1. Let us calculate the synchrotron radiation related problem in NSLS II. NSLS II adopts DBA lattice (separate function magnets). Here are the parameters:

Table 1: NSLS II parameters

Parameters	Values
Energy [GeV]	3.0
Circumference [m]	780
Number of dipoles	60
Dipole field [T]	0.4
Beam current [A]	0.5
RF frequency [MHz]	499.68
Harmonic number	1320

From the design parameters, we can calculated the following parameters:

- In DBA lattice, dispersion D and dispersion slope D' are zero at one end of dipoles and non-zero at the other end of the dipole. Find dispersion function inside the dipole magnet.
- What is the compaction factor α_c of the ring?
- The energy loss due to the dipole field.
- If the accelerating phase of the RF cavity is $\pi/6$, at least how much voltage is required? How much is the power needed?
- Actually the RF voltage is about 3MV. Find the longitudinal tune of NSLS II
- What is the critical radiation frequency of the dipole radiation.
- Find the partition number $\bar{D}$ due to synchrotron radiation in dipole.
- Find the longitudinal damping rate α_E and compare with the period of longitudinal oscillation.
- Find the equilibrium energy spread of NSLS II.

Answer:

NSLS II has 60 dipoles to form a closed loop, therefore each dipole bends 6 degree, which is $6/180 * \pi = 0.105$ rad. The radius of the dipole can be found as $P = eB\rho$, therefore $\rho = 3GeV/c/0.4m = 25m$. The length of each dipole is $L_D = 2\pi\rho/60 = 2.618m$.

Since at one end has $d = 0$ and $d' = 0$, we can calculate the dispersion function in the dipole from this end using small angle approximation:

$$\begin{pmatrix} d(s) \\ d'(s) \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & l & \rho\theta^2/2 \\ 0 & 1 & \theta \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} d(0) = 0 \\ d'(0) = 0 \\ 1 \end{pmatrix}$$

The dispersion function in dipole is $d(s) = s^2/2/\rho$.

The compaction factor is

$$\begin{aligned} \alpha_c &= \frac{1}{C} \oint \frac{D(s)}{\rho} ds \\ &= \frac{60}{C} \int_0^{L_D} s^2/2/\rho^2 ds \\ &= \frac{10}{C} \frac{L_D^3}{\rho^2} = 3.68 \times 10^{-4} \end{aligned}$$

The energy loss due to synchrotron radiation is

$$U_{SR} = C_\gamma E^4 \oint ds/\rho^2/2/\pi = \frac{C_\gamma E^4}{\rho} = 0.287 MeV$$

These energy must be compensated by the RF cavity

$$\begin{aligned} eV \sin\phi_s &= U_{SR} \\ V &= 0.57 MV \end{aligned}$$

therefore the RF cavity must provide at least 0.57 MV to compensate the synchrotron radiation loss in the dipoles.

The actual voltage is 3MV, the phase should be $\arcsin(0.287/3) = \pi - 0.095$, the synchrotron tune is:

$$\nu_s = \sqrt{\frac{h\eta eV \cos\phi_s}{2\pi E}} = 8.8 \times 10^{-3}$$

The time to finish one synchrotron oscillation is $2\pi/(\nu_s \omega_0) = 2.94 \times 10^{-4} s$.

The critical frequency of the dipole radiation is given by

$$\omega_c = \frac{3}{2} \frac{c\gamma^3}{\rho} = 3.64 \times 10^{18} Hz$$

We then calculate the radiation integrals, taking advantage that $K(s) = 0$ in dipoles:

$$I_2 = \oint 1/\rho^2 ds = 2\pi/\rho = 0.2513 m^{-1}$$

$$I_3 = \oint 1/\rho^3 ds = 2\pi/\rho^2 = 1.0 \times 10^{-2} m^{-2}$$

$$I_4 = \oint D/\rho^3 ds = 60 \int_0^{L_D} s^2/2/\rho^4 ds = 10L_D^3/\rho^4 = 4.59 \times 10^{-4} m^{-1}$$

Therefore the partition number $\bar{D} = I_4/I_2 = 1.828 \times 10^{-3}$

The longitudinal damping rate

$$\alpha_E = \frac{U_0}{2T_0 E}(2 + \bar{D}) = 36.79 s^{-1}$$

It is much slower than the synchrotron oscillation.

The equilibrium energy spread can be calculated as:

$$\frac{\delta E}{E} = \sqrt{\frac{C_q \gamma^2 I_3}{2I_2 + I_4}} = 5.12 \times 10^{-4}$$