

1491 **Chapter 4**
1492 **Relativistic Cyclotron**

1493 **Abstract** This Chapter is a brief introduction to the relativistic cyclotron accelerator,
1494 hands-on: by numerical simulation. It begins with a brief reminder of the historical
1495 context, and introduces the theoretical material needed for the simulation exercises.
1496 The Chapter relies on the basic charged particle optics and acceleration concepts
1497 introduced in Chapter 3, and further addresses
1498 - Thomas focusing and the AVF cyclotron,
1499 - positive focusing index,
1500 - isochronous optics,
1501 - separated sector cyclotrons,
1502 - spin dynamics in an AVF cyclotron.
1503 Simulations use optical elements met earlier: TOSCA, DIPOLE, CAVITE, SPNTRK,
1504 etc. They further develop on the modeling of sector dipoles, edge focusing and flutter,
1505 isochronous optics, separated sector ring cyclotrons, and simulation of these type of
1506 optics using DIPOLE, DIPOLES and CYCLOTRON keywords.

1507 Notations used in the Text

$B; B_0$	field value; at reference radius R_0
$\mathbf{B}; B_R; B_y$	field vector; radial component; axial component
$B\rho = p/q; B\rho_0$	particle rigidity; reference rigidity
$C; C_0$	orbit length, $C = 2\pi R$; reference, $C_0 = 2\pi R_0$
E	particle energy
EFB	Effective Field Boundary
f_{rf}	RF frequency
h	RF harmonic number
$k = \frac{R}{B} \frac{dB}{dR}$	geometric index, a global quantity
$n = \frac{\rho}{B} \frac{dB}{d\rho}$	focusing index, a local quantity
$m; m_0; M$	mass; rest mass; in units of MeV/c^2
$\mathbf{p}; p_0$	particle momentum vector; reference momentum
q	particle charge
R	orbital radius
$R_0; R_E$	Reference radius at a reference energy; at energy E
s	path variable
v	particle velocity
$V(t); \hat{V}$	oscillating voltage; its peak value
x, x', y, y'	radial and axial coordinates in moving frame
$\beta = v/c; \beta_0; \beta_s$	normalized particle velocity; reference; synchronous
$\gamma = E/m_0$	Lorentz relativistic factor
$\Delta p, \delta p$	momentum offset
ϵ_R	strength of a depolarizing resonance
ϵ_u	Courant-Snyder invariant (u: x, r, y, l, Y, Z, s, etc.)
$\phi; \phi_s$	particle phase at voltage gap; synchronous phase

1509 Introduction

1510 The loss of isochronism due to the relativistic increase of the mass limits the en-
 1511 ergy reach of the classical cyclotron. Vertical focusing in the latter imposes on the
 1512 other hand slowly decreasing field with radius, *i.e.*, a negative index $-1 < k < 0$.
 1513 Isochronism requires instead an increasing field with radius, field index $k > 0$, a
 1514 consequence of $B(R) \propto \gamma(R)$, a property addressed below. The classical cyclotron
 1515 technology eventually culminated with the construction at Berkeley in 1946 of a
 1516 184 in, 4000 ton cyclotron - soon turned into a synchrocyclotron, following the
 1517 discovery of longitudinal phase stability.

1518 Thomas azimuthally varying field (AVF) concept (Fig. 4.1), introduced in
 1519 1938¹ [3], allowed overcoming that conflict between vertical focusing and isochro-

¹ That was eight years after the Nature article on spin precession [2].

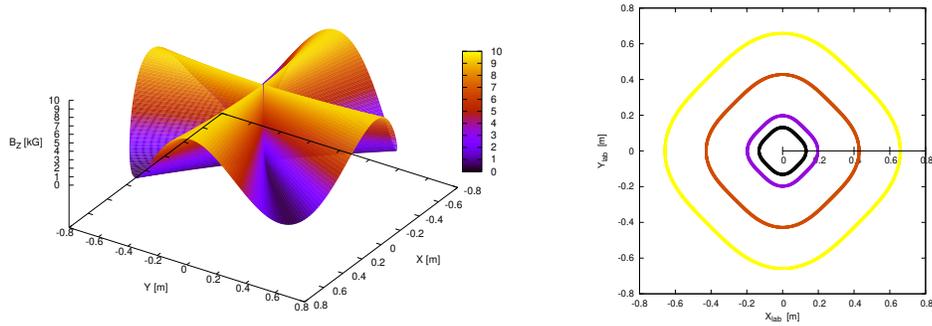
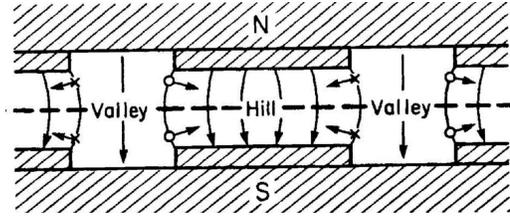


Fig. 4.1 Left: mid-plane magnetic field in a 4-periodic AVF cyclotron. Right: closed orbits around the cyclotron feature azimuthally varying curvature, greater on the hills, weaker in the field valleys

Fig. 4.2 Vertical azimuthal cross cut of an AVF cyclotron, showing the field hills (H) and valleys (V) [1]. Markers represent the radial component of the velocity vector, pointing alternately away from (cross) and toward the reader (circle); the resulting cross product with the magnetic field at the transition between the H and V regions is (positive charge is assumed) a force toward the median plane (arrows)



1520 nism, as an AVF results in vertical focusing (Fig. 4.2), or equivalently in a correction
1521 of the vertical wave number, namely

$$v_y = \sqrt{-k + F^2} = \sqrt{-\beta^2\gamma^2 + F^2} \quad (4.1)$$

1522 with F a parameter quantifying the field modulation (Eq. 4.8): strong enough field
1523 modulation results in $F > \beta\gamma$. Spiral pole geometry (Eq. 4.11, Fig. 4.9) came on the
1524 scene in 1954 [4], it allows increasing the axial focusing and thus higher energies.
1525 The first AVF cyclotron was operated in 1957, tens followed within the next decade,
1526 accelerating all sorts of ions, polarized beams, allowing variable energy [7], and
1527 application in a variety of domains as material science, radiobiology, production
1528 of beams of secondary particles. The concept of field sectors in the AVF cyclotron
1529 eventually led to the separated sector cyclotron in 1963: drifts between magnets
1530 allow room for higher efficiency extraction systems and thus higher beam current, for
1531 high-Q RF resonators resulting in greater turn separation, so facilitating the design of
1532 multiple stage high energy installations, and for beam monitoring instrumentation.
1533 Cyclotron energy and size subsequently increased, up to the present days GeV

1534 range. Cryogeny joined the scene in 1962, with the Michigan State University K500
 1535 superconducting coil cyclotron [6], allowing higher field and reduction of size.
 1536 Radioisotope production and proton therapy applications have seen a quick increase
 1537 in the number of AVF cyclotrons in the recent years.

