\left(-\frac{v_{i,\perp}^2}{2\sigma_\perp^2}\frac{1}{1+\tilde{q}} - \frac{v_{i,//}^2}{2\sigma_\perp^2}\frac{1}{\sigma_{//}^2/\sigma_\perp^2 + \tilde{q}}\right)}{(1+\tilde{q}) \sqrt{\left(\frac{\sigma_{//}^2}{\sigma_\perp^2} + \tilde{q}\right)^3}} d\tilde{q}$$

Cooling rate for non-magnetized electron cooling

$$\frac{1}{\tau_{v_i}} \equiv -\frac{1}{v_i}\frac{dv_i}{dt} = -\frac{1}{p_i}\frac{dE_i}{ds} = -\frac{F(v_i)}{p_i}$$

If τ_{v_i} is independent of t

$$v_i(t) = v_i(0)\exp\left(-\frac{t}{\tau_{v_i}}\right)$$

$$\vec{F} = \frac{4\pi n_e Z^2 e^4 L_c}{m_e (4\pi\varepsilon_0)^2}\int_{-\infty}^{\infty}\frac{\vec{v}_e - \vec{v}_i}{|\vec{v}_e - \vec{v}_i|^3} f(\vec{v}_e)\, d^3 v_e$$

$$f(\vec{v}_e) = \delta(\vec{v}_e)$$

In lab frame: $\tau_{v_i,lab} = \gamma_0 \tau_{v_i} = -\gamma_0 \dfrac{m_i v_i}{F(v_i)}$

For cold e- beam: $\vec{F} = -\dfrac{4\pi Z^2 e^4 n_e L_c}{m_e (4\pi\varepsilon_0)^2}\dfrac{\vec{v}_i}{|v_i|^3}$

$$F(v_i) = -\frac{4\pi Z^2 e^4 n_e L_c}{m_e (4\pi\varepsilon_0)^2}\frac{1}{|v_i|^2}$$

$$\tau_{v_i,lab} = -\gamma_0 \frac{m_i v_i}{F(v_i)} = \gamma_0{}^2 \frac{m_i m_e (4\pi\varepsilon_0)^2 |v_i|^3}{4\pi Z^2 e^4 n_{e,lab} L_c} = \gamma_0{}^2 \frac{m_i m_e (4\pi\varepsilon_0)^2 |v_i|^3 ecS}{4\pi Z^2 e^4 L_c}\frac{1}{S n_{e,lab} ec}$$

$$= \gamma_0{}^5 \frac{m_i m_e (4\pi\varepsilon_0)^2 c^3 ecS}{4\pi Z^2 e^4 L_c}\left(\frac{m_i{}^3 |v_i|^3}{m_i^3 c^3 \gamma_0{}^3}\right)\frac{1}{I_b}$$

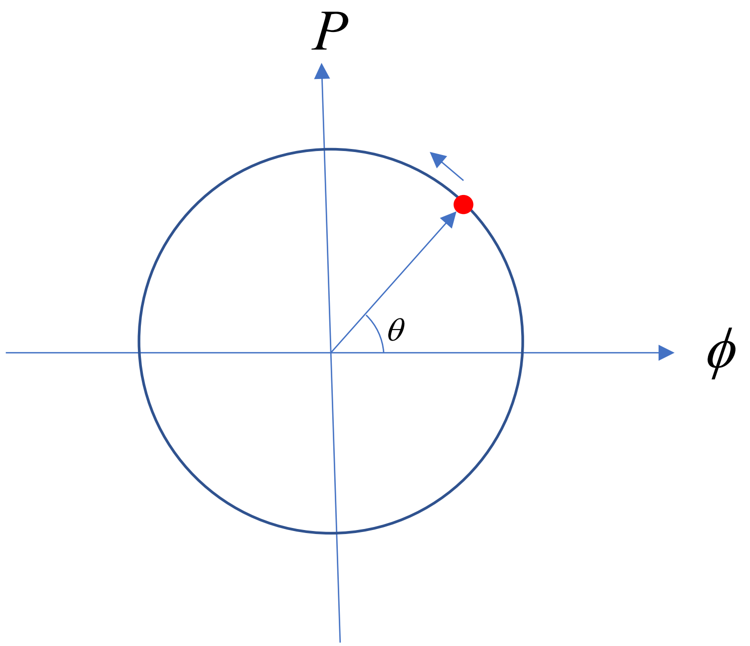
$$= \gamma_0{}^5 \frac{m_i m_e (4\pi\varepsilon_0)^2 c^4 Se}{4\pi Z^2 e^4 L_c}\theta_i^3 \frac{1}{I_b} = \gamma_0{}^5 \frac{A}{Z^2}\frac{Se}{4\pi r_e r_p L_c}\theta_i^3\frac{1}{I_b}$$

Cooling time increases dramatically with energy. Therefore, electron cooling is not efficient to cool high energy beam.

Angular spread of ions

S: area of beam cross section

Averaging longitudinal cooling over synchrotron oscillation (linear cooling force)



Action-angle variable: $P \equiv -h\frac{|\eta|}{\nu_s}\frac{\Delta p}{p} = \sqrt{2I}\sin\theta \qquad \phi \equiv \omega_{rf}\tau = \sqrt{2I}\cos\theta$

Reduction of action due to cooling: $\Delta I_c = \frac{1}{2}\Delta\left(P^2+\phi^2\right) = P\Delta P_c \qquad \Delta P_c = \frac{h|\eta|}{\nu_s\gamma}\Delta\delta\gamma_c$

Assuming cooling force is linear, $\Delta\delta\gamma_c = -\zeta_0 T_{rev}\delta\gamma \qquad \Delta P_c = -\zeta_0 T_{rev} P$

the action reduction becomes $\Delta I_c = -\zeta_0 T_{rev} P^2 = -2I\zeta_0 T_{rev}\sin^2\theta$

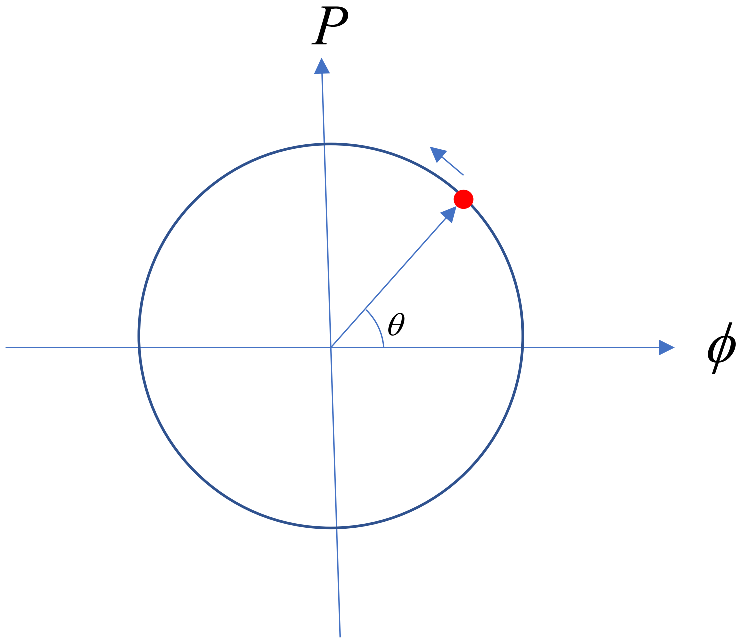
The average cooling rate is given by

$$\zeta(I) = -\frac{1}{I}\left\langle \frac{\Delta I_c}{T_{rev}} \right\rangle_{T_s} = 2\zeta_0\bar{\zeta}(I)$$

$$\bar{\zeta}(I) = \frac{1}{2\pi}\int_0^{2\pi}\sin^2\theta d\theta = \frac{1}{2}$$

For the linear cooling force, synchrotron motion reduces the averaged cooling rate by a factor of 2.

Averaging longitudinal cooling over synchrotron oscillation (linear cooling force)



Action-angle variable: $P \equiv -h\dfrac{|\eta|}{\nu_s}\dfrac{\Delta p}{p} = \sqrt{2I}\sin\theta \qquad \phi \equiv \omega_{rf}\tau = \sqrt{2I}\cos\theta$

Reduction of action due to cooling: $\Delta I_c = \dfrac{1}{2}\Delta\left(P^2+\phi^2\right) = P\Delta P_c \qquad \Delta P_c = \dfrac{h|\eta|}{\nu_s\gamma}\Delta\delta\gamma_c$

Assuming cooling force is linear, $\Delta\delta\gamma_c = -\zeta_0 T_{rev}\delta\gamma \qquad \Delta P_c = -\zeta_0 T_{rev} P$

the action reduction becomes $\Delta I_c = -\zeta_0 T_{rev} P^2 = -2I\zeta_0 T_{rev}\sin^2\theta$

The average cooling rate is given by

$$\zeta(I) = -\frac{1}{I}\left\langle\frac{\Delta I_c}{T_{rev}}\right\rangle_{T_s} = \zeta_0\bar{\zeta}(I)$$

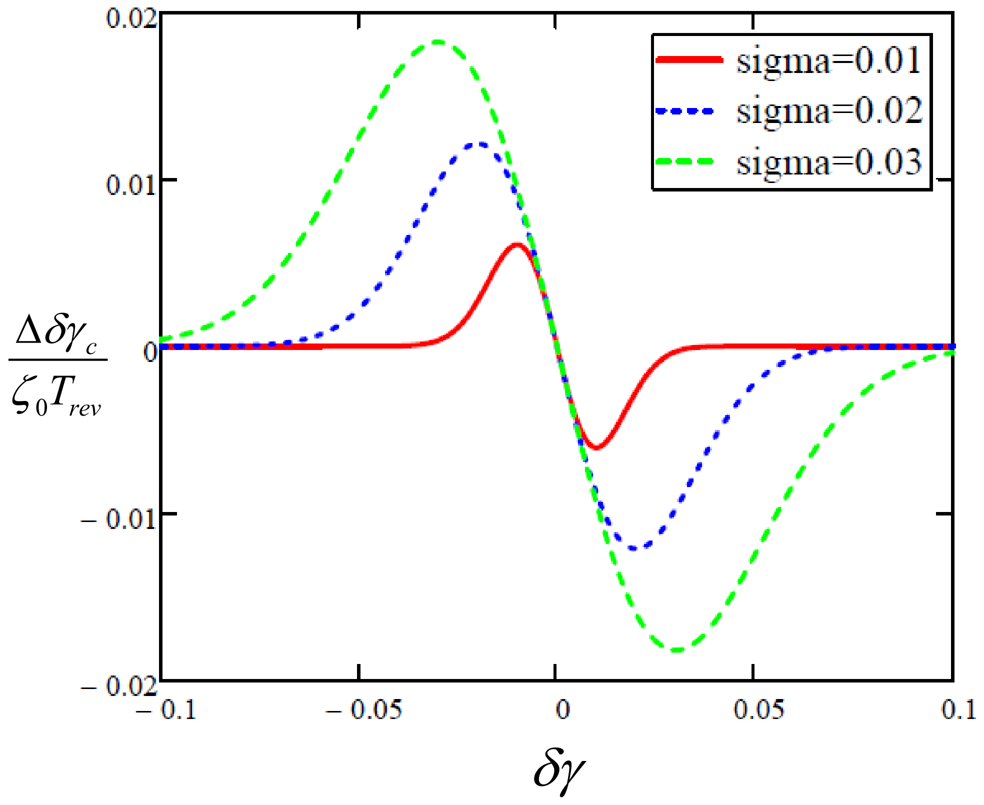
$$\bar{\zeta}(I) = \frac{1}{2\pi}\int_0^{2\pi}\sin^2\theta\, d\theta = \frac{1}{2}$$

For the linear cooling force, synchrotron motion reduces the averaged cooling rate by a factor of 2.

Averaging longitudinal cooling over synchrotron oscillation (Nonlinear cooling force: Gaussian model)

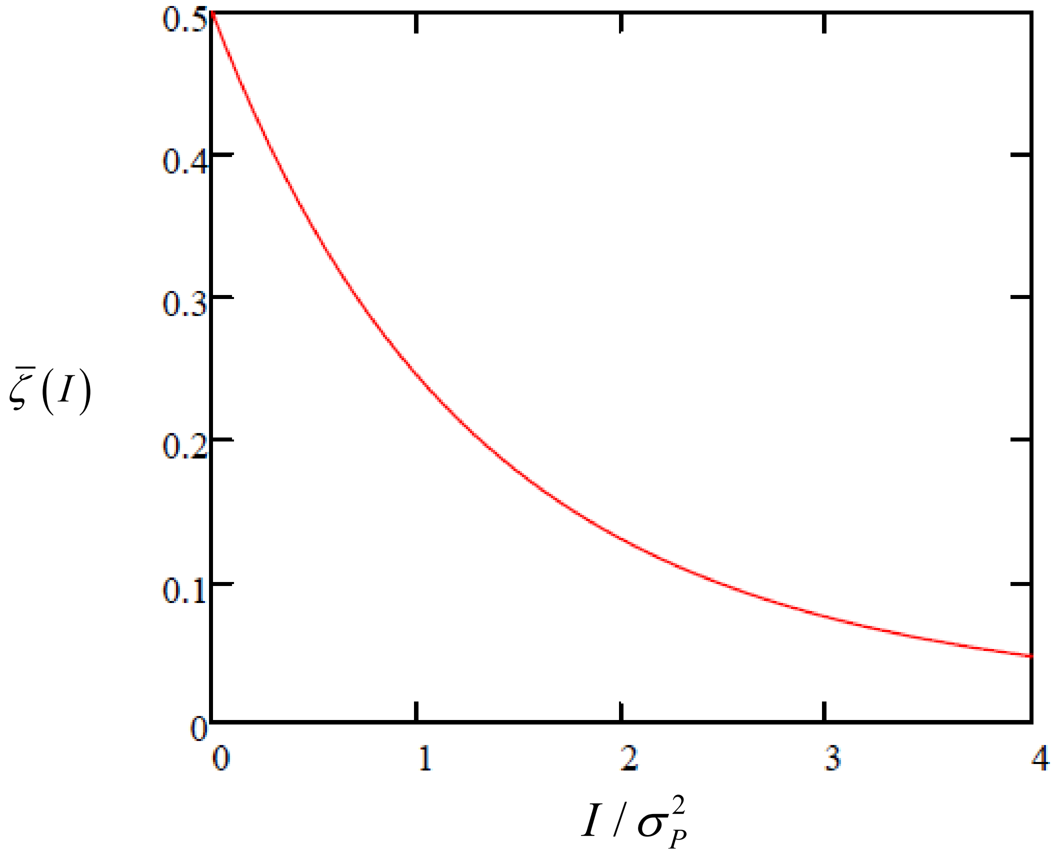
Non-linear cooling force model

$$\Delta\delta\gamma_c = -\zeta_0 T_{rev} \delta\gamma \exp\left(-\frac{\delta\gamma^2}{2\sigma_l^2}\right)$$



Reduction of cooling rate due to synchrotron oscillation

$$\bar{\zeta}(I) = \frac{1}{2}\exp\left(-\frac{I}{2\sigma_P^2}\right)\left[I_0\left(\frac{I}{2\sigma_P^2}\right) - I_1\left(\frac{I}{2\sigma_P^2}\right)\right] \qquad \sigma_P \equiv h\frac{|\eta|}{\nu_s}\frac{\sigma_l}{\gamma}$$



Characteristics of the Electron Cooling Force

$$\boxed{\vec{F}(\vec{v_i}) = -\frac{4\pi Q^2 e^4 n_e}{(4\pi\epsilon_0)^2 m_e} \int L_C(\vec{v}_{rel}) f(\vec{v_e}) \frac{\vec{v}_{rel}}{v_{rel}^3} d^3\vec{v_e}}$$

