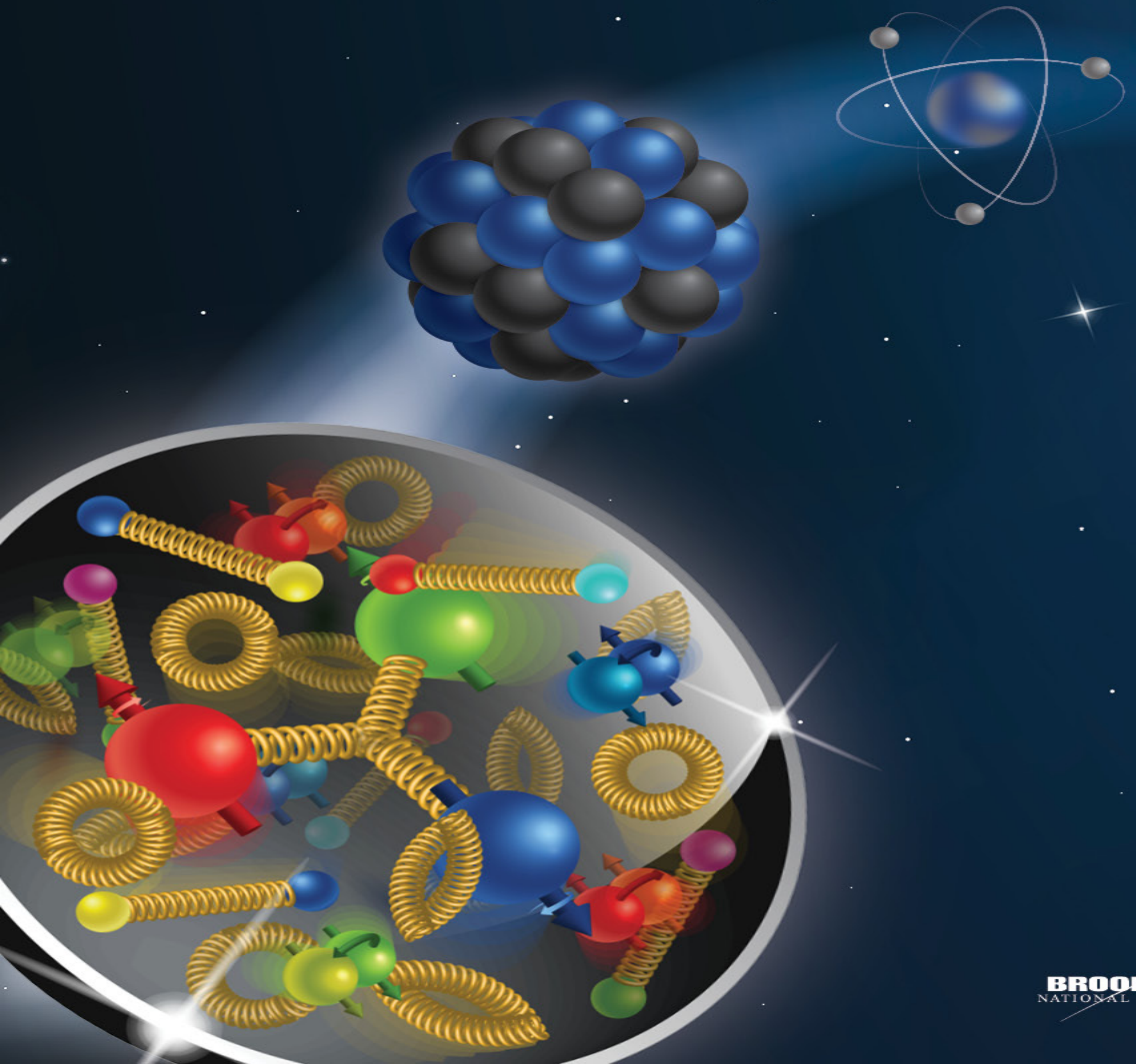

High Power RF Engineering -Coax (2)

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U.S. DEPARTMENT OF ENERGY | Office of Science

