High Power RF Engineering -Coax (1)

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Electron-Ion Collider

BROOKHAVEN NATIONAL LABORATORY

Jefferson Lab

U.S. DEPARTMENT OF ENERGY | Office of Science

Please register ASAP

- Here is the link for registration in order to have access to BNL this Nov:
- https://www.bnl.gov/guv/gis.php
-
- All students should register since you are going to have hands-on experience, even it is low power.
- In case there are questions, you may contact guvcenter@bnl.gov, and cc to DiFilippo, Lynanne difilip@bnl.gov, McKenzie, Gladys gmckenzie@bnl.gov and Binping Xiao binping@bnl.gov


- During the registration, if a point of contact is needed, please use:

Hammons, Lee

Email: hammons@bnl.gov

Work phone: 631 344 2891

Collider-Accelerator Department (AD)

Electron-Ion Collider

Note

- Speed of light in free space under vacuum c = 299 792 458 *m/s*
- Vacuum electric permittivity ε_0 = 8.854187 $\times 10^{-12}$ *F/m*
- Vacuum magnetic permittivity μ_0 = 1.256637 $\times 10^{-6}$ *N/A^2*
- $\varepsilon_0 \mu_0 = 1/c^2$
- Impedance of free space under vacuum $\eta = \sqrt{\mu_0/\varepsilon_0}$ = 376.730 313 Ω

Photon

- Quantum of the electromagnetic(EM) field.
- Wave-particle duality.
- Boson with spin 1.
- Massless.
- Speed c = 299792458 m/s in free space under vacuum.
- Electric charge 0.
- $c = f\lambda$, $E = hf$, $p = h/\lambda$, with f frequency, λ wavelength, E energy (of a photon), p momentum and h Planck constant.
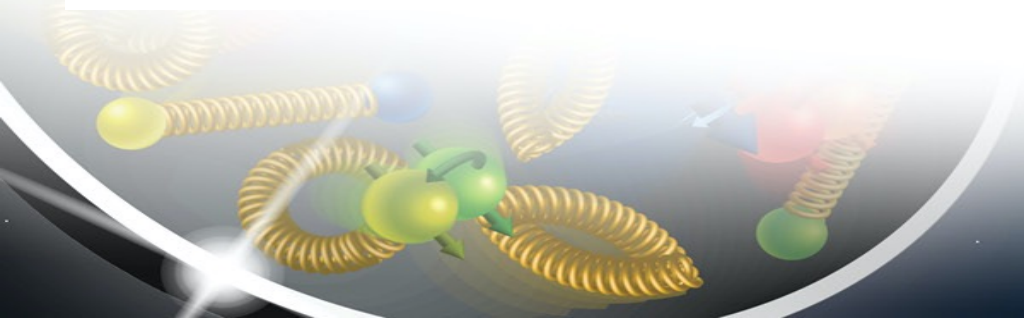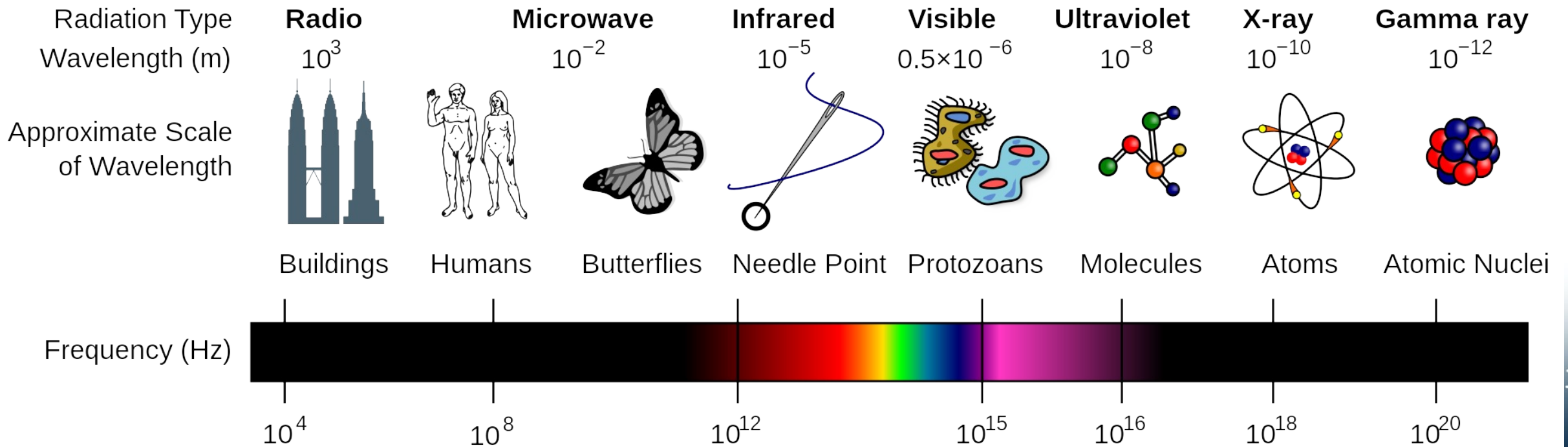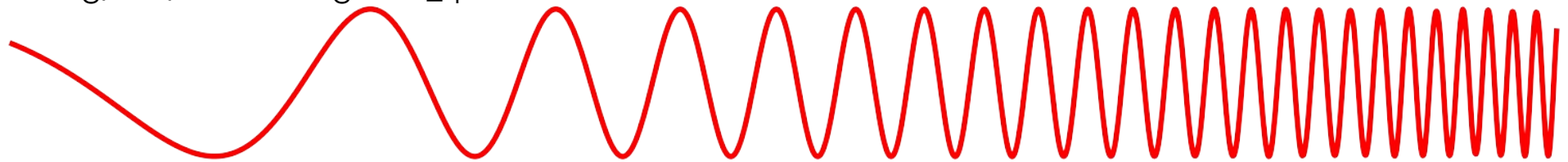
Radiofrequency (RF)

- Photon with ~ 20KHz to 300GHz frequency.

From https://en.wikipedia.org/wiki/Electromagnetic_spectrum

1kHz = 1000Hz
1MHz = 1000kHz
1GHz = 1000MHz
1THz = 1000GHz

Radiation Type	Radio	Microwave	Infrared	Visible	Ultraviolet	X-ray	Gamma ray
Wavelength (m)	10^3	10^{-2}	10^{-5}	0.5×10^{-6}	10^{-8}	10^{-10}	10^{-12}
Approximate Scale of Wavelength	Buildings	Humans	Butterflies	Needle Point	Protozoans	Molecules	Atoms	Atomic Nuclei

| Frequency (Hz) | 10^4 | 10^8 | 10^{12} | 10^{15} | 10^{16} | 10^{18} | 10^{20} |

Applications of RF

- Broadcast: Radio (AM/FM), TV, < 1GHz.

- Microwave oven, 2.45GHz.