Table 4.1 A comparison between an AVF and a separated sector cyclotron of same energy, 72 MeV (namely, former Injector 1 and present Injector 2 of PSI high power cyclotron, after Ref. [7, p. 126])

		AVF	separated sector
Injection energy	keV	14	870
Extraction energy	MeV	72	72
Magnet		single dipole	4 sectors
Weight	ton	470	4 × 180
Gap	mm	240 to 450	35
$\langle B \rangle$; B_{\max}	T	1.6; 2	0.36; 1.1
RF system		180° dees	2 resonators
Max accelerating voltage	kV	2 × 70	4 × 250
RF frequency	MHz	50	50
Normalized beam emittance, hor.; vert.	μm	2.4; 1.2	1.2; 1.2
Beam phase width	deg	16 - 40	12
Energy spread	%	0.3	0.2
Turn separation at extraction	mm	3	18

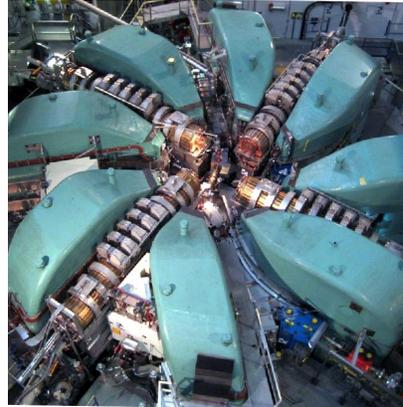


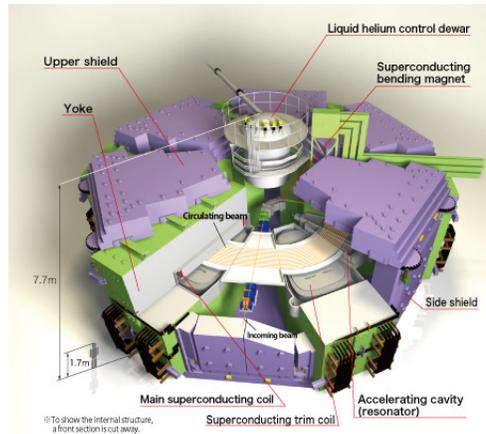
Fig. 4.3 PSI 590 MeV, 1.4 MW ring cyclotron, 15 m diameter. Proton acceleration takes ~180 turns. Beam extraction efficiency 99.99%. Delivers beams for secondary particle production including neutrons and muon beams [8]

1538 To this day, thousands of cyclotrons have been built, tens are produced each
 1539 year, applications include proton therapy, production of radio-isotopes, high power
 1540 proton beams, secondary particle beams. Cryogeny and high fields further allow
 1541 compactness (Fig. 4.4) [9] and higher rigidities (Fig. 4.5) [10]

Fig. 4.4 COMET at PSI. A 250 MeV, 500 nA, 4-sector isochronous AVF cyclotron, the spiral poles ensure vertical focusing. A 3 m diameter superconducting-coil provides the field. Delivers beams for proton therapy [9]



Fig. 4.5 RIKEN K1300, superconducting coil, separated-sector, ion cyclotron, 3.8 T field and 8 T m rigidity, 18.4 m in diameter, 8300 ton. Beam injection radius 3.56 m, extraction radius 5.36 m [11]



1542 4.1 Theory, Basic Concepts

1543 In the classical cyclotron, the relativistic increase of the mass slows down particles
 1544 as energy increases, causing loss of synchronism, and the necessary negative field
 1545 index for vertical focusing adds to that. Instead, constant $\omega_{\text{rev}} = qB/\gamma m_0$, given
 1546 $R = \beta c/\omega_{\text{rev}}$, leads to

$$k = \frac{R}{B} \frac{\partial B}{\partial R} = \frac{\beta}{\gamma} \frac{\partial \gamma}{\partial \beta} = \beta^2 \gamma^2 \quad (4.2)$$

1547 Thus isochronism of revolution motion requires k to be positive and follow the energy
 1548 increase: the weak focusing condition $-1 < k < 0$ can not be satisfied, transverse
 1549 periodic stability is lost.

1550 The revolution period on the equilibrium orbit, momentum $p = qBR$ and cir-
 1551 cumference C , is $T_{\text{rev}} = C/\beta c = 2\pi\gamma m_0/qB$. Isochronism requires p -independent
 1552 revolution period, $dT_{\text{rev}}/dp = 0$. Differentiating the previous expression, this yields

$$B(R) = \frac{B_0}{\gamma_0} \gamma(R) \quad (4.3)$$

1553 with B_0 and γ_0 some reference conditions,

1554 This led H.A. Bethe and M.E. Rose to conclude [12] “... *it seems useless to build*
 1555 *cyclotrons of larger proportions than the existing ones... an accelerating chamber*
 1556 *of 37 cm radius will suffice to produce deuterons of 11 MeV energy which is the*
 1557 *highest possible...*”. Frank Cole : “If you went to graduate school in the 1940s, this
 1558 inequality [$-1 < k < 0$] was the end of the discussion of accelerator theory.”

1559 Until...

1560 4.1.1 Thomas Focusing

1561 “It is shown in detail below that a variation of magnetic field proportional to $\cos 4\theta$,
 1562 [...] admits a stable family of closed orbits that are approximately circles about
 1563 the center of the cyclotron. Thus a variation of the magnetic field with angle, [...] *and*
 1564 *of order of magnitude v/c ; together with nearly the radial increase of relative*
 1565 *amount $\frac{1}{2}v^2/c^2$ of Bethe and Rose; gives stable orbits that are in resonance and*
 1566 *not defocused.” [3]; this was the transition to the isochronous cyclotron in 1938,
 1567 with the introduction by L.H. Thomas of the concept of polar variation of the axial
 1568 field, the “AVF” (Azimuthally Varying Field) cyclotron (Fig. 4.6). Off mid-plane,
 1569 the radial component of the velocity vector bends the trajectories toward the median
 plane (Fig. 4.2); this is sketched in the hard edge model in Fig. 4.7.*

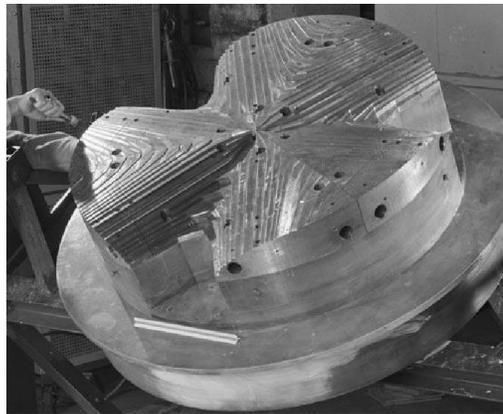
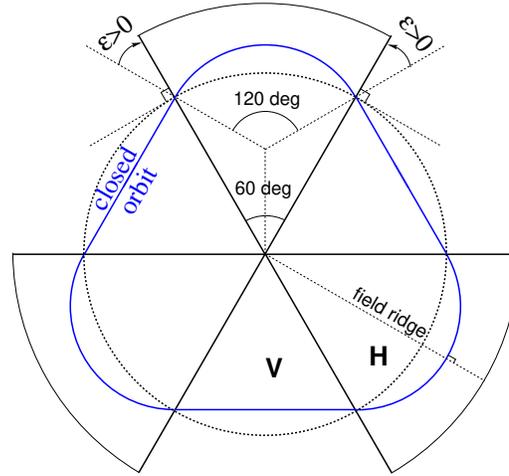