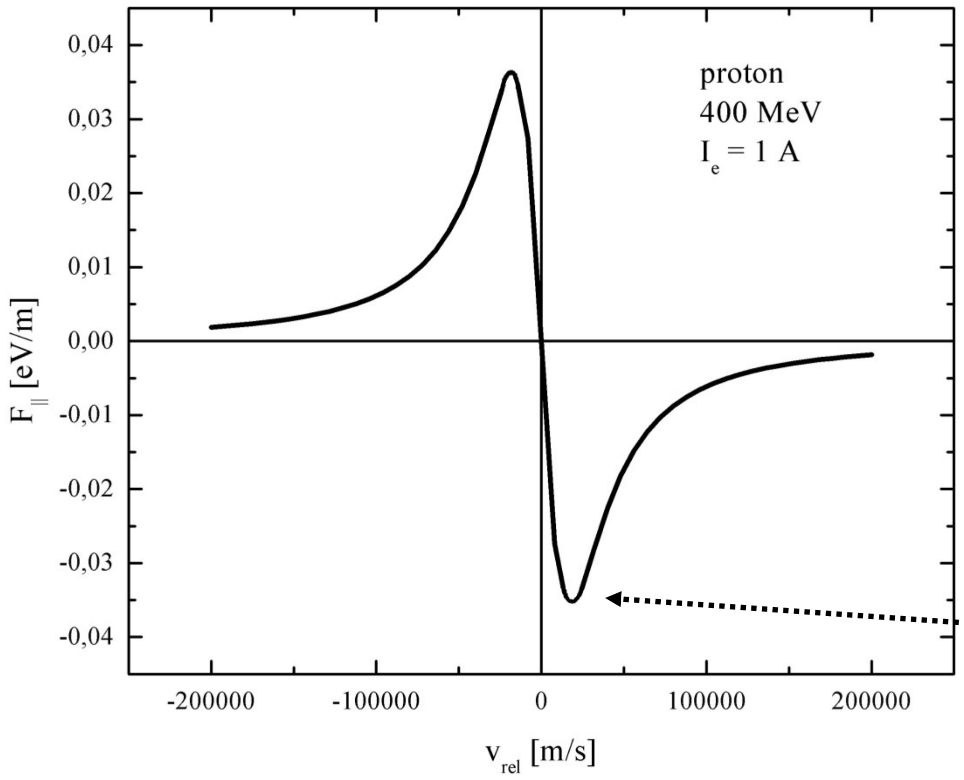
$$\vec{v}_{rel} = \vec{v_i} - \vec{v_e}$$

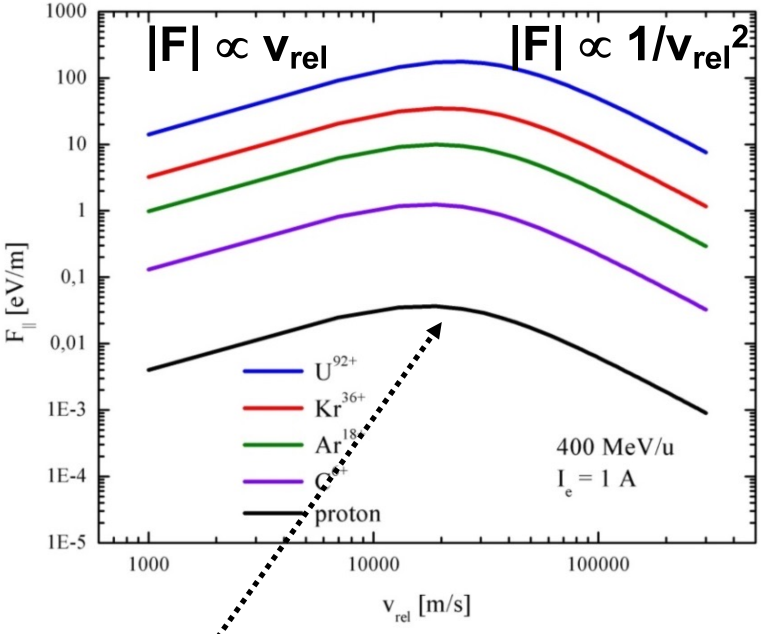
cooling force F

for small relative velocity: $\propto v_{rel}$

for large relative velocity: $\propto v_{rel}^{-2}$

increases with charge: $\propto Q^2$





maximum of cooling force at effective electron temperature

Electron Cooling Time

first estimate:
(Budker 1967)

$$\tau = \frac{3}{8\sqrt{2\pi} n_e Q^2 r_e r_i c L_C} \left(\frac{k_B T_e}{m_e c^2} + \frac{k_B T_i}{m_i c^2}\right)^{3/2}$$

for large relative velocities

cooling time $\tau_z \propto \frac{A}{Q^2} \frac{1}{n_e \eta} \beta^3 \gamma^5 \theta_z^3$

$$\begin{cases} \theta_{x,y} = \dfrac{v_{x,y}}{\gamma\beta c} \\ \theta_{\parallel} = \dfrac{v_{\parallel}}{\gamma\beta c} \end{cases}$$

cooling rate (τ^{-1}):

- slow for hot beams $\propto \theta^3$
- decreases with energy $\propto \gamma^{-2}$ ($\beta \cdot \gamma \cdot \theta$ is conserved)
- linear dependence on electron beam intensity n_e and cooler length $\eta = L_{ec}/C$
- favorable for highly charged ions Q^2/A
- independent of hadron beam intensity

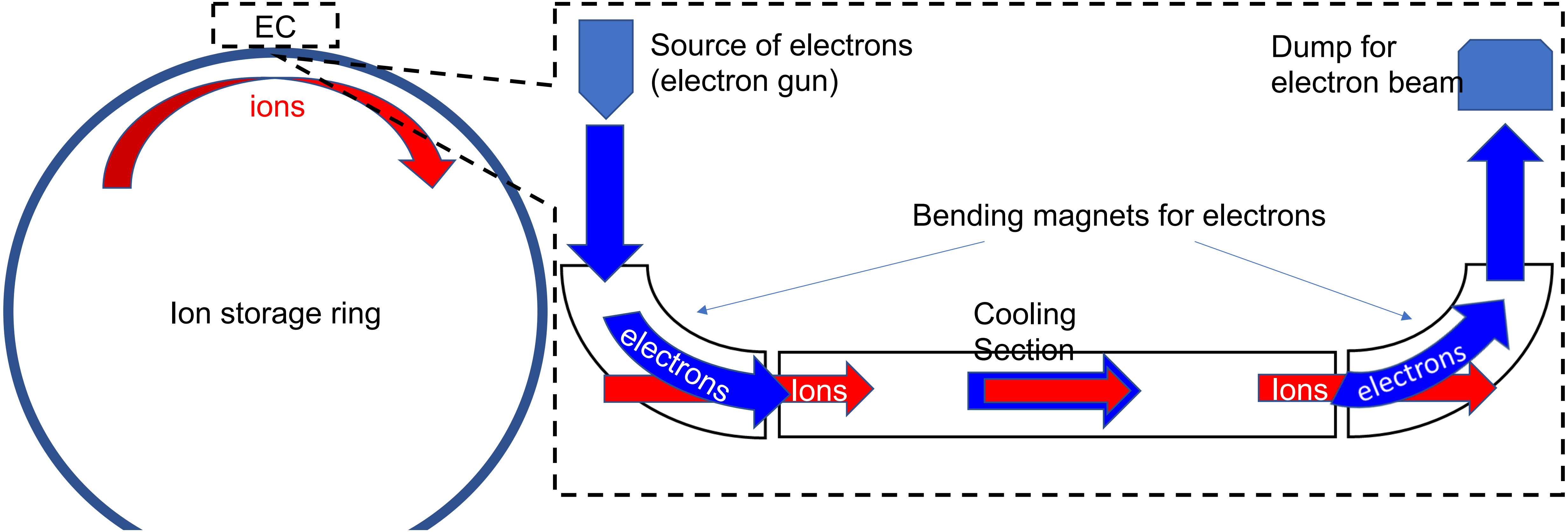
for small relative velocities

cooling rate is constant and maximum at small relative velocity

$F \propto v_{rel} \Rightarrow \tau = \Delta t = p_{rel}/F = \text{constant}$

A couple of technical details – fresh electron bunch

- We discard the electron bunch after each pass through the cooling section, so that on each turn the ions interact with a fresh cold electron bunch.



- Electrons are charged particles so we can inject them into the ion storage ring and extract them from the ring with bending magnets. Ions are charged particles too, but because they are much heavier than electrons the electron bending magnets do not affect them

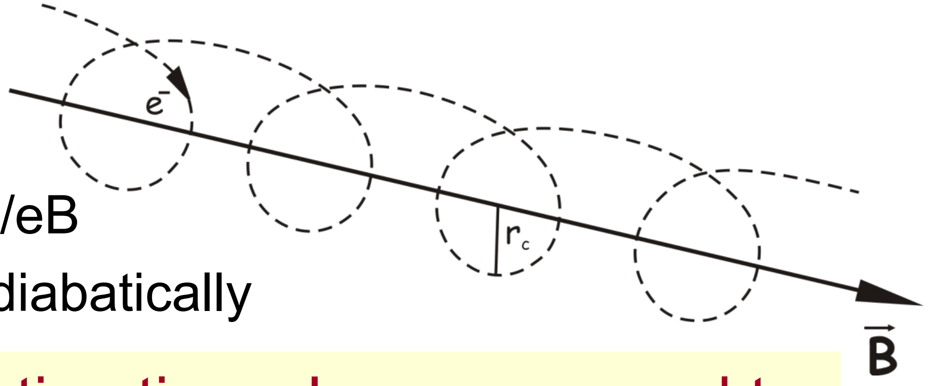
Electron Motion in Longitudinal Magnetic Field

single particle cyclotron motion

cyclotron frequency $\omega_c = eB/\gamma m_e$

cyclotron radius $r_c = v_\perp/\omega_c = (k_B T_\perp m_e)^{1/2}\ \gamma/eB$

electrons follow the magnetic field line adiabatically



important consequence: for interaction times long compared to the cyclotron period the ion does not sense the transverse electron temperature $\Rightarrow$ **magnetized cooling ($T_{eff} \approx T_\parallel \ll T_\perp$)**

electron beam space charge:

transverse electric field + B-field $\Rightarrow$ azimuthal drift

$$v_{azi} = r\omega_{azi} = r\frac{2\pi r_e n_e c^2}{\gamma \omega_c}$$

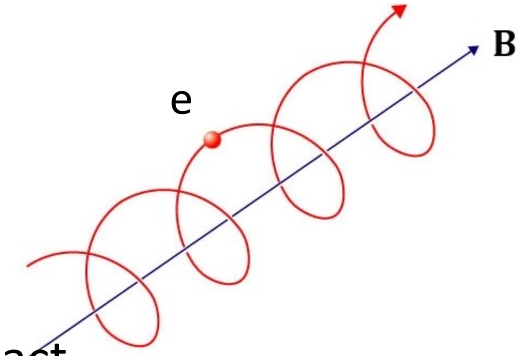
$\Rightarrow$ electron and ion beam should be well centered

Favorable for optimum cooling (small transverse relative velocity):

- high parallelism of magnetic field lines $\Delta B_\perp/B_0$
- large beta function (small divergence) in cooling section

Magnetized electron cooling

- In the presence of a longitudinal magnetic field, the electrons will rotate around their axis with the Cyclotron frequency (or Larmor frequency).



$$\omega_c = eB/\gamma m_e$$

- If the magnetic field is very strong such that the impact parameter b is much larger than the Cyclotron radius

$$r_c = v_\perp/\omega_c = (k_B T_\perp m_e)^{1/2}\,\gamma/eB$$

and the collision time is much longer than Cyclotron period, the transverse degree of freedom does not take part in the energy exchange between electrons and the ions.

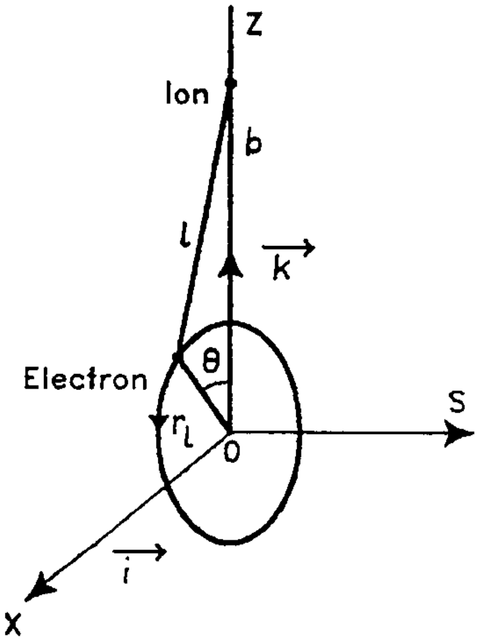
$$\vec{f} = \mathrm{Cst}\,\vec{\ell}\,/\,\ell^3$$

$$\vec{\ell} = -r_\ell \sin\theta\,\vec{i} + (b - r_\ell\cos\theta)\vec{k}$$

$$\ell^2 = b^2 + r_\ell^2 - 2\cdot r_\ell\, b\cos\theta$$

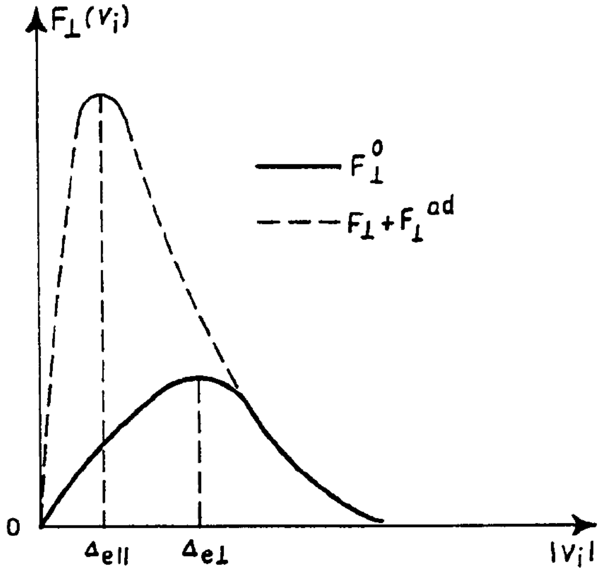
$$d\vec{\ell} = \left[-r_\ell\cos\theta\,\vec{i} + r_\ell\sin\theta\,\vec{k}\right]d\theta.$$

$$W = \int_{\theta=0}^{\theta=2\pi} \vec{f}\cdot d\vec{\ell} = 0.$$



Magnetized electron cooling force

- In the presence of such strong magnetic field, the transverse motion is 'frozen' and the electron cooling efficiency is determined by the electrons' longitudinal temperature which is a few orders of magnitude lower than the transverse one, leading to enhanced cooling.



