Cryogenics : The cool part of accelerators.

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Dhananjay K Ravikumar
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What and why?

Cryogenics: Science of producing and maintaining ultra cold temperatures.

Needed in accelerators for:

Superconducting components such as magnets and RF cavities

Lesser costs involved (Capital and operational) in commissioning superconducting machines

How?

Cryo R&D involves Thermodynamics, Fluid Mechanics and Heat Transfer

Managing heat loads (Static and Dynamic) are very real constraints on the feasibility of any design (RF/Magnet).

Studies are performed to solve heat transfer problems and to try and work around these constraints while keeping the physics performance in mind.

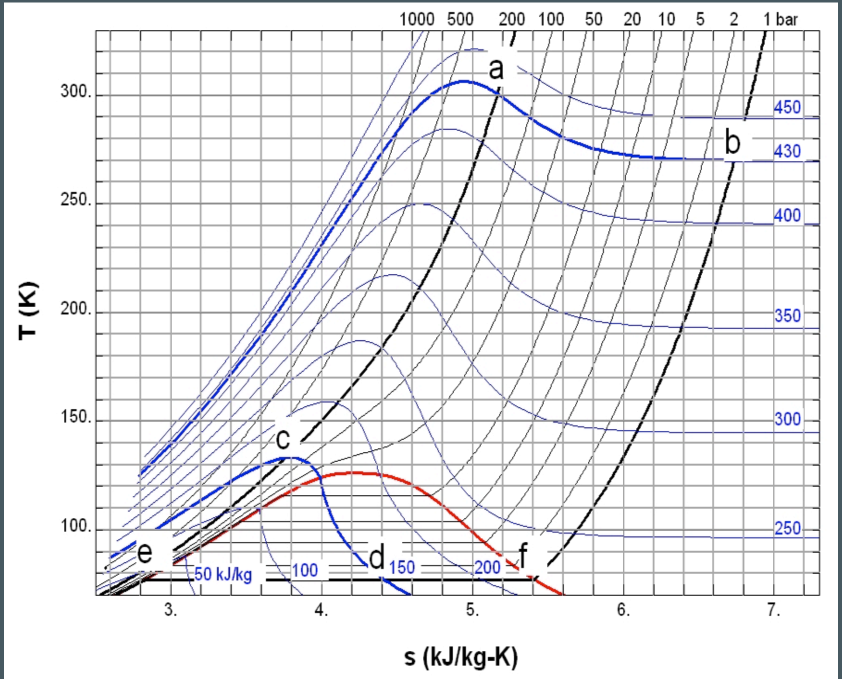
Thermodynamics

The laws of thermodynamics have to be satisfied for any realistic system design.

First and second law analysis are performed to come up with a feasible design of the refrigeration system used to cool the operational fluid to cryogenic temperatures.

A variety of refrigeration cycles can be employed with increasing complexity to improve the efficiency of the system.

How is cooling achieved?

