

Cryogenics : The cool part of accelerators.

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What and why?

Cryogenics: Science of producing and maintaining ultra cold temperatures.

Needed in accelerators for:

Superconducting components such as magnets and RF cavities

Lesser costs involved (Capital and operational) in commissioning superconducting machines

How?

Cryo R&D involves Thermodynamics, Fluid Mechanics and Heat Transfer

Managing heat loads (Static and Dynamic) are very real constraints on the feasibility of any design (RF/Magnet).

Studies are performed to solve heat transfer problems and to try and work around these constraints while keeping the physics performance in mind.

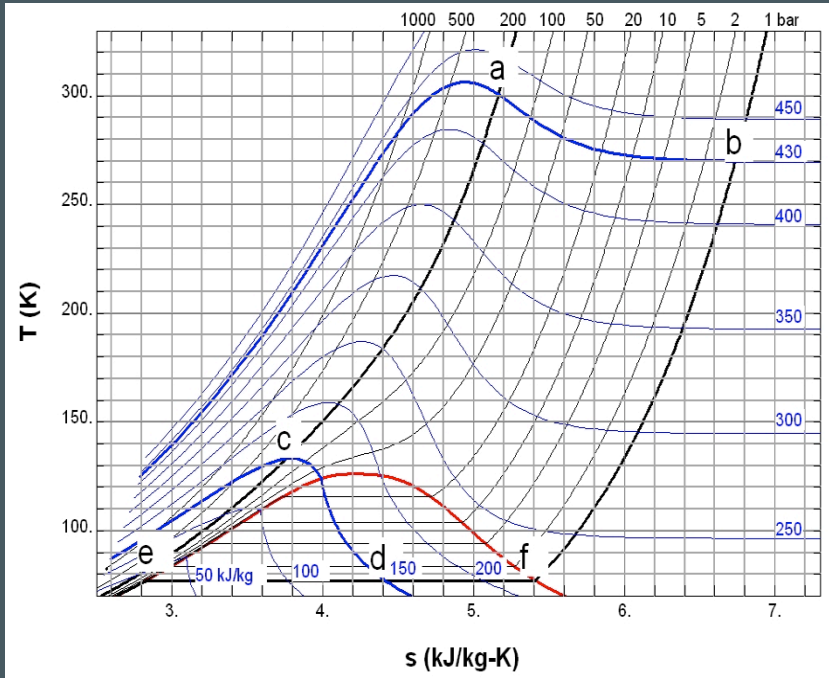
Thermodynamics

The laws of thermodynamics have to be satisfied for any realistic system design.

First and second law analysis are performed to come up with a feasible design of the refrigeration system used to cool the operational fluid to cryogenic temperatures.

A variety of refrigeration cycles can be employed with increasing complexity to improve the efficiency of the system.

How is cooling achieved?