Fig. 4.6 Azimuthal pole shaping in an AVF cyclotron, an electron model, here [1]. The focusing pattern is FfFfFf, an alternation of strong (hill regions) and weak (valleys) radial focusing

1570 AVF axial focusing is introduced in proper amount to compensate the axial
 1571 defocusing effect resulting from k following $\beta^2\gamma^2$ (Eq. 4.2), which in turn ensures
 1572 $T_{\text{rev}}=\text{constant}$. This is still today the technology of cyclotrons in the few tens of
 1573

Fig. 4.7 Sketch of a 3-periodic, 60 deg radial sector AVF cyclotron, in a hard-edge model (zero field between the poles, azimuthally constant field across the poles). The closed orbit is at an $\varepsilon = 30^\circ$ “wedge angle” from the sector edges. The wedge is closing, so introducing vertical focusing, and weakening the horizontal focusing. Note that the closed orbit is normal to the field ridge in the hill region, by symmetry



1574 MeV range, for isotope production mostly, and up to 200 MeV and beyond for proton
1575 therapy application.

1576 The single-magnet concept of the classical cyclotron remains, however the mag-
1577 net pole is shaped to introduce a $2\pi/N$ -periodical field modulation (Figs. 4.4, 4.6).
1578 A convenient analytical approach consists in assuming a sinusoidal azimuthal de-
1579 pendence of the field, writing

$$\mathcal{F}(\theta) = 1 + f \sin(N\theta) \quad (4.4)$$

1580 This modulation of the field results in a so-called “scalloping” of the orbit, around
1581 the reference circle (Fig. 4.7).

1582 The necessary radial increase of the field for preserving the isochronism of the
1583 orbits in the relativistic regime (Eq. 4.3) may be obtained by introducing a radial
1584 dependence of the pole profile (Fig. 4.6). The mid-plane field can thus be expressed
1585 under the form

$$B(R, \theta) = B_0 \mathcal{R}(R) \mathcal{F}(\theta) \quad (4.5)$$

1586 The orbit curvature varies along the $\frac{2\pi}{N}$ -periodic orbit. This requires distinguishing
1587 between the local focusing index

$$n = \frac{\rho(s)}{B(s)} \frac{dB}{d\rho} \quad (4.6)$$

1588 and a geometrical index

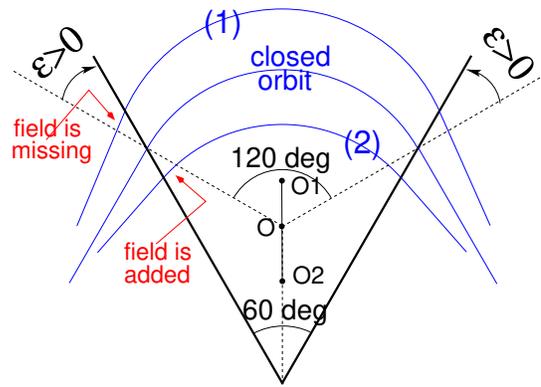
$$k = \frac{R}{B} \frac{dB}{dR} \quad (4.7)$$

1589 a global quantity which determines two others, namely the wave numbers of the
1590 transverse oscillatory particle motion (Eq. 4.10).

1591 **Edge Focusing**

1592 Edge focusing is a local effect of the entrance and exit ends of a dipole. In the AVF
 1593 cyclotron the wedge angle is positive (Fig.4.8), first order horizontal defocusing
 1594 results. First order vertical focusing results from the angle of the particle velocity
 1595 vector to the azimuthal component of the field at magnet edge; a first order correction
 1596 has to be added to account for the fringe field extent. More on edge focusing can be
 1597 found in Section 18.4.3.

Fig. 4.8 A 120 deg bending of the closed orbit (curvature center at O) is ensured by a 60 deg sector dipole. This results in $\varepsilon = 30$ deg “wedge angles” at the entrance and exit face of the 60 deg sector dipole. The angle of the trajectories to the azimuthal field component results in vertical focusing of off mid-plane trajectories. The wedge angle causes a decrease in the horizontal focusing: trajectory (1) (curvature center at O1) is bent less due to missing field, trajectory (2) (curvature center at O2) is bent more due to added field



1598 A “flutter” factor, F, can be introduced to quantify the focusing effect of the
 1599 azimuthal modulation of the field. For a given orbit, of average radius $R = C/2\pi$
 1600 and local curvature $\rho(s)$, it writes

$$F = \left(\frac{\langle (\mathcal{F} - \langle \mathcal{F} \rangle)^2 \rangle}{\langle \mathcal{F} \rangle^2} \right)^{1/2} \xrightarrow{\text{hard edge}} \left(\frac{R}{\rho} - 1 \right)^{1/2} \quad (4.8)$$

1601 wherein $\langle * \rangle = \oint (*) d\theta / 2\pi$. R/ρ is the value reached in the limit where the field
 1602 is null in the valleys, azimuthally constant in the hill regions (hard-edge sector field
 1603 model). If the scalloping of the orbit is small, *i.e.*, $C/2\pi \approx \rho$, and accounting for the
 1604 isochronism condition $k = \beta^2 \gamma^2$ then

$$v_R \approx \sqrt{1+k} = \gamma, \quad v_y \approx \sqrt{-k+F^2} \stackrel{\text{isochr.}}{=} \sqrt{-\beta^2 \gamma^2 + F^2} \quad (4.9)$$

$$v_R^2 + v_y^2 = 1 + F^2 \xrightarrow{\text{hard edge}} \frac{R}{\rho}$$

1605 The flutter results in $-k + F^2 > 0$ (whereas $k > 0$ as B increases with R to ensure
1606 isochronism) thus the vertical motion features periodic stability.

1607 Note that in the hypothesis of sinusoidal azimuthal field modulation of Eq. 4.4,
1608 one has $F = f/\sqrt{2}$ and

$$v_y \approx \sqrt{-k + f^2/2}, \quad v_R^2 + v_y^2 = 1 + f^2/2 \quad (4.10)$$

1609 4.1.2 Spiral Sector

1610 Spiral sector geometry was introduced in 1954 in the context of FFAG studies [4], and
1611 found application in cyclotrons (as seen in the COMET cyclotron, Fig. 4.4). Spiraling
1612 the edges results in stronger vertical focusing (Eq. 4.13) compared to a radial sector
1613 (Eq. 4.10), it also permits an increase of the wedge angle with momentum (*i.e.*, with
1614 the radius, Fig. 4.9), so maintaining proper compensation of the increase of $k(R)$.
1615 A convenient analytical approach consists in assuming an azimuthally sinusoidal
1616 modulation