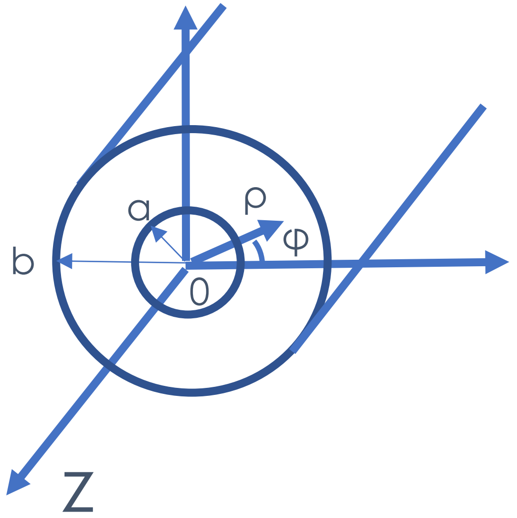
Note

- Speed of light in vacuum $c = 299\ 792\ 458\ m/s$
- Vacuum electric permittivity $\varepsilon_0 = 8.854187 \times 10^{-12}\ F/m$
- Vacuum magnetic permittivity $\mu_0 = 1.256637 \times 10^{-6}\ N/A^2$
- $\varepsilon_0 \mu_0 = 1/c^2$
- Impedance of free space $\eta = \sqrt{\mu_0/\varepsilon_0} = 376.730\ 313\ \Omega$

Coaxial cables

- Two major types of coaxial lines: RG (Radio Guide) and LMR®. Types of connectors: SMA, type-N, TNC, SMC, BNC, SMB, 7/16 etc.
- RG cables, normally 50 or 75Ω, are labeled "RG" with the number formerly standing for the diameter of the cable, i.e. RG59 refers to 75Ω cable with .059" diameter of the center pin.
- LMR® cables are owned by the company Times Microwave, they are labeled "LMR-" with a number roughly referring to the diameter of the cable in mil (0.001inch). Please note that diameter of the cable includes not only the inner conductor, the dielectric material and the outer conductor, but also the braid (signal shielding) and the jacket.
- We use LMR®-1700 as an example.

TEM Field Pattern



- Frequency independent
- $k_c = 0,\ k = \beta = \omega\sqrt{\mu\varepsilon}$, no cutoff frequency.