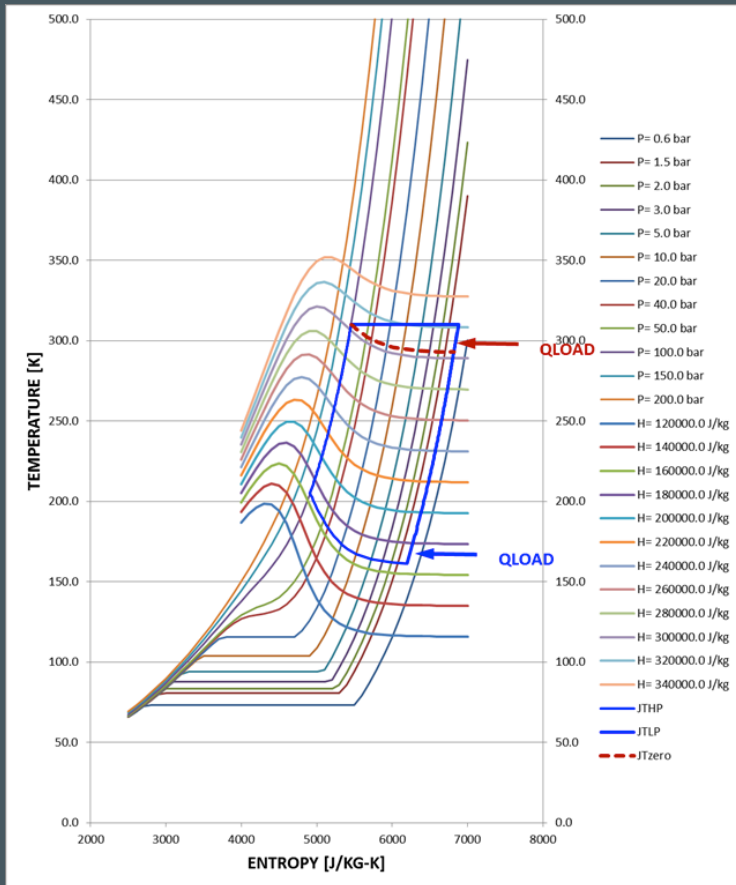


Applying basic energy conservation to an expansion device

$$Q_{\text{in}} + m(h_{\text{in}} + v_{\text{in}}^2/2 + gz_{\text{in}}) = W_{\text{out}} + m(h_{\text{out}} + v_{\text{out}}^2/2 + gz_{\text{out}})$$

There is no heat in and work done and changes in kinetic and potential energies are negligible which results in $h_1 = h_2$

Expansion is a constant enthalpy process



Thermodynamic properties of Fluids are available as Excel packages. (GASPAK and HePAK)

Entire cryogenic systems are “designed” in microsoft excel.

Closed Cycles. (He costs 22\$ / gallon)

Figure to the left shows a basic J-T expansion cycle with a HX stack to improve efficiency

Thermal Shields

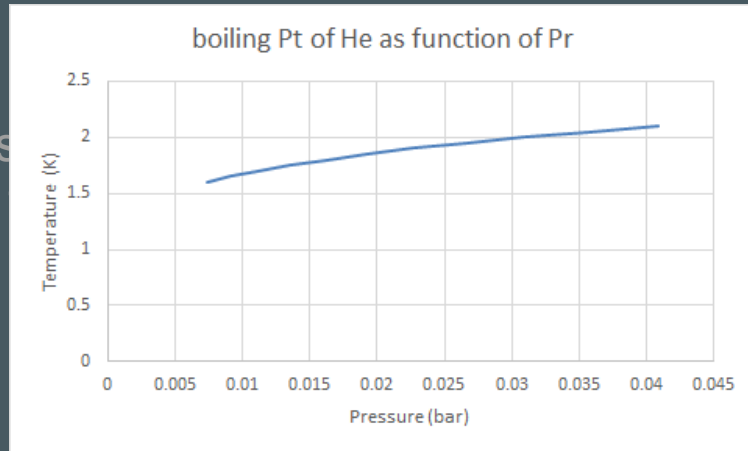
Colder than outer space

Boiling point of Liquid helium is set by the vapor pressure.

1 bar is atmospheric pressure.

2K boiling pt corresponds to a pressure of 30 mbar
Vapor pressure of evaporating He must correspond
30mbar if it should boil at 2K.

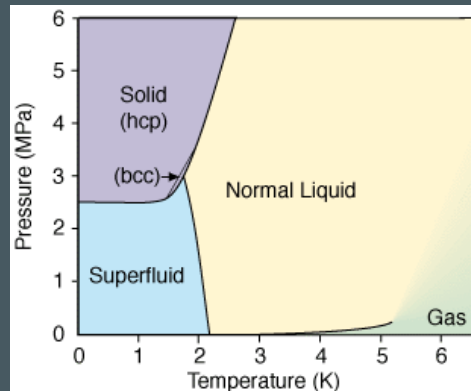
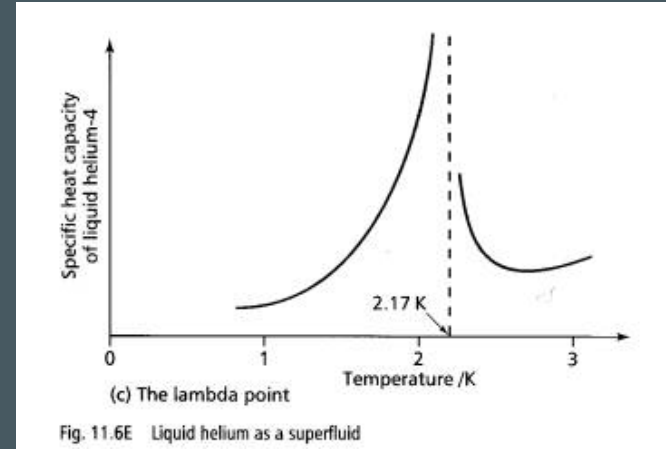
More fascinating things start to happen at these temperatures.