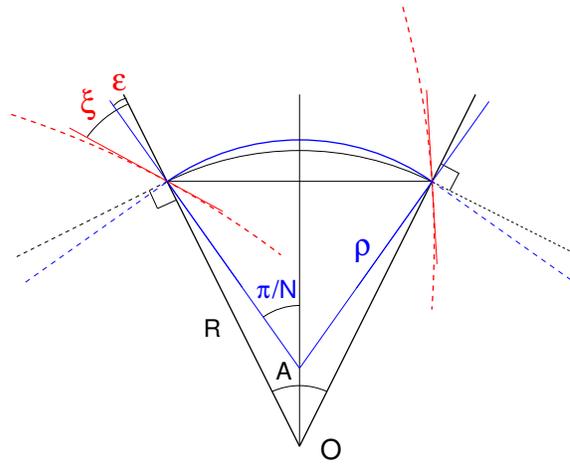
$$\mathcal{F}(R, \theta) = 1 + f \sin \left[N \left(\theta - \tan(\xi) \ln \frac{R}{R_0} \right) \right] \quad (4.11)$$

1617 whereas the mid-plane field writes under the form

$$B(R, \theta) = B_0 \mathcal{R}(R) \mathcal{F}(R, \theta) \quad (4.12)$$

The magnet edge geometry is given by (Fig. 4.9)

Fig. 4.9 Geometrical parameters of a $2\pi/N = A$ sector dipole of a N-sector cyclotron. 'O' is the center of the ring. The dashed lines figure the edges in the case of a spiral sector with spiral angle ξ at radius R (ξ increases with radius). A $p = qB\rho$ momentum closed orbit follows an arc with local curvature radius $\rho(\theta)$ (field varies along the arc). In the hard edge field model, an R-radius circle is a line of constant field inside the sector, null outside



$$r = r_0 \exp(\theta/\tan(\xi))$$

1618 a logarithmic spiral centered at the center of the ring, with ξ the spiral angle, the
 1619 angle that the tangent to the spiral edge does with the ring radius. This results in a
 1620 larger contribution of the flutter term in the vertical wave number,

$$\nu_y = \sqrt{-k + F^2(1 + 2 \tan^2 \xi)} \quad (4.13)$$

1621 Since the field index k increases, to ensure isochronism (Eq. 4.2), the spiral angle
 1622 has to follow, increase with radius, so to maintain $-k + F^2(1 + 2 \tan^2 \xi) > 0$. A
 1623 limitation here is the maximum spiral angle achievable: $\xi \rightarrow 90$ deg means magnet
 1624 edge going parallel to the closed orbit!

1625 As an illustration, in TRIUMF cyclotron in the 500 MeV region, ξ increases to
 1626 72 deg (from null in the 100 MeV region) whereas $1 + 2 \tan^2 \xi$ increases to 20 (from
 1627 1 in the 100 MeV region) and compensates a low $F < 0.07$ (down from $F = 0.3$).

1628 Most isochronous cyclotrons over a few tens of MeV use spiraled sectors [1], to
 1629 benefit from the more efficient vertical focusing.

1630 4.1.3 Separated Sector Cyclotron

1631 The separated sector ring cyclotron came on the scene in the early 1960s. PSI
 590 MeV ring is an example of a separated spiral sector lattice (Fig. 4.3). Its 70 MeV



Fig. 4.10 PSI injector II, four separated radial sectors, 0.87 MeV injection energy, accelerates protons to 72 MeV in 100 turns. The drifts house 50.7 MHz RF and a flattop cavity. Injection is from the top, in the central region

1632 injector is another example, a radial sector lattice (Fig. 4.10). The separated dipole
 1633 structure is akin to the mid-1950s separated sector radial and spiral FFAGs (Chap-
 1634 ter 10) (often considered part of the cyclotron family). The sector dipoles are sepa-
 1635 rated by iron-free spaces - not necessarily field-free owing to fringe fields.
 1636

1637 High energy cyclotrons in general use spiral sector optics, as it allows stronger
 1638 vertical focusing resulting from the spiral angle. For instance, ξ reaches 72° at
 1639 505 MeV in TRIUMF H^- 520 MeV cyclotron (from zero in the 100 MeV range); ξ
 1640 reaches 35° in PSI 500 MeV H^+ cyclotron.

1641 The limit in energy of the separated sector cyclotron resides in the achievable
 1642 field strength, magnet and overall cyclotron size, acceleration rate and achievable
 1643 beam separation between the last two turns for extraction.

1644 Fringe Field

1645 In separated sector cyclotrons, the flutter factor (Eqs. 4.4, 4.11) which characterizes
 1646 the azimuthal modulation of the field (Eqs. 4.5, 4.12) arises from the limited az-
 1647 imuthal extent of the dipoles and resulting fringe fields. A convenient model for the
 1648 latter is (see Sec. 18.5)

$$F(d) = \frac{1}{1 + \exp P(d)} \quad (4.14)$$

$$P(d) = C_0 + C_1 \left(\frac{d}{\lambda}\right) + C_2 \left(\frac{d}{\lambda}\right)^2 + C_3 \left(\frac{d}{\lambda}\right)^3 + C_4 \left(\frac{d}{\lambda}\right)^4 + C_5 \left(\frac{d}{\lambda}\right)^5$$

1649 For the record, d is the distance to the magnet EFB, λ is a length which characterizes
 1650 the extent of the fringe field (about the height of the dipole gap). The numerical
 1651 values of $C_0 - C_5$ coefficients are determined so to match the fringe field shape
 1652 (*e.g.*, constant in a simpler model, R -dependent in a fancier model accounting for
 1653 R -dependence of the fringe field shape and extent).

1654 4.1.4 Isochronism

In the hypothesis of isochronism, the revolution angular frequency satisfies

$$\omega_{\text{rev}} = \frac{c\beta(\gamma)}{R(\gamma)} = \text{constant}$$

1655 An orbital radius $R_\infty = c/\omega_{\text{rev}}$ is reached asymptotically as $\beta = v/c = R/R_\infty \rightarrow 1$.
 1656 In terms of the RF frequency and harmonic number,

$$R_\infty = h \frac{c}{\omega_{\text{rf}}} \quad (4.15)$$

1657 Given $BR_\infty = \gamma m_0 c/q$ and using $\gamma = (1 - (R/R_\infty)^2)^{-1/2}$, the field on the orbit can
 1658 be expressed in terms of R_∞ , namely,

$$B(R) = \gamma B_0 = \frac{B_0}{\sqrt{1 - (R/R_\infty)^2}} \quad \text{with } B_0 = \frac{m_0 \omega_{\text{rev}}}{q} = \frac{m_0}{q} \frac{\omega_{\text{rf}}}{h} \quad (4.16)$$

1659 and goes to infinity with $R \rightarrow R_\infty$.

1660 For protons, , with $m_0 = 1.6726 \times 10^{-27}$ kg, $q = 1.6021 \times 10^{-27}$ C, $BR_\infty [T m] =$
 1661 $\gamma m_0 c / q = 3.1\gamma$. A typical value for R_∞ can be obtained assuming for instance an
 1662 upper $\gamma = 1.64$ (600 MeV) in a region of upper field value $B = 1.64$ T, yielding
 1663 $R_\infty \approx 3.1$ m.