- Radar, normally between 0.4 and 36 GHz.

- Communications
  - Cellular (from sub-GHz to a few tens of GHz)
  - GPS (1575.42MHz, 1227.6MHz 1176MHz)
  - Bluetooth 2.4GHz ISM (industrial, scientific and medical) band from 2.402GHz to 2.480GHz.
  - WiFi (2.4GHz and 5GHz)
  - And more

- Particle accelerator: use EM field to accelerate charged particles (electrons, protons, heavy ions in the form of beams).

Types of Transmission Lines

- There are many types of transmission line structures that can transmit RF power, for examples parallel lines (ladder line, twisted pair, commonly used in 50Hz electric power transmission). coaxial line (coaxial cable), planar transmission lines (stripline, microstrip). We will focus on the coaxial cable/line.

- Two major types of coaxial lines: RG (Radio Guide) and LMR®. Types of connectors: SMA, type-N, TNC, SMC, BNC, SMB, 7/16 etc.

Coaxial Line

Stripline

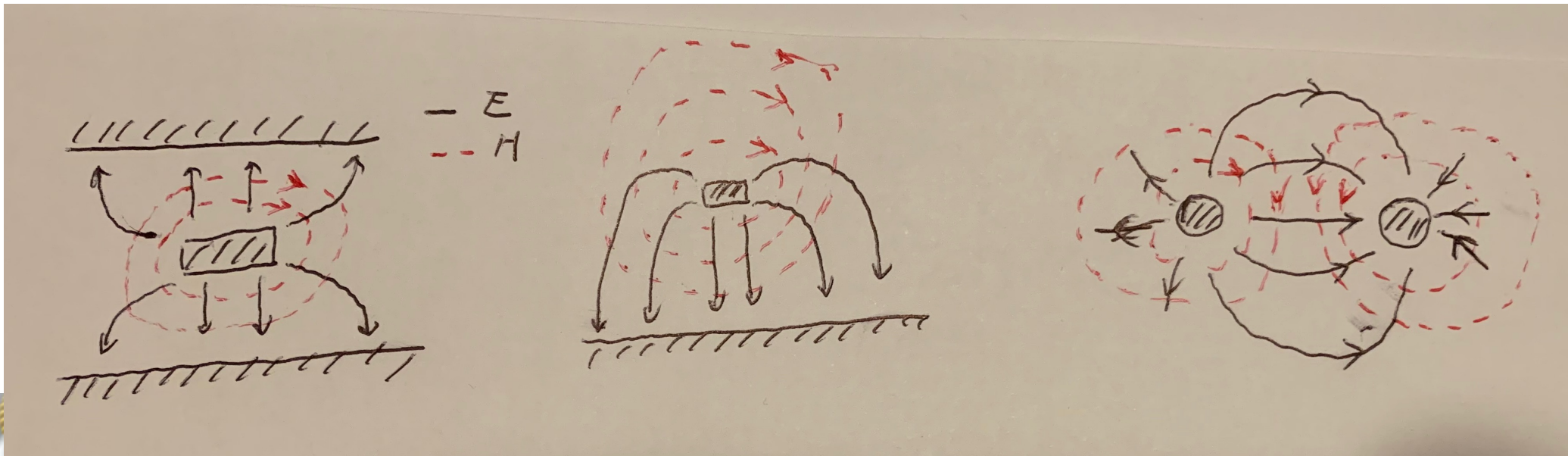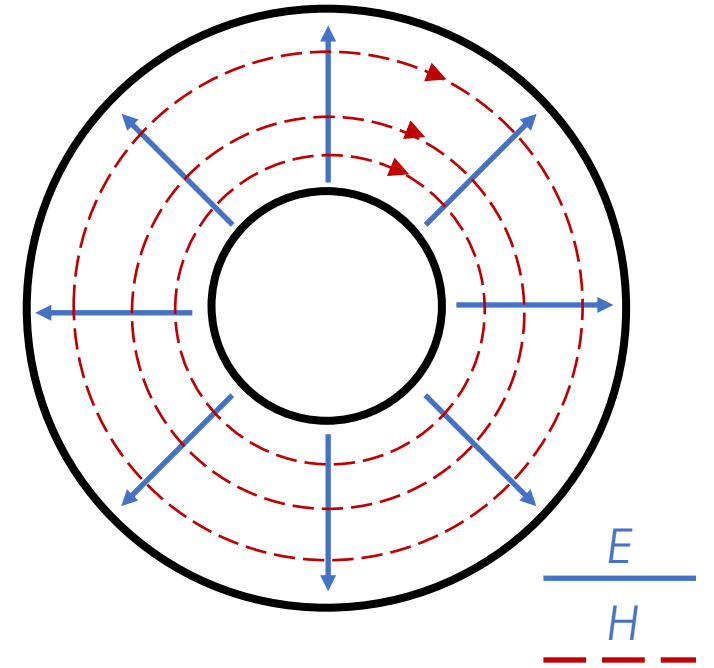Microstrip

Dielectric/Air/Vacuum

Metal

Cross section of transmission lines

Electron-Ion Collider

Guess the field pattern

We need mathematical derivation.

Maxwell Equations

$\nabla \cdot \boldsymbol{D} = \rho$

$\nabla \cdot \boldsymbol{B} = 0$

$\nabla \times \boldsymbol{E} = -\partial \boldsymbol{B}/\partial t$

$\nabla \times \boldsymbol{H} = \boldsymbol{J} + \partial \boldsymbol{D}/\partial t$

With material property

$\boldsymbol{D} = \varepsilon \boldsymbol{E}$

$\boldsymbol{B} = \mu \boldsymbol{H}$

$\boldsymbol{J} = \sigma(\boldsymbol{E} + \boldsymbol{E}_{ext})$

and boundary conditions TBD

In cylindrical coordinate, try to get E_ρ, E_φ, H_ρ, H_φ as a function of E_z & H_z, the reason is that EM waves are transverse wave and there is a chance that $E_z = 0$ and/or $H_z = 0$

Maxwell Equations - Right Side

Surface current exists for normal/super conductors, thus **J** is not 0 (next lecture)

In the waveguide ρ=0 & **J**=0 (ideal conductor)

Fields are time dependent $e^{j\omega t}$, $\partial/\partial t \rightarrow j\omega$

∂**H**$/\partial t = j\omega$**H** & ∂**E**$/\partial t = j\omega$**E**

$-\partial$**B**$/\partial t = -\mu\partial$**H**$/\partial t = - j\omega\mu$**H** & ∂**D**$/\partial t = -\varepsilon\partial$**E**$/\partial t = j\omega\varepsilon$**E**

We have ∇ x **E** = $-j\omega\mu$**H** & ∇ x **H** = $j\omega\varepsilon$**E**

From now on time dependent is omitted, and we assume it is under vacuum.