$$E_\rho = \frac{A}{\sqrt{\varepsilon}\rho} e^{-j\beta z}$$

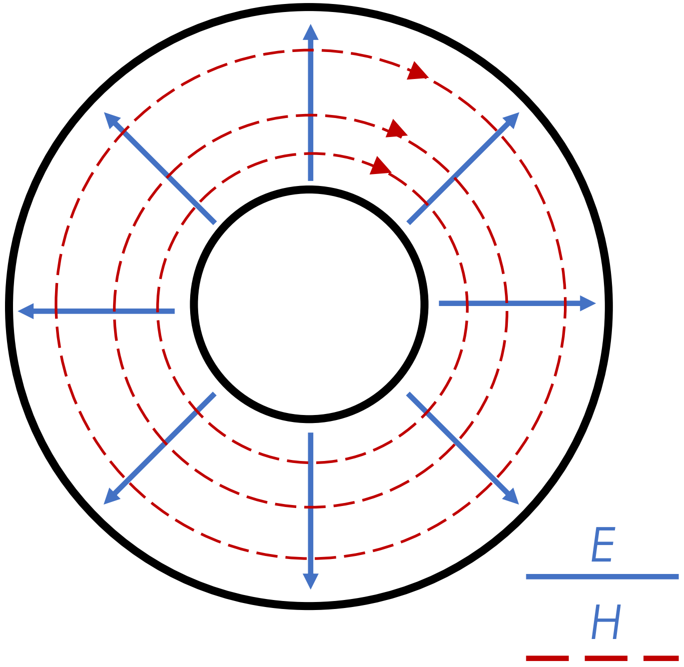
$$E_\varphi = 0$$

$$E_z = 0$$

$$H_\rho = 0$$

$$H_\varphi = \frac{A}{\sqrt{\mu}\rho} e^{-j\beta z}$$

$$H_z = 0$$



Or, times $\sqrt{\varepsilon}$

$$E_\rho = \frac{A}{\rho} e^{-j\beta z}$$

$$E_\varphi = 0$$

$$E_z = 0$$

$$H_\rho = 0$$

$$H_\varphi = \frac{A}{\eta\rho} e^{-j\beta z}$$

$$H_z = 0$$

Limiting factors

- High peak power in a short period of time that brings up the peak *E* field and causes RF breakdown by arcing (Pulsed mode).
- High average power that excessively dissipated on metal wall (which is related to the *H* field distribution) and other lossy materials (teflon, ferrite, SiO_2 etc) in the RF device, which causes the device not working properly, or even get damaged. (Pulsed/CW)

TEM Characteristic Impedance

- Wave impedance η.
- $I = \oint H_\varphi \, dl = \frac{2\pi A}{\sqrt{\mu}}$ (peak value, $e^{j\omega t - j\beta z}$ is not here)
- $V = \int_a^b E_\rho \, d\rho = \frac{A}{\sqrt{\varepsilon}} ln \frac{b}{a}$
- Characteristic impedance $Z_0 = \frac{V}{I} = \frac{\eta}{2\pi} ln \frac{b}{a}$
- Under vacuum without dielectric material (or air-filled) it is $\sim 60 ln \frac{b}{a}$ Ω

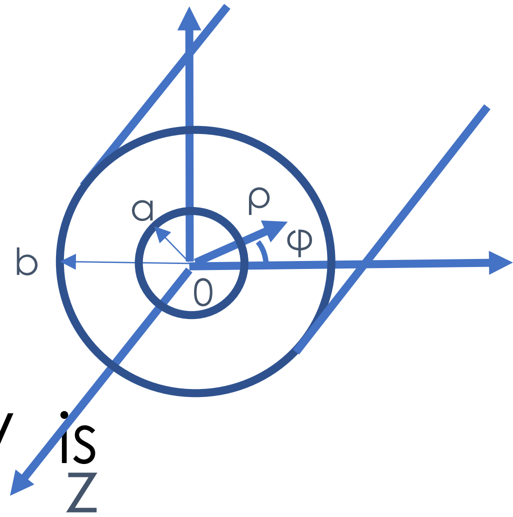
TEM LC

- Inductance per unit length $L = \frac{\mu}{I^2}\int_{r=a}^{r=b} \boldsymbol{H}\cdot\boldsymbol{H}^* dS = \frac{\mu}{I^2}\int_{r=a}^{r=b} \frac{A}{\sqrt{\mu\rho}} e^{j\omega t - j\beta z} \frac{A}{\sqrt{\mu\rho}} e^{-j\omega t + j\beta z} dS = \frac{\mu}{(2\pi)^2}\int_{r=a}^{r=b} \frac{1}{\rho^2} d(\pi\rho^2) = \frac{\pi\mu}{(2\pi)^2}(\ln b^2 - \ln a^2) = \frac{\mu}{2\pi} ln\frac{b}{a}$ H/m.
- Capacitance per unit length $C = \frac{\mathcal{E}}{V^2}\int_{r=a}^{r=b} \boldsymbol{E}\cdot\boldsymbol{E}^* dS = \frac{\mathcal{E}}{V^2}\int_{r=a}^{r=b} \frac{A}{\sqrt{\mathcal{E}\rho}} e^{j\omega t - j\beta z} \frac{A}{\sqrt{\mathcal{E}\rho}} e^{-j\omega t + j\beta z} dS = \frac{\mathcal{E}}{(ln\frac{b}{a})^2}\int_{r=a}^{r=b} \frac{1}{\rho^2} d(\pi\rho^2) = \frac{\pi\mathcal{E}}{(ln\frac{b}{a})^2}(\ln b^2 - \ln a^2) = \frac{2\pi\mathcal{E}}{ln\frac{b}{a}}$ F/m.
- $Z_0 = \sqrt{\frac{L}{C}} = \frac{\eta}{2\pi} ln\frac{b}{a}$

