Homework 2.

1 (8 point): Cooling time in LEReC

Use the beam parameter table for LEReC in lecture and estimate the cooling time/rates for Au ions. Compare with experimental measurements in year 2020 and 2021. Comment on the reasons for possible discrepancies.

2 (8 point): Cooling rate reduction for non-linear cooling force

Derive the cooling rate reduction by integrating the reduction of action (similar to what is done for the linear forces) over one synchrotron oscillation and show that for non-linear cooling force the reduction due to synchrotron motion is higher than a linear force.

Hint: one can assume the force is in Gaussian model

$$\Delta\delta\gamma_c = -\zeta_0 T_{rev}\delta\gamma \exp\left(-\frac{\delta\gamma^2}{2\sigma_l^2}\right)$$

and show that the reduction factor (over one oscillation period) is

$$\bar{\zeta}(I) = \frac{1}{2}\exp\left(-\frac{I}{2\sigma_P^2}\right)\left[I_0\left(\frac{I}{2\sigma_P^2}\right) - I_1\left(\frac{I}{2\sigma_P^2}\right)\right]$$

3 (9 point): Design an electron cooler ring for the future electron ion collider (EIC).

Fill the table at your choices with beam parameters (and give reasons) for an electron cooler ring for the future EIC.

Keep in mind (while choosing beam parameters) that the bunch charge is limited by the incoherent space charge tune shift (AKA, Laslett formula)

as

$$\Delta\nu_{e(x,y)} = \frac{I_e}{4\pi I_a \gamma^3} \int_0^C \frac{\beta(s)}{\sigma_e^2(s)} ds = \frac{I_e C}{4\pi I_a \gamma^3 \varepsilon_{x,y}},$$

which should be less than 0.2 to avoid particle loss), where Ia is Alfven current (17 kA) and C being the cooling section length. The goal is to have cooling time less than 5 hours (show calculation/justification).

parameter	**value**
p-bunch length σ_z [cm]	6
Proton beam energy (GeV)	275
protons: $\sigma_{\delta p}$	6.8e-4
protons: N_p	6.88e10
protons: $\varepsilon_{xp,yp}$ [nm]	11.3, 1.0
protons: $\beta_{xp,yp}$ [m]	168, 1900
Electron beam energy (MeV)	
Electrons per bunch: N_e	
Cooling section length (m)	
e- geo. Emittance (rms): $\varepsilon_{xe,ye}$ [nm]	

e- betatron function: $\beta_{xe,ye}$ [m]	
e- bunch length (rms): L_{ze} [cm]	
e- energy spread (frac.): $\sigma_{\delta e}$	
e- angular spread:	
Cooling times τ[min]	