Maxwell Equations - Helmholtz

∇ x **E** = $-j\omega\mu$**H** & ∇ x **H** = $j\omega\varepsilon$**E**

So we have ∇ x ∇ x **E** = $-j\omega\mu\nabla$ x **H** = $\omega^2\mu\varepsilon$**E**

and ∇ x ∇ x **E** = $\nabla(\nabla\cdot$**E**$)$ - ∇^2**E** = - ∇^2**E**

thus ∇^2**E** + $\omega^2\mu\varepsilon$**E** = 0, with (angular) wave number $k = \omega\sqrt{\mu\varepsilon} = \frac{\omega}{c} = \frac{2\pi}{\lambda}$ in vacuum, we will use vacuum later on, ∇^2**E** + k^2**E** = 0

This is Helmholtz equation

Instead of solving this Helmholtz equation directly, we express the transverse components (E_x, E_y, H_x, H_y) as a function of longitudinal components (E_z, H_z) and then combine with boundary condition to get the field distribution. It is still Helmholtz equation though.

Math – curl/rot/∇x

In Cartesian coordinate: $\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}$ unit vectors

$$\nabla \times \boldsymbol{A} = \begin{bmatrix} \boldsymbol{x} & \boldsymbol{y} & \boldsymbol{z} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ A_x & A_y & A_z \end{bmatrix} = \boldsymbol{x}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \boldsymbol{y}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \boldsymbol{z}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

In Cylindrical coordinate: $\boldsymbol{\rho}, \boldsymbol{\varphi}, \boldsymbol{z}$ unit vectors

$$\nabla \times \boldsymbol{A} = \frac{1}{\rho}\begin{bmatrix} \boldsymbol{\rho} & \rho\boldsymbol{\varphi} & \boldsymbol{z} \\ \dfrac{\partial}{\partial \rho} & \dfrac{\partial}{\partial \varphi} & \dfrac{\partial}{\partial z} \\ A_\rho & \rho A_\varphi & A_z \end{bmatrix}$$

$$= \boldsymbol{\rho}\left(\frac{1}{\rho}\frac{\partial A_z}{\partial \varphi} - \frac{\partial A_\varphi}{\partial z}\right) + \boldsymbol{\varphi}\left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho}\right) + \boldsymbol{z}\frac{1}{\rho}\left(\frac{\partial(\rho A_\varphi)}{\partial \rho} - \frac{\partial A_\rho}{\partial \varphi}\right)$$

Math - ∇^2

The divergence of the gradient $\nabla^2\boldsymbol{f} = \nabla\cdot\nabla\boldsymbol{f}$ represents the distribution of sources.

Laplace's equation: $\nabla^2\boldsymbol{f}(x,y,z) = 0$
Poisson's equation: $\nabla^2\boldsymbol{f}(x,y,z) = \boldsymbol{h}(x,y,z)$
Helmholtz equation: $(\nabla^2 + k^2)\boldsymbol{f}(x,y,z) = 0$

In Cartesian coordinate:
$$\nabla^2\boldsymbol{f}(x,y,z) = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\boldsymbol{f}(x,y,z)$$
In Cylindrical coordinate:
$$\nabla^2\boldsymbol{f}(\rho,\varphi,z) = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial f}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial\varphi^2} + \frac{\partial^2 f}{\partial z^2}$$

Propagation

- Fields travelling along +z direction $e^{-j\beta z}$
- Propagation constant β can be complex, i.e. in the form of $e^{-(\alpha + j\beta)z}$
  - Real part α is the decay (attenuation constant) and cannot be negative.
  - Imaginary part is the oscillation (phase constant), it is related to phase velocity $V_{ph} = \frac{\omega}{\beta} = f\lambda_{\text{guide}}$, with λ_{guide} guide wavelength, the distance between **two adjacent equal phase planes** along the waveguide.
- We call z direction longitudinal direction and x-y plane transverse plane.

Attenuation and dB

- RF waveguide without attenuation does not exist in real world, even when superconducting material is used.
- A good waveguide should have small attenuation
- Attenuation from metal boundary is usually calculated as "perturbation", meaning the field pattern in a waveguide (or cavity) is calculated first without considering loss on the metal, and then loss is calculated based on the field pattern.
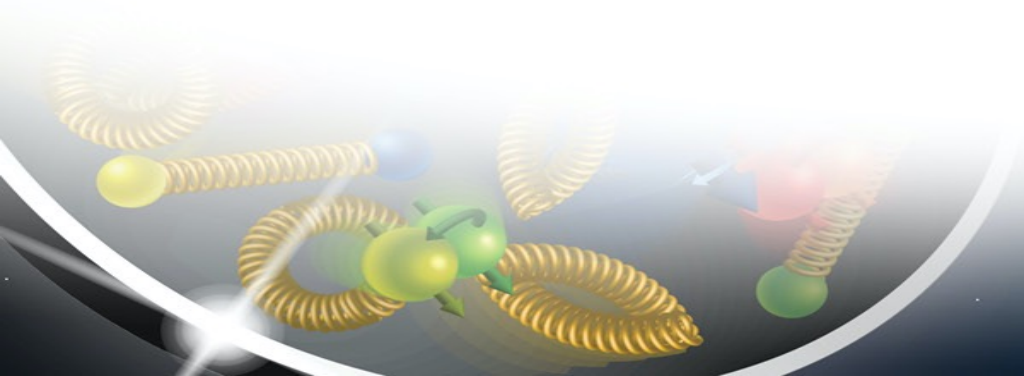
Attenuation and dB

- Decay in a waveguide is quantified using dB = 10log(P_{out}/P_{in})
  - power attenuated to 50% means ~-3dB, to 10% means -10dB.
  - We also use dB as the gain, power boost to 2x means 3dB, to 10x means 10dB.
  - Sometimes we say 3dB attenuation, it means -3dB, and 3dB gain (for a RF amplifier), it means +3dB.
  - Note dB represents power ratio not power. dBm (decibel mW) represents power (P_{out} relative to 1mW, 0dBm = 1mW)
- In propagation, we used αz [Np] with $\frac{P_{out}}{P_{in}} = e^{-2\alpha z}$ thus $\alpha z = \frac{1}{2} ln \frac{P_{in}}{P_{out}} =$

$\frac{1}{2} ln \frac{P_{in}}{P_{in} - P_{loss}} = \frac{1}{2} ln \left(1 - \frac{P_{loss}}{P_{in}} \right)^{-1} \approx \frac{P_{loss}}{2 P_{in}}$, with Z the length and α attenuation per meter [Np/m]
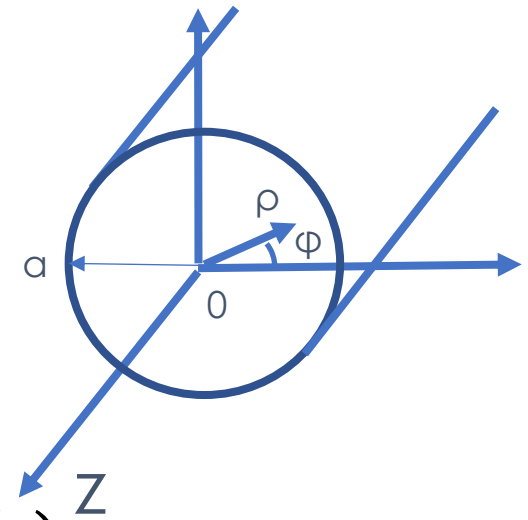
- $1[Np] = -8.686[dB]$

Attenuation α

- $\frac{P_{out}}{P_{in}} = e^{-2\alpha z}$ a factor of 2 appears because this is for power.

