# Today schedule:

1. Photography session (15 minutes)
2. Emittance measurements short lecture.(30 minutes)
3. 10 minutes break.
4. Group A  will go to ATF control (~50 min)
5. Group B will simulate RF acceleration (~50 min)
6. Short break for switch between computer and control rooms

# PHY542. Emittance Measurements

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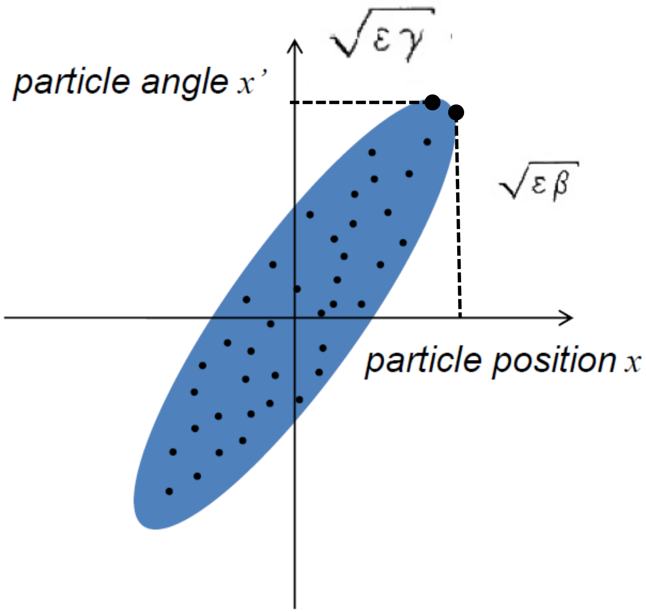
March. 30, 2015