LMR®-1700: Z_0, LC & Cutoff

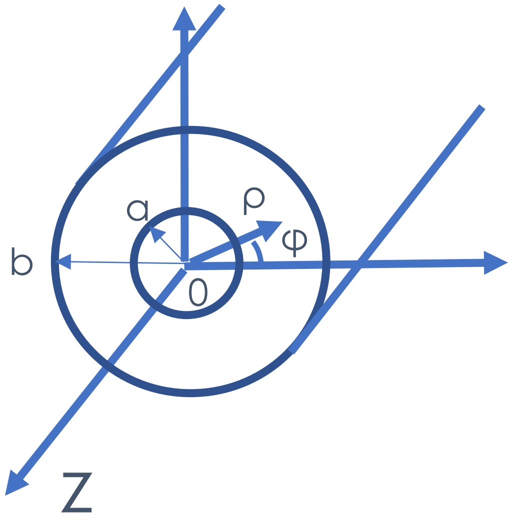
- Dielectric material Foam Polyethylene with 1.26 dielectric constant, thus velocity of propagation v is 89% ($1/\sqrt{\varepsilon_r}$) of c, time delay in the cable is 1/v = 3.75nS/m
- 0.527" ID (*a* = 6.7mm), Cu tube, 1.356" OD (*b* = 17.2mm), Aluminum. Characteristic impedance $Z_0 = \frac{\eta}{2\pi\sqrt{\varepsilon_r}} ln\frac{b}{a} = 50\Omega$.
- Inductance per unit length $L = \frac{\mu}{2\pi} ln\frac{b}{a} = 0.189$ µH/m. Capacitance per unit length $C = \frac{2\pi\mathcal{E}}{ln\frac{b}{a}} = 74.2$ pF/m.
- Cutoff frequency (TE_{11}) is $f_c = \frac{v}{\pi}\frac{1}{b+a} = 3.55\text{GHz}$, it is recommended to use it up to 70% of the cutoff frequency, at 2.5GHz.

TEM Transmitted Peak Power



- Transmitted power $P_t = \frac{V^2}{2Z_0} = \frac{2\pi}{\varepsilon\eta} A^2 ln \frac{b}{a}$ (1/2 because V is peak value)
- Another way is Poynting vector $P_t = \int_{r=a}^{r=b} \boldsymbol{E} \times \boldsymbol{H}^* dS = \int_{r=a}^{r=b} \frac{A}{\sqrt{\varepsilon}\rho} e^{j\omega t - j\beta z} \frac{A}{\sqrt{\mu}\rho} e^{-j\omega t + j\beta z} dS = \frac{A^2}{\varepsilon\eta} \int_{r=a}^{r=b} \frac{1}{\rho^2} d(\pi\rho^2) = \frac{\pi A^2}{\varepsilon\eta} (\ln b^2 - \ln a^2) = \frac{2\pi}{\varepsilon\eta} A^2 ln \frac{b}{a}$
- Power flows in the space between the inner and out conductors, unlike electric wires (parallel lines, twisted pair) that power flows on the metal surface.

TEM 30Ω for Peak Power



- Peak E field (which breakdown happens) is at $\rho = a$, with $E_a = \frac{A}{a\sqrt{\varepsilon}}$ in ρ direction. Here E_a is material and frequency dependent, it does not depend on a (with a in mm range and above). E_a and a determine the maximum A can be achieved. $E_a = \frac{A(a)}{a\sqrt{\varepsilon}}$ is a constant for a given material in certain condition under certain frequency.

- $P_t = \frac{2\pi}{\eta} {E_a}^2 a^2 ln\frac{b}{a}$
- The bigger the size, the more the peak transmitted power.
- (a^2*ln(1/a))'=(-a^2lna)'=-2alna-a=0 thus a=e^-0.5, b/a=e^0.5
- For a fixed size coaxial cable (fixed b value), maximum peak transmitted power happens at $\frac{b}{a} = e^{0.5}$(~1.65) and $Z_0 = 30\Omega$.