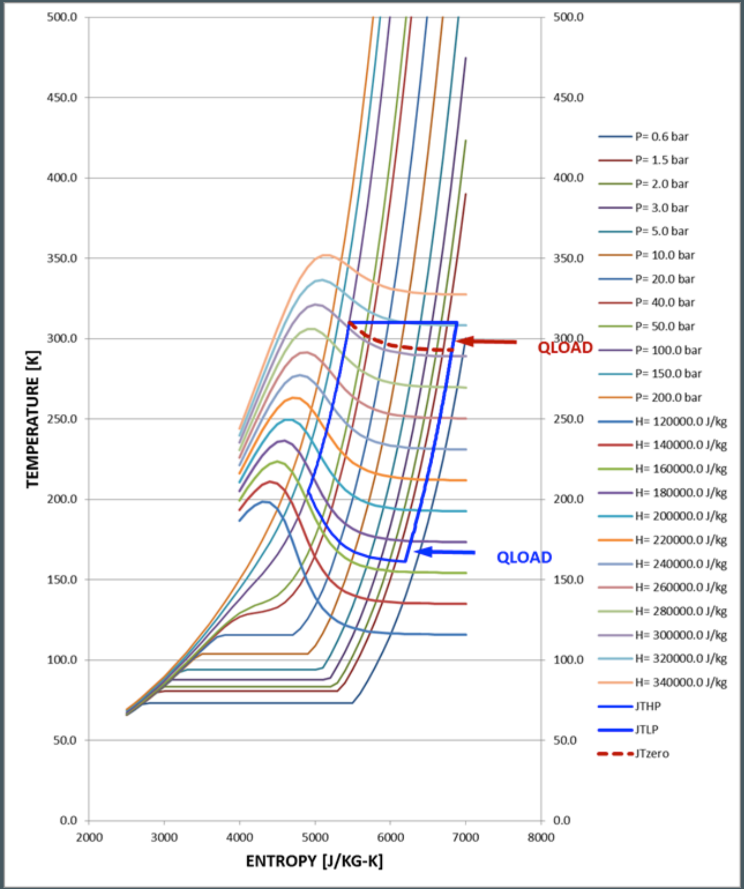


Applying basic energy conservation to an expansion device

$$Q_{in} + m(h_{in} + v_{in}^2/2 + gz_{in}) = W_{out} + m(h_{out} + v_{out}^2/2 + gz_{out})$$

There is no heat in and work done and changes in kinetic and potential energies are negligible which results in $h_1 = h_2$

Expansion is a constant enthalpy process



Thermodynamic properties of Fluids are available as Excel packages. (GASPAK and HePAK)

Entire cryogenic systems are “designed” in microsoft excel.

Closed Cycles. (He costs 22$ / gallon)

Figure to the left shows a basic J-T expansion cycle with a HX stack to improve efficiency

Thermal Shields

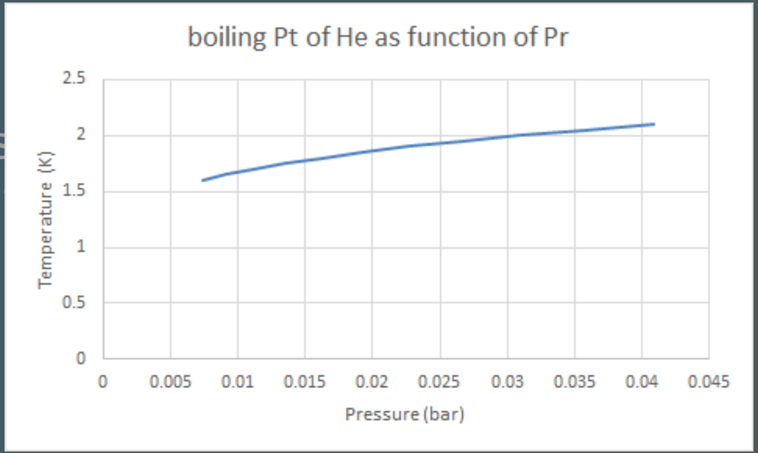
Colder than outer space

Boiling point of Liquid helium is set by the vapor pressure.

1 bar is atmospheric pressure.

2K boiling pt corresponds to a pressure of 30 mbar s
Vapor pressure of evaporating He must correspond
30mbar if it should boil at 2K.

More fascinating things start to happen at these temperatures.