# Emittance, what is it ?




ε = Area in *x, x'* plane occupied by beam particles divided by π

Beam ellipse and its orientation is described by 4 parameters

$$\varepsilon = \gamma x^2 + 2\,\alpha\, x\, x' + \beta\, x'^2$$

$\sqrt{\beta\varepsilon}$ Is the beam half width

$\sqrt{\gamma\varepsilon}$ Is the beam half divergence

$\alpha$ Describes how strong x and x' are correlated

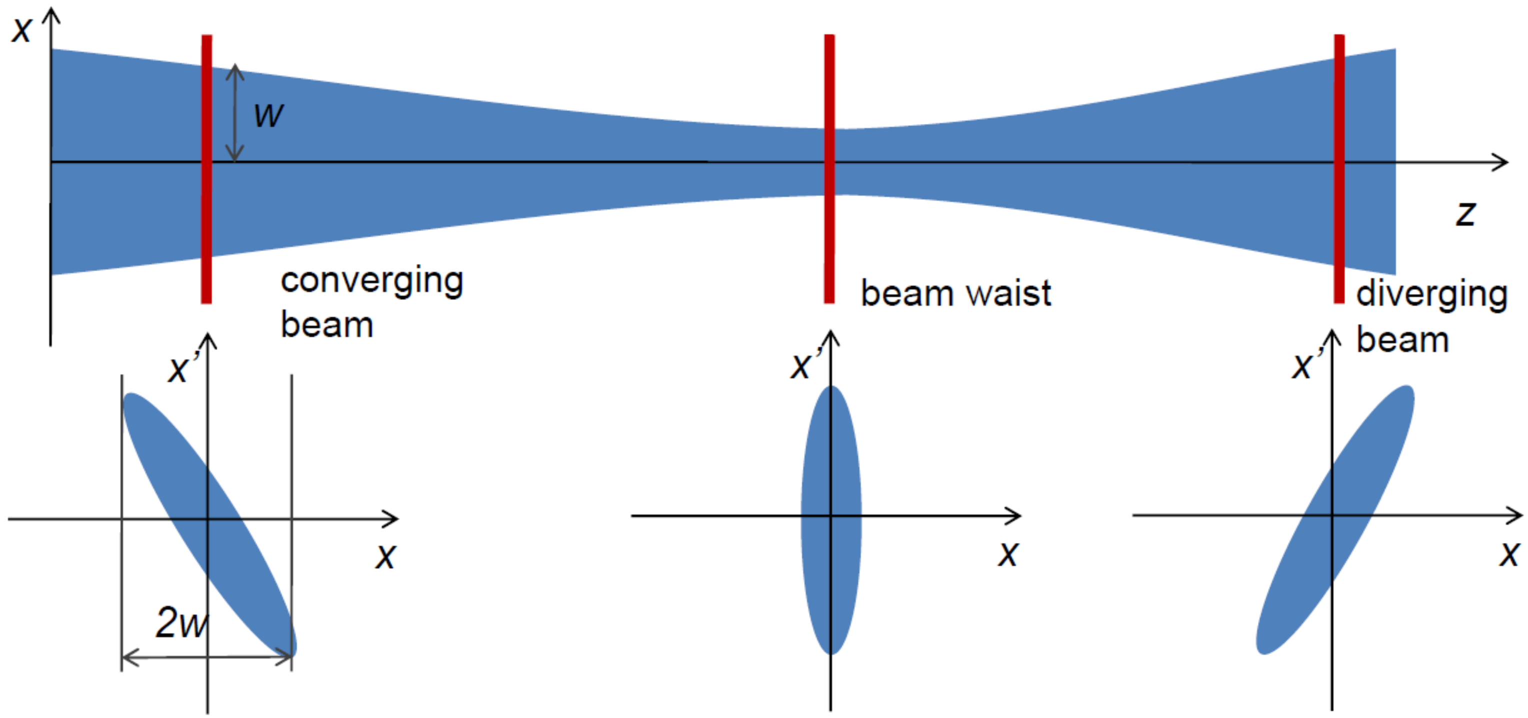
a<0 beam diverging

a>0 beam converging

a=0 beam size is maximum or minimum(waist)

The three orientation parameters are connected by the relation $$\gamma = \frac{1+\alpha^2}{\beta}$$

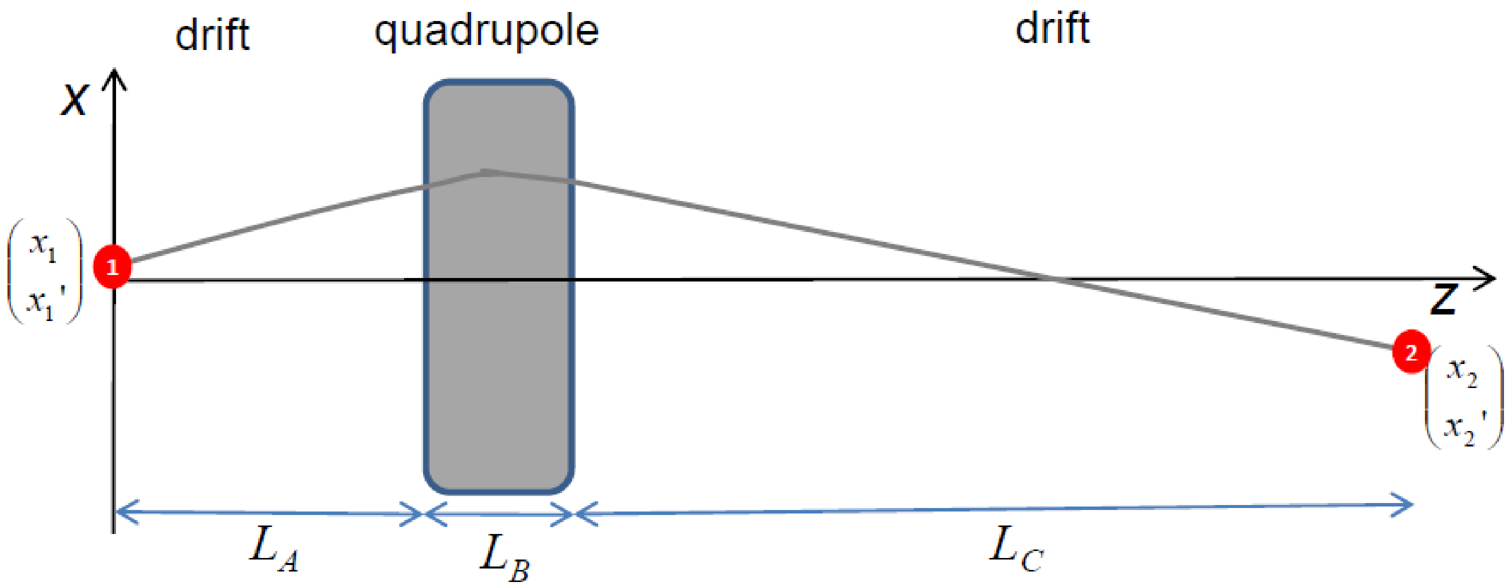
# *Beam envelope along beamline.*




Along a beamline the orientation and aspect ratio of the beam ellipse in x, x' changes, but area (emittance) remains constant.

Beam width along Z is described $$w(z) = \sqrt{\beta(z)\,\varepsilon}$$

# *Transport of single particle described with matrix*

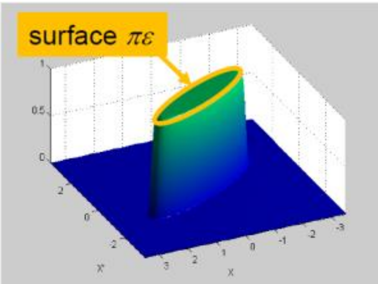



$$M_{Drift} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \qquad M_{Quadrupole} = \begin{pmatrix} \cos\left(\sqrt{k}L\right) & 1/\sqrt{k}\sin\left(\sqrt{k}L\right) \\ -\sqrt{k}\sin\left(\sqrt{k}L\right) & \cos\left(\sqrt{k}L\right) \end{pmatrix}$$

$$\begin{pmatrix} x_2 \\ x_2' \end{pmatrix} = M \cdot \begin{pmatrix} x_1 \\ x_1' \end{pmatrix}$$ There $$M = \begin{pmatrix} 1 & L_C \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos\left(\sqrt{k}L_B\right) & 1/\sqrt{k}\sin\left(\sqrt{k}L_B\right) \\ -\sqrt{k}\sin\left(\sqrt{k}L_B\right) & \cos\left(\sqrt{k}L_B\right) \end{pmatrix} \cdot \begin{pmatrix} 1 & L_A \\ 0 & 1 \end{pmatrix}$$

