Homework 1
Due: *Friday, February 11, 2022*

1. In class, we calculated the derivatives in a new comoving coordinate (ξ, t'):

$$\frac{\partial}{\partial t} \rightarrow \frac{\partial}{\partial t'} + c\frac{\partial}{\partial \xi}$$
$$\frac{\partial}{\partial z} \rightarrow -\frac{\partial}{\partial \xi}$$

It is more useful sometimes to use the coordinates (ξ, z') instead of (ξ, t'). Calculate how $\partial/\partial z$ and $\partial/\partial t$ are expressed in this new coordinate. What would be the expression for the quasi-static approximation in the new coordinate system (recall, in the one we worked out in class, it is $\partial_{t'} \ll c\partial_\xi$)?

2. In class, we derived $\vec{p}_\perp(\xi)$ and $\vec{p}_z(\xi)$ in a plane wave. In this problem we want to test these equations. Consider a Gaussian pulse, given by

$$a_0 = a_{00}\exp\left[-\frac{(\xi - \xi_0)^2}{2(c\tau)^2}\right].$$

Experimentally, even a Gaussian pulse approximates a plane wave near the peak if the pulse length is much longer than an oscillation period, i.e. $c\tau >> 1/k_0$, where $k_0 = 2\pi/\lambda$ is the wavenumber.
In this example, we want to look at a typical 100 fs Ti:sapphire laser, where $\lambda \sim 1\ \mu m$, $\sqrt{2}\tau = 100$ fs and choose the peak at the origin of ξ axis, i.e. $\xi_0 = 0$;

First, let's get a bit familiar with this laser

(a) Calculate the laser period in fs & show that $c\tau >> 1/k_0$

Then, use a programming language of your choice to compare the motion of an electron in these fields for a weak laser ($a_0 = 0.01$) and a relativistic laser ($a_0 = 4$). You can start from the momentum equations and numerically integrate

(b) Compare the relative motion of $k_0 x$ and $k_0 z$ (as a function of $k_0\xi$) in magnitude and frequency
(c) Compare the relative strength, frequency and phase of p_x/mc and p_z/mc
(d) Show that the drift velocity is equal to $\frac{a_0}{4(1+a_0^4/4)}$. *Hint: you can use the few cycles near the top of the laser pulse to find the drift velocity.*