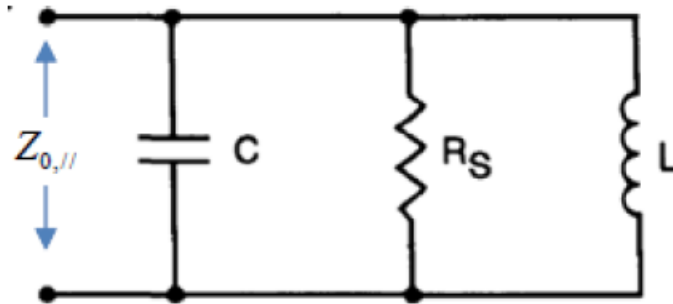


Solutions for HW # 17

1.



The impedance is determined by

$$\begin{aligned}
 \frac{1}{Z_{0, //}} &= \frac{1}{Z_R} + \frac{1}{Z_L} + \frac{1}{Z_C} \\
 &= \frac{1}{R_s} + \frac{1}{j\omega L} + j\omega C \\
 &= \frac{1 + jR_s\sqrt{\frac{C}{L}}\left(\omega\sqrt{LC} - \frac{1}{\omega\sqrt{LC}}\right)}{R_s}, \quad (1) \\
 &= \frac{1 + jQ\left(\frac{\omega}{\omega_R} - \frac{\omega_R}{\omega}\right)}{R_s} \\
 &= \frac{1 + iQ\left(\frac{\omega_R}{\omega} - \frac{\omega}{\omega_R}\right)}{R_s}
 \end{aligned}$$

i.e.

$$Z_{0, //} = \frac{R_s}{1 + iQ\left(\frac{\omega_R}{\omega} - \frac{\omega}{\omega_R}\right)}$$

where  $j = -i$ ,  $Q \equiv R_s\sqrt{\frac{C}{L}}$  and  $\omega_R \equiv \frac{1}{\sqrt{LC}}$ .

2.

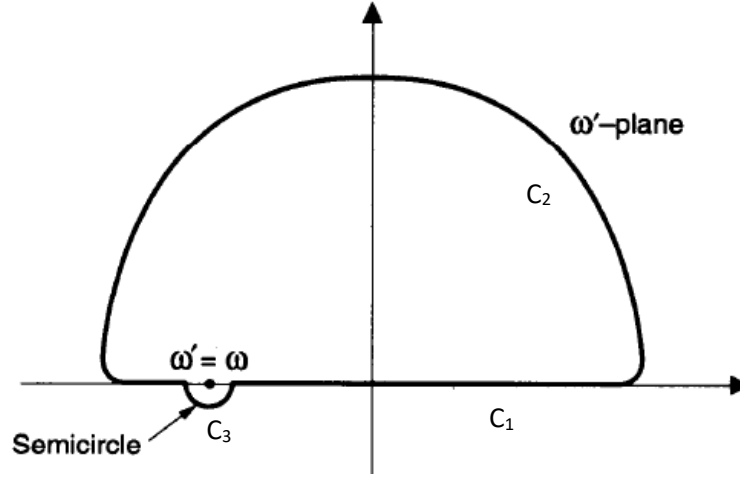


Figure 1: Integration contour in complex  $\omega'$  plane.

From Cauchy residue theorem, the contour integral can be calculated as

$$\int_C \frac{Z_{//}(\omega')}{\omega' - \omega} d\omega' = 2\pi i Z_{//}(\omega) . \quad (2)$$

The LHS of (2) can be split into the following form

$$\begin{aligned} \int_C \frac{Z_{//}(\omega')}{\omega' - \omega} d\omega' &= \int_{C_1} \frac{Z_{//}(\omega')}{\omega' - \omega} d\omega' + \int_{C_2} \frac{Z_{//}(\omega')}{\omega' - \omega} d\omega' + \int_{C_3} \frac{Z_{//}(\omega')}{\omega' - \omega} d\omega' \\ &= P.V. \int_{-\infty}^{\infty} \frac{Z_{//}(\omega')}{\omega' - \omega} d\omega' + 0 + \int_{e^{i\pi}}^{e^{i2\pi}} \frac{Z_{//}(\omega)}{e^{i\theta}} d e^{i\theta} , \quad (3) \\ &= P.V. \int_{-\infty}^{\infty} \frac{Z_{//}(\omega')}{\omega' - \omega} d\omega' + 0 + i\pi Z_{//}(\omega) \end{aligned}$$

where the integral along  $C_2$  vanishes since we assume  $Z_{//}(\omega')$  is well behaved at large  $|\omega'|$ . From eq. (2) and (3), it follows

$$Z_{//}(\omega) = -\frac{i}{\pi} P.V. \int_{-\infty}^{\infty} \frac{Z_{//}(\omega')}{\omega' - \omega} d\omega' . \quad (4)$$

Splitting eq. (4) into the real and imaginary part leads to

$$\text{Re}[Z_{//}(\omega)] = \frac{1}{\pi} P.V. \int_{-\infty}^{\infty} \frac{\text{Im}[Z_{//}(\omega')]}{\omega' - \omega} d\omega' , \quad (5)$$

and

$$\text{Im}[Z_{//}(\omega)] = -\frac{1}{\pi} P.V. \int_{-\infty}^{\infty} \frac{\text{Re}[Z_{//}(\omega')]}{\omega' - \omega} d\omega' . \quad (6)$$