The curious case of Superfluid He

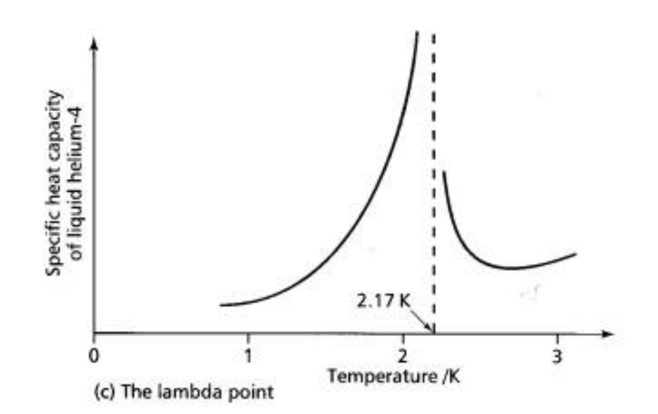
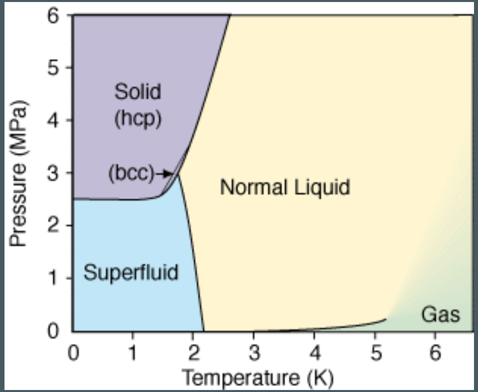


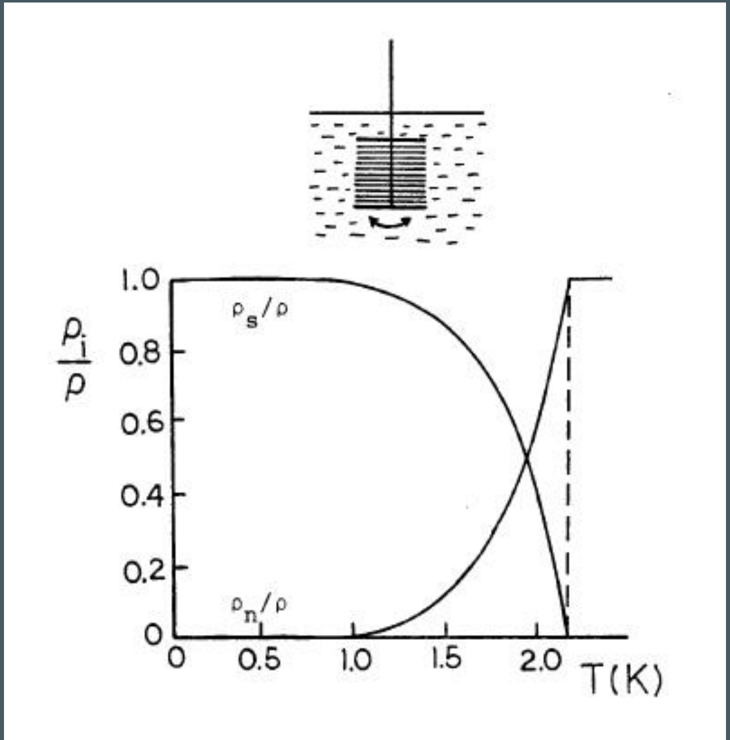
Fig. 11.6E Liquid helium as a superfluid



Why is the Helium boiling as
it is being cooled?

Phase diagram
Of Helium

The two fluid model and heat transfer



Superfluid Helium exhibits remarkable thermal properties.

Hence used in cooling to 2K. (not that we have other choices.)

Superfluid has zero entropy! How is it so efficient in heat transfer?

Superfluid pressure transducers.