- For fields we have $\frac{E_{out}}{E_{in}} = \frac{H_{out}}{H_{in}} = \sqrt{\frac{P_{out}}{P_{in}}} = e^{-\alpha z}$.

- αz is the total attenuation for a cable with length z, it is the real part of the field propagation factor $e^{-(\alpha + j\beta)z}$.
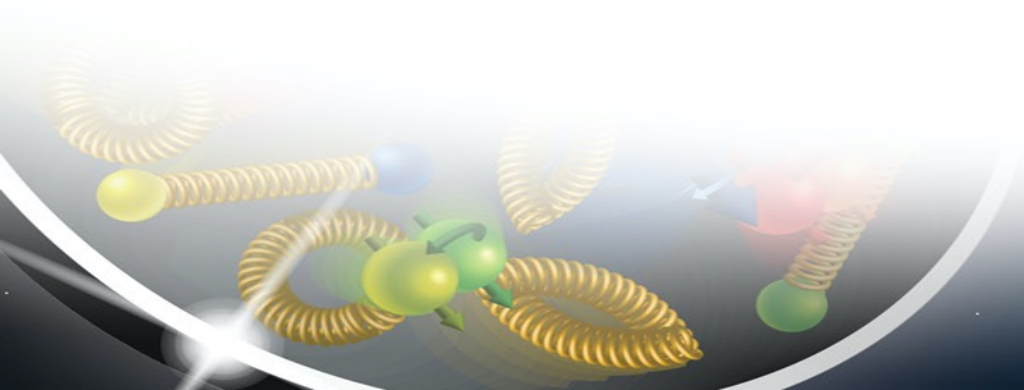
Assumption - lossless

- When we calculate the EM field of the coaxial line/waveguide/RF cavity, it is assumed first all metal walls are lossless ($\alpha = 0$).

- After the EM field is calculated using lossless metal walls (perfect conductor), the H field on the metal surface is then used to calculate the RF power dissipation – perturbation method.

- Fields travelling along +z direction $e^{-j\beta z}$, with β a real number and $\frac{\partial}{\partial z} = -j\beta$

Cylindrical coordinate

$\nabla \times \mathbf{E} = -j\omega\mu\mathbf{H}$ & $\nabla \times \mathbf{H} = j\omega\varepsilon\mathbf{E}$

$\nabla \times \mathbf{E} = \boldsymbol{\rho}\left(\frac{1}{\rho}\frac{\partial E_z}{\partial \varphi} - \frac{\partial E_\varphi}{\partial z}\right) + \boldsymbol{\varphi}\left(\frac{\partial E_\rho}{\partial z} - \frac{\partial E_z}{\partial \rho}\right) + \mathbf{z}\frac{1}{\rho}\left(\frac{\partial(\rho E_\varphi)}{\partial \rho} - \frac{\partial E_\rho}{\partial \varphi}\right)$

$= \boldsymbol{\rho}\left(\frac{1}{\rho}\frac{\partial E_z}{\partial \varphi} + j\beta E_\varphi\right) + \boldsymbol{\varphi}\left(-j\beta E_\rho - \frac{\partial E_z}{\partial \rho}\right) + \mathbf{z}\frac{1}{\rho}\left(\frac{\partial(\rho E_\varphi)}{\partial \rho} - \frac{\partial E_\rho}{\partial \varphi}\right)$

$= \boldsymbol{\rho}\left(-j\omega\mu H_\rho\right) + \boldsymbol{\varphi}\left(-j\omega\mu H_\varphi\right) + \mathbf{z}\left(-j\omega\mu H_z\right)$

Similarly

$\boldsymbol{\rho}\left(\frac{1}{\rho}\frac{\partial H_z}{\partial \varphi} + j\beta H_\varphi\right) + \boldsymbol{\varphi}\left(-j\beta H_\rho - \frac{\partial H_z}{\partial \rho}\right) + \mathbf{z}\frac{1}{\rho}\left(\frac{\partial(\rho H_\varphi)}{\partial \rho} - \frac{\partial H_\rho}{\partial \varphi}\right)$

$= \boldsymbol{\rho}\left(j\omega\varepsilon E_\rho\right) + \boldsymbol{\varphi}\left(j\omega\varepsilon E_\varphi\right) + \mathbf{z}\left(j\omega\varepsilon E_z\right)$

Maxwell Equations to Field Distribution

- Compare the ρ, φ, z components of **E** & **H**

- Get E_ρ, E_φ, H_ρ, H_φ as a function of E_z & H_z

- The reason is that EM waves are transverse wave and there is a chance that $E_z = 0$ (TE), or $H_z = 0$ (TM), or both are 0 (TEM).

- Modes in coaxial line with homogeneous dielectric material can be decomposed to TE, TM and TEM.

- Coaxial line with inhomogeneous dielectric material can have modes that have all six components (both E_z and H_z are non-zero) that cannot be decomposed to TE, TM and TEM, which need to be solved numerically and are not covered in this lecture.

Field Distribution

$$\frac{1}{\rho}\frac{\partial E_z}{\partial \varphi} + j\beta E_\varphi = -j\omega\mu H_\rho$$

$$-j\beta E_\rho - \frac{\partial E_z}{\partial \rho} = -j\omega\mu H_\varphi$$

$$\frac{1}{\rho}\left(\frac{\partial(\rho E_\varphi)}{\partial \rho} - \frac{\partial E_\rho}{\partial \varphi}\right) = -j\omega\mu H_z$$

$$\frac{1}{\rho}\frac{\partial H_z}{\partial \varphi} + j\beta H_\varphi = j\omega\varepsilon E_\rho$$

$$-j\beta H_\rho - \frac{\partial H_z}{\partial \rho} = j\omega\varepsilon E_\varphi$$

$$\frac{1}{\rho}\left(\frac{\partial(\rho H_\varphi)}{\partial \rho} - \frac{\partial H_\rho}{\partial \varphi}\right) = j\omega\varepsilon E_z$$

$$k_c^2 H_\rho = j\left(\frac{\omega\varepsilon}{\rho}\frac{\partial E_z}{\partial \varphi} - \beta\frac{\partial H_z}{\partial \rho}\right)$$

$$k_c^2 H_\varphi = -j\left(\omega\varepsilon\frac{\partial E_z}{\partial \rho} + \frac{\beta}{\rho}\frac{\partial H_z}{\partial \varphi}\right)$$

$$k_c^2 E_\rho = -j\left(\beta\frac{\partial E_z}{\partial \rho} + \frac{\omega\mu}{\rho}\frac{\partial H_z}{\partial \varphi}\right)$$

$$k_c^2 E_\varphi = j\left(-\frac{\beta}{\rho}\frac{\partial E_z}{\partial \varphi} + \omega\mu\frac{\partial H_z}{\partial \rho}\right)$$

$$k_c^2 = k^2 - \beta^2 \ \& \ k = \omega\sqrt{\mu\varepsilon}$$

Note: there are two equations that have not been used yet.