# R.m.s emittance

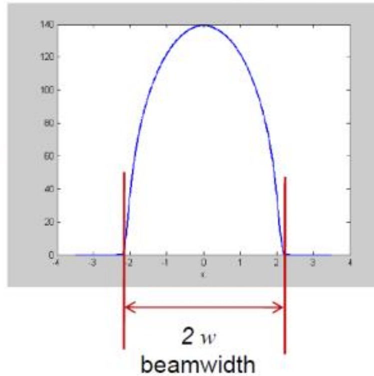
particle density in $x, x'$ space




projection on $x$ axis

$$f(x) = \int_{-\infty}^{\infty} \rho(x, x')\, dx'$$

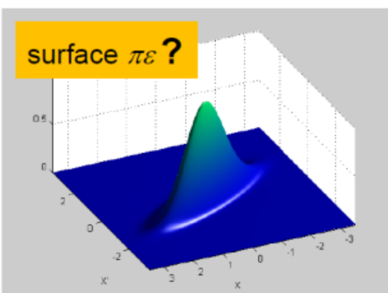
transverse beam profile




$$w = \sqrt{\beta\varepsilon}$$

In reality beam density in x, x' space is rarely a area with sharp boundary

particle density in x, x' space

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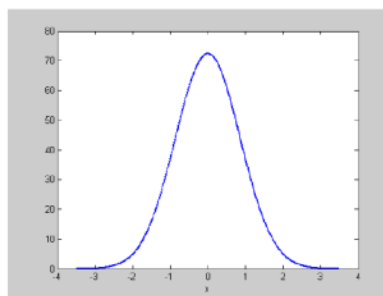


beamwidth ?

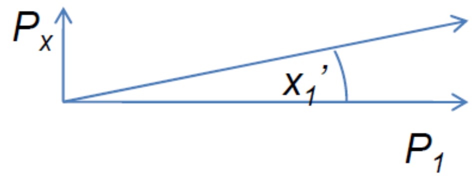
$$w_{RMS} = \sqrt{\frac{\int_{-\infty}^{+\infty} (x - x_{COG})^2 f(x)\, dx}{\int_{-\infty}^{+\infty} f(x)\, dx}}$$

For Gauss beam distribution

$$w_{rms} = \sigma_X$$

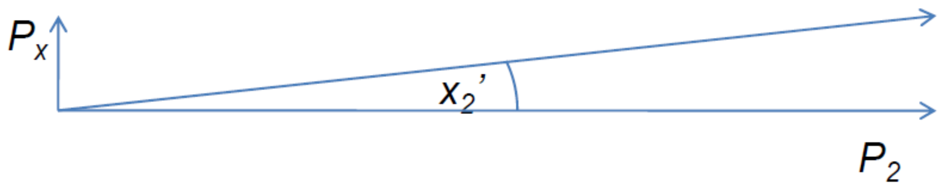
$\varepsilon_{rms}$ , 1 sigma r.m.s. emittance $\Leftrightarrow$ $w_{RMS} = \sqrt{\beta\varepsilon}$

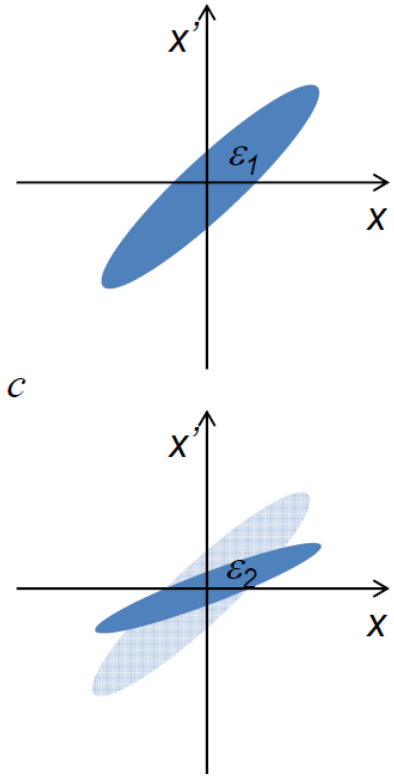
*Geometrical emittance is only constant in beamlines without acceleration*




acceleration

$$\varepsilon_2 = \varepsilon_1 \frac{P_1}{P_2} \quad \text{with} \quad P = m\beta_{rel}\gamma_{rel}\ c$$



