Problem 1. **4x5 points.** Matrix of an ideal solenoid.

Consider particles with momentum $p_0$ propagating along the axis of idealized solenoid with

$$B_s = \begin{cases} 
0, s < 0 \\
B_o, 0 \leq s \leq l \\
0, s > 1
\end{cases}$$

All other components of the field are zero, e.g. $s=z$, not curvature.

(a) Use Sylvester formula and calculate 4x4 transport matrix of the solenoid;

(b) Show that resulting matrix can be presented in form of focusing matrix in each direction and a rotation

$$M_s = \begin{bmatrix}
I \cos \varphi & I \sin \varphi \\
-I \sin \varphi & I \cos \varphi
\end{bmatrix}
\begin{bmatrix}
F & 0 \\
0 & F
\end{bmatrix}$$

where

$$I = \begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}; F = \begin{bmatrix}
a & b \\
c & a
\end{bmatrix}; ab - cd = 1$$

are 2x2 matrices and $F$ is focusing one. Write expressions for $\varphi, F$ through $p_0, B_o, l, \ldots$.

(c) Finally use one tricks available for you since we can use torsion and decouple $x$ and $y$ motion:

$$\tilde{h}_n = \frac{\pi^2_2 + \pi^2_3}{2} + f \frac{x^2}{2} + g \frac{y^2}{2} + L(x \pi_1 - y \pi_1)$$

$$f = \left(\frac{e B_o}{2 p_o c}\right)^2; g = \left(\frac{e B_o}{2 p_o c}\right)^2; L = \kappa + \frac{e}{2 p_o c} B_s;$$

by choosing $\kappa = -\frac{e}{2 p_o c} B_s$. Show that matrix in this coordinates system is block diagonal (e.g. de-coupled)

$$M_s = \begin{bmatrix}
F & 0 \\
0 & F
\end{bmatrix}$$

with $F$ identical to that in the problem (b) above. Show also that rotation is angle around $z$-axis is $\kappa l = -\varphi$.

(d) Finally, explain why a simple trajectory $x=$const and $y=$const (which intuitively is trajectory parallel to the magnetic lines) is not a solution?

$$v_{x,y} = 0; \to \tilde{v} = \hat{\tilde{v}}_o; \hat{f} = \frac{e}{c} \hat{\tilde{v}}_o \times \hat{\tilde{B}}_o = 0$$

**Hint:** consider what is happening at the entrance and exit to the solenoid.