1664 *Radial field law*

1665 From Eq, 4.16 it results that the radial field form factor of Eqs. 4.5, 4.12 can be
 1666 written

$$\mathcal{R}(R) = \left(1 - \left(\frac{R}{R_\infty} \right)^2 \right)^{-1/2} \quad (4.17)$$

1667 with $R_\infty = c/\omega_{\text{rev}}$. A possible approach consists in using the Taylor expansion of
 1668 $\mathcal{R}(R)$ (within the limits of convergence of that series), namely

$$\mathcal{R}(R) = 1 + \frac{1}{2} \left(\frac{R}{R_\infty} \right)^2 + \frac{3}{8} \left(\frac{R}{R_\infty} \right)^4 + \frac{5}{16} \left(\frac{R}{R_\infty} \right)^6 + \dots \quad (4.18)$$

1669 The coefficients in this polynomial in R/R_∞ are the field index and its derivatives,
 1670 they can be a starting point for further refinement of the isochronism, including for
 1671 instance side effects of the azimuthal field form factor $\mathcal{F}(R, \theta)$ (Eqs. 4.4, 4.11).

1672 *Wave numbers*

1673 It follows from Thomas focusing that the radial field index in the AVF can be set
 1674 positive and increase with radius to satisfy $k = \frac{R}{B} \frac{\partial B}{\partial R} = \beta^2 \gamma^2$ (Eq. 4.2). Limiting any
 1675 phase slip to substantially less $\pm\pi/2$ requires a tolerance below 10^{-5} on field error.
 1676 Adjusting $k(R)$ to ensure such accuracy is a long process and includes pole ma-
 1677 chining, shimming, and other correction coil strategies. The isochronism constraint
 1678 determines the wave numbers, namely

$$\nu_R = \gamma \quad \text{and} \quad \nu_y = \sqrt{-\beta^2 \gamma^2 + F^2 (1 + 2 \tan^2 \xi)} \quad (4.19)$$

1679 wherein flutter F and spiral angle ξ are R-dependent quantities.

1680 **4.1.5 Fast Acceleration**

1681 Fixed field and fixed RF frequency in a cyclotron allow fast acceleration, a limitation
 1682 is in the amount of voltage which can be installed. In high power cyclotrons, the
 1683 voltage per turn reaches megavolts, about 4 MV for instance at the PSI 590 MeV ring

1684 cyclotron, where bunches are accelerated from 72 MeV to 590 MeV in less than 200
1685 turns.

1686 Fast acceleration results in fast crossing of harmful betatron resonances. For
1687 instance the “Walkinshaw resonance” $\nu_R = 2\nu_y$, which may lie across the beam
1688 path in the wave number diagram, as $\nu_R \approx \gamma$ whereas the vertical wave number
1689 takes its value in the $\nu_y \approx 1^- \sim 1.5$ region. This resonance causes the horizontal
1690 motion to couple to the vertical motion, resulting in increased vertical beam size and
1691 subsequently particle losses in the the vacuum chamber walls.

1692 Fast acceleration improves extraction efficiency, as the turn separation dR/dn is
1693 proportional to the energy gain per turn (Sec. 4.1.6).

1694 4.1.6 Cyclotron Extraction

1695 From $R = p/qB$ and assuming constant field (which may be legitimate in the
1696 presence of a very small field index), one draws

$$\frac{dR}{R} = \frac{dp}{p} = \frac{EdE}{E^2 - M^2} \quad (4.20)$$

1697 The minimum radial distance between the last two turns, where the extraction septum
1698 is located, is imposed by beam loss tolerances. Space charge in particular matters, as
1699 it increases the energy spread, and thus the radial extent of a bunch. In the relativistic
1700 cyclotron the separation between two consecutive turns satisfies

$$\Delta R \approx \frac{\gamma}{\gamma + 1} \frac{\Delta E}{E} \frac{R}{v_R^2} \quad (4.21)$$

with ΔE the effective acceleration rate per turn. In particular, the latter can be shown
to constrain the current limit, namely [13]

$$\text{current limit} \propto (\Delta E)^3$$

1701 Equation 4.15 indicates that a greater RF harmonic allows greater extraction
1702 radius, benefits extraction efficiency. It results from Eq. 4.21 that a large ring has
1703 similar effect, whereas on the contrary size is a limitation to intensity in small
1704 cyclotrons.

1705 In low energy cyclotrons (γ close to 1), extraction efficiency may also be increased
1706 by moving the wave number $\nu_R \approx \gamma$ close to $\nu_R = 1$ resonance.

1707 4.1.7 Resonant Spin Motion

1708 Spin precession during the cyclic motion of a particle in the field of a cyclotron
1709 dipole satisfies (Sec. 18.6.1)

$$\frac{d\mathbf{S}}{dt} = \frac{q}{m} \mathbf{S} \times \boldsymbol{\omega}_{\text{sp}} \quad (4.22)$$

1710 with $\boldsymbol{\omega}_{\text{sp}}$ the precession vector. By analogy with betatron wave numbers, a precession
1711 frequency can be introduced, $\nu_{\text{sp}} = G\gamma$, the number of spin precessions per turn.

If an ion travels in the bend plane (the median plane of the cyclotron dipole), it only experiences a magnetic field normal to its velocity, namely, $\mathbf{B}|_{y=0} \equiv \mathbf{B}_y$, thus its spin precession vector

$$\boldsymbol{\omega}_{\text{sp}} = \frac{q}{m_0\gamma} [(1 + G)\mathbf{B}_{\parallel} + (1 + G\gamma)\mathbf{B}_{\perp}]$$

1712 reduces to $\boldsymbol{\omega}_{\text{sp}} = \frac{q}{m_0\gamma} (1 + G\gamma)\mathbf{B}_{\perp}$, the spin precesses around $\mathbf{B}_{\perp} \equiv \mathbf{B}_y$, the angle
1713 $(\mathbf{S}, \mathbf{B}_y)$ remains unchanged during particle motion.

1714 In the AVF cyclotron, due to the azimuthal field modulation (Eq. 4.4) and to the
1715 radial index (Eq. 4.2), the azimuthal and radial field components B_{θ} and B_R are non-
1716 zero out of the median plane (Maxwell equations). Thus the local spin precession
1717 axis is not vertical, and the precession vector that the ion experiences changes as
1718 it oscillates vertically about the median plane. Resonance between spin precession
1719 (characterized by spin tune $\nu_{\text{sp}} = G\gamma$) and vertical particle oscillation motion (wave
1720 number ν_y) occurs if the two motions feature coinciding frequencies. This condition
1721 can be written under the form

$$\nu_{\text{sp}} \pm \nu_y = \text{integer} \quad (4.23)$$

1722 Near the resonance, at fixed energy (constant spin tune $\nu_{\text{sp}} = G\gamma$), the torque that
1723 the magnetic field exerts periodically causes the spins to tilt away from the vertical,
1724 the spin precession axis is at an angle to the vertical. Exactly on the resonance ($\nu_{\text{sp}} \pm$
1725 $\nu_y = \text{integer}$) the spin precession axis lies in the median plane. As a consequence,
1726 when a particle crosses the resonant condition (Eq. 4.23) during acceleration, its spin
1727 precesses away from the initial precession direction. Assuming an isolated resonance,
1728 the initial ($S_{y,i}$) and final ($S_{y,f}$) values of the vertical spin components, respectively far
1729 upstream and far downstream of the resonance, satisfy the Froissart-Stora rule [14]

$$\frac{S_{y,f}}{S_{y,i}} = 2e^{-\frac{\pi}{2} \frac{|\epsilon_R|^2}{\dot{\Delta}}} - 1 \quad (4.24)$$