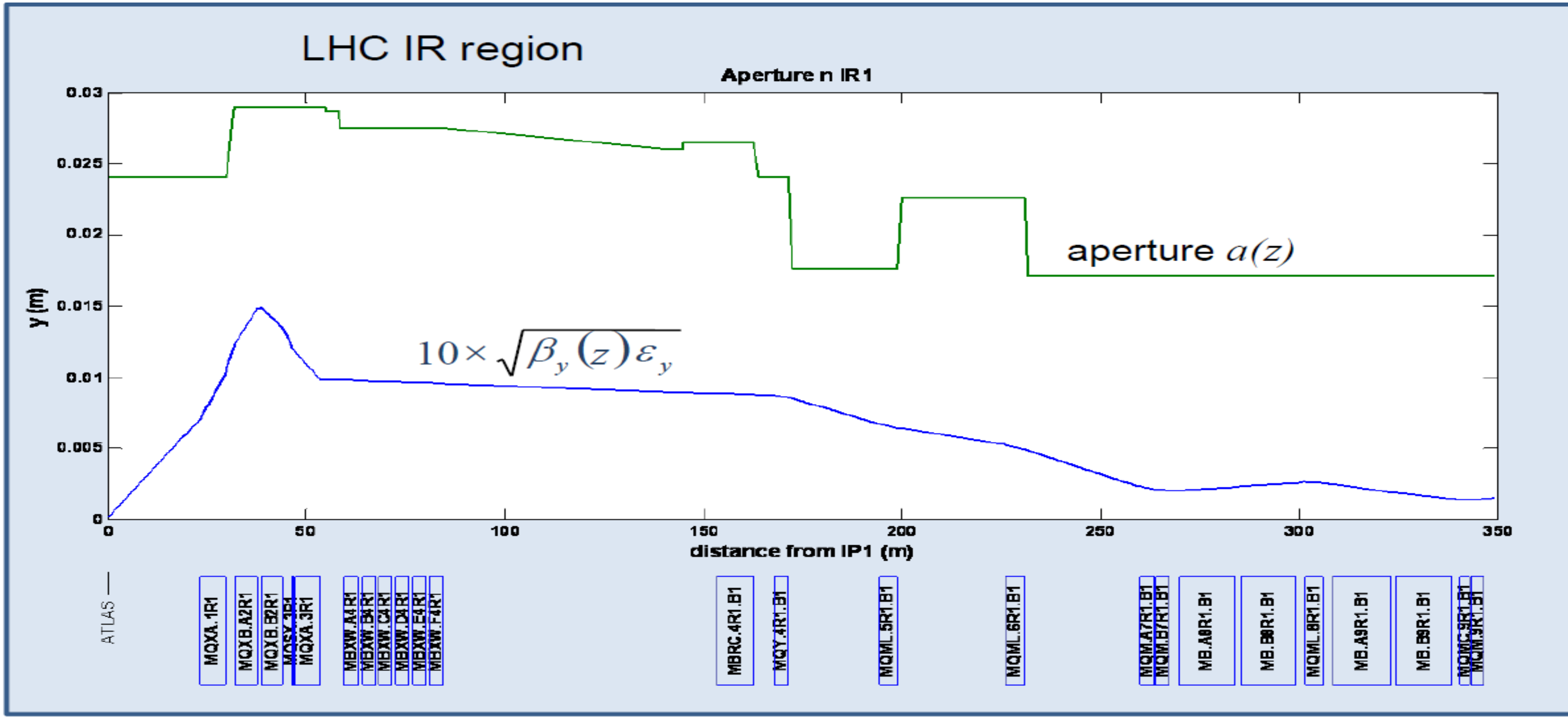




Normalized emittance preserved with acceleration

$$\boxed{\varepsilon_N = \beta_{rel}\gamma_{rel}\ \varepsilon}$$

# *Emittance, why measuring it ?*

The emittance tells if a beam **fits** in the vacuum chamber or not $w(z) = \sqrt{\beta(z)\,\varepsilon} \;<\; a(z)$




***Emittance is one of key parameters for overall performance of an accelerator:***

- *Luminosity of colliders for particle physics*
- *Brightness of synchrotron radiation sources*
- *Wavelength range of free electron lasers*
- *Resolution of fixed target experiments*