- In the limits of a very strong magnetic field, where transverse motion of electrons is completely suppressed, the magnetized friction force was derived as

$$\boxed{\vec{F} = \frac{2\pi Z^2 e^4 n_e}{m} \frac{\partial}{\partial \vec{V}} \int \left[\frac{V_\perp^2}{U^3} L_M + \frac{2}{U} \right] f(v_e) dv_e \,,}$$

$$L_M = \ln\left(\frac{\rho_{\max}}{\rho_L}\right)$$

$\vec{V} = (V_\perp, V_\parallel)$ is the ion velocity $\qquad U = \sqrt{V_\perp^2 + (V_\parallel - v_e)^2}$ $\qquad \rho_L = \frac{cm\Delta_\perp}{eB}$

Empirical formula for magnetized cooling force

- In case of finite values of the magnetic field, an empirical expression for the magnetized cooling force can be used:

Maximal impact parameter

$$F = C\frac{4\pi n_e Z^2 e^4}{m_e}\frac{V}{(V^2+\Delta_{eff}^2)^{3/2}}L_M \qquad L_M=\ln(\rho_{max}/\rho_L+1)$$

Fitting parameter

Effective longitudinal velocity spread of the electrons

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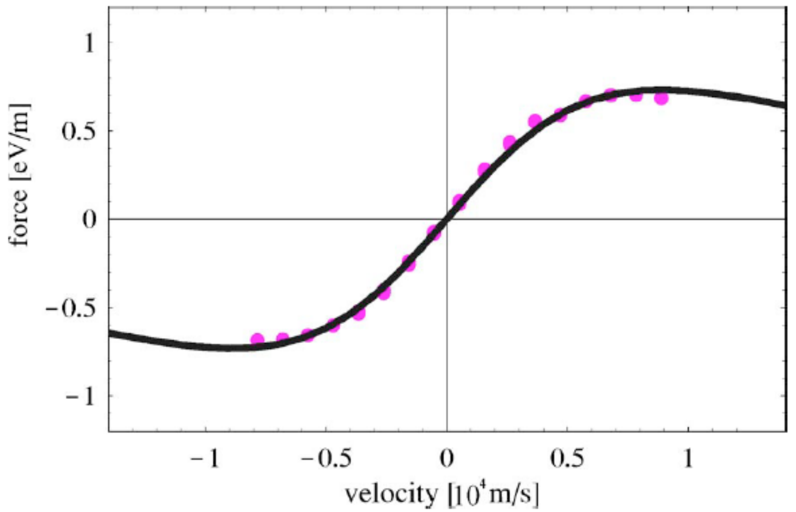


FIG. 9. (Color online) Measured force (dots) in [eV/m] vs velocity [$\times 10^4$ m/s] for an electron current of 300 mA (B=0.12 T) and the fitting curve according to the Eq. (4) with an experimentally determined effective velocity.

- Comparison with measured friction force for an ion beam

$$F = C\frac{4\pi Z^2 e^4 n_e}{m\sqrt{2\pi}\Delta_\perp^2\Delta_\parallel}\int_{-\infty}^{\infty}\frac{v_\parallel L_M(v_\parallel,v_\perp,\Delta_{eff})}{(v_\perp^2+v_\parallel^2+\Delta_{eff}^2)^{3/2}}$$

$$\times\exp\left(-\frac{v_\perp^2}{2\Delta_\perp^2}-\frac{(v_\parallel-v_0)^2}{2\Delta_\parallel^2}\right)v_\perp dv_\perp dv_\parallel,$$

Models of the Electron Cooling Force

- binary collision model

description of the cooling process by successive collisions of two particles and integration over all interactions
analytic expressions become very involved, various regimes
(multitude of Coulomb logarithms)

- dielectric model

interaction of the ion with a continuous electron plasma
(scattering off of plasma waves)
fails for small relative velocities and high ion charge

- an empiric formula (Parkhomchuk) derived from experiments:

$$\boxed{\vec{F} = -4\frac{n_e}{m_e}\frac{(Qe^2)^2}{(4\pi\epsilon_0)^2}\ln\left(\frac{b_{max}+b_{min}+r_c}{b_{min}+r_c}\right)\frac{\vec{v}_{ion}}{(v_{ion}^2+v_{eff}^2)^{3/2}}}$$

$$b_{min} = \frac{Qe^2/4\pi\epsilon_0}{m_e v_{ion}^2}; \quad b_{max} = \frac{v_{ion}}{min(\omega_{pe}, 1/T_{cool})}, \quad v_{eff}^2 = v_{e,\parallel}^2 + v_{e,\perp}^2$$

Electron Beam Properties

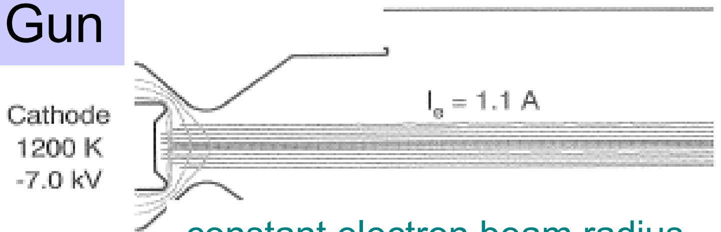
electron beam temperature

transverse $k_B T_\perp = k_B T_{cat}$, with transverse expansion ($\propto B_c/B_{gun}$)

longitudinal $k_B T_{//} = (k_B T_{cat})^2/4E_0 \ll k_B T_\perp$ lower limit : $k_B T_\parallel \geq 2e \frac{n_e^{1/3}}{4\pi\epsilon_0}$

typical values: $k_B T_\perp \approx 0.1$ eV (1100 K), $k_B T_{//} \approx 0.1$ - 1 meV

Gun

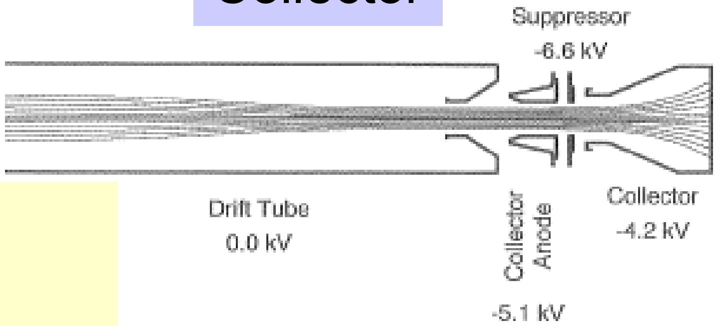


constant electron beam radius

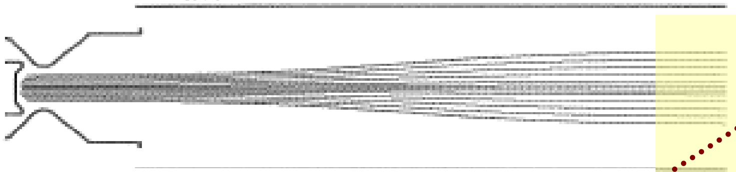
electron beam confined by longitudinal magnetic field (from gun to collector)

Cooling Section

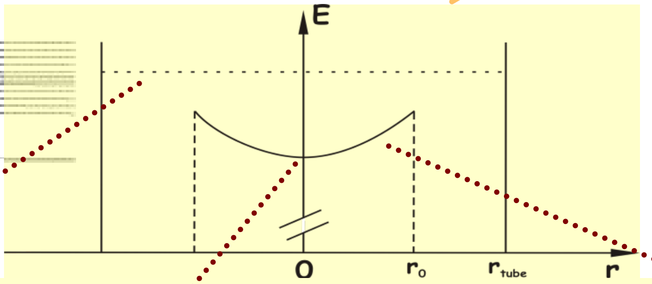
Collector



$I_e = PU_{an}^{3/2}$



transversely expanded electron beam



radial variation of electron energy due to space charge:

$$E(r) = eU_{cat} - \underline{n_e}\pi r_0^2 r_e m_e c^2 [1 + 2\ln(r_{tube}/r_0)] + \underline{n_e}\pi r_e m_e c^2 r^2$$

Imperfections and Limiting Effects in Electron Cooling

technical issues:

ripple of accelerating voltage

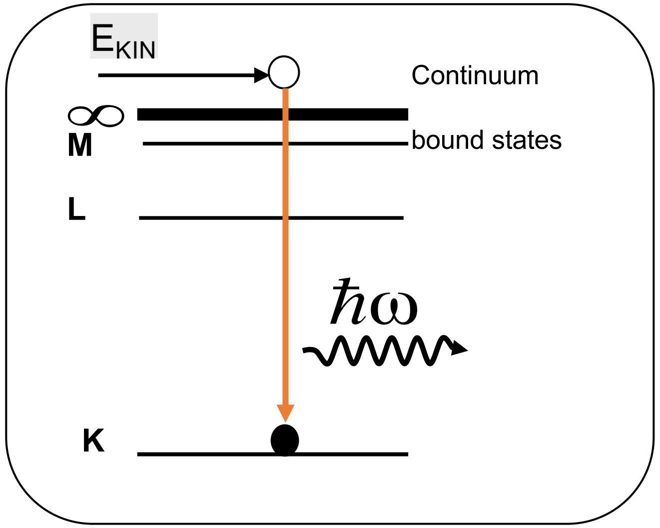
magnetic field imperfections

beam misalignment

space charge of electron beam and compensation

physical limitation:

Radiative Electron Capture (REC)



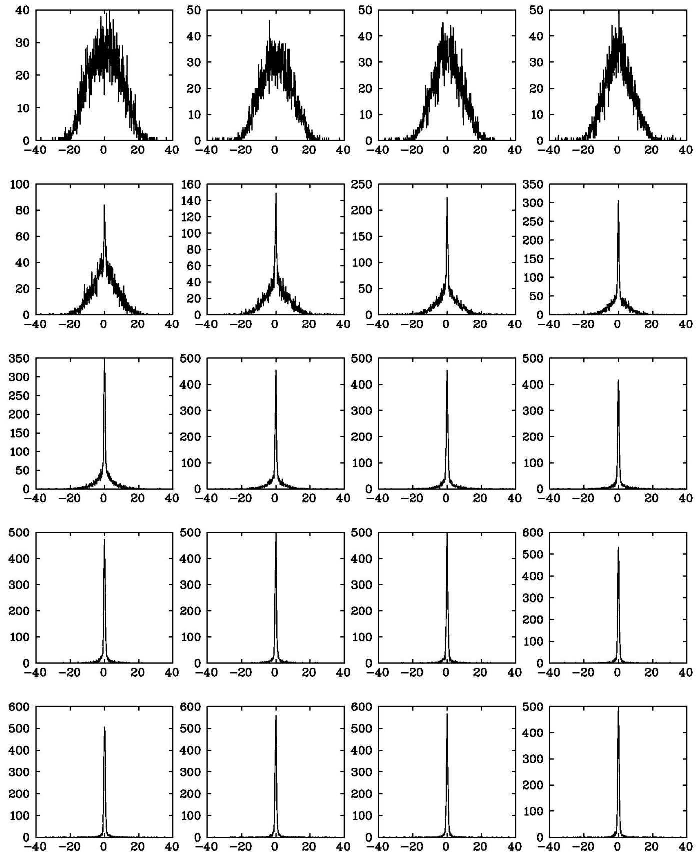
losses by recombination (REC)

loss rate $\tau^{-1} = \gamma^{-2}\alpha_{REC} n_e \eta$

$$\alpha_{REC} = \frac{1.92\times 10^{-13} Q^2}{\sqrt{k_B T}} \left(\ln \frac{5.66 Q}{\sqrt{k_B T}} + 0.196 \left(\frac{k_B T}{Q^2}\right)^{1/3} \right) [cm^3 s^{-1}]$$

Examples of Electron Cooling

fast transverse cooling at TSR, Heidelberg



profile every 0.1 s. [mm]