Heat Transfer

Three modes of heat transfer

- Conduction
- Convection
- Radiation

Heat Transfer analysis is performed to estimate heat loads in accelerator structures such as Waveguides, RF cavities

Done to estimate heat loads that have to be handled by the cryogenic system

Conduction

Mainly involves solving the diffusion equation in 3D with various boundary conditions (to suit the problem at hand)

Heat Diffusion Equation: $\nabla^2 T + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$,

where $\alpha = \frac{k}{\rho C_P V}$ is the thermal diffusivity

Very hard to solve analytically especially when K is f (T) and when there are multiple materials (Ex: Cu coated Stainless steel)

Numerical heat transfer

3 ways of discretizing the PDE

Finite difference, Finite element & Finite volume

Finite Difference Approximation

Heat Diffusion Equation: $\nabla^2 T + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$,

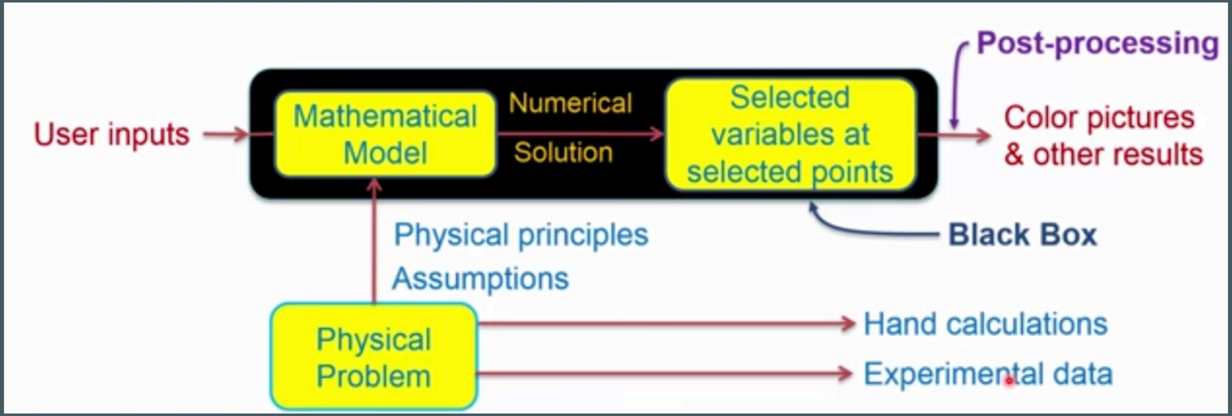
where $\alpha = \frac{\mathrm{k}}{\rho C_P V}$ is the thermal diffusivity

No generation and steady state: $\dot{q}=0$ and $\frac{\partial}{\partial t} = 0, \Rightarrow \nabla^2 T = 0$

First, approximated the first order differentiation at intermediate points (m+1/2,n) & (m-1/2,n)

$$\left.\frac{\partial T}{\partial x}\right|_{(m+1/2,n)} \approx \left.\frac{\Delta T}{\Delta x}\right|_{(m+1/2,n)} = \frac{T_{m+1,n} - T_{m,n}}{\Delta x}$$

$$\left.\frac{\partial T}{\partial x}\right|_{(m-1/2,n)} \approx \left.\frac{\Delta T}{\Delta x}\right|_{(m-1/2,n)} = \frac{T_{m,n} - T_{m-1,n}}{\Delta x}$$

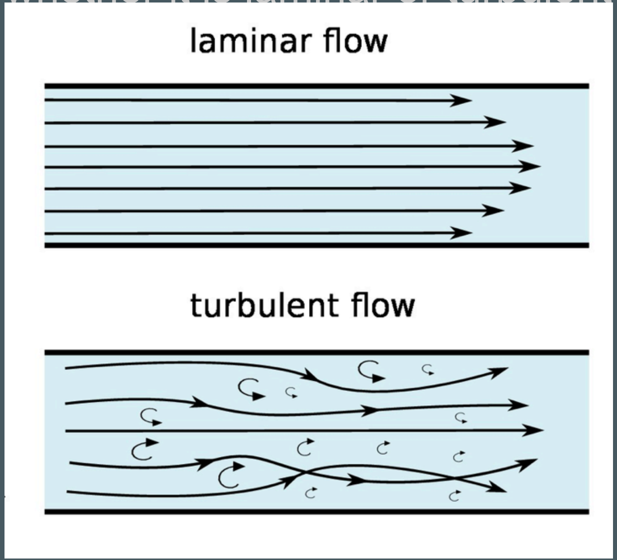


Only

best!