LMR®-1700: Breakdown field

- Foam Polyethylene normally has a breakdown E field of 20MV/m or higher.
- The breakdown E field of a cable is normally restricted by the weakest link, for example, the connections where a small air gap exists.
- For clean, dry air at one atmosphere, the breakdown E field is ~3MV/m. We use 1.5MV/m (50%) considering temperature, pressure, moisture etc.

LMR®-1700: Breakdown

- $E_\rho = \frac{A}{\rho} e^{-j\beta z}$, peak field is at $\rho = a$, with $E_{pk} = \frac{A}{a}$ =1.5 MV/m, thus A = 10 kV.
- Breakdown voltage is thus $V = \int_a^b E_\rho \, d\rho = A \ln \frac{b}{a}$ =9.45kV.
- Breakdown power is $P_t = \frac{V^2}{2Z_0} =$ 0.9MW with a matched load at 50Ω.

LMR®-1700: Breakdown – full reflection

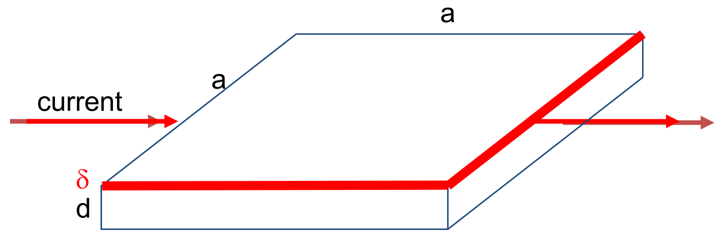
- In the case of full reflection, *E* and *H* field travelling along z ($e^{-j\beta z}$) is reflected back (along –z, $e^{+j\beta z}$) with a certain phase difference φ determined by the boundary condition, thus $e^{+j(\beta z+\varphi)}$,
- *E* field now becomes $\frac{A}{\rho}e^{-j\beta z} + \frac{A}{\rho}e^{+j(\beta z+\varphi)}$, E_{pk} is thus twice the previous value in the worst case.
- The breakdown power is 25% of the previous value, 225kW.

R_s

- $\boldsymbol{J} = \sigma \boldsymbol{E}$ on metal surface with σ the DC conductivity.
- $\nabla \times \boldsymbol{E} = -j\omega\mu\boldsymbol{H}$ & $\nabla \times \boldsymbol{H} = (j\omega\varepsilon+\sigma)\boldsymbol{E} = \sigma\boldsymbol{E}$ for a good conductor at microwave frequency range (true for Cu below 10^{18}Hz) that $\omega\varepsilon \ll \sigma$
- $\nabla \times \nabla \times \boldsymbol{E} = -\nabla^2\boldsymbol{E} = -j\omega\mu\, \nabla \times \boldsymbol{H} = -j\omega\mu\sigma\boldsymbol{E} \rightarrow \nabla^2\boldsymbol{E} = j\omega\mu\sigma\boldsymbol{E}$
- Wave along z (perpendicular to metal surface with metal at z=0 and after), **E** has x component, H has y component
- $E_x = E_0 e^{-(1+j)\frac{z}{\delta}}$ field along metal thickness z & skin depth $\delta = \sqrt{\frac{2}{\omega\mu_0\sigma}}$

  & $H_y = \frac{1-j}{\omega\mu_0\delta} E_0 e^{-(1+j)\frac{z}{\delta}}$
- Surface impedance $Z_s = \frac{E_x}{H_y} = (1+j)\sqrt{\frac{\mu_0\omega}{2\sigma}}$ with its real part R_s

