Influence of energy jitter to PCA-based CeC

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Estimate Cooling Force for PCA-based CeC: Longitudinal Electric Field in the Kicker Section

The electric potential induced by the line density perturbation is determined by the following equations

$$\frac{1}{r}\left[\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\varphi(r,z)\right)\right]+\frac{\partial^2}{\partial z^2}\varphi(r,z)=\frac{1}{\varepsilon_0}\rho_2(z)f_\perp(r)$$

If we take the transverse distribution of the electrons as

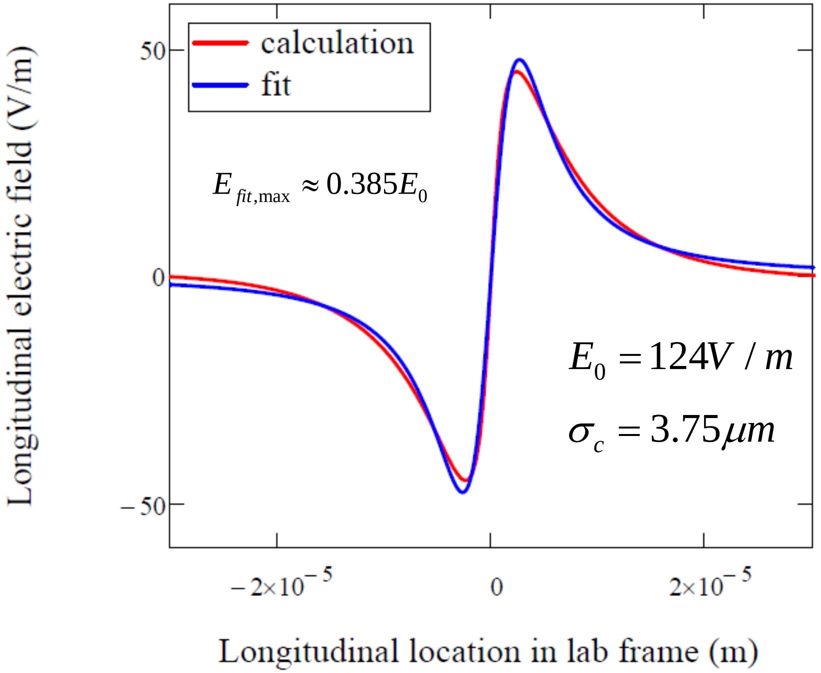
$$f_\perp(r)=\frac{1}{\pi a^2}H(a-r)$$

The electric field can be solved as

$$E_z(r,z)=-\frac{\partial\varphi}{\partial z}=\frac{1}{2\pi}\int_{-\infty}^{\infty}\tilde{E}_z(r,k_z)e^{ik_z z}dk_z$$

$$\tilde{E}_z(r)=-ik_z\frac{\tilde{\rho}_2(k_z)}{\pi\varepsilon_0}$$

$$\times\left[I_0(k_z r)\int_{r/a}^{1}\eta K_0(k_z a\cdot\eta)d\eta+K_0(k_z r)\int_0^{r/a}\eta I_0(k_z a\cdot\eta)d\eta\right]$$



For easy implementation into ion tracking code, we use the fitting formula:

$$E_{fit}(z)=E_0\cdot\frac{z}{\sigma_c}\left[1+\frac{z^2}{\sigma_c^2}\right]^{-3/2}$$

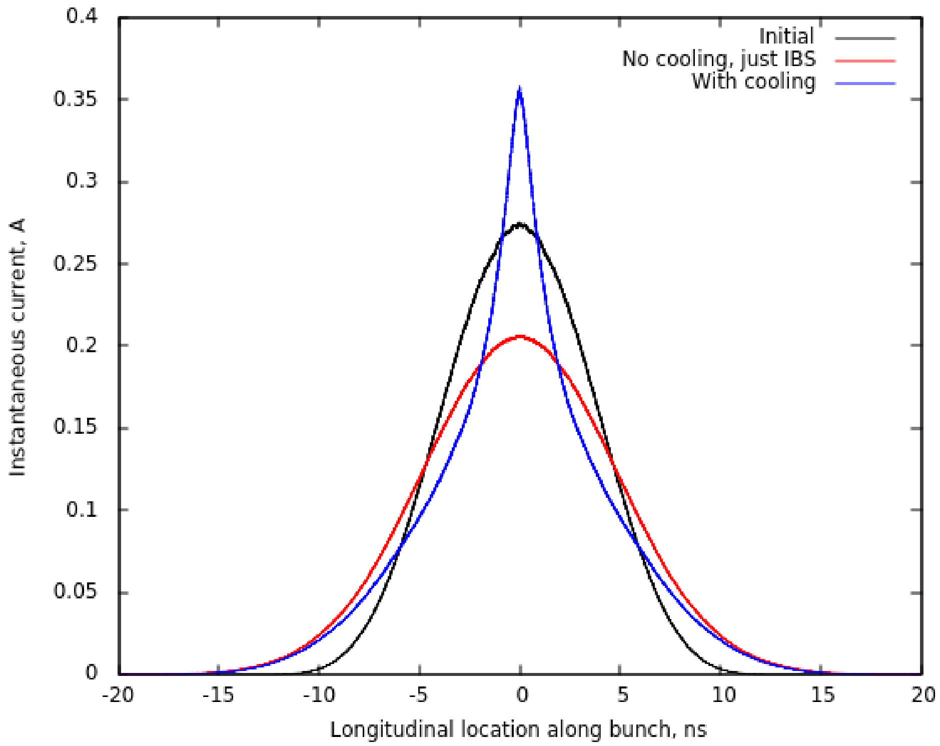
Single-pass Kick and Tracking Results for PCA-based CeC

$$\Delta\gamma_{j,N} = -g_\gamma \frac{\left(D\cdot\delta_{j,N}\right)}{\sigma_c}\left[1+\frac{\left(D\cdot\delta_{j,N}\right)^2}{\sigma_c^2}\right]^{-3/2} + g_\gamma\sqrt{\frac{3\pi}{8}\rho_{ion}\left(z_{j,N}\right)\sigma_c}\cdot X_{j,N} + \frac{g_\gamma}{Z_i}\sqrt{\frac{3\pi}{8}\rho_e\left(z_{j,N}\right)\sigma_c}\cdot Y_{j,N}$$

$$g_\gamma = Z_i e E_0 L_k \big/ \left(A_i m_u c^2\right)$$

Parameters used in ion tracking	
E_0*	62 V/m
σ_c	3.75 μm
Z_{ion}	79
Ion bunch intensity	2E8
D (R_{56})	1.2 cm

*E_0 is reduced by a factor of 2 to account for reduced cooling for ions with large betatron amplitude



Influence of energy jitter

$$\Delta\gamma_{j,N} = -g_\gamma \frac{D\cdot\left(\delta_{j,N}+\Delta\delta_N\right)}{\sigma_c}\left[1+\frac{\left(\left[D\cdot\left(\delta_{j,N}+\Delta\delta_N\right)\right]\right)^2}{\sigma_c^2}\right]^{-3/2}$$

$$+g_\gamma\sqrt{\frac{3\pi}{8}\rho_{ion}\left(z_{j,N}\right)\sigma_c}\cdot X_{j,N}+\frac{g_\gamma}{Z_i}\sqrt{\frac{3\pi}{8}\rho_e\left(z_{j,N}\right)\sigma_c}\cdot Y_{j,N}$$

$\Delta\delta_N$ is the energy jitter for turn N, which is a random number uniformly distributed in the interval $\left[-\frac{\Delta\delta_{\max}}{2} : \frac{\Delta\delta_{\max}}{2}\right]$