The curious case of Superfluid He

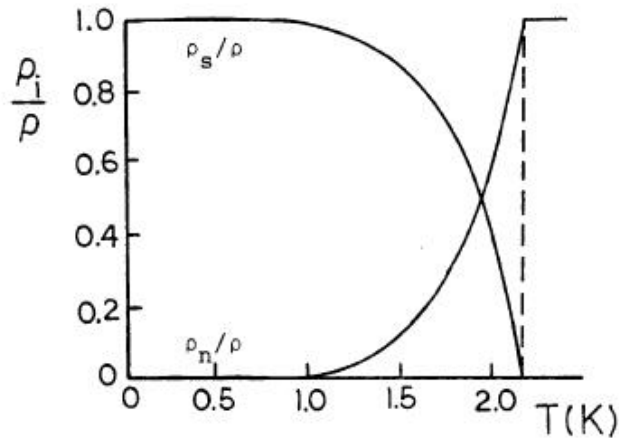
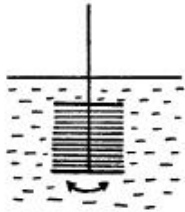


Why is the Helium boiling as it is being cooled?



Phase diagram
Of Helium

The two fluid model and heat transfer



Superfluid Helium exhibits remarkable thermal properties.

Hence used in cooling to 2K. (not that we have other choices.)

Superfluid has zero entropy! How is it so efficient in heat transfer?

Superfluid pressure transducers.

Heat Transfer

Three modes of heat transfer

Conduction

Convection

Radiation

Heat Transfer analysis is performed to estimate heat loads in accelerator structures such as Waveguides, RF cavities

Done to estimate heat loads that have to be handled by the cryogenic system

Conduction

Mainly involves solving the diffusion equation in 3D with various boundary conditions (to suit the problem at hand)

$$\text{Heat Diffusion Equation: } \nabla^2 T + \frac{\dot{q}}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t},$$

$$\text{where } \alpha = \frac{k}{\rho C_p V} \text{ is the thermal diffusivity}$$

Very hard to solve analytically especially when K is $f(T)$ and when there are multiple materials (Ex: Cu coated Stainless steel)

Numerical heat transfer

3 ways of discretizing the PDE

Finite difference, Finite element & Finite volume

Finite Difference Approximation

$$\text{Heat Diffusion Equation: } \nabla^2 T + \frac{\dot{q}}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t},$$

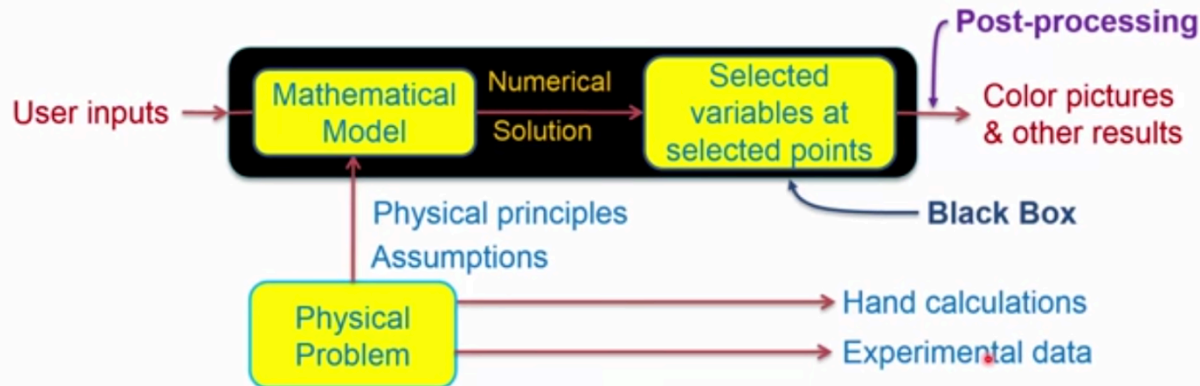
where $\alpha = \frac{k}{\rho C_p V}$ is the thermal diffusivity

No generation and steady state: $\dot{q}=0$ and $\frac{\partial}{\partial t} = 0, \Rightarrow \nabla^2 T = 0$