Fluid flow and convection

Fluid flow can be classified in many different ways. For forced convection , our main concern is whether it is laminar or turbulent



Turbulence is chaotic

Major characteristics are enhanced rates of momentum and energy diffusion

Downside for accelerator applications is that the pressure fluctuations induce structural vibrations

Vortex shedding and the plight Of Tacoma Narrows bridge.

The Navier Stokes Equations

Navier–Stokes Equations
3 – dimensional – unsteady

Coordinates: (x,y,z) Time : t Density: ρ Pressure: p Reynolds Number: Re
Velocity Components: (u,v,w) Stress: τ Heat Flux: q Prandtl Number: Pr

Continuity:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

X – Momentum:

$$\frac{\partial(\rho u)}{\partial t} + \frac{\partial(\rho u^2)}{\partial x} + \frac{\partial(\rho uv)}{\partial y} + \frac{\partial(\rho uw)}{\partial z} = -\frac{\partial p}{\partial x} + \frac{1}{Re_r}\left[\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z}\right]$$

Y – Momentum:

$$\frac{\partial(\rho v)}{\partial t} + \frac{\partial(\rho uv)}{\partial x} + \frac{\partial(\rho v^2)}{\partial y} + \frac{\partial(\rho vw)}{\partial z} = -\frac{\partial p}{\partial y} + \frac{1}{Re_r}\left[\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{yz}}{\partial z}\right]$$

Z – Momentum:

$$\frac{\partial(\rho w)}{\partial t} + \frac{\partial(\rho uw)}{\partial x} + \frac{\partial(\rho vw)}{\partial y} + \frac{\partial(\rho w^2)}{\partial z} = -\frac{\partial p}{\partial z} + \frac{1}{Re_r}\left[\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z}\right]$$

Total Energy – Et:

$$\frac{\partial(E_T)}{\partial t} + \frac{\partial(uE_T)}{\partial x} + \frac{\partial(vE_T)}{\partial y} + \frac{\partial(wE_T)}{\partial z} = -\frac{\partial(up)}{\partial x} - \frac{\partial(vp)}{\partial y} - \frac{\partial(wp)}{\partial z}$$

$$+ \frac{1}{Re_r}\left[\frac{\partial}{\partial x}(u\,\tau_{xx} + v\,\tau_{xy} + w\,\tau_{xz}) + \frac{\partial}{\partial y}(u\,\tau_{xy} + v\,\tau_{yy} + w\,\tau_{yz}) + \frac{\partial}{\partial z}(u\,\tau_{xz} + v\,\tau_{yz} + w\,\tau_{zz})\right]$$

$$- \frac{1}{Re_r Pr_r}\left[\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z}\right]$$

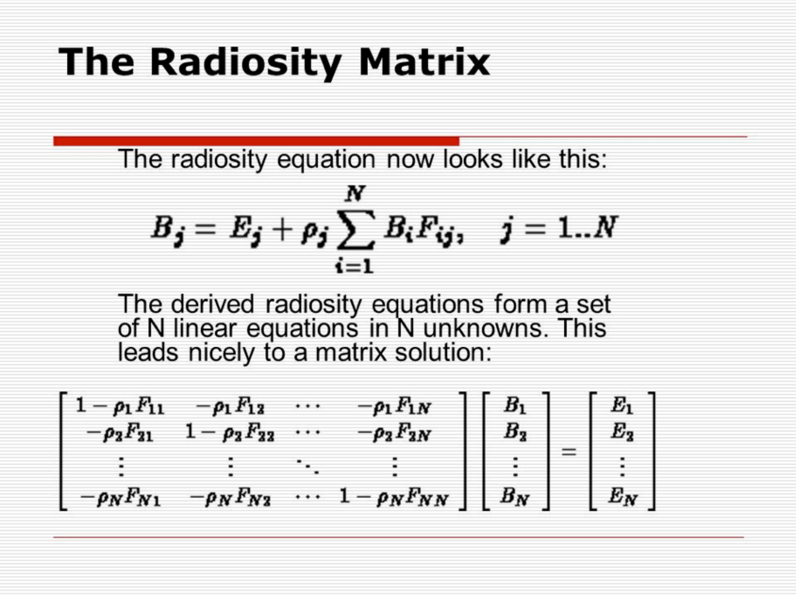
Remain unsolved in the 200 years since they were written.