1730 The magnitude of the tilt (*i.e.*, the amount that the $(\mathbf{S}, \mathbf{B}_y)$ angle is changed under
1731 the effect of the resonance crossing, from $(\mathbf{S}_{y,i}, \mathbf{B}_y)$ to $(\mathbf{S}_{y,f}, \mathbf{B}_y)$) depends on the
1732 strength of the resonance, $|\epsilon_R|$, and on the crossing speed, *i.e.*, the rate of change of

1733 the distance to the resonance $\Delta = \nu_{\text{sp}} \pm \nu_y - \text{integer}$,

$$\dot{\Delta} = \frac{d\nu_{\text{sp}}}{dt} \pm \frac{d\nu_y}{dt} \quad (4.25)$$

1734 In particular, if the resonance is crossed very slowly, the spin will follow the motion of
1735 the precession axis from vertical before the resonance to vertical after the resonance.

1736 If the crossing is very fast, spins maintain their initial angle to the vertical.

1737 **4.2 Exercises**1738 **Preliminaries**

- 1739 • Zgoubi users' guide at hand, when setting up the input data files to work out
 1740 the exercises, is a must-have. PART B of the guide in particular, details the
 1741 formatting of the input data lists following keywords (a few keywords only, for
 1742 instance FAISCEAU, MARKER, YMY, do not require additional data), and gives
 1743 the units to be used.
- 1744 • About keywords: by "keyword" it is meant, the name of the optical elements,
 1745 or I/O procedures, or commands, as they appear in simulation input data file.
 1746 Keywords are most of the time referred to without any additional explanation: it
 1747 is understood that the users' guide is at hand, and details regarding the use and
 1748 functioning to be sought there: in PART A of the guide, as to what a particular
 1749 keyword does and how it does it; in PART B as to the formatting of the data
 1750 list under a particular keyword. The users' guide INDEX is a convenient tool to
 1751 navigate through the keywords.
- 1752 – The notation KEYWORDS[ARGUMENT1, ARGUMENT2, ...]: it uses the
 1753 nomenclature found in the Users' Guide, Part B. Consider a couple of exam-
 1754 ples:
 1755 · OBJET[KOBJ=1] stands for keyword OBJET, and the value of KOBJ=1
 1756 retained here;
 1757 · OPTIONS[CONSTY=ON] stands for keyword OPTIONS, and the option
 1758 retained here, CONSTY, switched ON.
- 1759 – The keyword INCLUDE is used in many simulation input data files. The reason
 1760 is mostly to reduce the length of these files. It may always be replaced by the
 1761 sequence that it INCLUDEs.
- Coordinate Systems: two sets of coordinate notations are used in the exercises,

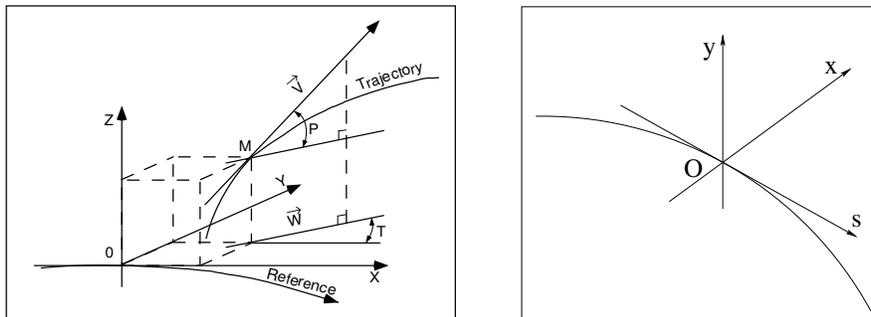


Fig. 4.11 Zgoubi Cartesian frame (O;X,Y,Z), and moving frame (O;s,x,y)

- 1763 – on the one hand (and, in the Solutions Section mostly), zgoubi's (Y,T,Z,P,X,D)
 1764 coordinates in the optical element reference frame (O;X,Y,Z), the very frame
 1765 in which the optical element field $\mathbf{E}(X, Y, Z)$ and/or $\mathbf{B}(X, Y, Z)$ is defined (the
 1766 origin for X depends on the optical element). Particle coordinates in this frame
 1767 can be
- 1768 · either Cartesian, in which case X, Y (angle T) and Z (angle P) denote
 - 1769 respectively the longitudinal, transverse horizontal and vertical coordinates,
 - 1770 · or cylindrical, in which case, given m the projection of particle position M
 - 1771 in the $Z=0$ plane, Y denotes the radius: $Y = |\mathbf{Om}|$, whereas X denotes the
 - 1772 **OX-Om** angle (and, yes, the nature of the variables named X and Y in the
 - 1773 source code does change);

Note: the sixth zgoubi's coordinate above is

$$D = \frac{\text{particle rigidity}}{BORO}$$

- 1774 with BORO a reference rigidity, the very first numerical datum to appear in
 1775 any zgoubi sequence, as part of the definition of initial particle coordinates by
 1776 OBJET or MCOBJET. BORO may sometimes be denoted $B\rho_{\text{ref}}$, depending
 1777 upon the context. Note that D-1 identifies with the above $\delta p/p$.
- 1778 – on the other hand (and, in the exercise assignments mostly), the conventional
 1779 $(x, x', y, y', \delta l, \delta p/p)$ coordinates in the moving frame (O;s,x,y) or close variants.
- 1780 Comments are introduced wherever deemed necessary (hopefully, often enough)
 1781 in an effort to lift potential ambiguities regarding coordinate notations.

1782 4.1 Modeling Thomas AVF Cyclotron

1783 In this exercise a field map model of Thomas cyclotron is built (Fig. 4.1) and used
 1784 to derive parameters of this 4-period AVF cyclotron. The field map is 2-dimensional,
 1785 it describes the magnetic field in the median plane (the symmetry plane of the dipole
 1786 magnet), in a polar coordinate system. TOSCA is used to handle and raytrace through
 1787 that map [15].

1788 A 2-dimensional $m(R, \theta)$ polar meshing of the median plane is used (as in
 1789 Fig. 3.18, page 36). The median plane field map provides $B_Z(R, \theta)$ values of the
 1790 field component normal to the (X, Y) plane, at the nodes of the mesh. Computation
 1791 of the field along (R, θ) particle trajectories in the (O;X,Y,Z) frame is performed
 1792 from the field map data, using interpolation techniques [15].