First, approximated the first order differentiation at intermediate points $(m+1/2, n)$ & $(m-1/2, n)$

$$\left. \frac{\partial T}{\partial x} \right|_{(m+1/2, n)} \approx \frac{\Delta T}{\Delta x} \bigg|_{(m+1/2, n)} = \frac{T_{m+1, n} - T_{m, n}}{\Delta x}$$

$$\left. \frac{\partial T}{\partial x} \right|_{(m-1/2, n)} \approx \frac{\Delta T}{\Delta x} \bigg|_{(m-1/2, n)} = \frac{T_{m, n} - T_{m-1, n}}{\Delta x}$$



Only

best!

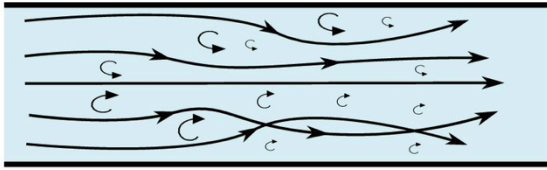
Fluid flow and convection

Fluid flow can be classified in many different ways. For forced convection, our main concern is whether it is laminar or turbulent.

laminar flow



turbulent flow

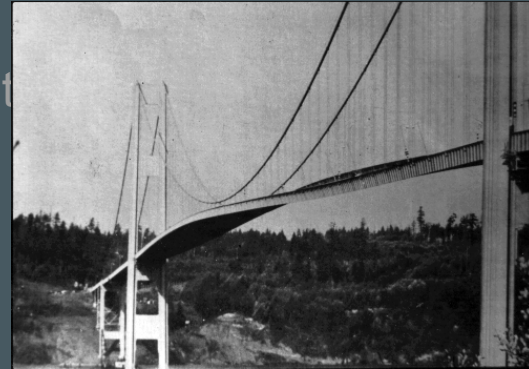


Turbulence is chaotic

Major characteristics are enhanced rates of momentum and energy diffusion

Downside for accelerator applications is that the pressure fluctuations induce structural vibrations

Vortex shedding and the plight
Of Tacoma Narrows bridge.



The Navier Stokes Equations



Navier-Stokes Equations 3 - dimensional - unsteady



Coordinates: (x,y,z) Time: t Density: ρ Pressure: p Reynolds Number: Re
Velocity Components: (u,v,w) Stress: τ Heat Flux: q Prandtl Number: Pr

Continuity:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

X - Momentum:

$$\frac{\partial(\rho u)}{\partial t} + \frac{\partial(\rho u^2)}{\partial x} + \frac{\partial(\rho uv)}{\partial y} + \frac{\partial(\rho uw)}{\partial z} = -\frac{\partial p}{\partial x} + \frac{1}{Re_r} \left[\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} \right]$$

Y - Momentum:

$$\frac{\partial(\rho v)}{\partial t} + \frac{\partial(\rho uv)}{\partial x} + \frac{\partial(\rho v^2)}{\partial y} + \frac{\partial(\rho vw)}{\partial z} = -\frac{\partial p}{\partial y} + \frac{1}{Re_r} \left[\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} \right]$$

Z - Momentum:

$$\frac{\partial(\rho w)}{\partial t} + \frac{\partial(\rho uw)}{\partial x} + \frac{\partial(\rho vw)}{\partial y} + \frac{\partial(\rho w^2)}{\partial z} = -\frac{\partial p}{\partial z} + \frac{1}{Re_r} \left[\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z} \right]$$

Total Energy - Et:

$$\begin{aligned} \frac{\partial(E_T)}{\partial t} + \frac{\partial(uE_T)}{\partial x} + \frac{\partial(vE_T)}{\partial y} + \frac{\partial(wE_T)}{\partial z} = & -\frac{\partial(up)}{\partial x} - \frac{\partial(vp)}{\partial y} - \frac{\partial(wp)}{\partial z} \\ & + \frac{1}{Re_r} \left[\frac{\partial}{\partial x} (u \tau_{xx} + v \tau_{xy} + w \tau_{xz}) + \frac{\partial}{\partial y} (u \tau_{xy} + v \tau_{yy} + w \tau_{yz}) + \frac{\partial}{\partial z} (u \tau_{xz} + v \tau_{yz} + w \tau_{zz}) \right] \\ & - \frac{1}{Re_r Pr_r} \left[\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z} \right] \end{aligned}$$

Remain unsolved in the 200 years since they were written.