Cutoff

- $k_c^2 = k^2 - \beta^2 \ \& \ k = \omega\sqrt{\mu\varepsilon}$

- If $k < k_c$, it forces β to be imaginary so that above equation holds, which means the real positive part α appears again even considering metal wall loss as perturbation, and with a smaller k (below k_c), α is larger, attenuation is stronger.

- Transmission lines/Waveguides with cutoff frequency can be used as a high pass filter.

- An example will be shown later in circular/rectangular waveguides.

Transverse Electric (TE)

- $E_z = 0$ & $H_z \neq 0$.
- Wave impedance

$Z_{TE} = E_\rho/H_\varphi = -E_\varphi/H_\rho = \omega\mu/\beta = k\eta/\beta$

- $k_c^2 = k^2 - \beta^2$ & $k = \omega\sqrt{\mu\varepsilon}$

$$H_\rho = \frac{-j\beta}{k_c^2}\frac{\partial H_z}{\partial \rho}$$

$$H_\varphi = \frac{-j\beta}{\rho k_c^2}\frac{\partial H_z}{\partial \varphi}$$

$$E_\rho = \frac{-j\omega\mu}{\rho k_c^2}\frac{\partial H_z}{\partial \varphi}$$

$$E_\varphi = \frac{j\omega\mu}{k_c^2}\frac{\partial H_z}{\partial \rho}$$

$$\frac{\partial(\rho H_\varphi)}{\partial \rho} = \frac{\partial(H_\rho)}{\partial \varphi}$$

$$\frac{1}{\rho}\left(\frac{\partial(\rho E_\varphi)}{\partial \rho} - \frac{\partial E_\rho}{\partial \varphi}\right) = -j\omega\mu H_z$$

Transverse Magnetic (TM)

- $E_z \neq 0$ & $H_z = 0$.
- Wave impedance

  $Z_{TM} = E_\rho / H_\varphi = -E_\varphi / H_\rho = \beta/\omega\varepsilon = \beta\eta/k$

- $k_c^2 = k^2 - \beta^2$ & $k = \omega\sqrt{\mu\varepsilon}$

$$H_\rho = \frac{j\omega\varepsilon}{\rho k_c^2} \frac{\partial E_z}{\partial \varphi}$$

$$H_\varphi = \frac{-j\omega\varepsilon}{k_c^2} \frac{\partial E_z}{\partial \rho}$$

$$E_\rho = \frac{-j\beta}{k_c^2} \frac{\partial E_z}{\partial \rho}$$

$$E_\varphi = -\frac{j\beta}{\rho k_c^2} \frac{\partial E_z}{\partial \varphi}$$

$$\frac{\partial(\rho E_\varphi)}{\partial \rho} = \frac{\partial E_\rho}{\partial \varphi}$$

$$\frac{1}{\rho}\left(\frac{\partial(\rho H_\varphi)}{\partial \rho} - \frac{\partial H_\rho}{\partial \varphi}\right) = j\omega\varepsilon E_z$$

Transverse Electromagnetic (TEM)

- $E_z = 0$ & $H_z = 0$.
- $k_c = 0,\ k = \beta = \omega\sqrt{\mu\varepsilon}$
- Wave impedance

$Z_{TEM} = E_\rho/H_\varphi = -E_\varphi/H_\rho = \eta$

$$\sqrt{\varepsilon}E_\varphi = -\sqrt{\mu}H_\rho$$

$$\sqrt{\varepsilon}E_\rho = \sqrt{\mu}H_\varphi$$

$$\frac{\partial(\rho E_\varphi)}{\partial\rho} = \frac{\partial E_\rho}{\partial\varphi}$$

$$\frac{\partial(\rho H_\varphi)}{\partial\rho} = \frac{\partial H_\rho}{\partial\varphi}$$

Field pattern is frequency independent

Coaxial cables

TEM (1)

$$\sqrt{\varepsilon}E_\varphi = -\sqrt{\mu}H_\rho$$
$$\sqrt{\varepsilon}E_\rho = \sqrt{\mu}H_\varphi$$
$$\frac{\partial(\rho E_\varphi)}{\partial\rho} = \frac{\partial E_\rho}{\partial\varphi}$$
$$\frac{\partial(\rho H_\varphi)}{\partial\rho} = \frac{\partial H_\rho}{\partial\varphi}$$
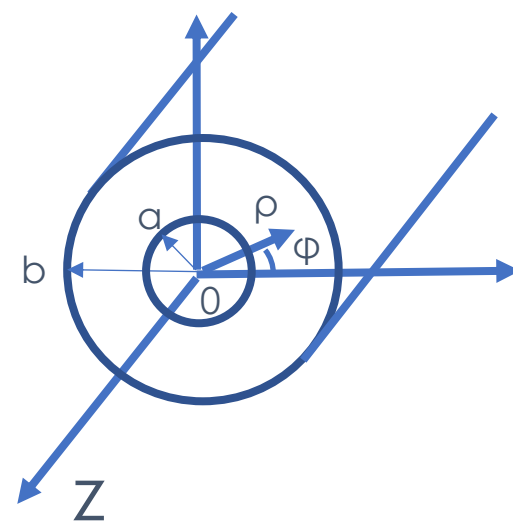
$\frac{\partial(\rho H_\varphi)}{\partial\rho} = \frac{\partial H_\rho}{\partial\varphi}$ with $\sqrt{\varepsilon}E_\varphi = -\sqrt{\mu}H_\rho$ & $\sqrt{\varepsilon}E_\rho = \sqrt{\mu}H_\varphi$

$$\rightarrow \frac{\partial(\rho E_\rho)}{\partial\rho} = -\frac{\partial E_\varphi}{\partial\varphi} \rightarrow -\frac{\partial^2 E_\varphi}{\partial\rho\partial\varphi} = \frac{\partial^2(\rho E_\rho)}{\partial\rho^2}$$

and $\frac{\partial(\rho E_\varphi)}{\partial\rho} = \frac{\partial E_\rho}{\partial\varphi} \rightarrow \frac{\partial^2 E_\rho}{\partial\varphi^2} = \frac{\partial^2(\rho E_\varphi)}{\partial\rho\partial\varphi} = \frac{\partial}{\partial\varphi}\left(E_\varphi + \rho\frac{\partial E_\varphi}{\partial\rho}\right) = \frac{\partial E_\varphi}{\partial\varphi} + \rho\frac{\partial^2 E_\varphi}{\partial\rho\partial\varphi}$