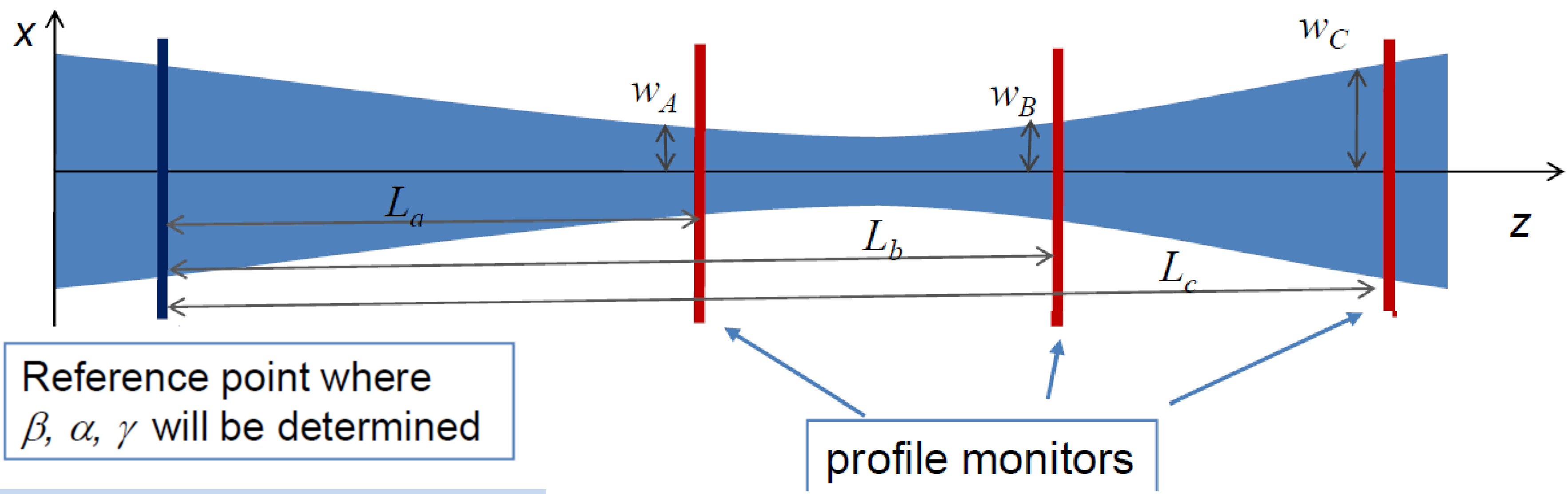
# *Emittance, how to measure it ?*

- Methods based on transverse beam profile measurements
  - Quadrupole scan
  - Different location beam profile measurements
- Slit and pepper pot methods
- Other

# Emittance measurement in transfer line or linac

Twiss parameters $\alpha,\beta,\gamma$ are a priori not known, they have to be determined together with emittance ε