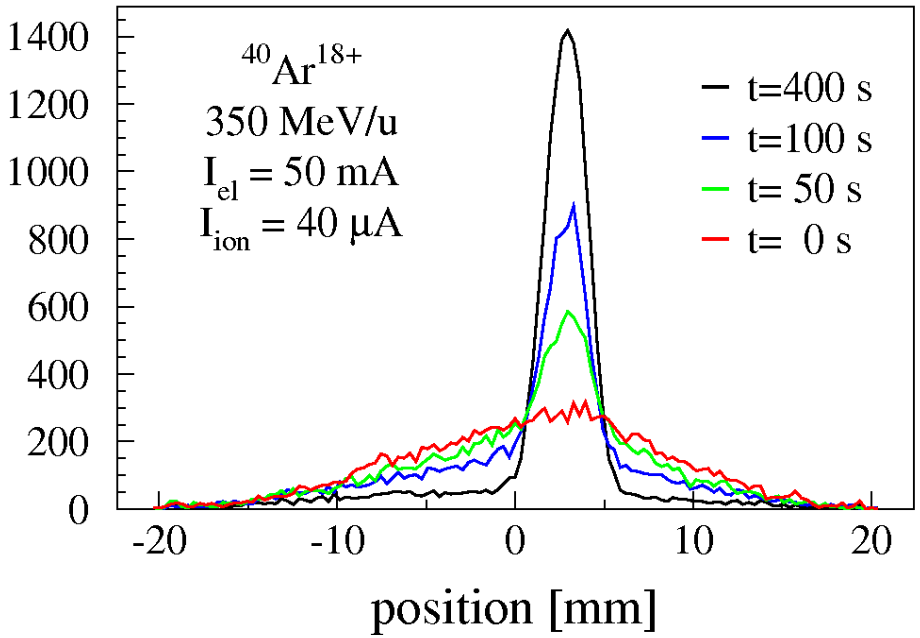
cooling of 6.1 MeV/u C^{6+} ions

0.24 A, 3.4 keV electron beam

$n_e = 1.56 \times 10^7\ cm^{-3}$

measured with residual gas ionization beam profile monitor

transverse cooling at ESR, Darmstadt



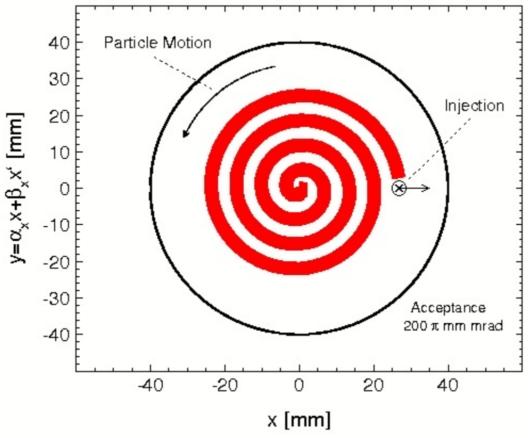
cooling of 350 MeV/u Ar^{18+} ions

0.05 A, 192 keV electron beam

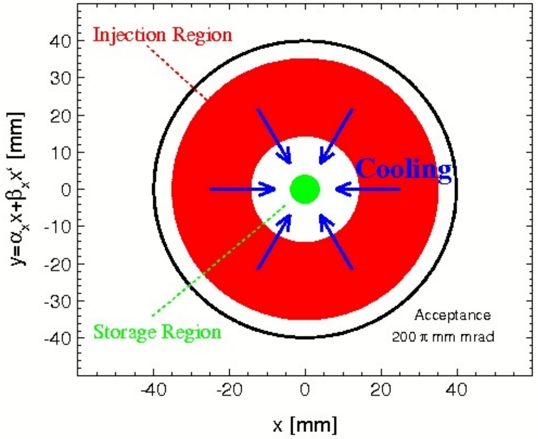
$n_e = 0.8 \times 10^6\ cm^{-3}$

Accumulation of Heavy Ions by Electron Cooling

standard multiturn injection

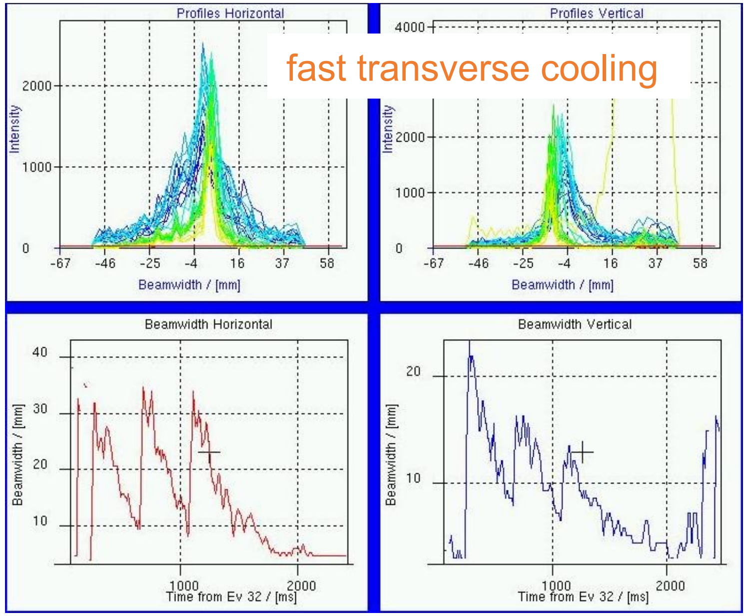


fast accumulation by repeated multiturn injection with electron cooling



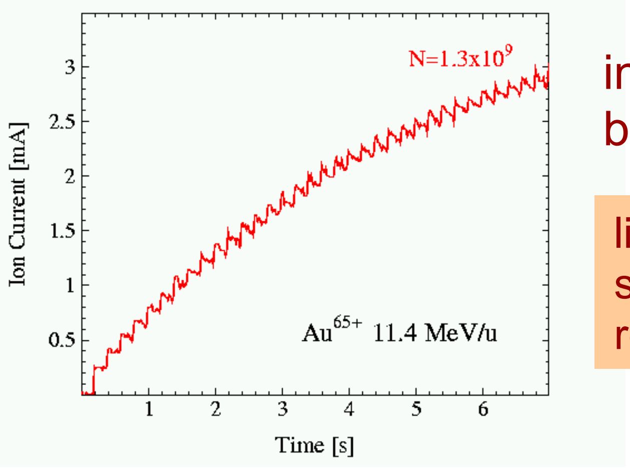
horizontal

vertical



profile

beam size

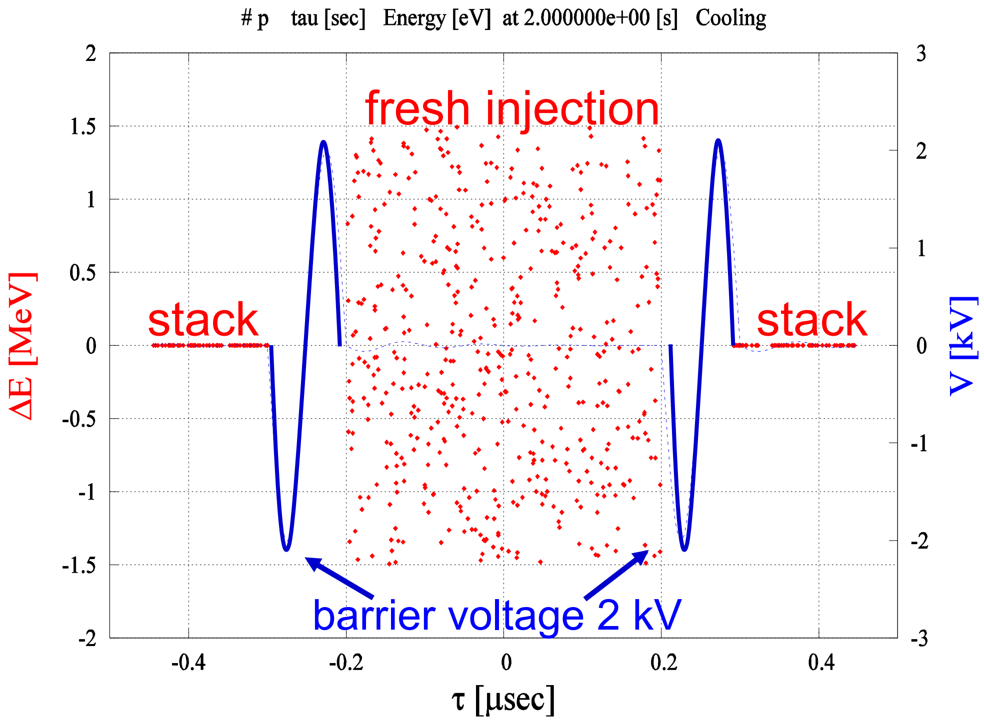


intensity increase in 5 s by a factor of ≈ 10

limitations:
space charge tune shift
recombination (REC)

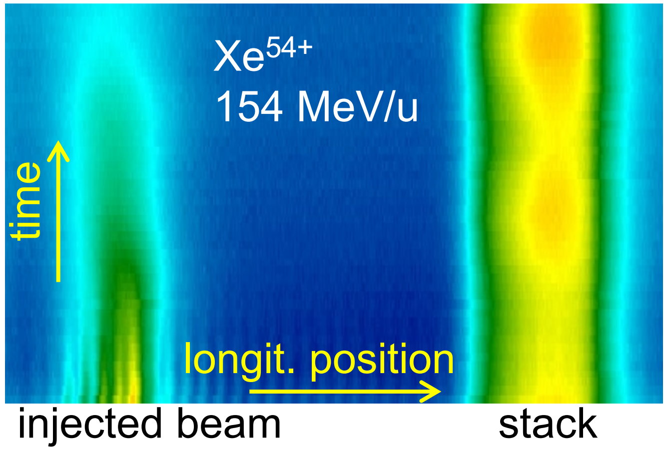
Accumulation of Secondary Particles

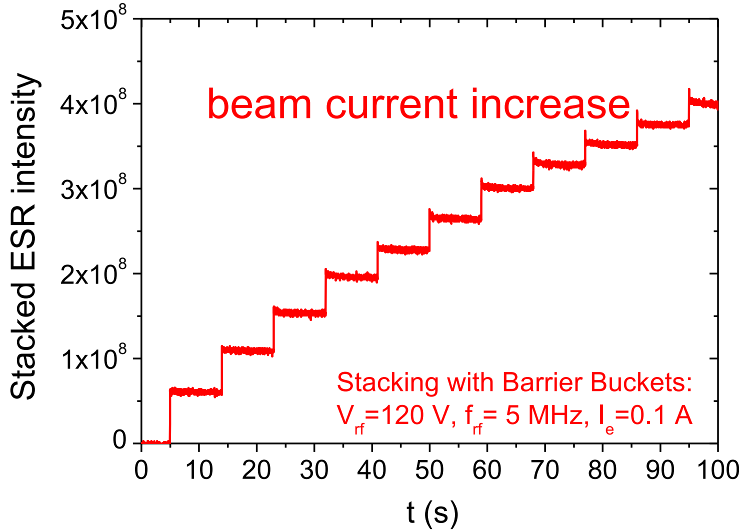
basic idea: confine stored beam to a fraction of the circumference, inject into gap and apply cooling to merge the two beam components
⇒ fast increase of intensity (for secondary beams)



simulation of longitudinal stacking
with barrier buckets and electron cooling

experimental verification at ESR

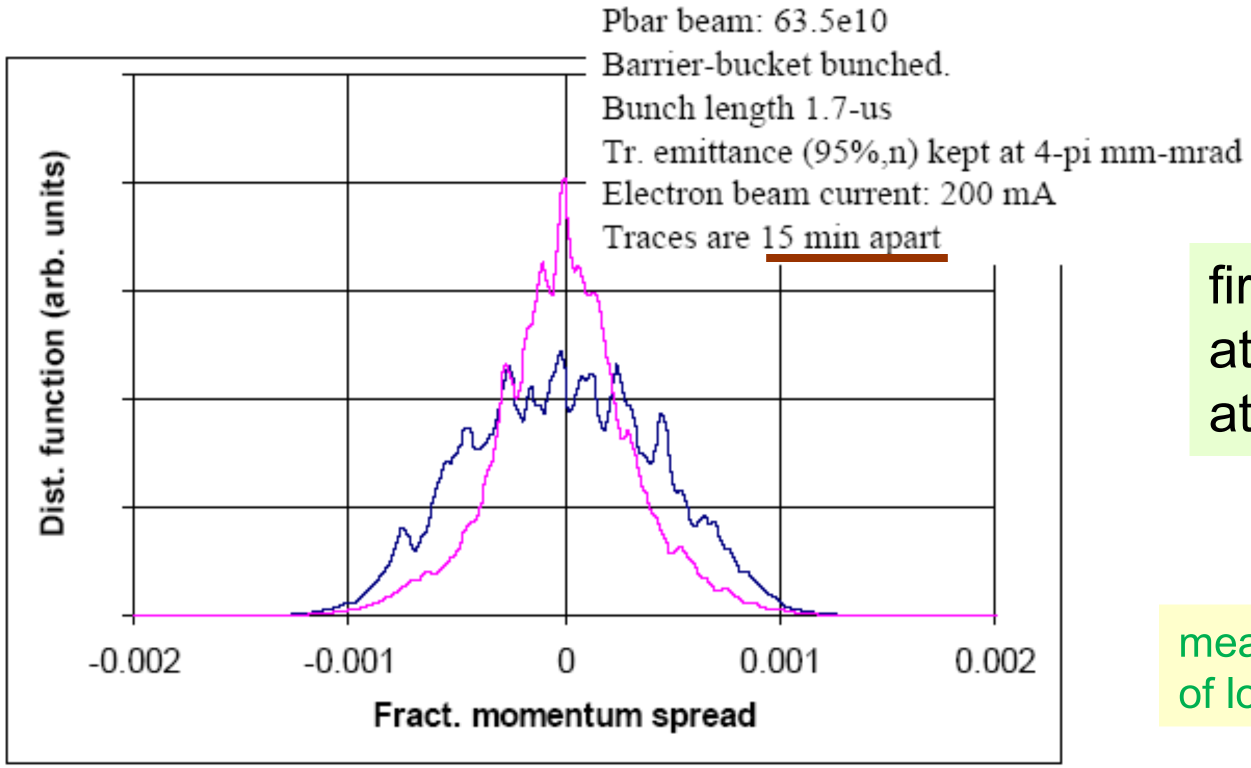




Examples of Electron Cooling

high energy electron cooling of 8 GeV antiprotons
longitudinal cooling with 0.2 A, 4.4 MeV electron beam

First e-cooling demonstration - 07/15/05

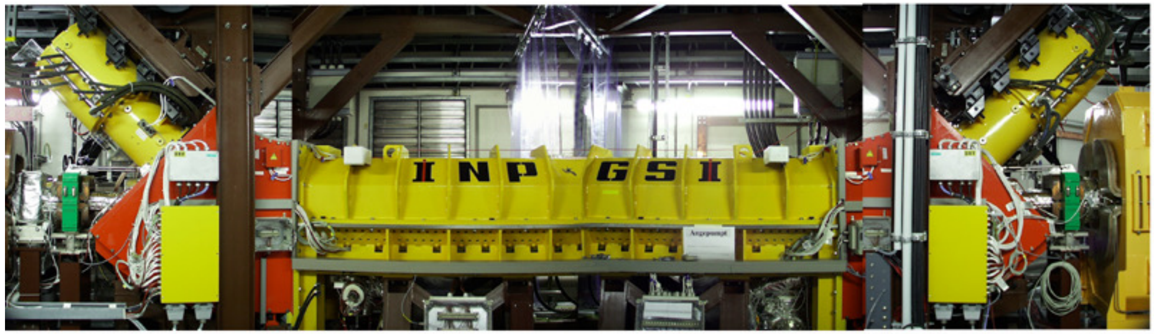


first electron cooling
at relativistic energy
at Recycler, FNAL

measured by detection
of longitudinal Schottky noise

Electron Cooling Systems

Low Energy: 35 keV SIS/GSI

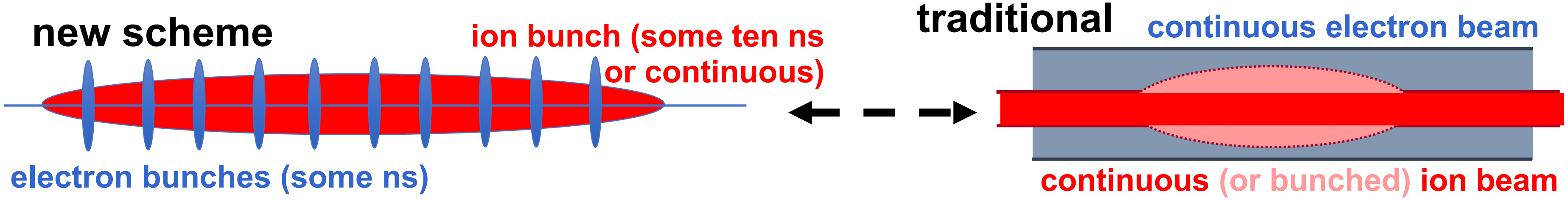


Medium Energy:
300 keV
ESR/GSI

High Energy:
4.3 MeV Recycler/FNAL

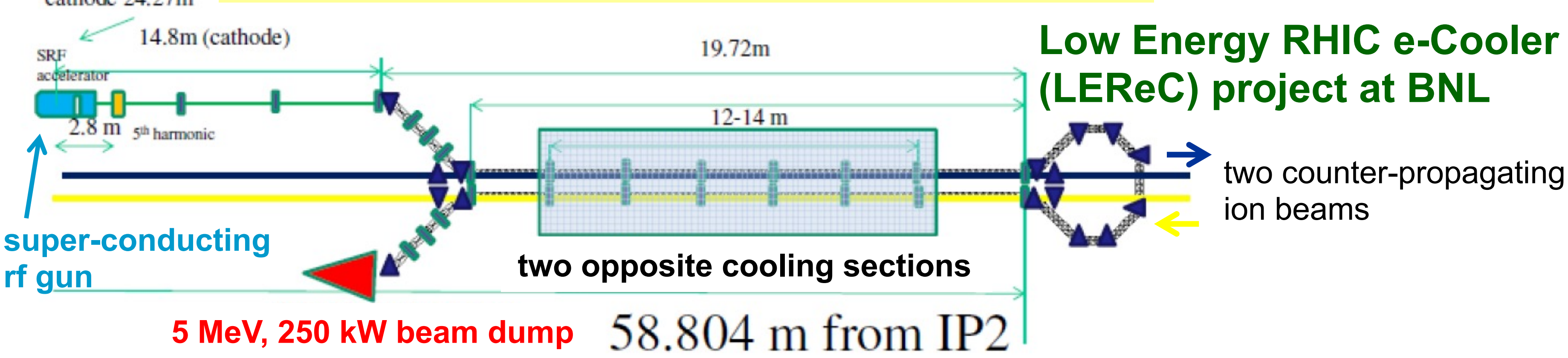
Bunched Beam Electron Cooling

Electron cooling with electrostatic acceleration is limited in energy (5-10 MeV). A bunched electron beam offers the extension of the electron cooling method to higher energy (linear rf accelerator).