Cancelling the term E_φ, we get,

$$-\frac{\partial^2 E_\rho}{\partial\varphi^2} = \frac{\partial(\rho E_\rho)}{\partial\rho} + \rho\frac{\partial^2(\rho E_\rho)}{\partial\rho^2} = \frac{\partial\left(\rho\frac{\partial(\rho E_\rho)}{\partial\rho}\right)}{\partial\rho}$$

and then use "separation of variables" $E_\rho(\rho,\varphi) = P(\rho)\Phi(\varphi)e^{-j\beta z}$

$$-\frac{\partial^2\Phi}{\Phi\partial\varphi^2} = \frac{\partial\left(\rho\frac{\partial(\rho P)}{\partial\rho}\right)}{P\partial\rho} = k_a^2$$

So $P = \frac{A}{\rho}$ & $k_a = 0$, and $\Phi(\varphi) = B$, thus $E_\rho(\rho,\varphi) = \frac{A}{\sqrt{\varepsilon}\rho}e^{-j\beta z}$ and $H_\varphi(\rho,\varphi) = \frac{A}{\sqrt{\mu}\rho}e^{-j\beta z}$

TEM (2)
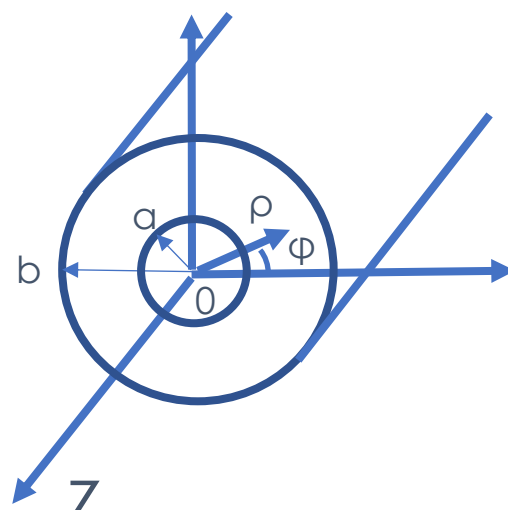
$$\frac{\partial(\rho H_\varphi)}{\partial\rho} = \frac{\partial H_\rho}{\partial\varphi} \text{ with } \sqrt{\varepsilon}E_\varphi = -\sqrt{\mu}H_\rho \ \& \ \sqrt{\varepsilon}E_\rho = \sqrt{\mu}H_\varphi$$

$$\rightarrow \frac{\partial(\rho E_\rho)}{\partial\rho} = -\frac{\partial E_\varphi}{\partial\varphi} \rightarrow -\frac{\partial^2 E_\varphi}{\partial\varphi^2} = \frac{\partial^2(\rho E_\rho)}{\partial\rho\partial\varphi} = \frac{\partial}{\partial\varphi}\left(E_\rho + \rho\frac{\partial E_\rho}{\partial\rho}\right) = \frac{\partial E_\rho}{\partial\varphi} + \rho\frac{\partial^2 E_\rho}{\partial\rho\partial\varphi}$$

and $\frac{\partial(\rho E_\varphi)}{\partial\rho} = \frac{\partial E_\rho}{\partial\varphi} \rightarrow \frac{\partial^2 E_\rho}{\partial\rho\partial\varphi} = \frac{\partial^2(\rho E_\varphi)}{\partial\rho^2}$, applying these into above,

$$-\frac{\partial^2 E_\varphi}{\partial\varphi^2} = \frac{\partial(\rho E_\varphi)}{\partial\rho} + \rho\frac{\partial^2(\rho E_\varphi)}{\partial\rho^2} = \frac{\partial\left(\rho\frac{\partial(\rho E_\varphi)}{\partial\rho}\right)}{\partial\rho}$$

and then use "separation of variables" $E_\varphi(\rho,\varphi) = P(\rho)\Phi(\varphi)e^{-j\beta z}$

$$-\frac{\partial^2\Phi}{\Phi\partial\varphi^2} = \frac{\partial\left(\rho\frac{\partial(\rho P)}{\partial\rho}\right)}{P\partial\rho} = k^2, \text{ or simply } \frac{\partial(\rho E_\rho)}{\partial\rho} = -\frac{\partial E_\varphi}{\partial\varphi} \text{ with } E_\rho(\rho,\varphi) = \frac{A}{\sqrt{\varepsilon}\rho}e^{-j\beta z}$$

Boundary conditions: $E_\varphi(a,\varphi) = E_\varphi(b,\varphi) = 0$ thus $E_\varphi(\rho,\varphi) = 0$ & $H_\rho(\rho,\varphi) = 0$

TEM Field Pattern

- Frequency independent
- $k_c = 0$, $k = \beta = \omega\sqrt{\mu\varepsilon}$, no cutoff frequency.

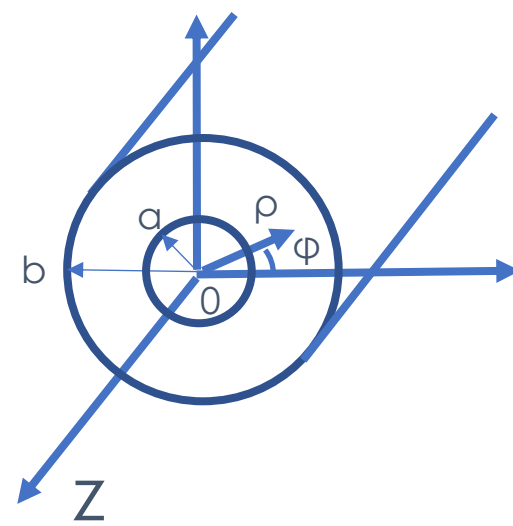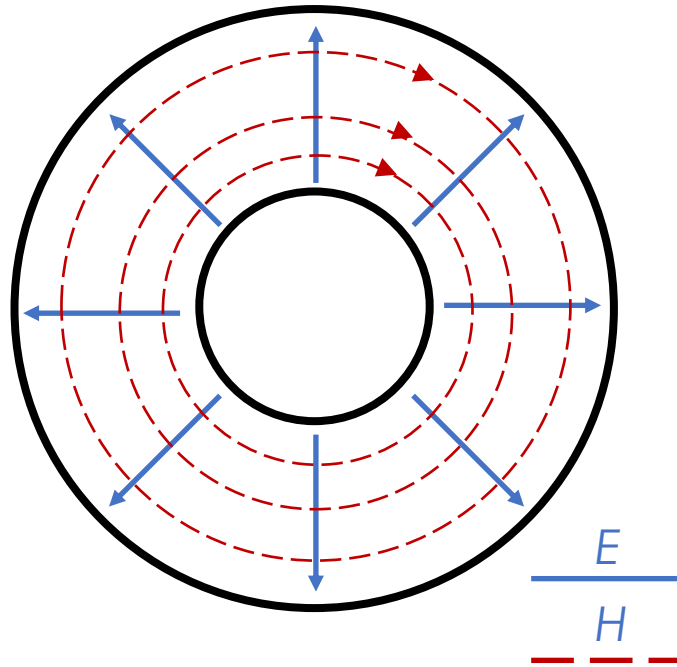$$E_\rho = \frac{A}{\sqrt{\varepsilon}\rho} e^{-j\beta z}$$