1 of 7 problems for which the clay mathematical institute offers a prize 1 million dollars.

Problem is not even to solve the equations. it is to either prove it can be or cannot be solved.

Radiation

Radiation heat transfer is more geometry than heat transfer.



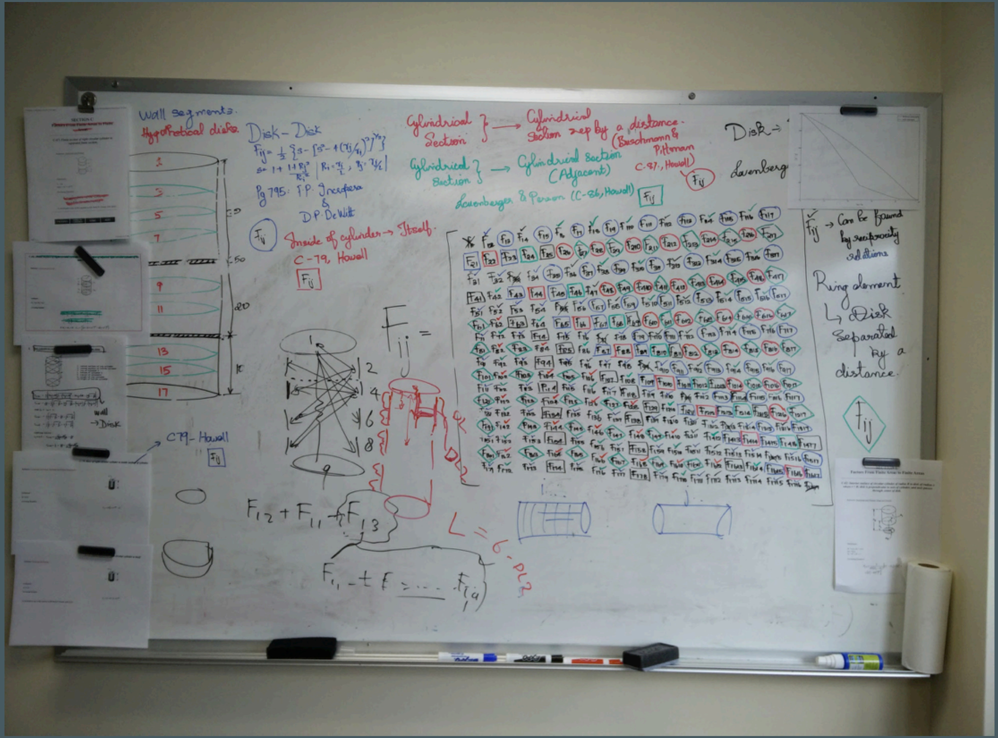
The Radiosity Matrix

The radiosity equation now looks like this:

$$B_j = E_j + \rho_j \sum_{i=1}^{N} B_i F_{ij}, \quad j = 1..N$$

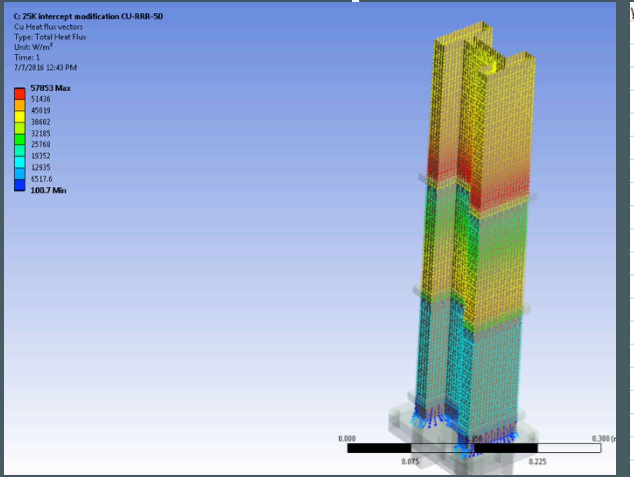
The derived radiosity equations form a set of N linear equations in N unknowns. This leads nicely to a matrix solution:

$$\begin{bmatrix} 1-\rho_1 F_{11} & -\rho_1 F_{12} & \cdots & -\rho_1 F_{1N} \\ -\rho_2 F_{21} & 1-\rho_2 F_{22} & \cdots & -\rho_2 F_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ -\rho_N F_{N1} & -\rho_N F_{N2} & \cdots & 1-\rho_N F_{NN} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_N \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_N \end{bmatrix}$$



Sample form factor calc for a cylindrical geometry (beam pipe)

Case study - Thermal analysis for the HOM



y location on the waveguide	Temperature (K)	H actual	Rho (T) Ohm-M	Rho_surface(T) (Ohm)	Q/A (W/m^2)	A = 0.013692, Q =	h'
0	2	100	8E-10	0.001432494	7.162468247	0.124197199	-0.021836793
0.03	4.05	76.931	8E-10	0.001432494	4.239019995	0.073504607	-0.013005123
0.06	4.969	58.86	8E-10	0.001432494	2.481436838	0.043028115	-0.007634462
0.09	6.5015	45.1	8E-10	0.001432494	1.456853204	0.025261835	-0.004503431
0.12	9.425	34.52	8E-10	0.001432494	0.85350149	0.014799716	-0.002662408
0.15	13.571	26.44	8E-10	0.001432494	0.500709246	0.008682298	-0.001582375
0.18	17.541	20.25	8.10164E-10	0.001441565	0.295565842	0.005125112	-0.000945935
0.21	21.333	15.5	8.3333E-10	0.00146203	0.175626328	0.003045361	-0.000568983
0.24	25.005	11.85	8.7009E-10	0.001493928	0.104890585	0.001818803	-0.000343909
0.27	33.484	9.07	1.05755E-09	0.001647019	0.067745922	0.001174714	-0.000228473
0.3	45.758	6.96	1.43032E-09	0.001915421	0.046393037	0.000804455	-0.000163217
0.33	57.8	5.323	1.912E-09	0.002214581	0.03137434	0.000544031	-0.000115262
0.36	69.603	4.9	2.57618E-09	0.002570608	0.030860146	0.000535115	-0.000118512
0.39	81.085	3.1171	3.3651E-09	0.002937966	0.014273096	0.000247495	-5.73412E-05
0.42	97.405	2.3915	4.7405E-09	0.003487063	0.009971731	0.00017291	-4.28716E-05
0.45	132.72	1.83	6.9632E-09	0.00422622	0.007076595	0.000122708	-3.58708E-05
0.48	167.51	1.4	9.11068E-09	0.004834183	0.0047375	8.21482E-05	-2.91556E-05
0.51	201.69	1.071	1.1169E-08	0.005352478	0.003069756	5.32296E-05	-2.39245E-05
0.54	235.2	0.81	1.4216E-08	0.006038602	0.001980963	3.43499E-05	-2.08962E-05
0.57	267.97	0.6291	1.71376E-08	0.00663014	0.001311995	2.275E-05	-2.1151E-05
0.6	300	0.4813	0.00000002	0.007162468	0.000829592	1.43851E-05	-2.76531E-05
						0.303271336	

Temperature profiles along the length of the waveguide