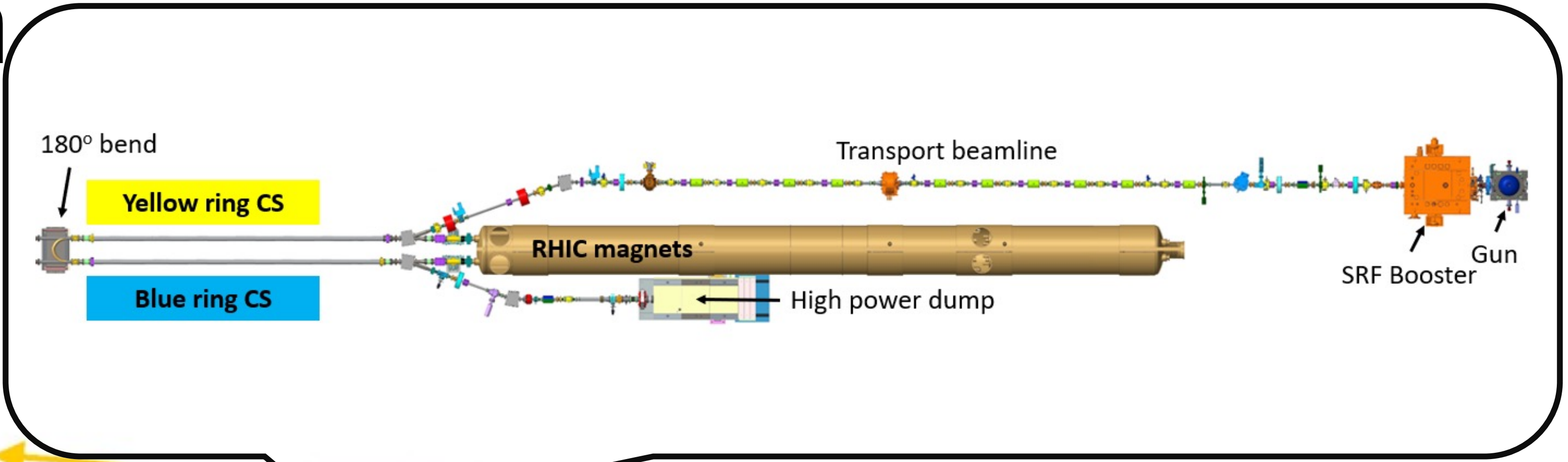


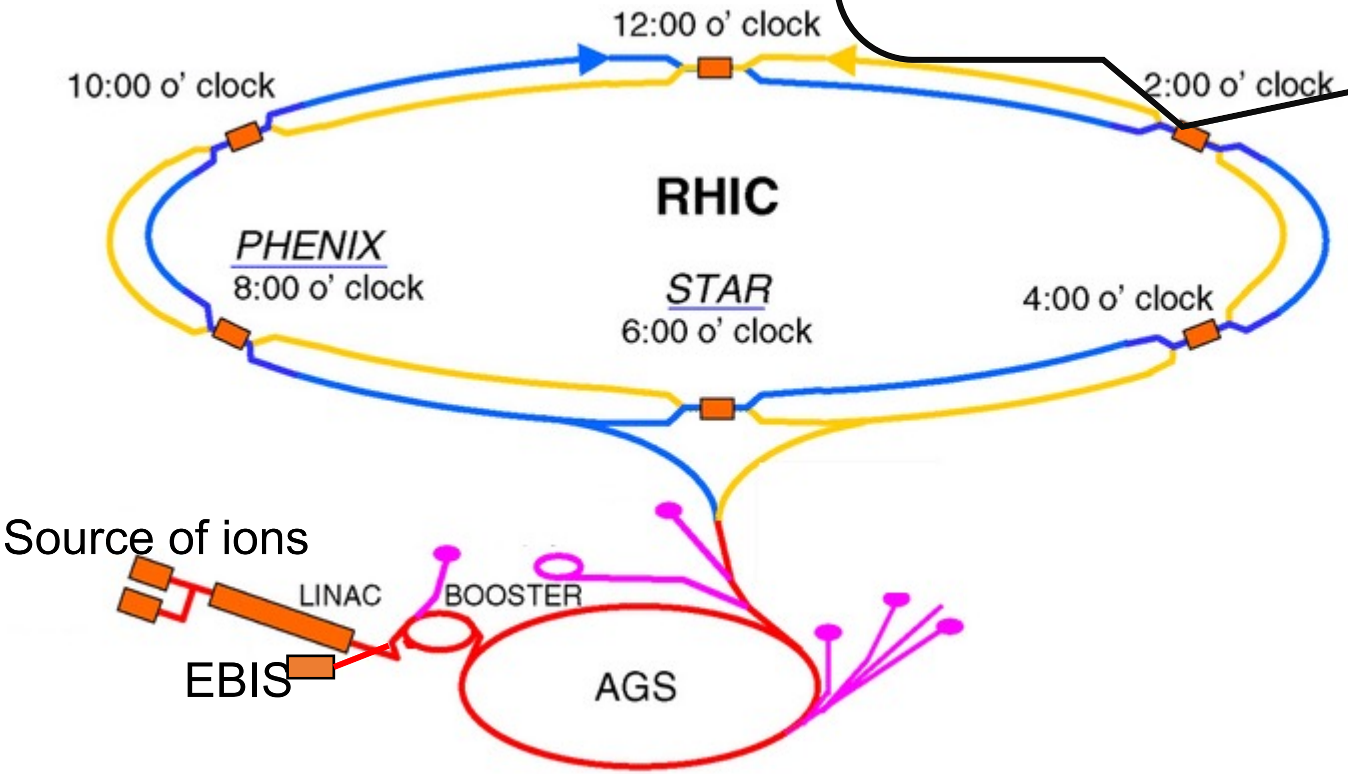
<u>Issues</u>:
- high intensity bunches (production, transport)
- momentum spread and emittance of bunches
- beam alignment
- magnetized ↔ non-magnetized (magnetic shielding)
- synchronization

Low Energy RHIC e-Cooler (LEReC) project at BNL



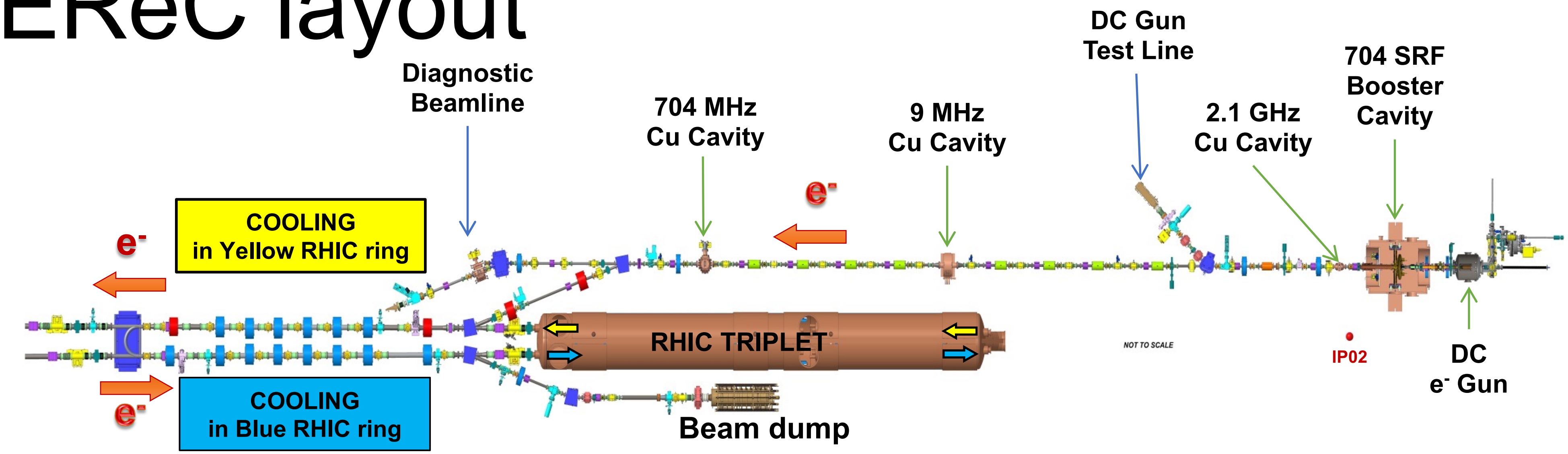
LEReC location





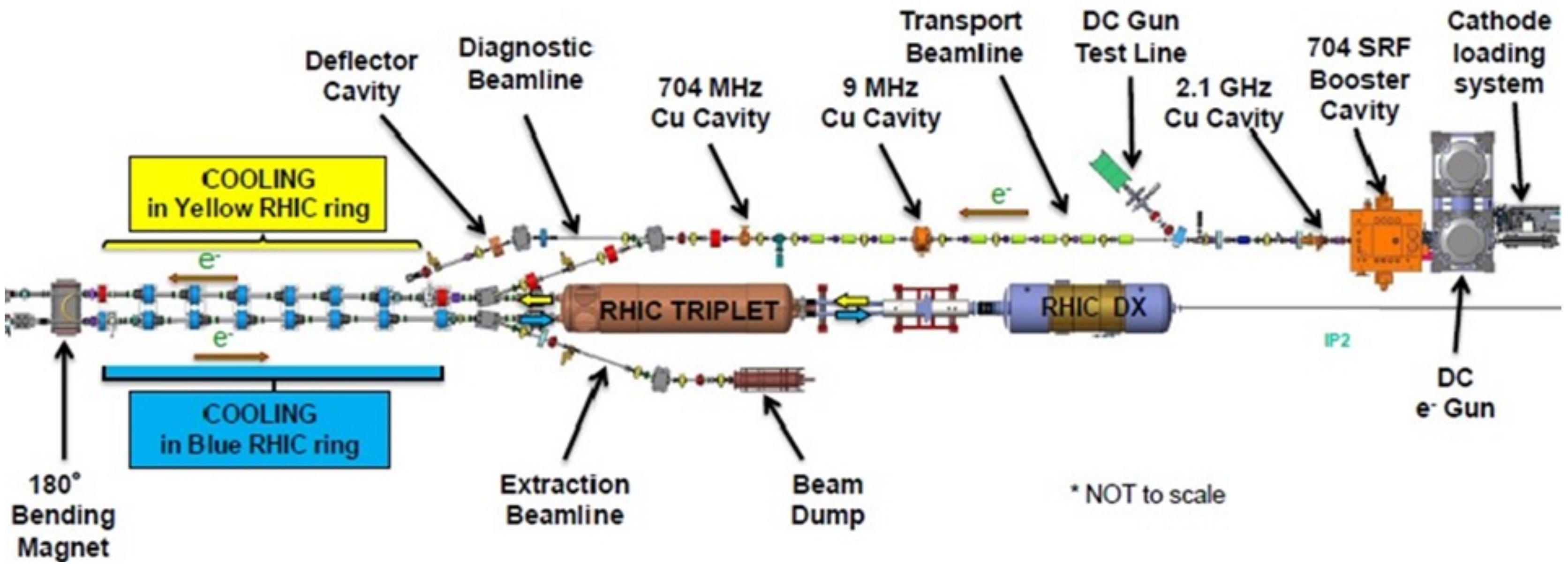
- LEReC is located inside the RHIC tunnel in “2 o’clock” region
- LEReC supplies electron bunches to the two cooling sections - both in the “yellow” storage ring and the “blue” storage ring.

LEReC layout



- Electron bunches are produced in an Electron Gun
- After exiting the Gun the electron bunches are accelerated by the accelerating cavity called the Booster.
- After the Booster the bunches are sent first to the Cooling section in the Yellow RHIC ring then into the Blue RHIC ring
- Then the electron bunches are extracted and sent to the beam dump.

Distinctive features of LEReC

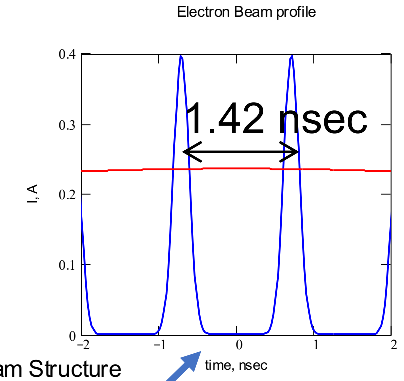


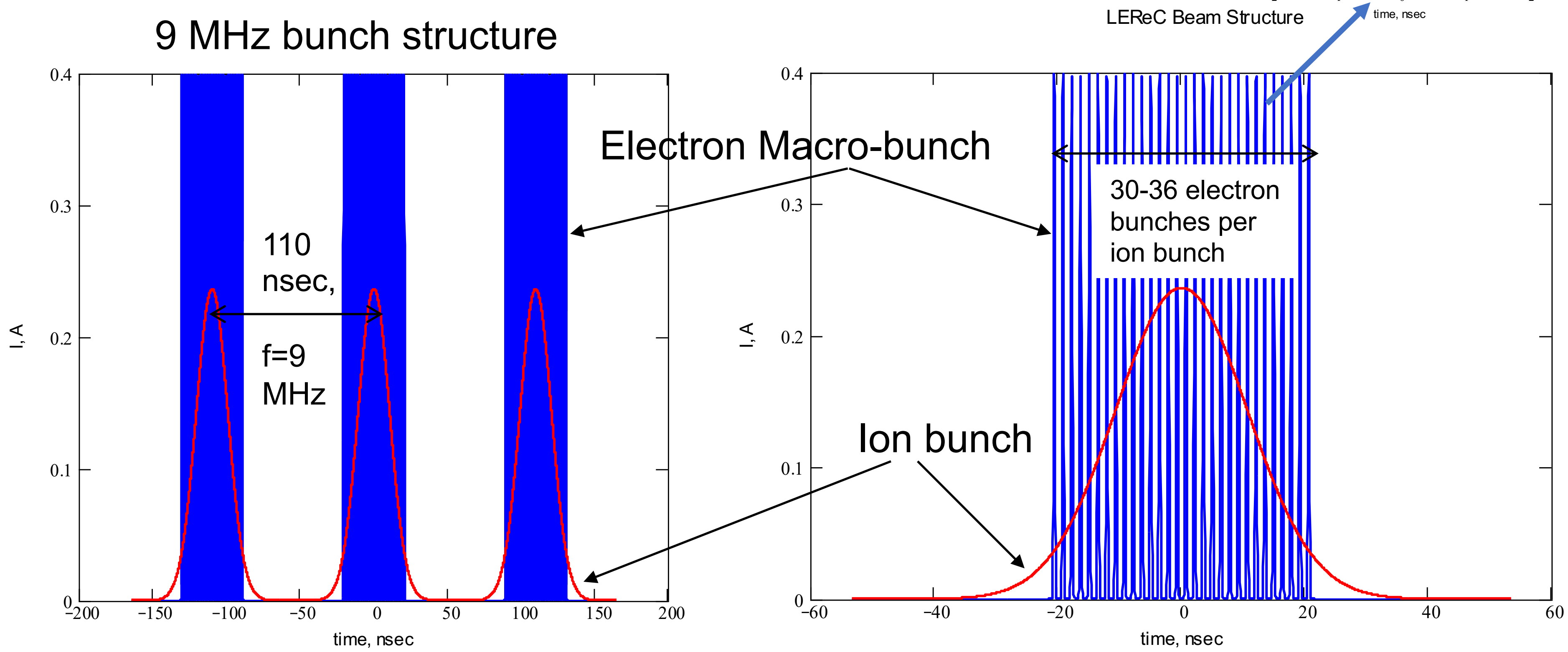
- LEReC is fully operational electron cooler which:
  - utilizes RF-accelerated electron bunches
  - uses non-magnetized electron beam (there is no magnetization at the cathode and no continuous solenoidal field in the cooling section)
  - after cooling ions in one RHIC ring ("Yellow") the same electron beam is used one more time to cool ions in the other RHIC ring ("Blue")
- LEReC approach is directly scalable to high energies (10's of MeV)