$$E_\varphi = 0$$

$$E_z = 0$$

$$H_\rho = 0$$

$$H_\varphi = \frac{A}{\sqrt{\mu}\rho} e^{-j\beta z}$$

$$H_z = 0$$

$$\frac{E}{H}$$

Or, times $\sqrt{\varepsilon}$

$$E_\rho = \frac{A}{\rho} e^{-j\beta z}$$

$$E_\varphi = 0$$

$$E_z = 0$$

$$H_\rho = 0$$

$$H_\varphi = \frac{A}{\eta\rho} e^{-j\beta z}$$

$$H_z = 0$$

TEM Field Pattern (2)

- Both E_ρ & H_φ are inversely proportional to ρ.

- *E* and *H* on the inner conductor are higher than those on the outer conductor.

- *E* field determines arcing, *H* field determine RF power dissipation on metal.

- Arcing is likely to happen first on the inner conductor surface, and RF power dissipation on the inner conductor surface is also higher.
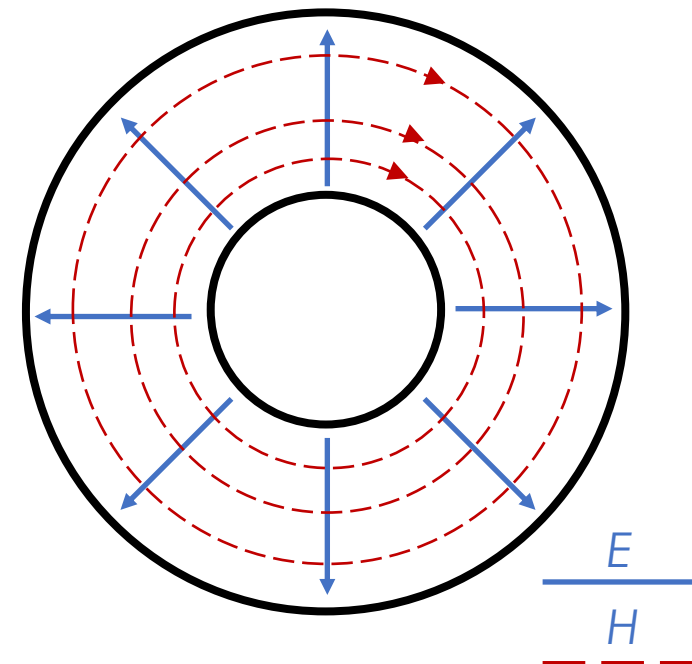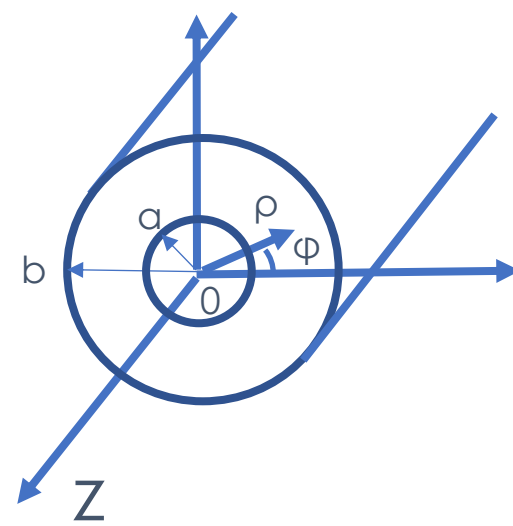
TEM Field Pattern - E
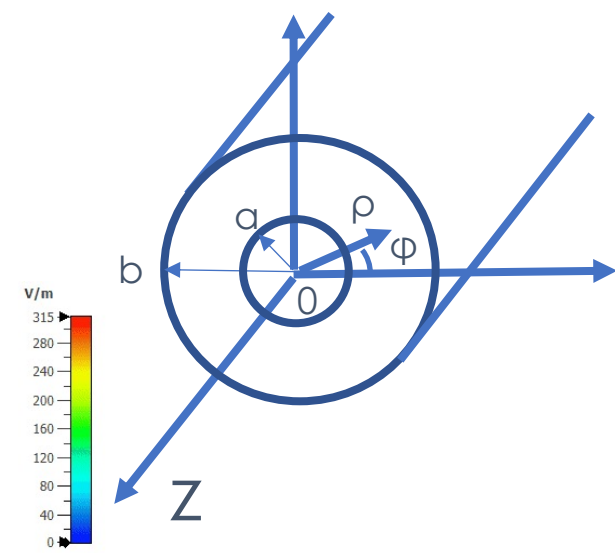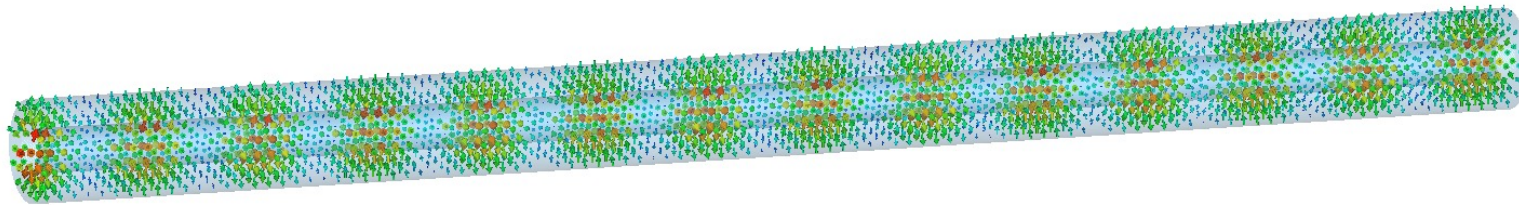
e-field (f=1) [1(1)]
Frequency 1 GHz
Phase 0 °
Maximum (Plot) 315.34 V/m

