# PCA amplifier and PCA with wigglers

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# Introduction

Outline

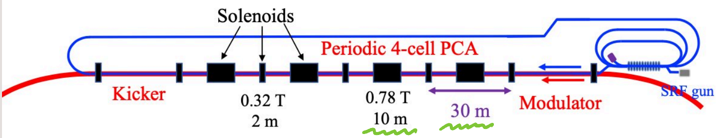
- Analysis of PCA for EIC amplifier
- PCA with wigglers
- Frequency characteristic of the modulator and kicker
- Summary/Discussion

We analyze PCA using the model of slices (hadron and electron point charges are replaced by thin slices with a Gaussian surface charge distribution) developed for MBEC[1].

[1] G. Stupakov, "Cooling rate for microbunched electron cooling without amplification", PRAB **21**, 114402 (2018).

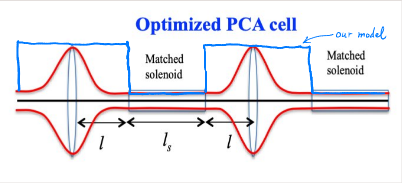
# PCA parameters

## EIC CeC with PCA




| Name | Current experiment | CeC cooler for EIC |
|---|---|---|
| PCA Lattice | Periodic, 4 cells, regular | Periodic, 4 cells, optimized |
| $\gamma$ | 28.5 | 293 |
| Hadrons | Au ions | Protons |
| $E_h$, GeV | 26.5 | 275 |
| $E_e$, MeV | 14.56 | 150 |
| l, m | 2x1 | 2x15 |
| $a_0$, mm | 0.2 | 0.15 ✓ |
| Q, nC | 1.5 | 1.5 |
| $I_0$, A | 75 | 150 ✓ |
| $\varepsilon_{norm}$, m | $5\ 10^{-6}$ | $5\ 10^{-6}$ |
| Frequency, THz | 25 | 500 ✓ |
| PCA gain | 100 | 400 |
| Lattice | regular | 1:2 |
| 3D emittance Cooling time, min | 15-20 | <5 |

# PCA cell



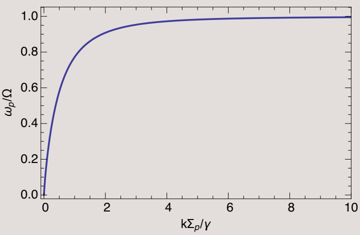

The model: consider a cell as a sequence of two drifts: one with a small beam size (0.15 mm inside the matched solenoids) and the rest of the cell with a large beam radius.

The plasma frequency, $\omega_p \propto 1/\Sigma_p$ ($\Sigma_p$ is the rms transverse size of the beam). We assume $\omega_p = 0$ outside of the solenoids (in the region of length $2\ell$).

In our model $\omega_p(k) = \Omega f(k\Sigma_p/\gamma)$

$$\Omega = \frac{c}{\Sigma_p}\sqrt{\frac{I_e}{I_A\gamma^3}}$$

For EIC CeC parameters, $\lambda_p = 2\pi/\Omega = 50.4$ m.



# Plasma oscillations

When the transverse size of the beam varies with $s$, $\Sigma_p(s) \propto \sqrt{\beta(s)}$, we have $\omega_p(k, s)$. The equation for the Fourier component $\delta\hat{n}_k(s)$ reads:

$$\frac{d^2\delta\hat{n}_k}{ds^2} + k_p^2(s, k)\delta\hat{n}_k = 0$$

where $k_p = \omega_p/c$. Here $\delta\hat{n}_k = \int \delta n(z) e^{-ikz} dz$.
Outside of the long solenoid (length $\ell_s$) we have

$$\frac{d^2\delta\hat{n}_k}{ds^2} = 0$$

From these equations we can obtain the matrix formalism for the transformation of the vector $(\delta\hat{n}_k, d\delta\hat{n}_k/ds)^T$

$$\begin{pmatrix} \delta\hat{n}_k \\ \delta\hat{n}'_k \end{pmatrix}_s = M(s, k_p) \begin{pmatrix} \delta\hat{n}_k \\ \delta\hat{n}'_k \end{pmatrix}_{s=0}$$

# Matrix formalism

In the solenoid

$$M(s, k_p) = \begin{pmatrix} \cos(k_p s) & \frac{1}{k_p}\sin(k_p s) \\ -k_p \sin(k_p s) & \cos(k_p s) \end{pmatrix}$$