LEReC beam structure in cooling section

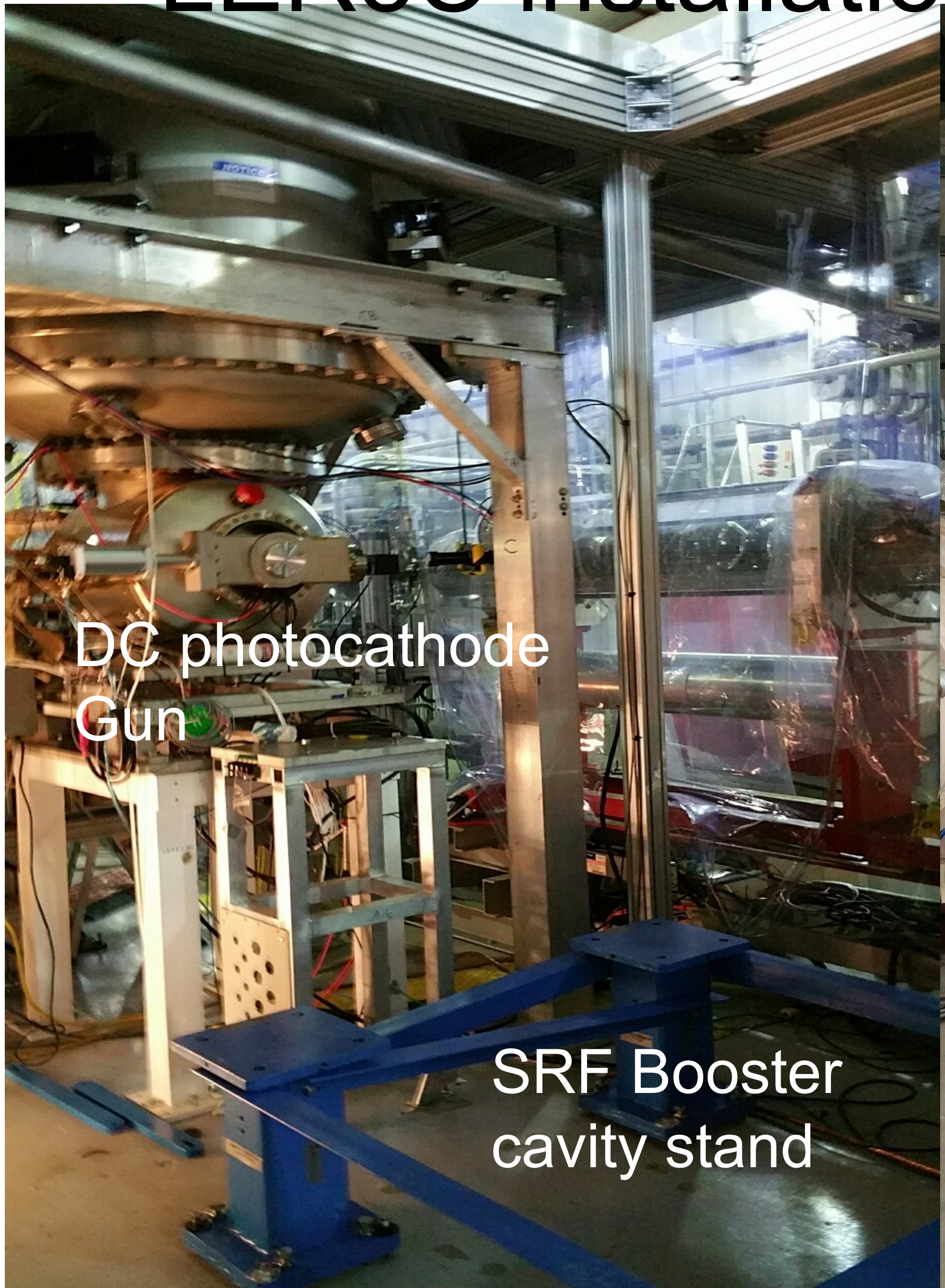
Ions structure:
120 bunches
f_rep=9.1 MHz
N_ion=5e8, I_peak=0.24 A
Rms length=3.2 m

Electrons:
f_SRF=704 MHz
Q_e=100 pC, I_peak=0.4 A
Rms length=3 cm

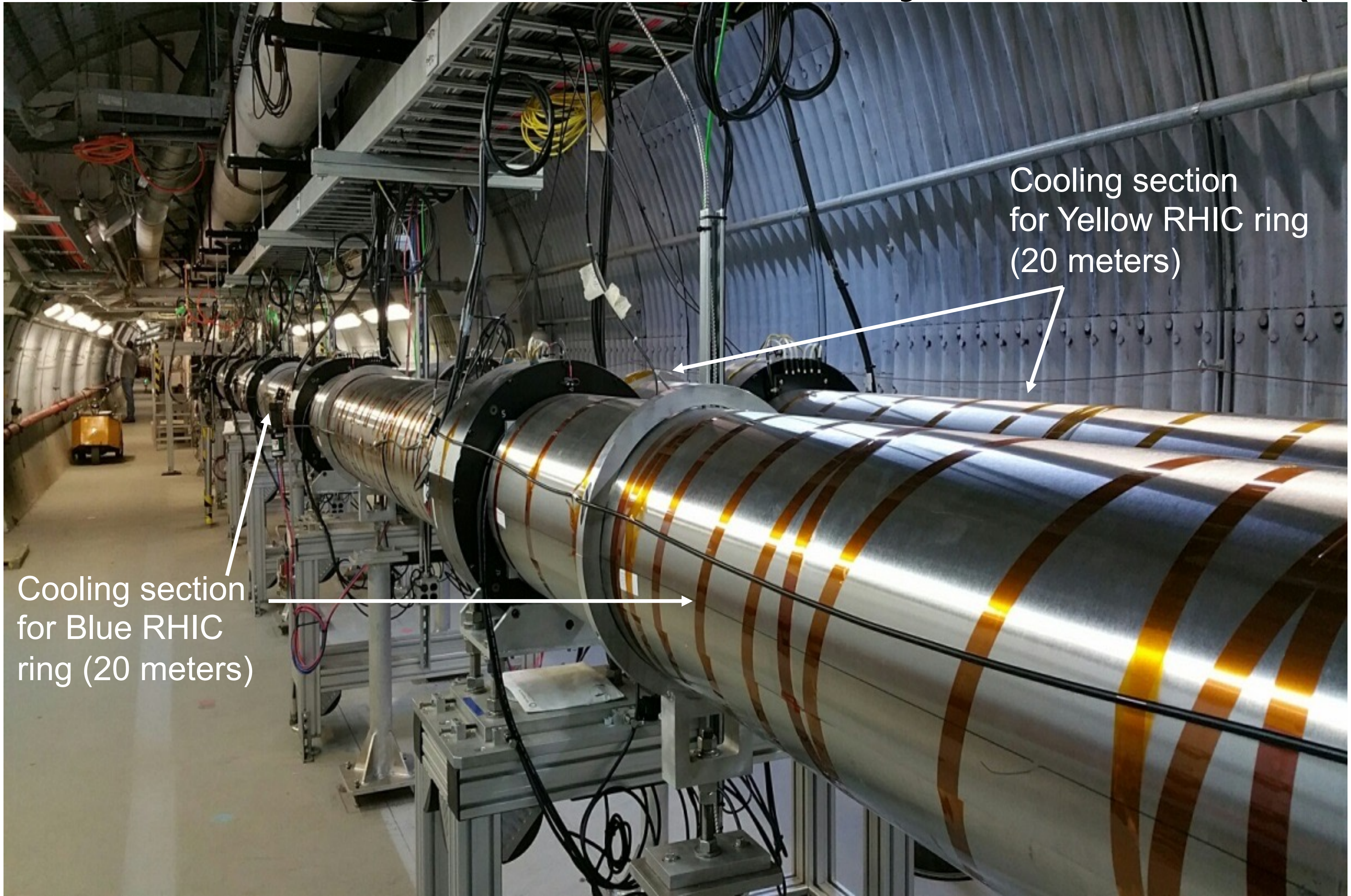




LEReC installation (October 2017)



LEReC cooling sections fully installed (2018)



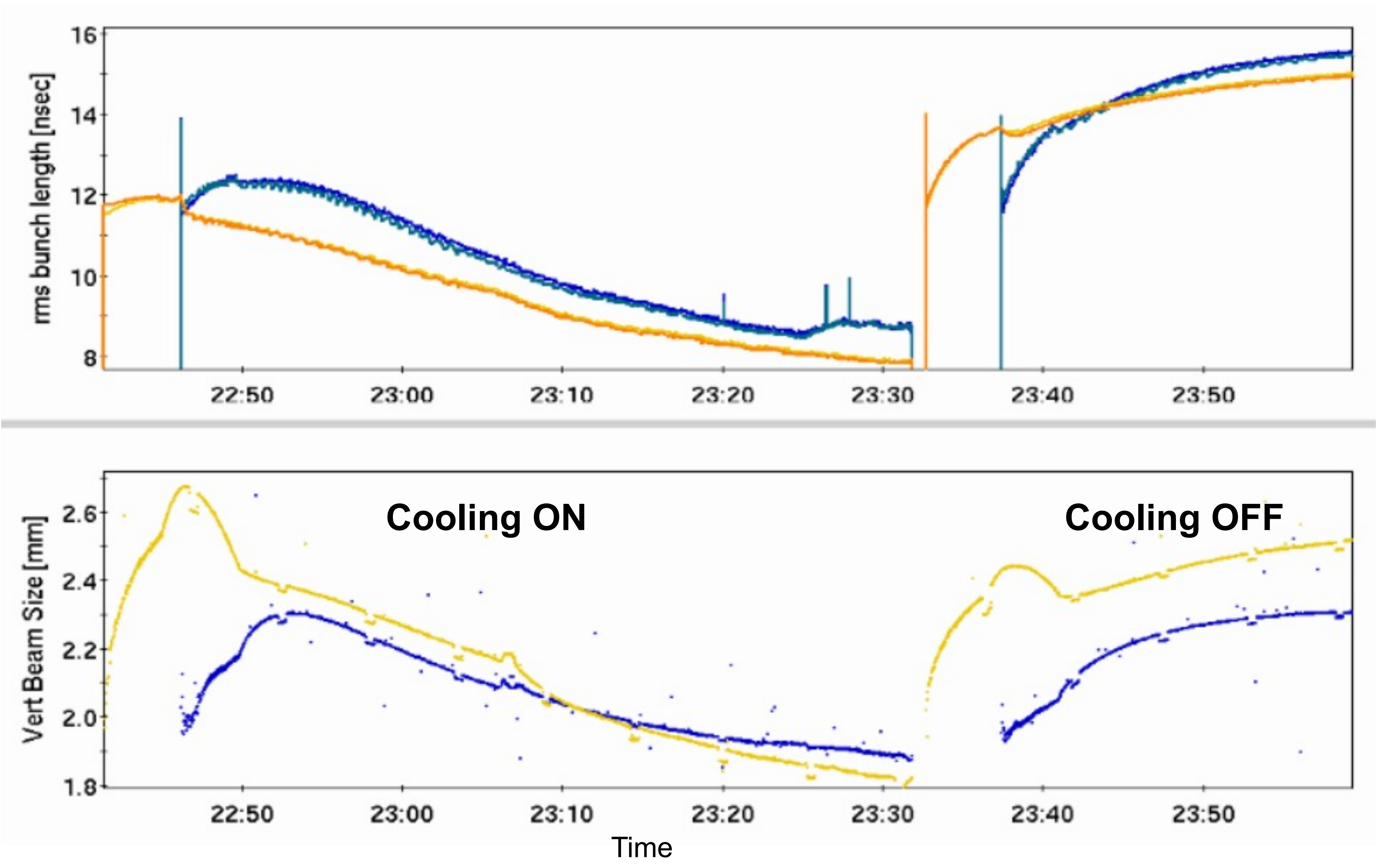
LEReC electron beam parameters

Cooler parameters used for RHIC operations		
Au ions beam energy, GeV/nucleon	3.85	4.6
Electrons kinetic energy, MeV	1.6	2.0
Cooling section length, m	20	20
Electron bunch (704MHz) charge, pC	67	67
Bunches per macrobunch (9 MHz)	30	30
Charge in macrobunch, nC	2	2
RMS normalized emittance, um	1.5	1.5
Average current, mA	18	18
RMS energy spread	< 4e-4	< 4e-4
RMS angular spread	<150 urad	<150 urad

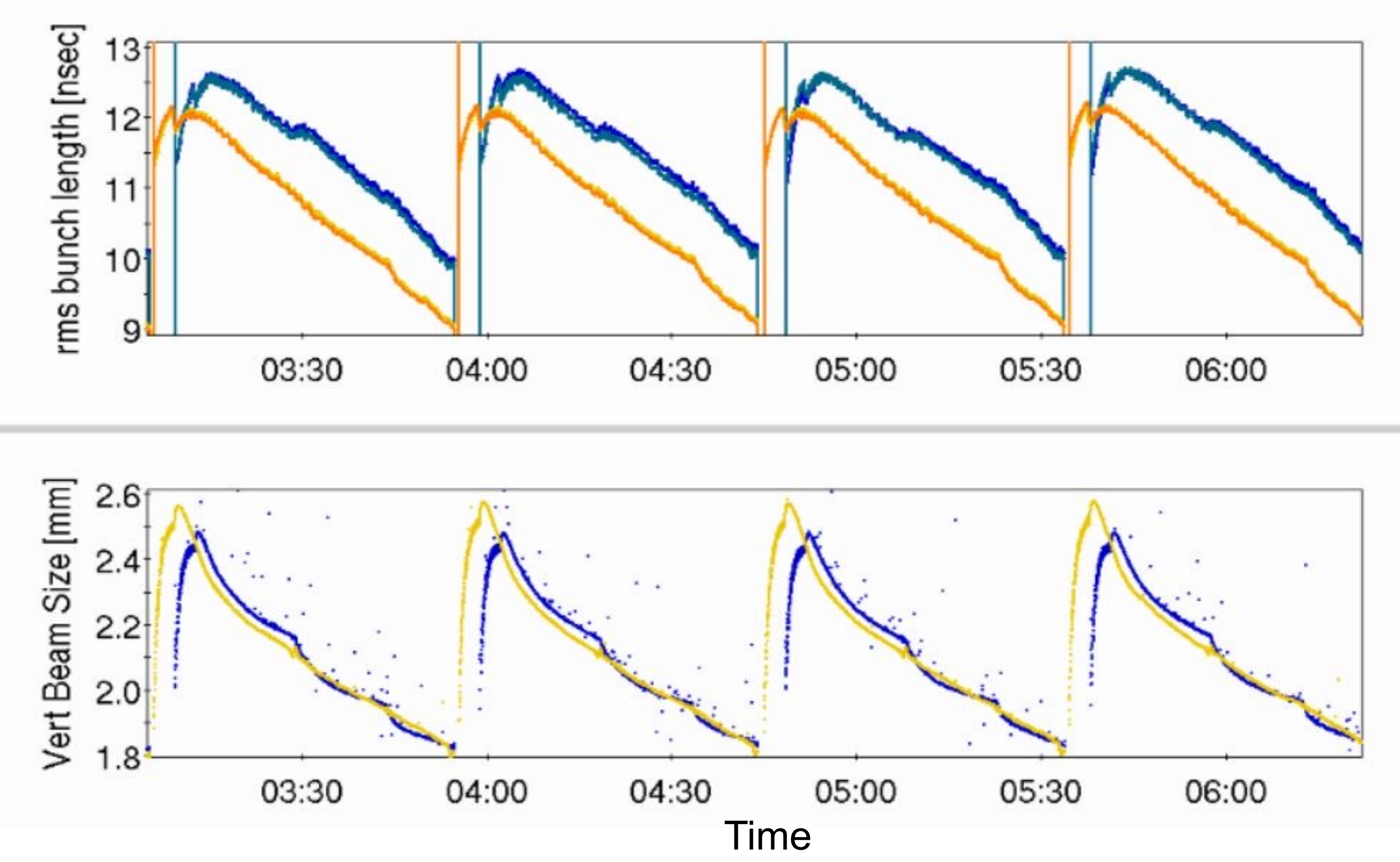
$$\sigma_\theta = \frac{\Delta_t}{\gamma\beta c}$$

LEReC operated for RHIC physics program using 1.6 and 2 MeV electron beam (LEReC was designed to operate with 1.6, 2.0 and 2.6 MeV electrons. Operation with 2.6 MeV electrons was not needed due to sufficient collider luminosity at that energy).