(a) Construct a 360° 2-dimensional map of the median plane field $B_Z(R, \theta)$,
 simulating the field in the 4-period Thomas cyclotron of Fig. 4.1 and following
 (Eq. 4.4)

$$B_Z(R, \theta) = B_0 [1 + f \sin(4(\theta - \theta_i))]$$

1793 with θ_i some arbitrary origin of the azimuthal angle, to be determined (hint: depend-
 1794 ing on θ_i value, the closed orbit may be at an angle to the polar radius, as seen in
 1795 Fig. 4.1; in that case TOSCA would require non-zero in and out positioning angles
 1796 TE and TS, to be determined and stated using KPOS option [15]; instead, a proper

1797 choice of θ_i value allows TE=TS=0). Assume an average axial field $B_0 = 0.5$ T,
 1798 $B_z > 0$ and $0 < f < 1$ modulation. Use a uniform mesh in a polar coordinate system
 1799 (R, θ) as sketched in Fig. 3.18, covering $R=1$ to 100 cm. Take a radial increment of
 1800 the mesh $\Delta R = 0.5$ cm, azimuthal increment $\Delta\theta = 0.5$ cm/RM, with RM a reference
 1801 radius as required in this process, say $RM = 50$ cm.

1802 The appropriate 6-column formatting of the field map data for TOSCA to read
 1803 them is the following:

1804 $R \cos \theta, Z, R \sin \theta, BY, BZ, BX$

1805 with θ varying first, R varying second in that list. Z is the vertical direction (normal
 1806 to the map mesh), so $Z \equiv 0$ in this 2-dimensional mesh.

1807 Plot $B_z(R, \theta)$ over the extent of the field map.

1808 (b) Raytrace a few concentric closed trajectories centered on the center of the
 1809 dipole, ranging in $10 \leq R \leq 80$ cm. Plot these concentric trajectories in the $(O; X, Y)$
 1810 laboratory frame. Initial coordinates can be defined using OBJET, particle coordi-
 1811 nates along trajectories during the stepwise raytracing can be logged in zgoubi.plt
 1812 by setting IL=2 under TOSCA.

1813 (c) Check the effect of the integration step size on the accuracy of the trajectory
 1814 and time-of-flight computation, by considering some Δs values in $[0.1, 10]$ cm, and
 1815 energies in a range from 200 keV to a few tens of MeV (assume proton).

1816 (d) Produce a graph of the R-dependence of wave numbers.

1817 (e) Calculate the numerical value of the axial wave number, ν_y , from the flutter
 1818 (Eq. 4.8). Compare with the numerical values: some discrepancy may be found:
 1819 repeat (d) for $f=0.1, 0.2, 0.3, 0.6$, check the evolution of this discrepancy, find its
 1820 origin.

1821 4.2 Isochronism

1822 (a) Demonstrate equation 4.2.

1823 (b) Demonstrate equation 4.3.

1824 (c) Devise numerical simulations proper to illustrate these relationships.

1825 4.3 Designing an Isochronous AVF Cyclotron

1826 (a) Introduce an R-dependent field index $k(R)$ in the AVF cyclotron designed in
 1827 exercise 4.1, proper to ensure R-independent revolution period, in three different
 1828 cases of modulation: $f=0$ (no modulation), $f=0.2$ and $f=0.9$.

1829 Check this property by computing the revolution period T_{rev} as a function of
 1830 kinetic energy E_k , or radius R . On a common graph, display both T_{rev} and $dT_{\text{rev}}/T_{\text{rev}}$
 1831 as a function of radius, including for comparison a fourth case: $B=\text{constant}=5$ kG.

1832 (b) Plot the energy dependence of wave numbers.

1833 4.4 Acceleration in an AVF Cyclotron

1834 Produce an acceleration cycle of a single proton, from 0.2 to 100 MeV, in the
 1835 AVF cyclotron designed in exercise 4.3. Assume proper modulation (coefficient f)
 1836 for vertical focusing all the way to 300 MeV about. Assume a double-dee design, and
 1837 400 keV gap voltage, peak, use CAVITE[IOPT=7] for acceleration to account for RF
 1838 phase.

1839 **4.5 Thomas-BMT Spin Precession in Thomas Cyclotron**

1840 This exercise uses the field maps and input data file of exercise 4.4. Dependence
1841 of energy boost on RF phase is removed by using CAVITE[IOPT=3] [15].

1842 (a) Find the $G\gamma$ value for which the resonant condition (Eq. 4.23) is satisfied.

1843 (b) Consider a helion particle with non-zero vertical motion, so that it experiences
1844 non-vertical magnetic field as it cycles around. Track its spin through the resonance,
1845 take initial spin vertical $\mathbf{S} \equiv \mathbf{S}_Z$. Plot S_Z as a function of $G\gamma$ and energy.

1846 Calculate the crossing speed (Eq. 4.25).

1847 (c) Simulate a series of spin tilts in the range $-1 < S_{y,f}/S_{y,i} < 1$, by varying
1848 the vertical motion amplitude, plot the ration $S_{y,f}/S_{y,i}$ of the final to initial value of
1849 the vertical spin component, as a function of the initial vertical coordinate of the
1850 particle.

1851 (d) From a match of this $S_{y,f}/S_{y,i}$ series with Eq. 4.24, show that the resonance
1852 strength changes in proportion to the square root of the vertical motion invariant.

1853 Produce a plot of the series of resonance crossing $S_Z(\text{turn})$, for this series of Z_0
1854 values.

1855 (e) Repeat (c) by changing the resonance crossing speed instead (leaving Z_0
1856 unchanged).

1857 Show that this $S_{y,f}/S_{y,i}$ series can be matched with the expression of Eq. 4.24.

1858 **4.6 Edge Focusing, Flutter**

1859
1860 In this exercise DIPOLES is used to simulate a 30 deg sector dipole of a 90 deg-
1861 periodic cyclotron, and reproduce the principle geometry of the isochronous AVF
1862 cyclotron of exercise 4.3 (PSI 4-sector 72 MeV injector II, Fig. 4.10, is an example
1863 of such 4-cell configuration with short dipoles). DIPOLES is chosen rather than
1864 DIPOLE as it allows radial field indices up to high order [15, Eq. 6.3.19], as needed to
1865 devise proper radial field profile, whereas DIPOLE does not go beyond a $\partial^3 B_Z/\partial R^3$
1866 index [15, Eq. 6.3.17].

1867 Take fringe fields into account (Eq. 4.15), with

1868 - $\lambda = 7$ cm the fringe extent (changing λ changes the flutter, Eq. 4.8),

1869 - $C_0 = 0.1455$, $C_1 = 2.2670$, $C_2 = -0.6395$, $C_3 = 01.1558$ and $C = 0$, for a
1870 realistic fringe field model.

1871 (a) Assume $k = 0$, here. Produce a model of the 4-period AVF, accounting for a
1872 field extent λ as allowed by DIPOLE.

1873 Plot closed orbits across a period for a few different particle rigidities - FIT can
1874 be used to find the closed orbits. Plot the field along these orbits.