*Method A*



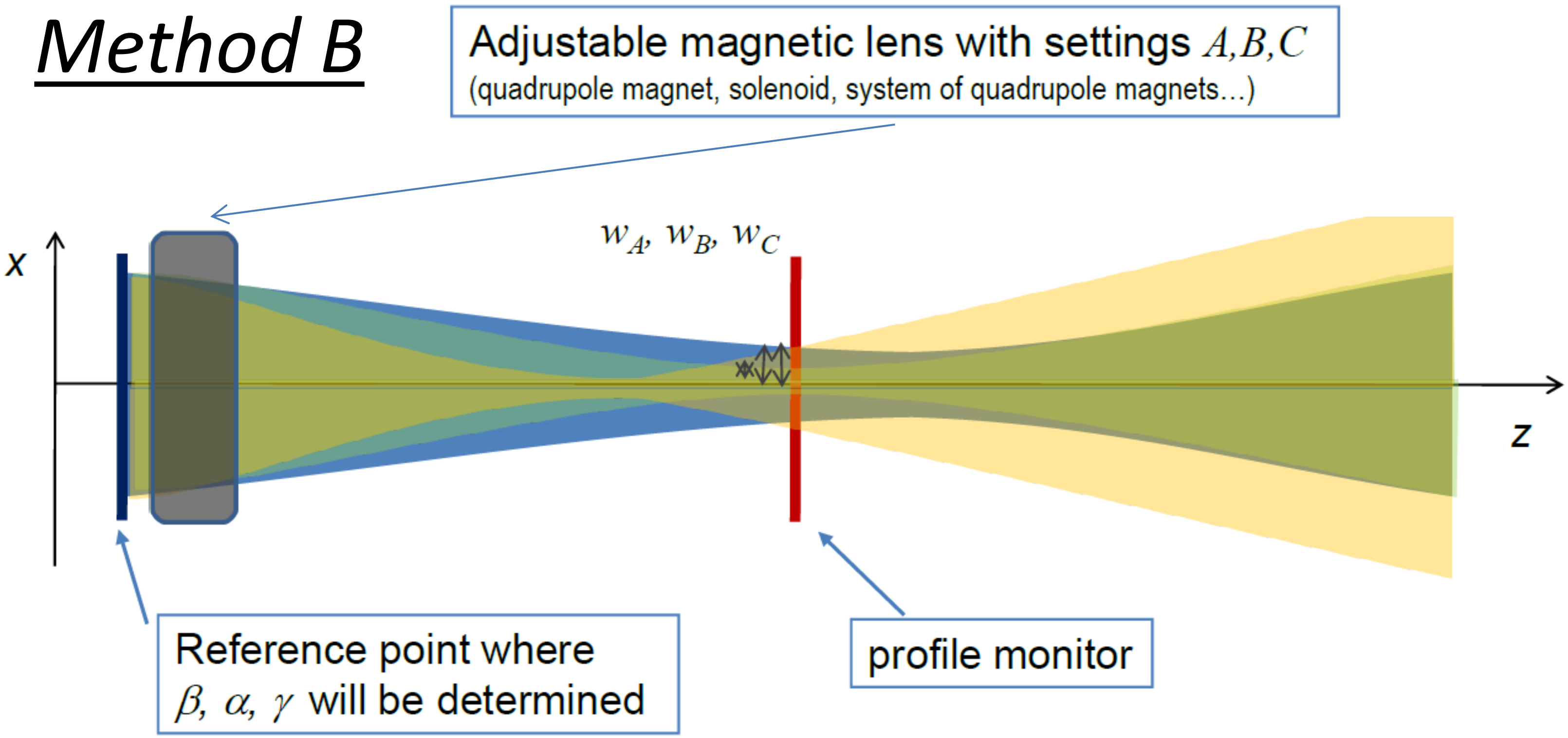

$$w_A^2 = \beta\,\varepsilon - 2\,L_A\,\alpha\,\varepsilon + L_A^2\,\gamma\,\varepsilon$$
$$w_B^2 = \beta\,\varepsilon - 2\,L_B\,\alpha\,\varepsilon + L_B^2\,\gamma\,\varepsilon$$
$$w_C^2 = \beta\,\varepsilon - 2\,L_C\,\alpha\,\varepsilon + L_C^2\,\gamma\,\varepsilon$$

3 linear equations, 3 independent variable
Solved by inverting matrix.

$$\beta\varepsilon\cdot\gamma\varepsilon - (\alpha\varepsilon)^2 = \varepsilon^2\left(\beta\cdot\gamma - \alpha^2\right) = \varepsilon^2 \quad \Rightarrow \quad \sqrt{\beta\varepsilon\cdot\gamma\varepsilon - (\alpha\varepsilon)^2} = \varepsilon, \quad \beta = \frac{\beta\varepsilon}{\varepsilon}, \quad \alpha = \frac{\alpha\varepsilon}{\varepsilon}$$

# Emittancemeasurement in transfer line or linac, (count.)

- ***Method B***




$$w^2 = c^2\ \beta\varepsilon - 2cs\ \alpha\ \varepsilon + s^2\ \gamma\varepsilon, \qquad \begin{pmatrix} c & s \\ c' & s' \end{pmatrix} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} m_{11}(I_{mag}) & m_{12}(I_{mag}) \\ m_{21}(I_{mag}) & m_{22}(I_{mag}) \end{pmatrix}$$

$$w_A^2 = c_A^2 \beta\varepsilon - 2c_A s_A\ \alpha\ \varepsilon + s_A^2\ \gamma\varepsilon$$
$$w_B^2 = c_B^2 \beta\varepsilon - 2c_B s_B\ \alpha\ \varepsilon + s_B^2\ \gamma\varepsilon$$
$$w_C^2 = c_C^2 \beta\varepsilon - 2c_C s_C\ \alpha\ \varepsilon + s_C^2\ \gamma\varepsilon$$

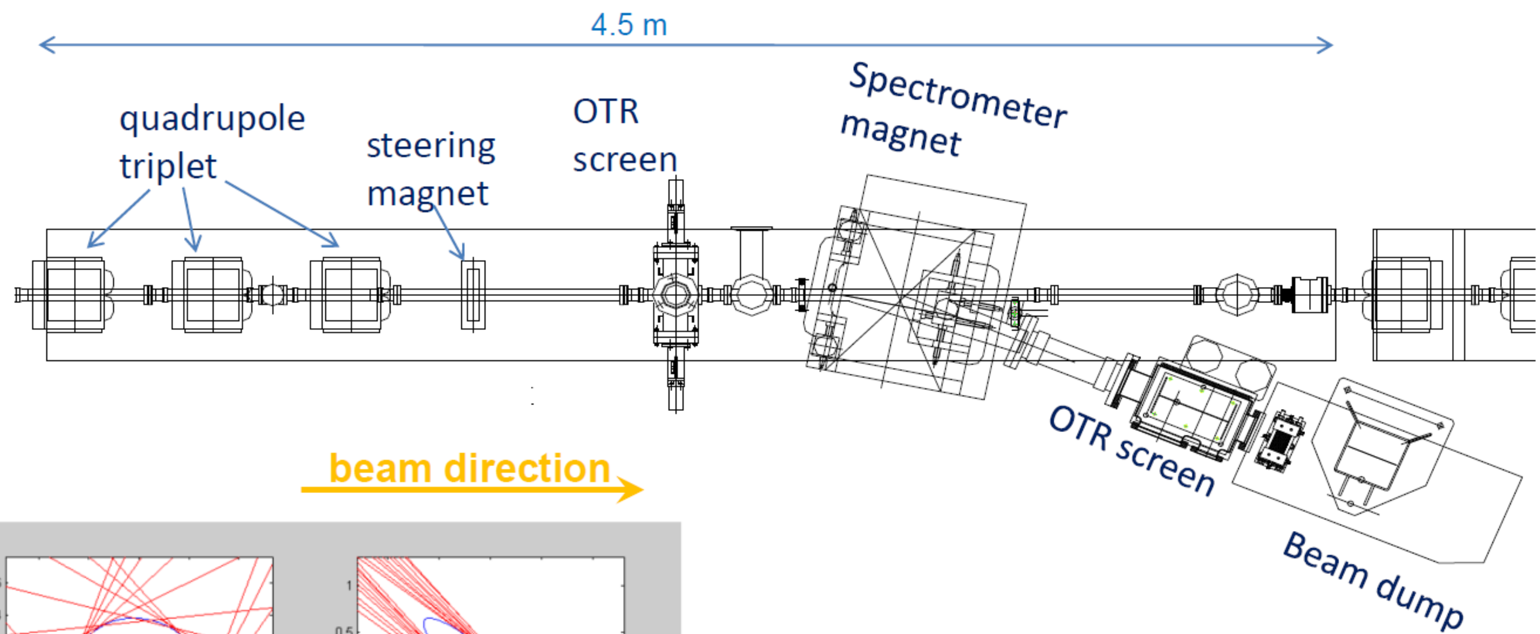
3 linear equations, 3 independent variable
Solved by inverting matrix.