Input port

Output port
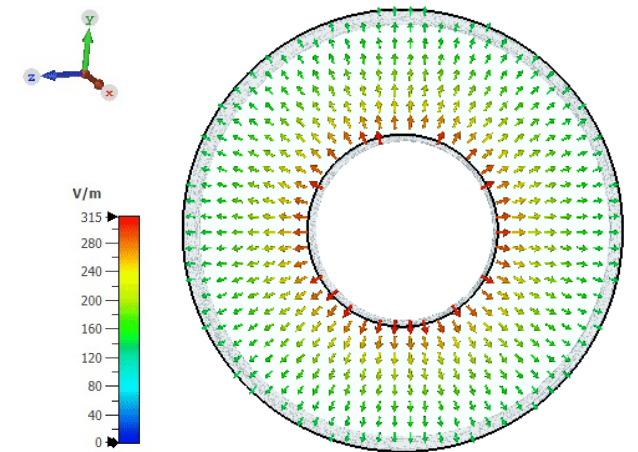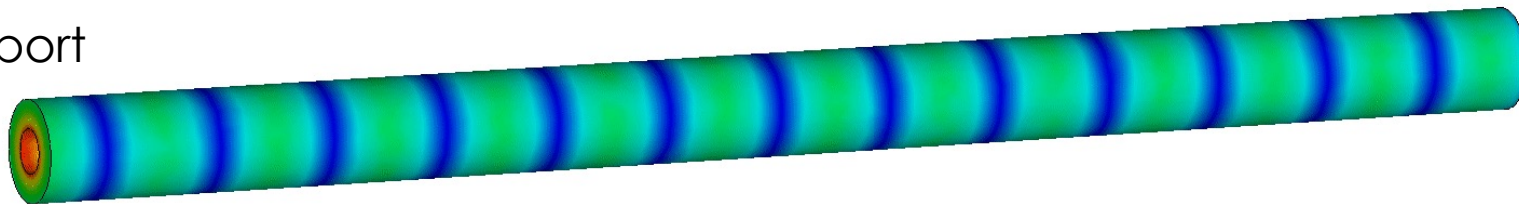
e-field (f=1) [1(1)]
Orientation Outside
Component Abs
Frequency 1 GHz
Phase 0 °
Maximum (Plot) 315.34 V/m

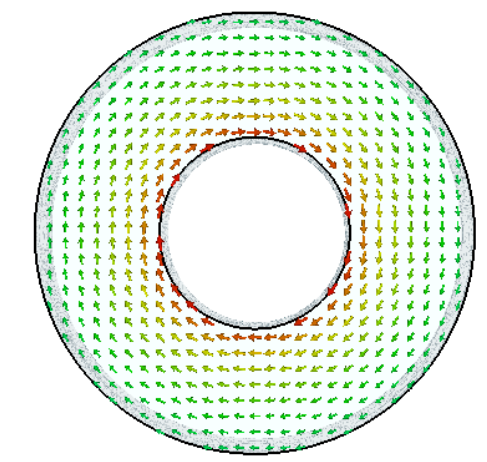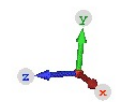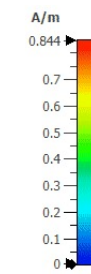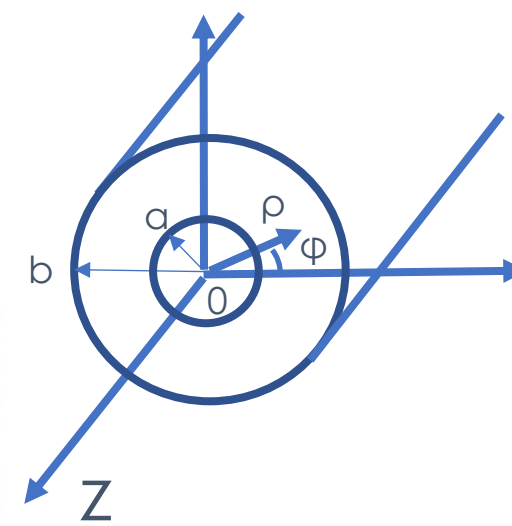The guide wavelength and phase velocity can be clearly observed.

tron-Ion Collider

TEM Field Pattern - H

h-field (f=1) [1(1)]
Frequency   1 GHz
Phase   0 °
Maximum (Plot)   0.844063 A/m

A/m
0.844
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
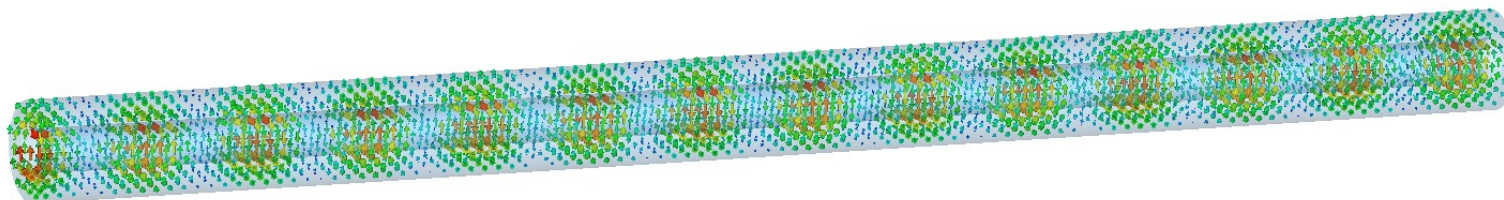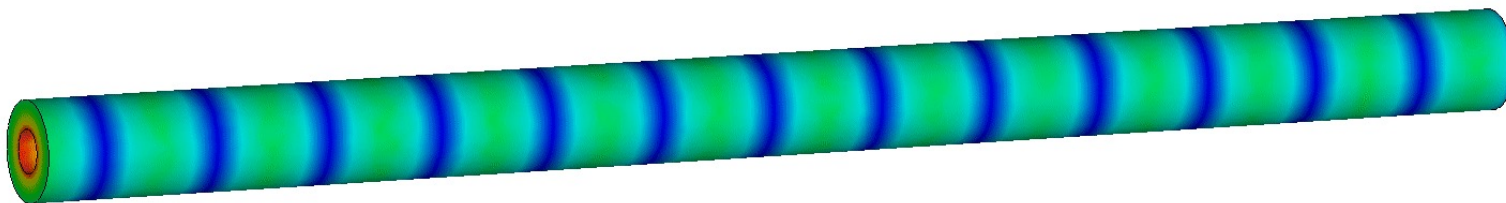
h-field (f=1) [1(1)]
Orientation   Outside
Component   Abs
Frequency   1 GHz
Phase   0 °
Maximum (Plot)   0.844063 A/m
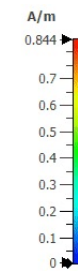
Other modes

- We will study the TE and TM modes in detail in the rectangular/circular waveguide section.

- TE_{11} cutoff frequency (approximate solution): $k_c = \frac{2}{b+a}$, it is the TE mode with the lowest cutoff frequency.

- TM_{01} cutoff frequency (approximate solution): $k_c = \frac{\pi}{b-a}$, it is the TM mode with the lowest cutoff frequency.

- Cutoff of TM_{01} is higher than the cutoff of TE_{11}.

- Coax cables are working with TEM mode below TE_{11} cutoff frequency.