Homework 10.

Problem 1. 3x5 points. Beam envelope in straight section.
For a one-dimensional motion consider beam propagating in a straight section starting as $\boldsymbol{s_o}$ and having length L. Let's eigen vector (beam envelope) at $\boldsymbol{s_o}$ is given by:

$$Y(\mathrm{s}_o) = \begin{bmatrix} \mathrm{w}_o \\ \mathrm{w}'_o + \dfrac{i}{\mathrm{w}_o} \end{bmatrix}; \beta_o \equiv \mathrm{w}_o^2;\ \ \alpha_o = -\frac{\beta'}{2} \equiv -\mathrm{w}_o \mathrm{w}'_o; \tag{1}$$

(a) Propagate the eigen vector along the straight section. Show that β-function can be expressed as

$$\beta(\mathrm{s}) = \beta^* + \frac{(s-s^*)^2}{\beta^*};$$

where β^*, s^* can be found from initial conditions (1). Hint: use derivative of β-function to find s^*. β^*(beta-star) is frequently used in colliders to describe the beam envelope in the collision point (detectors).

(b) Calculate the (betatron) phase advance acquired in the straight section. Express the phase advance as function of β^*, s^*. Write expression for x(s) and x'(s). Show that x'=const.

(c) What is the maximum possible betarton phase advance in a straight section (e.g. when s_o ,L are unlimited)?