1 of 7 problems for which the clay mathematical institute offers a prize 1 million dollars.

Problem is not even to solve the equations. it is to either prove it can be or cannot be solved.

Radiation

Radiation heat transfer is more geometry than heat transfer.

The Radiosity Matrix

The radiosity equation now looks like this:

$$B_j = E_j + \rho_j \sum_{i=1}^N B_i F_{ij}, \quad j = 1..N$$

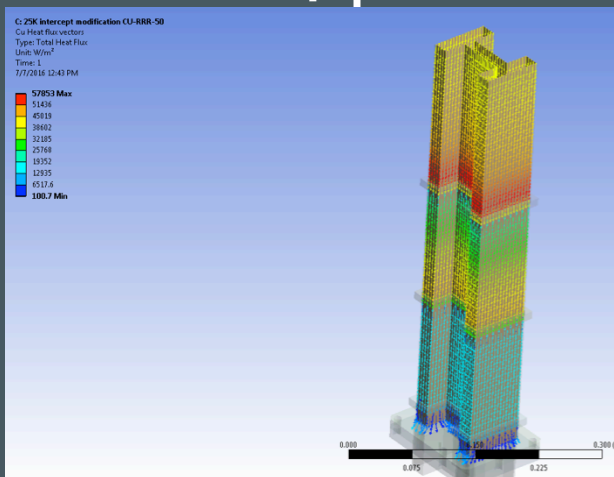
The derived radiosity equations form a set of N linear equations in N unknowns. This leads nicely to a matrix solution:

$$\begin{bmatrix} 1 - \rho_1 F_{11} & -\rho_1 F_{12} & \cdots & -\rho_1 F_{1N} \\ -\rho_2 F_{21} & 1 - \rho_2 F_{22} & \cdots & -\rho_2 F_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ -\rho_N F_{N1} & -\rho_N F_{N2} & \cdots & 1 - \rho_N F_{NN} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_N \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_N \end{bmatrix}$$

Sample form factor calc for a cylindrical geometry (beam pipe)



11

[illegible]

Temperature profiles along the length of the waveguide

total	ss	cu	total	ss	cu	total	ss	cu	total	ss	cu	total
	0.00057	9.30656E-06		0.00057	9.31E-06		0.00057	9.31E-06		0.00057	9.31E-06	
293.0 K to	293	293 293.0 K to	293	293 293.0 K to	293	293 293.0 K to	293	293 293.0 K to	293	293 293.0 K to	293	293 293.0 K to
	80	80	70	70	60	60	45	45	40	40	30	30
	2573.559	89389.89996	2651.347	95018.54	2721.811	101671.5	2811.128	115787.9	2835.841	122512.6	2876.591	140644.9

