Homework 9 **Due Dec. 4, 2023**

1. (5 points) The gain of a FEL oscillator (i.e. FEL working in low gain regime) depends on the energy of the electron beam. Using the results derived in the lecture, find the electron beam energy for maximal gain in a FEL oscillator. Assume that the undulator length is l_w, the undulator period is λ_w, the undulator parameter is K and the radiation wavelength is λ_0.

2. (5 points) Assuming the saturation of a FEL takes place at the condition (slides 32)

$$\Omega_{p,sat} L_G \approx 1,$$

where $\Omega_{p,sat} = \sqrt{\frac{eE_{sat}\theta_s\omega}{\gamma_z^2 c\mathcal{E}_0}}$ is the small-amplitude angular frequency of an electron oscillating in the radiation fields, E_{sat} is the amplitude of the radiation field at saturation and $L_G = \frac{1}{\sqrt{3}\Gamma}$ is the 1-D gain length of the radiation power, show that the radiation power at saturation is given by

$$P_{sat} = \varepsilon_0 c E_{sat}{}^2 A = \chi \cdot \rho \cdot \frac{\mathcal{E}_0}{e} I_e,$$

where A is the cross-section of the radiation fields (which is equal to the cross-section of the electron beam for 1-D model) and I_e is the peak current of the electron beam, find the numerical coefficient χ.