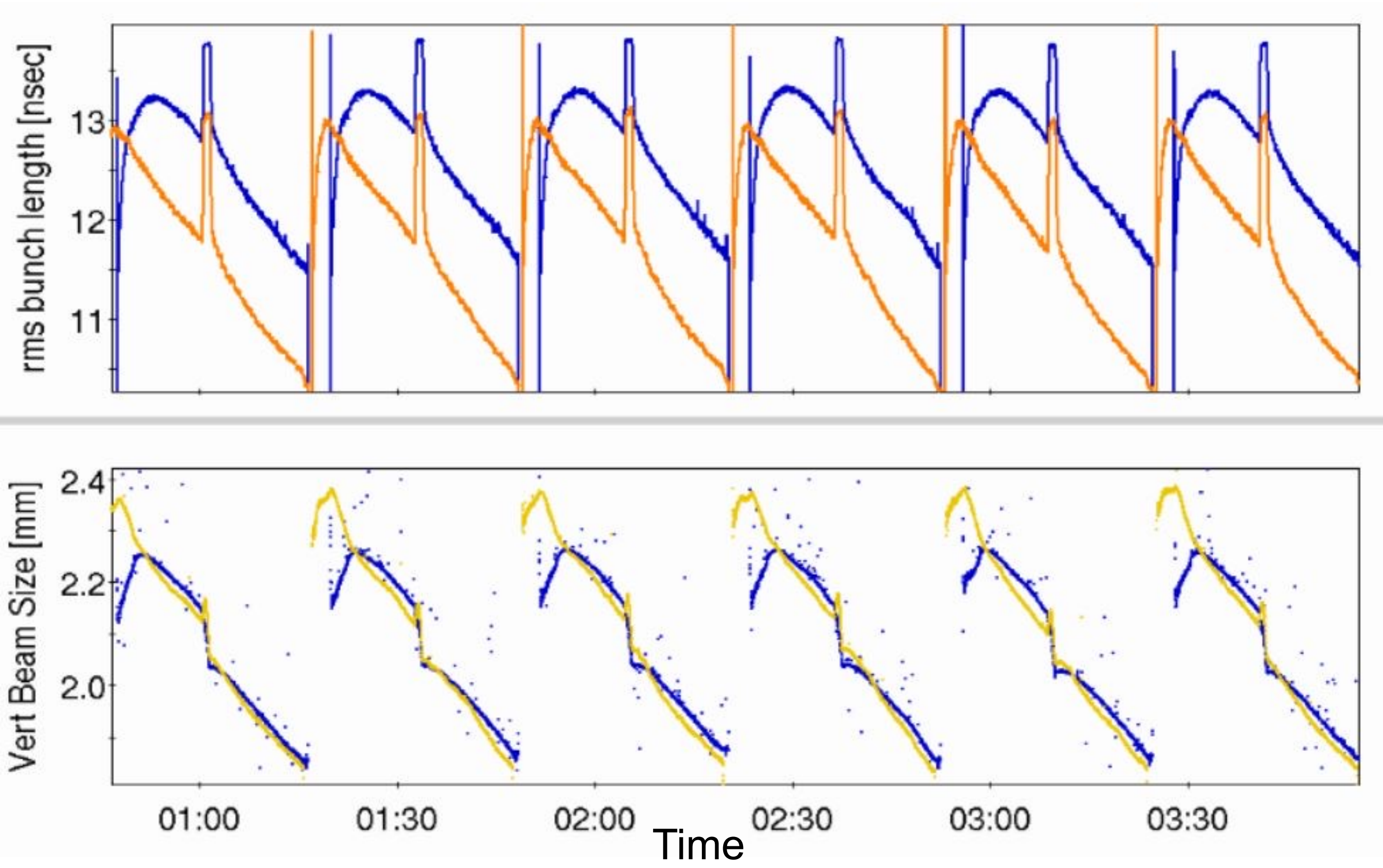
Physics stores with and without cooling of ions in Yellow and Blue RHIC rings - rms bunch length (top) and rms beam size (bottom)



2020 operation for physics (several physics stores)
2 MeV electrons, 111x111 Au ion bunches at 4.6 GeV/n, rms bunch length (top) and rms beam size (bottom)



2021 operation for physics (several physics stores),
1.6 MeV electrons, 111x111 Au ion bunches at 3.85 GeV/n,
rms bunch length (top) and rms beam size (bottom)



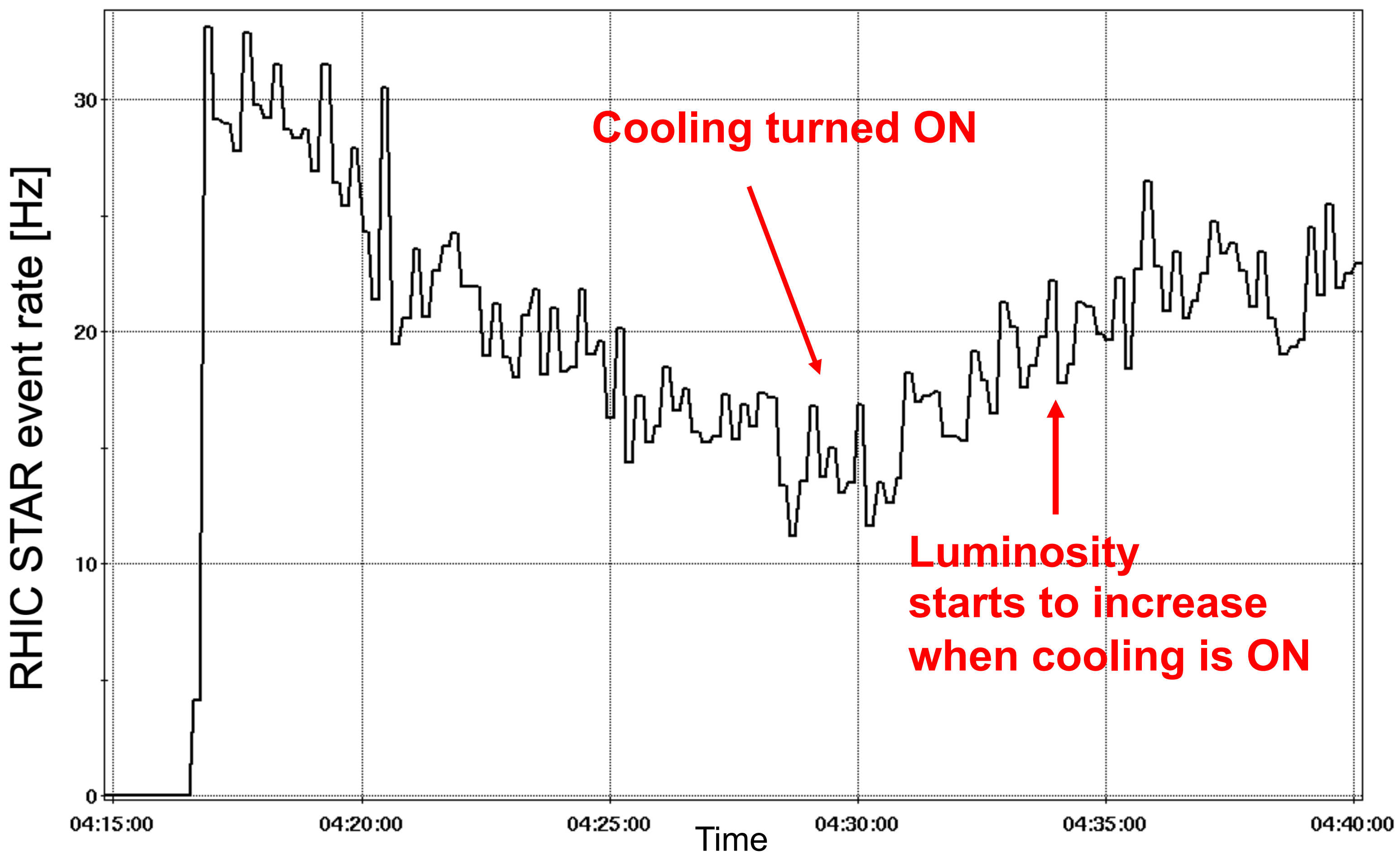
Spikes in bunch length are due to ion-beam RF manipulation to alleviate space-charge effects during the beta-squeeze.

Cooling optimization for luminosity

Luminosity optimization with cooling included:

- Finding optimum angular spread of electrons in cooling sections to provide sufficient transverse cooling.
- Optimization of electron and ion beam sizes in the cooling sections.
- Finding optimum working point in tune space for colliding beam in the presence of electron beam.
- Finding optimum electron current to reduce effects on ion beam from electrons and at the same time still provide sufficient cooling.
- Longer stores with cooling.
- With cooling counteracting longitudinal IBS and preventing debunching from the RF bucket, the ion's RF voltage was reduced resulting in smaller momentum spread of ions and improving ion lifetime.
- Once the transverse beam sizes were cooled to small values, the dynamic squeeze of ion beta-function at the collision point was established.

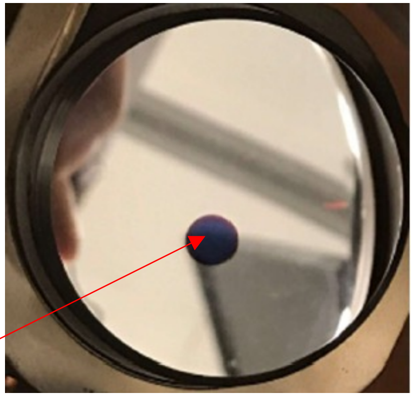
Effects of cooling on luminosity



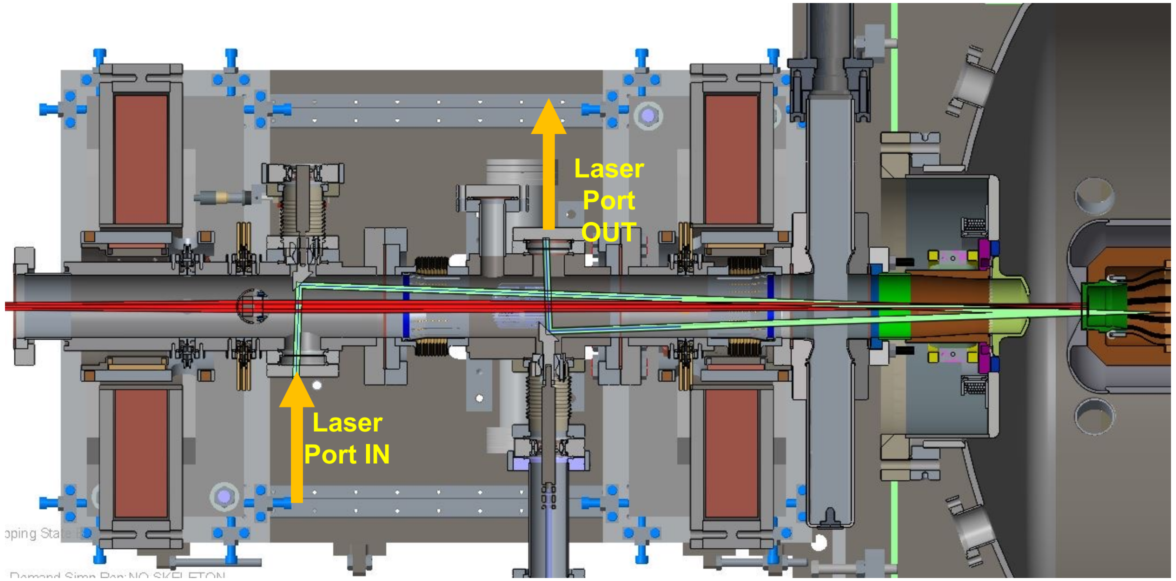
In addition to counteracting IBS, cooling of ion beam sizes results in luminosity increase.

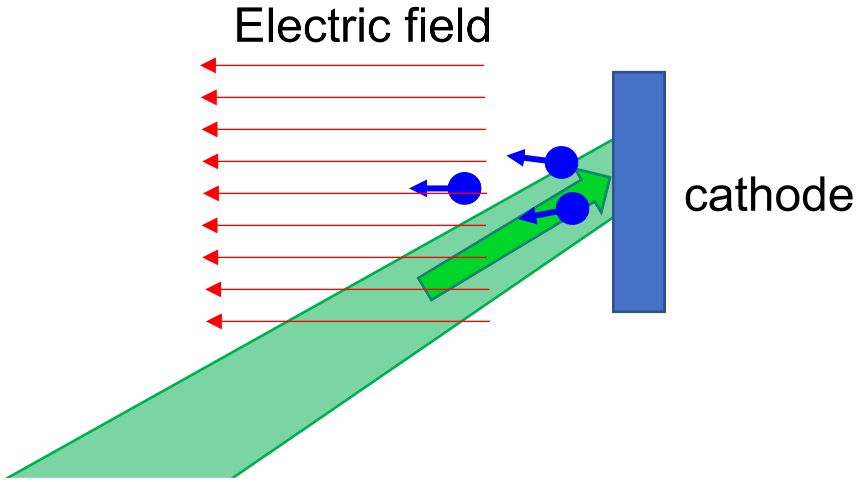
LEReC Gun

- Electrons are created when a laser impulse hits the surface of the cathode (a piece of metal covered with some active material)
- The laser “kicks” the electrons out of the active material
- The electric field of the gun accelerates electrons