220.384	146.70	83.19	229.891	151.134	88.430	239.563	167.563	94.621	262.184	160.242	107.759	268.001
110.192	73.35	41.60	114.945	75.567	44.215	119.782	83.781	47.311	131.092	80.121	53.879	134.000
73.461	48.90	27.73	76.630	50.378	29.477	79.854	55.854	31.540	87.395	53.414	35.920	89.334
55.096	36.67	20.80	57.4									
44.077	29.34	16.64	45.9									
36.721	24.45	13.07	38.3									
31.483	20.96	11.88	32.8									
27.548	18.34	10.40	28.7									
24.487	16.30	9.24	25.5									
22.038	14.67	8.32	22.9									
20.025	13.34	7.56	20.8									
18.365	12.22	6.93	19.1									
16.953	11.28	6.40	17.6									
15.742	10.48	5.94	16.4									
14.692	9.78	5.55	15.3									
13.774	9.17	5.20	14.3									
12.944	8.63	4.89	13.5									
12.244	8.15	4.62	12.7									
11.599	7.72	4.38	12.1									
11.019	7.33	4.16	11.4									
10.494	6.99	3.96	10.9									
10.017	6.67	3.78	10.4									
9.582	6.38	3.62	9.9									
9.183	6.11	3.47	9.5									
8.815	5.87	3.33	9.1									
8.476	5.64	3.20	8.8									
8.162	5.43	3.08	8.5									
7.871	5.24	2.97	8.2									
7.599	5.06	2.87	7.9									
7.346	4.89	2.77	7.6									
7.109	4.73	2.68	7.4									
6.887	4.58	2.60	7.1									
6.678	4.45	2.52	6.9									
6.482	4.31	2.45	6.7									
6.297	4.19	2.38	6.5									
6.122	4.07	2.31	6.3									
5.956	3.96	2.25	6.2									
5.800	3.86	2.19	6.0									
5.651	3.76	2.13	5.8									
5.510	3.67	2.08	5.7									
5.375	3.59	2.03	5.607	3.686	2.157	5.843	4.087	2.308	6.395	3.908	2.628	6.537
5.247	3.48	1.98	5.474	3.598	2.105	5.704	3.950	2.253	6.242	3.815	2.566	6.381
5.126	3.38	1.93	5.351	3.516	2.057	5.571	3.807	2.200	6.093	3.722	2.505	6.233
5.011	3.29	1.88	5.228	3.435	2.012	5.442	3.668	2.150	5.948	3.637	2.450	6.091
4.901	3.20	1.83	5.105	3.356	1.968	5.317	3.533	2.102	5.808	3.552	2.400	5.953
4.795	3.12	1.78	4.982	3.280	1.926	5.195	3.402	2.056	5.672	3.469	2.353	5.820
4.693	3.04	1.73	4.860	3.206	1.885	5.075	3.274	2.011	5.540	3.389	2.308	5.691
4.594	2.96	1.68	4.739	3.134	1.845	4.958	3.150	1.967	5.410	3.311	2.264	5.566
4.500	2.88	1.63	4.618	3.063	1.805	4.843	3.028	1.924	5.282	3.234	2.221	5.444
4.410	2.80	1.58	4.497	2.993	1.766	4.730	2.908	1.882	5.157	3.159	2.179	5.324
4.324	2.72	1.53	4.376	2.924	1.728	4.619	2.789	1.841	5.034	3.085	2.138	5.206
4.241	2.64	1.48	4.257	2.856	1.690	4.510	2.671	1.799	4.913	3.012	2.097	5.090
4.161	2.56	1.43	4.139	2.789	1.652	4.402	2.554	1.758	4.794	2.940	2.057	4.975
4.083	2.48	1.38	4.022	2.723	1.614	4.295	2.438	1.717	4.677	2.869	2.017	4.861
4.007	2.40	1.33	3.906	2.657	1.576	4.189	2.322	1.676	4.561	2.798	1.977	4.748
3.933	2.32	1.28	3.791	2.592	1.538	4.084	2.207	1.635	4.446	2.728	1.937	4.635
3.861	2.24	1.23	3.676	2.527	1.500	3.980	2.092	1.594	4.332	2.658	1.897	4.522
3.791	2.16	1.18	3.562	2.462	1.462	3.877	1.977	1.553	4.219	2.588	1.857	4.410
3.722	2.08	1.13	3.449	2.397	1.424	3.774	1.862	1.512	4.107	2.518	1.817	4.298
3.654	2.00	1.08	3.336	2.332	1.386	3.671	1.747	1.471	3.995	2.448	1.777	4.186
3.587	1.92	1.03	3.223	2.267	1.348	3.568	1.632	1.430	3.883	2.378	1.737	4.074
3.521	1.84	0.98	3.110	2.202	1.310	3.465	1.517	1.389	3.771	2.308	1.697	3.962
