

1. The energy loss per turn is given by

$$U_0 = \frac{e^2 \beta^3 \gamma^4}{3\epsilon_0 \rho} . \quad (1)$$

With $\rho = 2.22m$ and $\gamma = 1GeV / 0.511MeV = 1957$, eq. (1) yields

$$U_0 = \frac{e^2 \beta^3 \gamma^4}{3\epsilon_0 \rho} = 39.6KeV = 6.33 \times 10^{-15} J . \quad (2)$$

The critical photon energy is given by

$$E_c = \hbar \omega_c , \quad (3)$$

where \hbar is the denoted Planck constant and

$$\omega_c = \frac{3}{2} \gamma^3 \frac{c}{\rho} \approx 1.512 \times 10^{18} \text{ rad / s} \quad (4)$$

is the critical angular frequency of the synchrotron radiation. Inserting eq. (4) into eq. (3) yields

$$E_c \approx 0.996KeV = 1.594 \times 10^{-16} J . \quad (5)$$

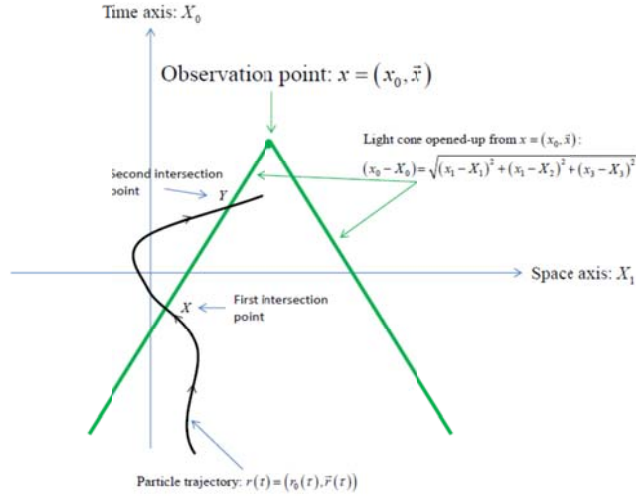
The total synchrotron radiation power for a beam is given by the 1-turn energy loss of all particles in the ring divided by the time it takes for one circulation (i.e. the revolution period)

$$P_{beam} = \left(U_0 \cdot N_{ring} \right) \frac{1}{T_{rev}} = \left(U_0 \cdot \frac{I_b T_{rev}}{e} \right) \frac{1}{T_{rev}} = U_0 \frac{I_b}{e} . \quad (6)$$

where $N_{ring} = I_b T_{rev} / e$ is the total number of electrons in the ring. Inserting eq. (2) and $I_b = 200mA$ into eq. (6) give

$$P_{beam} \approx 7.91KW . \quad (7)$$

2.



Since the two intersection points are on the light-cone opened-up by $x = (x_0, \vec{x})$, they satisfy the following equation:

$$(x_0 - X_0) - \sqrt{(X_1 - x_1)^2 + (X_2 - x_2)^2 + (X_3 - x_3)^2} = 0, \quad (8)$$

and

$$(x_0 - Y_0) - \sqrt{(Y_1 - x_1)^2 + (Y_2 - x_2)^2 + (Y_3 - x_3)^2} = 0. \quad (9)$$

Subtracting eq. (9) with eq. (8) yields

$$Y_0 - X_0 = \sqrt{(X_1 - x_1)^2 + (X_2 - x_2)^2 + (X_3 - x_3)^2} - \sqrt{(Y_1 - x_1)^2 + (Y_2 - x_2)^2 + (Y_3 - x_3)^2}. \quad (10)$$

The three points \vec{x} , \vec{X} and \vec{Y} form a triangle and since the difference in the length of any two sides of a triangle is always smaller than the length of the third side, it follows from eq. (10)

$$Y_0 - X_0 \leq \sqrt{(X_1 - Y_1)^2 + (X_2 - Y_2)^2 + (X_3 - Y_3)^2}. \quad (11)$$

The time it takes for the particle to get from \vec{X} to \vec{Y} is given by

$$\Delta t = \frac{Y_0 - X_0}{c}, \quad (12)$$

and hence the average velocity of the particle during its travelling from \vec{X} to \vec{Y} is

$$\langle v_{particle} \rangle = \frac{\sqrt{(X_1 - Y_1)^2 + (X_2 - Y_2)^2 + (X_3 - Y_3)^2}}{\Delta t} = \frac{c\sqrt{(X_1 - Y_1)^2 + (X_2 - Y_2)^2 + (X_3 - Y_3)^2}}{Y_0 - X_0} . \quad (13)$$

According to eq. (11) , the following relation holds

$$\frac{\sqrt{(X_1 - Y_1)^2 + (X_2 - Y_2)^2 + (X_3 - Y_3)^2}}{Y_0 - X_0} \geq 1 , \quad (14)$$

and inserting eq. (14) into eq. (13) yields

$$\langle v_{particle} \rangle = c \frac{\sqrt{(X_1 - Y_1)^2 + (X_2 - Y_2)^2 + (X_3 - Y_3)^2}}{Y_0 - X_0} \geq c . \quad (15)$$

Eq. (15) violates special relativity and hence the trajectory of a particle cannot intersect a light-cone twice.

3. The angular distribution of radiation power is given by

$$\frac{dP(t_r)}{d\Omega} = \frac{1}{4\pi\epsilon_0} \frac{e^2}{4\pi c} \frac{\dot{\beta}^2}{(1-\beta\cos\theta)^3} \left[1 - \frac{\sin^2\theta\cos^2\phi}{\gamma^2(1-\beta\cos\theta)^2} \right]. \quad (1)$$

For $\frac{1}{\gamma^4} \ll \theta \ll 1$ and $\gamma \gg 1$, we can use the following approximation

$$\begin{aligned} 1 - \beta\cos\theta &\approx 1 - \beta \left(1 - \frac{1}{2}\theta^2 \right) \\ &= 1 - \beta + \frac{1}{2}\beta\theta^2 \\ &= \frac{1}{\gamma^2(1+\beta)} + \frac{1}{2}\theta^2 \\ &= \frac{1}{\gamma^2} \left[\frac{1}{2-(1-\beta)} \right] + \frac{1}{2}\theta^2, \quad (2) \\ &\approx \frac{1}{2\gamma^2} \left[1 + \frac{1-\beta}{2} \right] + \frac{1}{2}\theta^2 \\ &\approx \frac{1}{2\gamma^2} \left[1 + \frac{1}{4\gamma^2} + \dots \right] + \frac{1}{2}\theta^2 \\ &\approx \frac{1}{2\gamma^2} + \frac{1}{2}\theta^2 \end{aligned}$$

and eq. (1) becomes

$$\frac{dP(t_r)}{d\Omega} \approx \frac{1}{4\pi\epsilon_0} \frac{2e^2}{\pi c} \frac{\gamma^6\dot{\beta}^2}{(1+\gamma^2\theta^2)^3} \left[1 - \frac{4\gamma^2\theta^2\cos^2\phi}{(1+\gamma^2\theta^2)^2} \right]. \quad (3)$$

Since the factor inside the square bracket is between 0 and 1, the angular width of eq. (3) is determined by the factor $(1+\gamma^2\theta^2)^{-3}$, i.e. the radiation power drops substantially when $\theta \geq \frac{1}{\gamma}$.