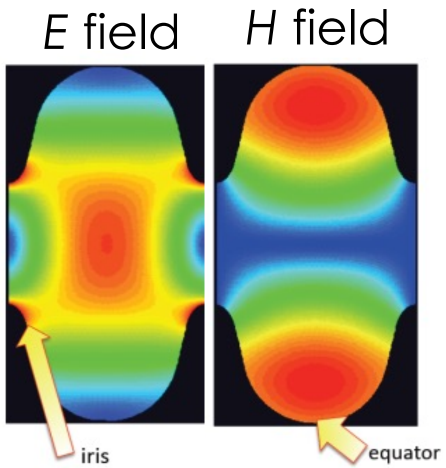
R_s



- For DC current passing through a conductor with certain surface area (*a* x *a*) and *d* thickness, with current perpendicular to cross section *a* x *d*, $R_{DC} = \frac{\rho a}{da} = \frac{\rho}{d}$
- For RF current, the RF field penetrates into the conductor by a characteristic length called skin depth δ, and $R_s = \sqrt{\frac{\mu_0 \omega}{2\sigma}} = \frac{1}{\sigma\delta} = \frac{\rho}{\delta}$
- For superconductors R_{DC} is zero and R_s is small but non-zero.
- R_s is material (and surface treatment of material) dependent and is frequency dependent because skin depth is frequency dependent.

RF loss

- RF loss can be calculated using transverse (x, y, in parallel with metal wall) E and/or H components (Poynting vector $P_{rf} = \frac{1}{2} Re \iint \boldsymbol{E_t} \times \boldsymbol{H_t}^* \cdot \boldsymbol{n} dS$) with n unit vector perpendicular to the metal surface S.
- $\boldsymbol{E_t} \times \boldsymbol{H_t}^* \cdot \boldsymbol{n} = \boldsymbol{n} \times \boldsymbol{E_t} \cdot \boldsymbol{H_t}^* = Z_s \boldsymbol{H_t} \cdot \boldsymbol{H_t}^*$
- RF loss is then calculated using $P_{rf} = \frac{1}{2} \iint |H_t|^2 R_s dS$.
- It can also be achieved using surface current $\boldsymbol{J_s} = \boldsymbol{n} \times \boldsymbol{H}$
- High RF power loss is associated with high H field NOT high E field.
- On the metal wall, considering the finite conductivity, we have $H_z \ll H_y$ & $E_z \gg E_x$ thus high H field contains mainly H_y and high E field mainly E_z.
- Moreover, we use "perturbation method" introduced previously to calculate the EM field, with E field perpendicular to, and H field in parallel with, metal (perfect conductor) wall. There is no transverse E component in this method, thus it cannot be used to calculate RF power loss.



Red high, blue low

Attenuation – metal wall loss

- Power loss per unit length $P_{loss} = \frac{1}{2}\left|H_\varphi\right|^2_{at\ a} R_s 2\pi a + \frac{1}{2}\left|H_\varphi\right|^2_{at\ b} R_s 2\pi b = \frac{A^2}{\mu} 2\pi R_s(\frac{1}{a} + \frac{1}{b})$, R_s the surface resistance of the metal wall.
- Attenuation α[Np] (per unit length) $= \frac{P_{loss}}{2P_t} = \frac{R_s}{2\eta} \frac{\frac{1}{a}+\frac{1}{b}}{ln\frac{b}{a}}$.

LMR®-1700: metal wall loss

- Try to calculate the attenuation of LMR®-1700 at 900MHz for 100m.
- At 900MHz, surface resistance of Cu 0.0077Ω, of Al 0.0097 Ω.
- For inner conductor and outer conductor with different material, it changes to $\frac{1}{2\eta}\frac{\frac{R_{S\_a}}{a}+\frac{R_{S\_b}}{b}}{ln\frac{b}{a}} = 0.0024\ \mathrm{Np}$, it corresponds to -2.08dB per 100m

Attenuation – dielectric loss

- Dielectric constant may contain an imaginary part which causes dielectric loss: $\varepsilon = \varepsilon' - j\varepsilon'' = \varepsilon'(1 - tan\delta)$, with loss tangent $tan\delta = \frac{\varepsilon''}{\varepsilon'}$ a number much smaller than 1 for dielectric material used in cables.
- Attenuation from dielectric loss can be calculated directly from propagation $e^{-j\beta z}$.
- For TEM, $k = \beta = \omega\sqrt{\mu\varepsilon} = \omega\sqrt{\mu\varepsilon'(1 - jtan\delta)} \approx \omega\sqrt{\mu\varepsilon'}(1 - jtan\delta/2)$, thus attenuation α[Np] (per unit length) $= \frac{1}{2}\omega\sqrt{\mu\varepsilon'}tan\delta$