3.455	1.76	0.93	3.000	2.137	1.272	3.362	1.402	1.348	3.660	2.238	1.657	3.850
3.390	1.68	0.88	2.890	2.072	1.234	3.259	1.287	1.307	3.550	2.168	1.617	3.738
3.325	1.60	0.83	2.781	2.007	1.196	3.156	1.172	1.266	3.440	2.098	1.577	3.626
3.260	1.52	0.78	2.672	1.942	1.158	3.053	1.057	1.225	3.330	2.028	1.537	3.514
3.195	1.44	0.73	2.563	1.877	1.120	2.950	0.942	1.184	3.220	1.958	1.497	3.402
3.130	1.36	0.68	2.454	1.812	1.082	2.847	0.827	1.143	3.110	1.888	1.457	3.290
3.065	1.28	0.63	2.345	1.747	1.044	2.744	0.712	1.102	3.000	1.818	1.417	3.178
3.000	1.20	0.58	2.236	1.682	1.006	2.641	0.597	1.061	2.890	1.748	1.377	3.066
2.935	1.12	0.53	2.127	1.617	0.968	2.538	0.482	1.020	2.780	1.678	1.337	2.954
2.870	1.04	0.48	2.018	1.552	0.930	2.435	0.367	0.979	2.670	1.608	1.297	2.842
2.805	1.00	0.46	1.909	1.487	0.892	2.332	0.252	0.938	2.560	1.538	1.257	2.730
2.740	0.96	0.44	1.800	1.422	0.854	2.229	0.137	0.897	2.450	1.468	1.217	2.618
2.675	0.92	0.42	1.691	1.357	0.816	2.126	0.022	0.856	2.340	1.398	1.177	2.506
2.610	0.88	0.40	1.582	1.292	0.778	2.023	-0.093	0.815	2.230	1.328	1.137	2.394
2.545	0.84	0.38	1.473	1.227	0.740	1.920	-0.208	0.774	2.120	1.258	1.097	2.282
2.480	0.80	0.36	1.364	1.162	0.702	1.817	-0.323	0.733	2.010	1.188	1.057	2.170
2.415	0.76	0.34	1.255	1.097	0.664	1.714	-0.438	0.692	1.900	1.118	1.017	2.058
2.350	0.72	0.32	1.146	1.032	0.626	1.611	-0.553	0.651	1.790	1.048	0.977	1.946
2.285	0.68	0.30	1.037	0.967	0.588	1.508	-0.668	0.610	1.680	0.978	0.937	1.834
2.220	0.64	0.28	0.928	0.902	0.550	1.405	-0.783	0.569	1.570	0.908	0.897	1.722
2.155	0.60	0.26	0.819	0.837	0.512	1.302	-0.898	0.528	1.460	0.838	0.857	1.610
2.090	0.56	0.24	0.710	0.772	0.474	1.199	-1.013	0.487	1.350	0.768	0.817	1.498
2.025	0.52	0.22	0.601	0.707	0.436	1.096	-1.128	0.446	1.240	0.698	0.777	1.386
1.960	0.48	0.20	0.492	0.642	0.398	0.993	-1.243	0.405	1.130	0.628	0.737	1.274
1.895	0.44	0.18	0.383	0.577	0.360	0.890	-1.358	0.364	1.020	0.558	0.697	1.162
1.830	0.40	0.16	0.274	0.512	0.322	0.787	-1.473	0.323	0.910	0.488	0.657	1.050
1.765	0.36	0.14	0.165	0.447	0.284	0.684	-1.588	0.282	0.800	0.418	0.617	0.938
1.700	0.32	0.12	0.056	0.382	0.246	0.581	-1.703	0.241	0.690	0.348	0.577	0.826
1.635	0.28	0.10	-0.053	0.317	0.208	0.478	-1.818	0.200	0.580	0.278	0.537	0.714
1.570	0.24	0.08	-0.162	0.252	0.170	0.375	-1.933	0.159	0.470	0.208	0.497	0.602
1.505	0.20	0.06	-0.271	0.187	0.132	0.272	-2.048	0.118	0.360	0.138	0.457	0.490
1.440	0.16	0.04	-0.380	0.122	0.094	0.169	-2.163	0.077	0.250	0.068	0.417	0.378
1.375	0.12	0.02	-0.489	0.057	0.056	0.066	-2.278	0.036	0.140	-0.002	0.377	0.266
1.310	0.08	0.00	-0.598	-0.008	0.018	-0.037	-2.393	-0.009	0.030	-0.072	0.337	0.154
1.245	0.04	0.00	-0.707	-0.073	-0.020	-0.146	-2.508	-0.049	-0.080	-0.142	0.297	0.042
1.180	0.00	0.00	-0.816	-0.138	-0.042	-0.255	-2.623	-0.089	-0.170	-0.216	0.257	-0.070
1.115	0.00	0.00	-0.925	-0.203	-0.064	-0.364	-2.738	-0.129	-0.260	-0.286	0.217	-0.158
1.050	0.00	0.00	-1.034	-0.268	-0.086	-0.473	-2.853	-0.169	-0.350	-0.356	0.177	-0.246
0.985	0.00	0.00	-1.143	-0.333	-0.108	-0.582	-2.968	-0.209	-0.440	-0.426	0.137	-0.334
0.920	0.00	0										