1875 (b) Proper R-dependence of the mid-plane magnetic field is now introduced, to
1876 ensure revolution period closest to energy independent. In that aim, DIPOLES field
1877 indices b_i are used.

1878 Assume a peak field value $\hat{B} = 1.1$ T in the dipoles, at radius $r=3.5$ m. Find the
1879 average orbit radius R , and average field B (such that $BR = p/q$), at an energy of
1880 72 MeV.

1881 Determine a series of index values, $b_{i=1,n}$, in the model [15, Eq. 6.3.19]

$$B_Z(R, \theta) = B_0 \mathcal{F}(R, \theta) \left(1 + b_1 \frac{R - RM}{RM} + b_2 \left(\frac{R - RM}{RM} \right)^2 + \dots \right) \quad (4.26)$$

1882 proper to bring the revolution period closest to being R-independent, in the energy
 1883 range 0.9 to 72 MeV (hint: use a limited Taylor development of Eq. 4.17 and identify
 1884 with the R-dependent factor in Eq. 4.26).

1885 (c) Play with the value of λ , concurrently to maintaining isochronism with appropriate
 1886 transverse field index values. Check the evolution of horizontal and vertical
 1887 focusing - OBJET[KOBJ=5] and MATRIX can be used to get the wave numbers.

1888 Check the following, from raytracing trials:

1889 (i) the effect of λ on horizontal focusing is weak (how does it contribute in the
 1890 matrix formalism?),

1891 (ii) greater (smaller) λ results in smaller (greater) flutter and weaker (stronger)
 1892 vertical focusing. Note: the integration step size in DIPOLE has to be made consistent
 1893 with the value of λ , in order to ensure that the numerical integration is converged.

1894 (d) For some reasonable value of λ (normally, about the height of a magnet gap,
 1895 say, a few centimeters), compute $F^2 = \left\langle \left(\frac{B(\theta) - \langle B \rangle}{\langle B \rangle} \right)^2 \right\rangle$, check $\nu_y = -\beta^2 \gamma^2 + F^2$
 1896 (Eq. 4.19). MATRIX can be used to compute ν_y , or otherwise Fourier analysis,
 1897 however not as straight forward as it requires multiturn raytracing.

1898 (e) Check the rule $F^2 \xrightarrow{\text{hard edge}} \frac{R}{\rho} - 1$ (Eq. 4.8), from the field $B(\theta)$ delivered by
 1899 DIPOLES. Give a theoretical demonstration of that rule.

1900 4.7 PSI Ring Cyclotron, Using its Measured Field Map

PSI field map, named POLARMES_PSI_drift.H, can be found at

<https://sourceforge.net/p/zgoubi/code/HEAD/tree/trunk/examples/KEYWORDS/POLARMES/>

1901 POLARMES_PSI_drift.H is an ascii file, it contains the mid-plane field at the nodes
1902 of a two-dimensional cylindrical mesh. The mesh is 136×140 nodes, respectively
1903 azimuthally \times radially.

1904 (a) Explain why POLARMES is used rather than TOSCA[MOD=25] [15].

1905 Set up a simulation data file to find a few closed orbits over PSI cyclotron energy
1906 range, 72 to 590 MeV. Hint: FIT can be used to find the periodic orbit coordinates.
1907 Assume a single particle defined in OBJET, REBELOTE can thus be used, following
1908 FIT, to repeat that sequence for a series of values of the relative rigidity of that particle.
1909 Use IL=2 under POLARMES to have step-by-step particle data logged in zgoubi.plt.

1910 Plot these periodic orbits in a laboratory frame.

1911 Plot the vertical component of the magnetic field along these closed orbits.

1912 (b) Plot beam envelopes at 72 MeV, 590 MeV, and some intermediate energies.

1913 (c) Do a dense orbit scan: 200 orbits over 72 – 590 MeV. Get from this scan a plot
1914 of the revolution time as a function of radius.

1915 (d) Plot the beam path in the wave number diagram (consider paraxial wave
1916 numbers).

1917 (e) Simulate a complete acceleration cycle, from 72 to 590 MeV, for a 200-particle
1918 6D bunch, with initial transverse emittances 5π mm mrad, initial length 2 cm and *rms*
1919 momentum spread 10^{-3} , Gaussian. Assume 4 MV per turn, accelerating gaps located
1920 in four drift spaces as in Fig. 4.3.

1921 (f) Repeat (e) with a initially 100% vertically polarized bunch. Check the trans-
1922 mission of the polarization (final polarization over initial polarization).

1923 4.8 A Model of PSI Ring Cyclotron Using CYCLOTRON

1924 The simulation input data file in Tab. 4.2 is based on the use of CYCLOTRON,
1925 to simulate a period of the eight-sector PSI ring cyclotron.

1926 This file is the starting point of the present exercise.

1927 (a) With zgoubi users' guide at hand, explain the signification of the data in that
1928 simulation input data file.

1929 (b) Compute and plot the radius dependence of the revolution period. Comment
1930 on the isochronism.

1931 (c) The field indices are aimed at realizing the isochronism (equal revolution
1932 period at all energies). Four are involved in (a) and (b), B1 to B4, they have been
1933 drawn from the PSI cyclotron cell field map data (exercise 4.7). Question (b) proves
1934 this small set of indices to result in poor isochronism.

1935 Add higher order indices, until a sufficient number is found that FIT is able to
1936 reach a final isochronism improved by more than an order of magnitude.

Table 4.2 Simulation input data file: a period of an eight-sector cyclotron. The data file is set up for a scan of the periodic orbits, from radius $R=204.1171097$ cm to $R=383.7131468$ cm, in 15 steps

```

PSI CYCLOTRON
'OBJET'
1249.382414
2
1 1
204.1171097 8.915858372 0. 0. 0. 1. 'o'
1
'PARTICUL'
PROTON
'CYCLOTRON'
2
1 45.0 276. 1.0
0. 0. 0.99212277 51.4590015 0.5 800. -0.476376328 2.27602517e-03 -4.8195589e-06 3.94715806e-09
18.3000E+00 1. 28. -2.0
8 1.1024358 3.1291507 -3.14287154 3.0858059 -1.43545 0.24047436 0. 0. 0.
11.0 3.5 35.E-3 0.E-4 3.E-8 1. 1. 1.
18.3000E+00 1. 28. -2.0
8 0.70490173 4.1601305 -4.3309575 3.540416 -1.3472703 0.18261076 0. 0. 0.
-8.5 2. 12.E-3 75.E-6 0.E-6 1. 1. 1.
0. -1
0 0. 0. 0. 0. 0. 0. 0.
0. 0. 0. 0. 0. 0.
2 10.
0.4
2 0. 0. 0. 0.
'FIT2'
2
1 31 0 [-300.,100]
1 35 0 [.,1.,3.]
2
3.1 1 2 #End 0. 1. 0
3.1 1 3 #End 0. 1. 0
'FAISCEAU'
'FAISTORE'
orbits.fai
1
'REBELOTE'
14 0.2 0 1
1
OBJET 30 221.065356:383.7131468
'SYSTEM'
1
gnuplot <./gnuplot_orbits.gnu
'END'

```

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