LMR®-1700: dielectric loss

- $\frac{1}{2}\omega\sqrt{\mu\varepsilon'}tan\delta$ = 0.001 Np per meter for PE with loss tangent 0.0001, corresponds to -0.87dB per 100m.
- Total -2.95dB per 100m.
- Dielectric loss for 100m (-0.87dB) causes 18% power loss, metal wall loss for 100m (-2.08dB) causes 38% power loss, dielectric loss is ~50% of the wall loss – this is why for high power CW application, cable without (or with minimum) dielectric material is normally used, without dielectric, turbulent air cooling can also be provided.

https://www.megaind.com/coaxial/elbows/

Mega coaxial elbow

Average power

- Breakdown defines the peak power (NOT the average power) the cable can handle.
- The limiting factor that defines the average power is normally the power dissipation (per unit length) the cable can handle.
- We can experimentally measure the max average power under certain frequency, and then use the attenuation at that frequency to get the power dissipation (per unit length) the cable can handle.
- Detailed thermal (and sometimes thermal-mechanical-RF analysis) can be done to help understanding, and improving, the max average power.
- The average power is different for different frequencies.

Average power (2)

- Using LMR1700 as an example, at 900MHz, the attenuation is 3.1dB/100m (0.031dB/m), slightly higher than the number we calculated previously. The percentage of the power loss per meter is then 1-10^(-0.031/10)=0.71%
- (Assuming) experimentally, we demonstrated that the cable can handle 3.23kW power, the power loss per meter is then 3.23kW x 0.71% = 23W.
- Table below shows that the max average power is limited by the power dissipation the cable can handle, which is ~23W/m

Freq [MHz]	30	50	150	220	450	900	1500	1800	2000	2500
Attenuation [dB/100m]	0.5	0.6	1.1	1.4	2.1	3.1	4.1	4.6	4.9	5.7
Avg. Power [kW]	20.27	15.55	8.72	7.09	4.79	3.23	2.4	2.15	2.02	1.76
Power dissipation [W/m]	23.32	21.47	22.06	22.82	23.11	22.97	22.55	22.65	22.66	22.95

TEM 75Ω for Average Power/Low Loss

- Power loss per unit length $P_{loss} = \frac{1}{2}\left|H_\varphi\right|^2_{at\ a} R_s 2\pi a + \frac{1}{2}\left|H_\varphi\right|^2_{at\ b} R_s 2\pi b = \frac{A^2}{\mu} 2\pi R_s(\frac{1}{a} + \frac{1}{b})$, R_s the surface resistance of the metal wall.
- Attenuation α[Np] (per unit length) $= \frac{P_{loss}}{2P_t} = \frac{R_S}{2\eta}\frac{\frac{1}{a}+\frac{1}{b}}{ln\frac{b}{a}}$.
- $\left(\frac{\frac{1}{a}+\frac{1}{b}}{ln\frac{b}{a}}\right)' = -\frac{1}{a^2 ln\frac{b}{a}} + \frac{\frac{1}{a}+\frac{1}{b}}{\frac{b}{a}ln^2\frac{b}{a}}\frac{b}{a^2} = -\frac{1}{a^2 ln\frac{b}{a}} + \frac{\frac{1}{a}+\frac{1}{b}}{a ln^2\frac{b}{a}} = 0 \rightarrow ln\frac{b}{a} = 1 + \frac{a}{b}$, thus $\frac{b}{a} = 3.59$. Here ' is d/da.
- For a fixed size coaxial cable (fixed b value), minimum attenuation happens at $\frac{b}{a} = \sim 3.59$ and $Z_0 = \sim 76\Omega$.

TEM 50Ω, commonly used

- 50Ω coaxial cable is widely used as a compromise between maximum transmitted power and minimum attenuation.
- For a 50Ω coaxial cable, $\frac{b}{a} = \sim 2.3023$.