Homework 2
Due: *Friday, February 18, 2022*

1. In class, we derived $\vec{p}_\perp(\xi)$ and $p_z(\xi)$ in a plane wave ($\vec{A} = \vec{A}_\perp(\xi)$) with initial conditions of $\vec{p} = 0$ when $\vec{A}_\perp(\xi) = 0$. Derive the quantities $\vec{p}_\perp(\xi)$ and $p_z(\xi)$ when the initial values are p_{z0} and $\vec{p}_{\perp 0}$ instead. What maximum energy might we expect compared to the particle that starts at rest, if a particle is initiated with $\gamma_0 = \sqrt{1 + \frac{p_{z0}^2}{m^2c^2}}$? What may limit this energy gain in practice?

2. Derive the E & B fields of a bi-Gaussian electron bunch with $v_z = v_b \approx c$; that is for a bunch that has the following density profile:

$$\rho(r, \xi) = \rho_0 \exp\left[-\frac{r^2}{2\sigma_r^2} - \frac{\xi^2}{2\sigma_z^2}\right].$$

Assume that $\sigma_r \ll \sigma_z$ if needed.
At what position (r, ξ) does the E field reaches its maximum? What is that maximum value?