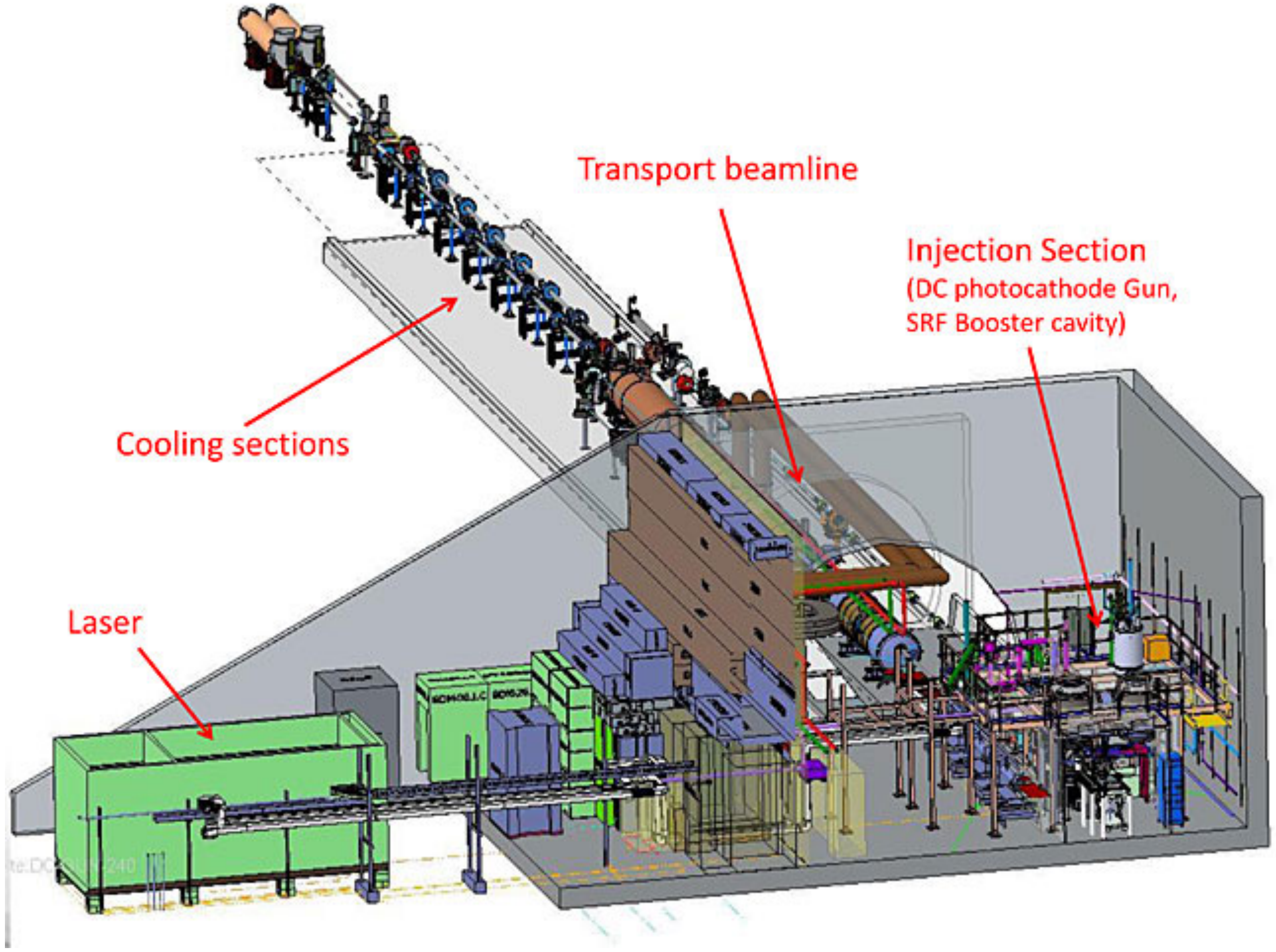


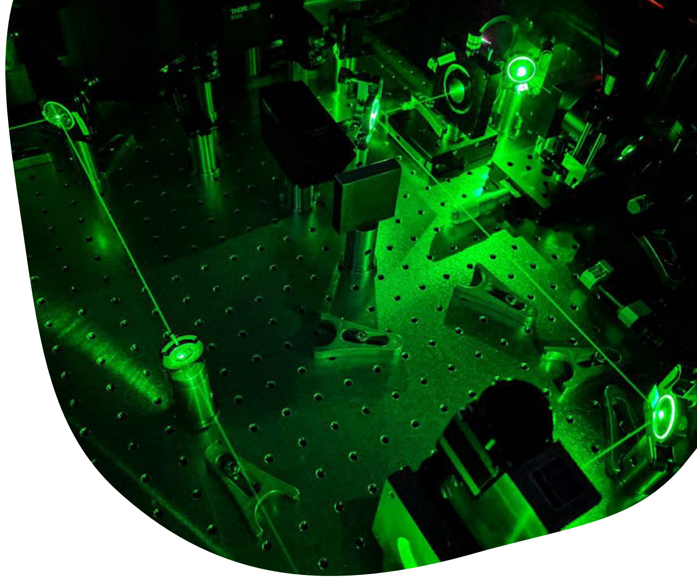
Active area of the cathode

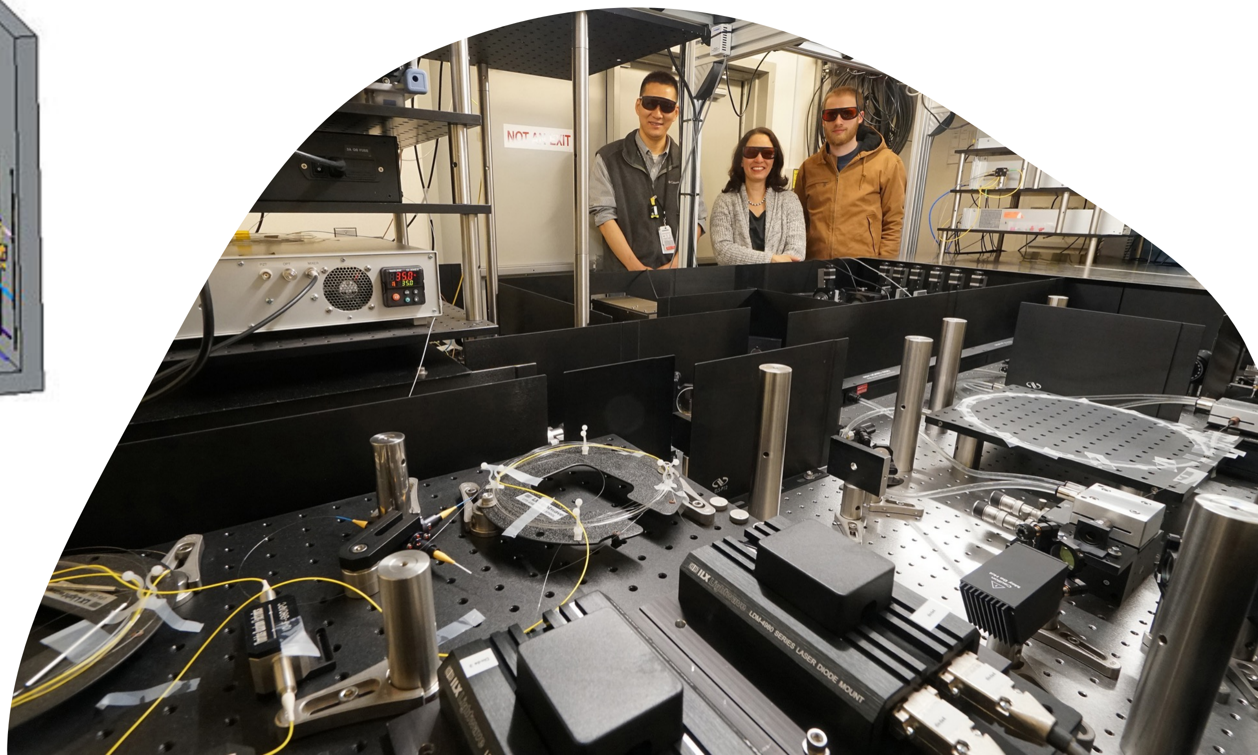




LEReC Laser

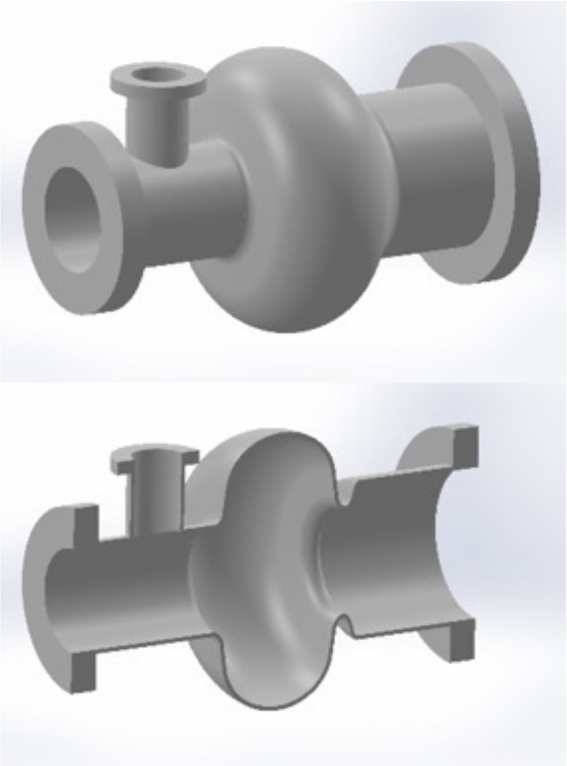

Transport beamline
Injection Section
(DC photocathode Gun,
SRF Booster cavity)
Cooling sections
Laser

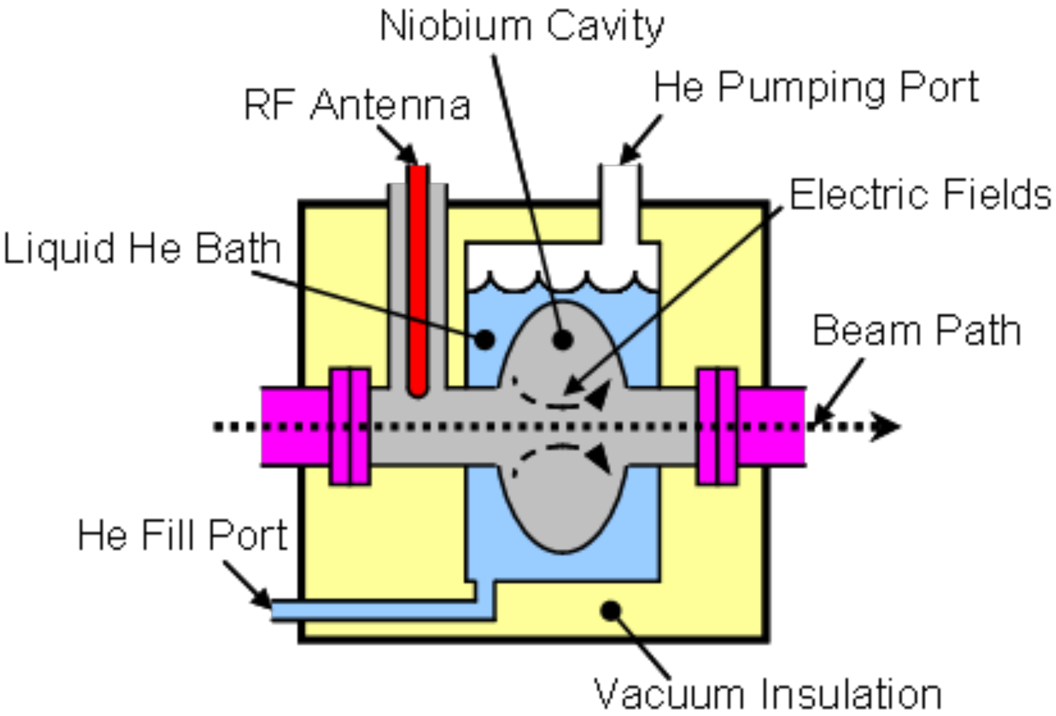


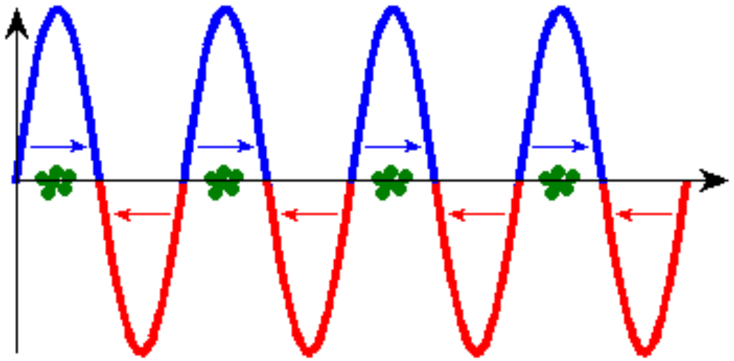


SRF Cavity

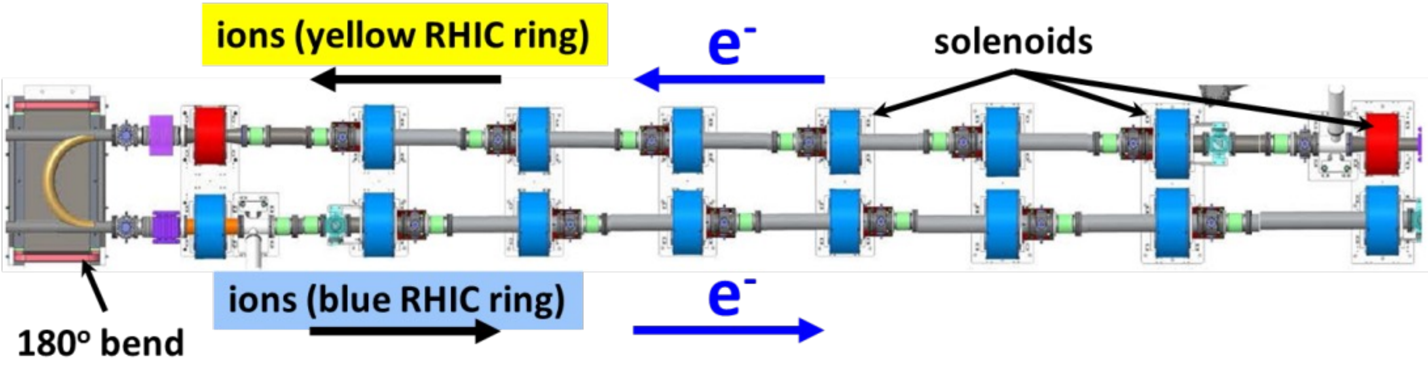
- The electrons are accelerated to the needed energy by a Superconducting Radio Frequency (SRF) Cavity
- Electromagnetic fields are excited in the cavity by coupling in an RF source with an antenna. Charged particles passing through apertures in the cavity are then accelerated by the electric fields

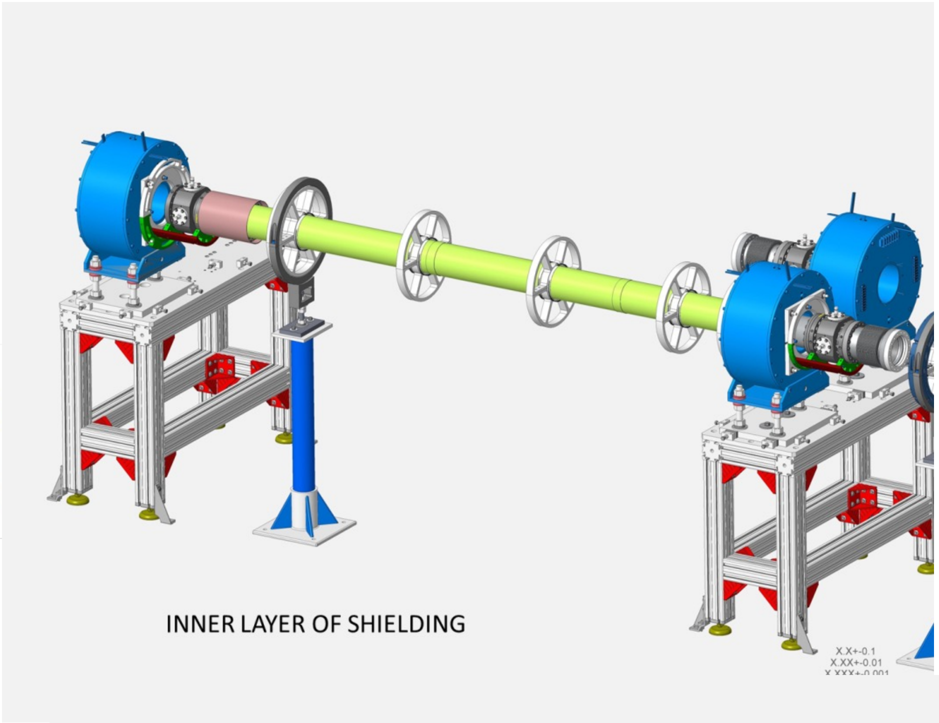


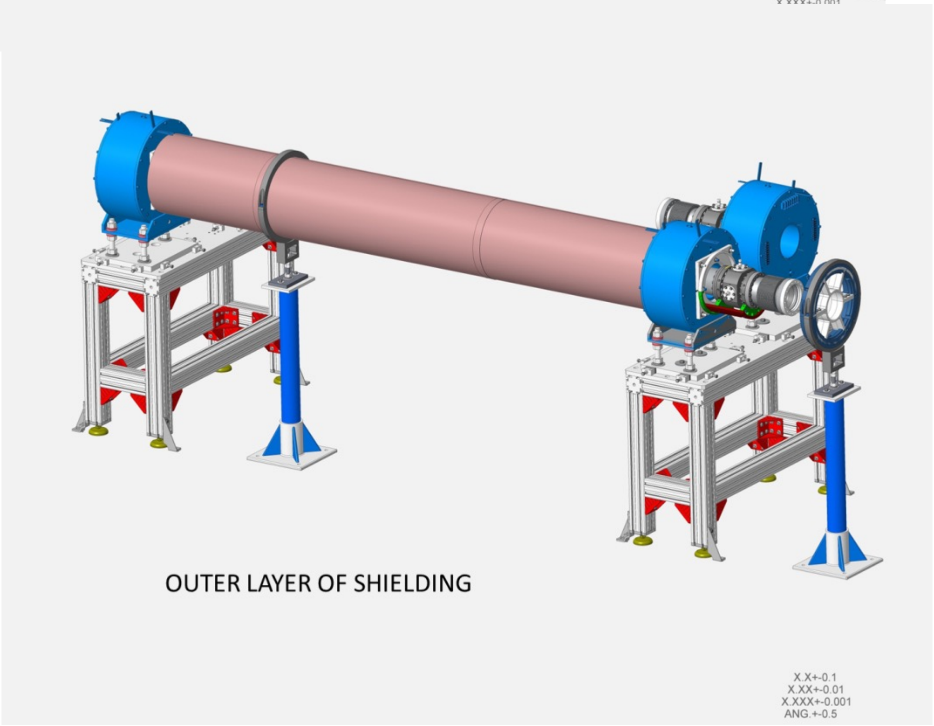




Cooling sections







- Each Cooling Section is 20 m long
- Each section contains 8 focusing lenses (solenoids)
- The distance between solenoids must be shielded by a special metal, to isolate a cooling section from the ambient magnetic fields.

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