In the drift we set $k_p = 0$

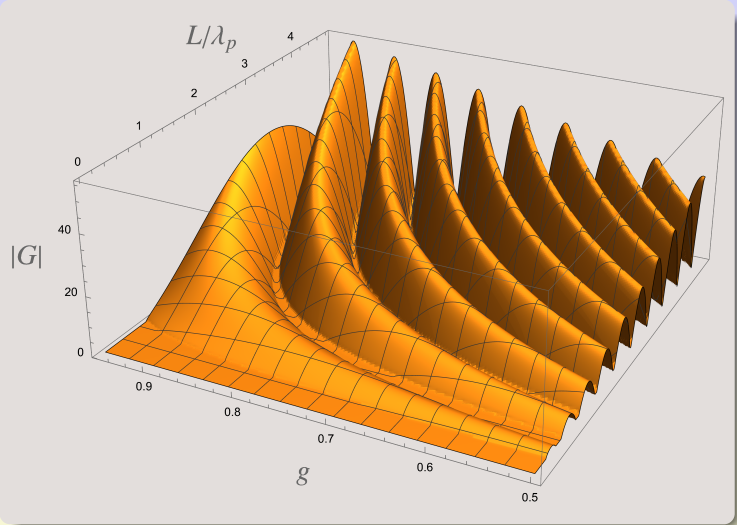
$$M(s, 0) = \begin{pmatrix} 1 & s \\ 0 & 1 \end{pmatrix}$$

Define the length of the cell $2L = \ell_s + 2\ell$ and $g = \ell/L$ (relative length of drift) then the total matrix is

$$M_{\text{cell}} = M(gL, 0) \cdot M(2(1-g)L, k_p) \cdot M(gL, 0)$$

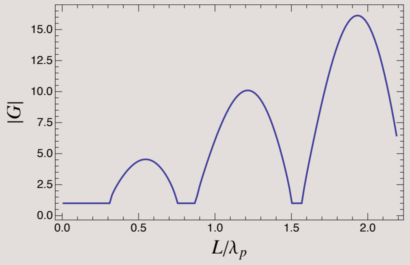
We calculate the eigenvalues $G$ of this matrix. If $|G| > 1$ then this is amplification per cell.

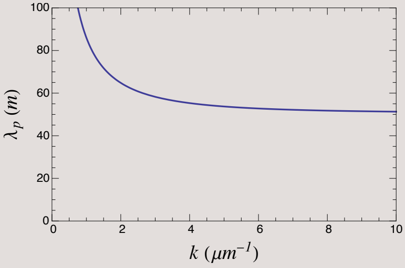
# One cell gain




Here $\lambda_p = 2\pi/k_p$. From the table of EIC PCA parameters, $g = 2/3$.

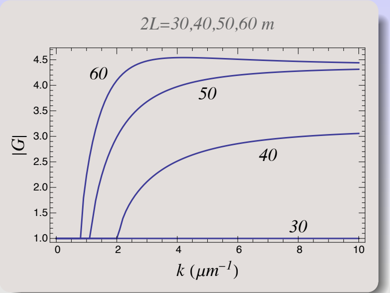
# Gain versus $\lambda_p$




We now specify the length $2L$, which gives the dependence $|G(k)|$.

# PCA amplifier



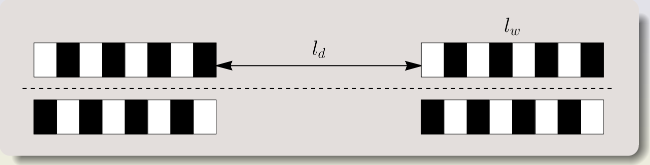

For the cell length $2L = 50$ m, the amplification of four sections is $4.3^4 = 342$. Remarkably, the amplification curve extends to high frequencies (high values of $k$), in contrast to MBEC.

The discrepancy with the EIC PCA parameter table is probably due to slightly different models of the cell.

# PCA with wigglers (WEPA[2])

Inside a wiggler electrons travel with a smaller $z$-velocity, $\gamma_z = \gamma/\sqrt{1+K^2/2}$, $K$ is the wiggler parameter. The plasma frequency in the wiggler is larger than in the drift (plasma period is smaller)

$$\omega_p^{(w)}(k=\infty) = \frac{c}{\Sigma_p \gamma_z}\sqrt{\frac{I_e}{I_A \gamma}}$$




We can have PCA in a wiggler-drift periodic system.

The hope is that the amplifier can be shorter. Wigglers also focus electrons in the vertical direction—a smaller transverse beam size. We analyzed such amplifier assuming $\omega_p = 0$ in the drift and $\omega_p^{(w)} > 0$ in the wiggler—mathematically this is very similar to the model considered above.

