Homework for Synchrotron Light Source, due Apr. 21, 2014

1. Let us calculate the synchrotron radiation related problem in NSLS II. NSLS II adopts DBA lattice (separate function magnets). Here are the parameters:

Table 1: NSLS II parameters

Parameters	Values
Energy [GeV]	3.0
Circumference [m]	780
Number of dipoles	60
Dipole field [T]	0.4
Beam current [A]	0.5
RF frequency [MHz]	499.68
Harmonic number	1320

From the design parameters, we can calculated the following parameters:

- In DBA lattice, dispersion D and dispersion slope D' are zero at one end of dipoles and non-zero at the other end of the dipole. Find dispersion function inside the dipole magnet.
- What is the compaction factor α_c of the ring?
- The energy loss due to the dipole field.
- If the accelerating phase of the RF cavity is $\pi/6$, at least how much voltage is required? How much is the power needed?
- Actually the RF voltage is about 3MV. Find the longitudinal tune of NSLS II
- What is the critical radiation frequency of the dipole radiation.
- Find the partition number $\bar{D}$ due to synchrotron radiation in dipole.
- Find the longitudinal damping rate α_E and compare with the period of longitudinal oscillation.
- Find the equilibrium energy spread of NSLS II.

Answer:

NSLS II has 60 dipoles to form a closed loop, therefore each dipole bends 6 degree, which is $6/180 * \pi = 0.105$ rad. The radius of the dipole can be found as $P = eB\rho$, therefore $\rho = 3GeV/c/0.4m = 25m$. The length of each dipole is $L_D = 2\pi\rho/60 = 2.618m$.

Since at one end has $d = 0$ and $d' = 0$, we can calculate the dispersion function in the dipole from this end using small angle approximation:

$$\begin{pmatrix} d(s) \\ d'(s) \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & l & \rho\theta^2/2 \\ 0 & 1 & \theta \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} d(0) = 0 \\ d'(0) = 0 \\ 1 \end{pmatrix}$$

The dispersion function in dipole is $d(s) = s^2/2/\rho$.

The compaction factor is

$$\begin{aligned} \alpha_c &= \frac{1}{C} \oint \frac{D(s)}{\rho} ds \\ &= \frac{60}{C} \int_0^{L_D} s^2/2/\rho^2 ds \\ &= \frac{10}{C} \frac{L_D^3}{\rho^2} = 3.68 \times 10^{-4} \end{aligned}$$

The energy loss due to synchrotron radiation is

$$U_{SR} = C_\gamma E^4 \oint ds/\rho^2/2/\pi = \frac{C_\gamma E^4}{\rho} = 0.287 MeV$$

These energy must be compensated by the RF cavity

$$\begin{aligned} eV \sin\phi_s &= U_{SR} \\ V &= 0.57 MV \end{aligned}$$

therefore the RF cavity must provide at least 0.57 MV to compensate the synchrotron radiation loss in the dipoles.

The actual voltage is 3MV, the phase should be $\arcsin(0.287/3) = \pi - 0.095$, the synchrotron tune is:

$$\nu_s = \sqrt{\frac{h\eta eV \cos\phi_s}{2\pi E}} = 8.8 \times 10^{-3}$$

The time to finish one synchrotron oscillation is $2\pi/(\nu_s \omega_0) = 2.94 \times 10^{-4} s$.

The critical frequency of the dipole radiation is given by

$$\omega_c = \frac{3}{2} \frac{c\gamma^3}{\rho} = 3.64 \times 10^{18} Hz$$

We then calculate the radiation integrals, taking advantage that $K(s) = 0$ in dipoles:

$$I_2 = \oint 1/\rho^2 ds = 2\pi/\rho = 0.2513 m^{-1}$$

$$I_3 = \oint 1/\rho^3 ds = 2\pi/\rho^2 = 1.0 \times 10^{-2} m^{-2}$$

$$I_4 = \oint D/\rho^3 ds = 60 \int_0^{L_D} s^2/2/\rho^4 ds = 10L_D^3/\rho^4 = 4.59 \times 10^{-4} m^{-1}$$

Therefore the partition number $\bar{D} = I_4/I_2 = 1.828 \times 10^{-3}$

The longitudinal damping rate

$$\alpha_E = \frac{U_0}{2T_0 E}(2 + \bar{D}) = 36.79 s^{-1}$$

It is much slower than the synchrotron oscillation.

The equilibrium energy spread can be calculated as:

$$\frac{\delta E}{E} = \sqrt{\frac{C_q \gamma^2 I_3}{2I_2 + I_4}} = 5.12 \times 10^{-4}$$