[2] WEPA - Wiggler Enabled Parametric Amplifier (tentative name)

# WEPA matrix analysis

In WEPA we write matrices for the vector $(\delta\hat{n}_k, (\gamma_z^2/\gamma^2)d\delta\hat{n}_k/ds)^T$ which is continuous through the transition from the wiggler to drift[3]. In the wiggler

$$M_w(s, k_p^{(w)}) = \begin{pmatrix} \cos(k_p^{(w)}s) & \dfrac{\gamma^2}{\gamma_z^2 k_p^{(w)}}\sin(k_p^{(w)}s) \\ -\dfrac{\gamma_z^2 k_p^{(w)}}{\gamma^2}\sin(k_p^{(w)}s) & \cos(k_p^{(w)}s) \end{pmatrix}$$

In the drift

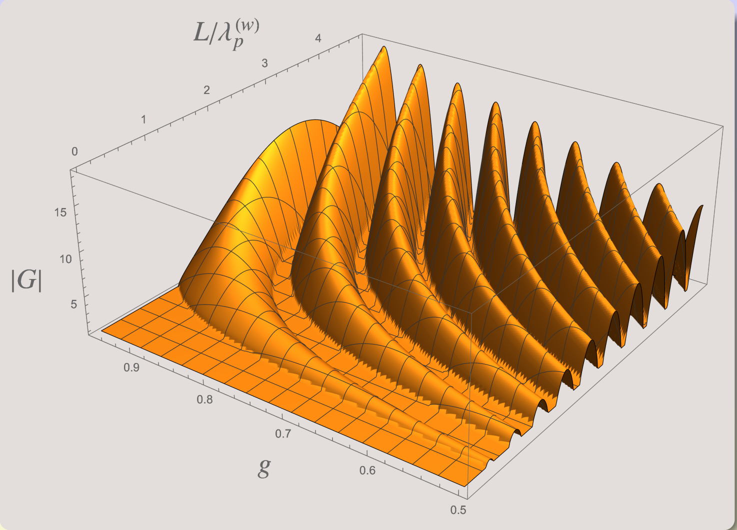
$$M_d(s) = \begin{pmatrix} 1 & s \\ 0 & 1 \end{pmatrix}$$

Define the length of the cell $2L = \ell_d + \ell_w$ and $g = \ell_d/2L$ then the total matrix is

$$M_{\rm cell} = M_d(gL) \cdot M_w(2(1-g)L, k_p^{(w)}) \cdot M_d(gL)$$

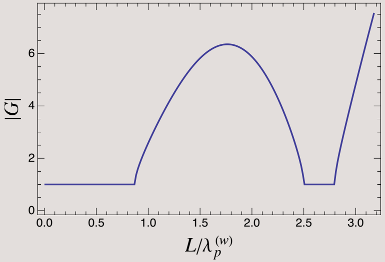
[3] This is because $(\gamma_z^2/\gamma^2)d\delta\hat{n}_k/ds \propto \Delta\hat{\eta}_k$—the energy perturbation.

# One cell gain for WEPA, $K = 2$



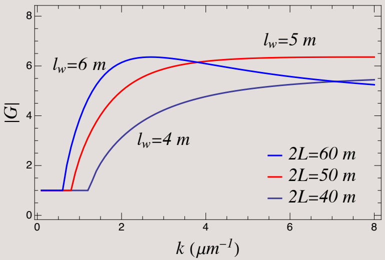

To increase gain we choose 90% drift, $g = 0.9$. We also choose $\Sigma_p = 70$ μm and the same $I_e = 150$ A.

# WEPA amplifier




Amplification versus $L/\lambda_p^{(w)}$ for $g = 0.9$.

One amplification cell




With four cells, the amplification is $\sim 6^4 \approx 1300$ and it is broadband, but the cells are long (45 m drift + 5 m wiggler). They however can be made considerably shorter.

# What is the role of the drifts in WEPA?

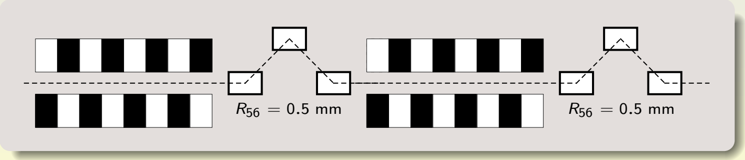
In the drift

$$M_d(s) = \begin{pmatrix} 1 & s \\ 0 & 1 \end{pmatrix}$$

Analysis shows that the drift provides $R_{56}$ that converts an energy perturbation in the plasma oscillation into the density perturbation:

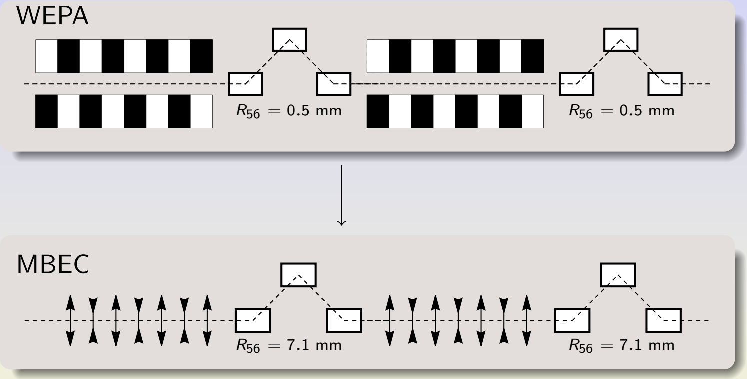
$$R_{56} = \frac{1}{\gamma^2} \times 45\ \mathrm{m} = 0.52\ \mathrm{mm}$$

We can replace 45 m drift by a short chicane with $R_{56} = 0.52\ \mathrm{mm}$ with the length less then 1 m. Then the cell length is $\sim 6$ m.



# Comment: connection between PCA and MBEC

If we replace wigglers by drifts we convert PCA with wiggler to MBEC.




MBEC is a parametric amplifier "on steroids" (large $R_{56}$ chicanes). This, however, suppresses the gain at large values of $k$.

# Landau damping and CSR wakefields

In our analysis we assumed a cold beam and hence ignored the Landau damping. We can estimate this effect as smearing of the density perturbations with wave number $k$ due the slippage caused by the energy spread in the beam $\sigma_\eta$

$$\propto \exp\left[-\frac{1}{2}k^2 R_{56}^2 \sigma_\eta^2\right]$$

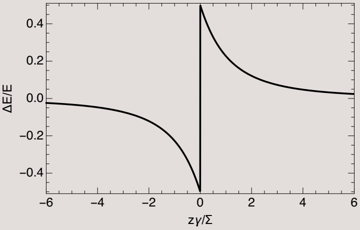
Assume $\sigma_\eta = 2 \times 10^{-4}$. This will suppress the amplification at $k \sim \sqrt{2}/R_{56}\sigma_\eta \approx 13.5\ \mu\mathrm{m}^{-1}$ (this corresponds to the frequency 640 THz).

There are also CSR wakefields in the wiggler that add to the space charge forces. They should be taken into account in the dynamics of plasma oscillations (it is possible that they can add to the signal amplification.).

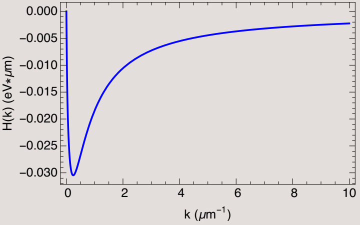
# Matching the amplifier to the modulator/kicker (M/K)

In MBEC, we assume that M/K are much shorter than $\lambda_p$ and ignore the Debye shielding.

Energy perturbation induced in electron by one ion as a function of distance from the slice ($\Sigma$ is the transverse size of the beam in M/K.)




Spectrum of the energy perturbation in the electron beam induced by one proton in the modulator. The wavenumber $k = 1\ \mu\mathrm{m}^{-1}$ corresponds to the frequency of $\approx 50$ Thz.



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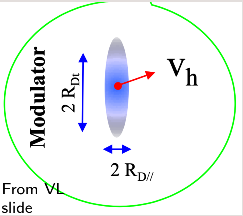


In PCA, the assumption is that the slice has dimensions of the order of the Debye radii. For the PCA beam, $r_{D,\parallel} \sim 7$ nm, $r_{D,\perp} \sim 230$ μm (assuming $\beta = 10$ m).

This relies on the dynamic Debye shielding of protons in the electron beam which requires M/K length of several plasma wavelength[4]. It is not clear how robust this mechanism is.

[4] G. Wang and M Blaskiewicz, "Dynamics of ion shielding in an anisotropic electron plasma", PRE **78**, 026413 (2008).

# Summary/Discussion

- Our analysis confirms the properties of EIC PCA *a*mplifier (within a factor of $\sim 2$) claimed by the PCA group.
- WEPA with small chicanes can provide and alternative to PCA with a considerably shorter cell length. However, adding small chicanes would require an adjusting the path length of the hadron beam (as we do in MBEC).
- PCA and WEPA are broadband amplifiers. However, there is a question of how well this amplifier matches to the spectral properties of M/K.
- Noise effects in the proton electron beams and the diffusion induced by this noise need to be calculated. The energy diffusion $D_e$ from[5]

$$D_e \propto \frac{I_e V_0^2}{\Delta f}$$

with $V_0$ the amplitude of the energy kick, $\Delta f$ the spectral width associated with the energy kick.

[5] S. Nagaitsev, V. Lebedev, G. Stupakov, E. Wang, and W. Bergan, "Cooling and diffusion rates in coherent electron cooling concepts", arXiv:2102.10239 (Feb. 2021).

# Summary/Discussion

- Going to higher frequencies in cooling (MBEC—$30-50$ THz, PCA—500 THz) means a much tighter control of the electron and hadron paths length ($< 1$ μm MBEC, $< 0.1$ μm PCA)[6].

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[6] S. Seletskiy, A. Fedotov, and D. Kayran. "Effect of coherent excitation in coherent electron cooler". May 2021.