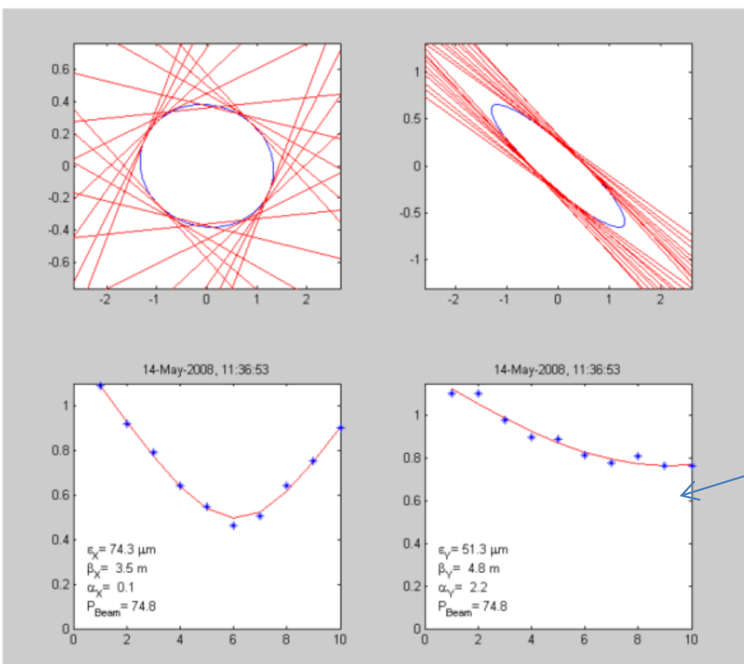
$$\beta\varepsilon \cdot \gamma\varepsilon - (\alpha\varepsilon)^2 = \varepsilon^2\left(\beta \cdot \gamma - \alpha^2\right) = \varepsilon^2 \quad \Rightarrow \quad \sqrt{\beta\varepsilon \cdot \gamma\varepsilon - (\alpha\varepsilon)^2} = \varepsilon, \quad \beta = \frac{\beta\varepsilon}{\varepsilon}, \quad \alpha = \frac{\alpha\varepsilon}{\varepsilon}$$

# Summary beam profile technics

- To determine ε, β, α at a reference point in a beamline one needs  at least three *w* measurements with different transfer matrices between the  reference point and the *w* measurements location.
- Different transfer matrices can be achieved with different profile monitor locations, different focusing magnet settings or combinations of both.
- Once β, α at one reference point is determined the values of β, α at every point in the beamline can be calculated.

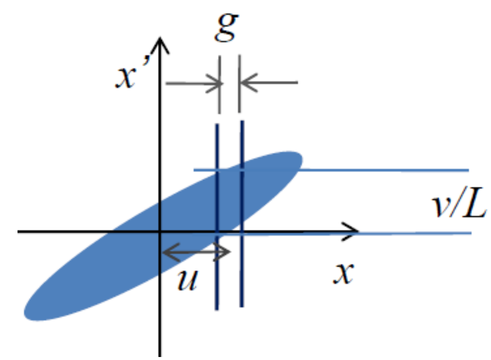
# Emittance measurement example:

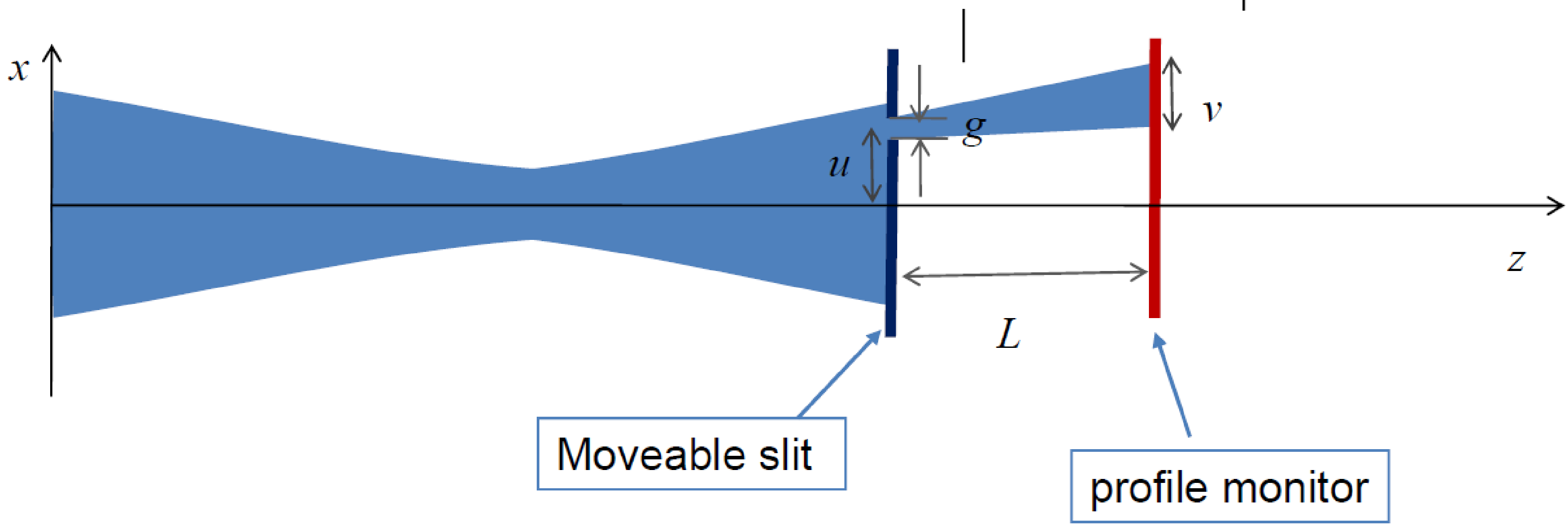



Parabola fit for quadrupole scan

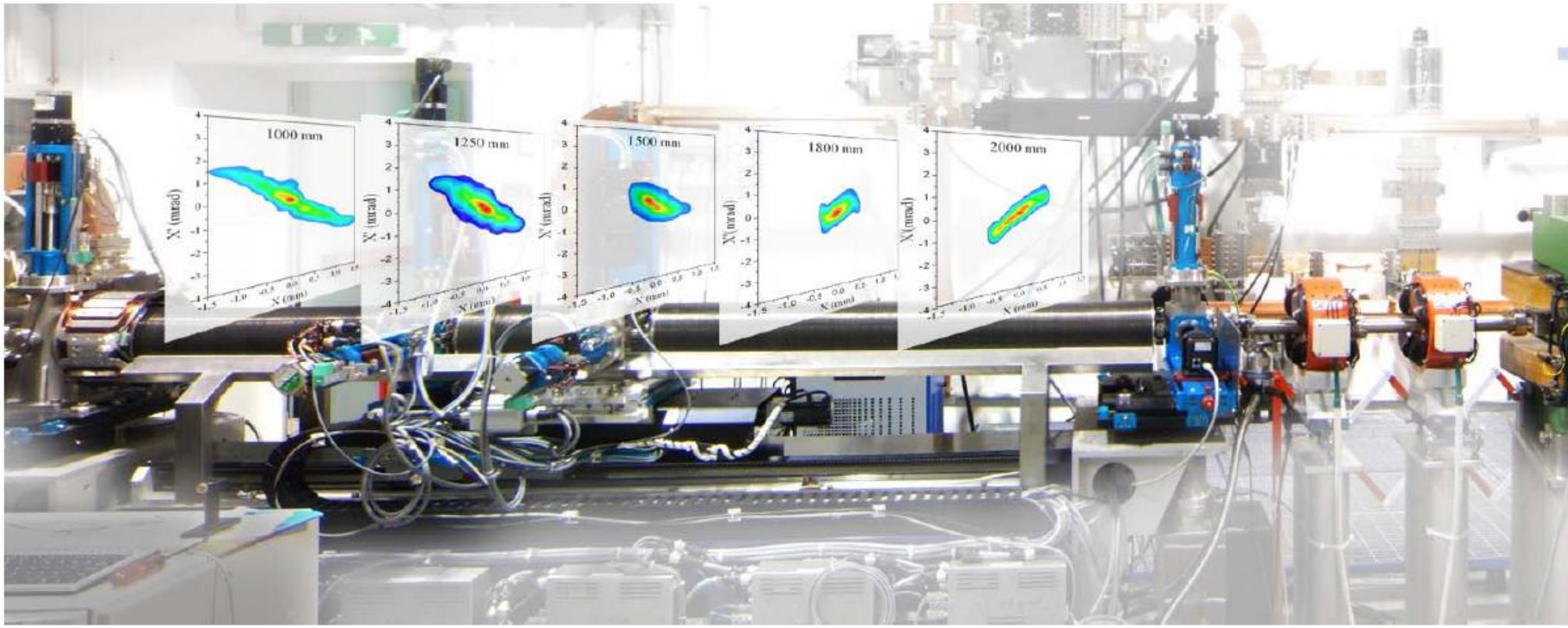
# Emittance measurement with moveable slit

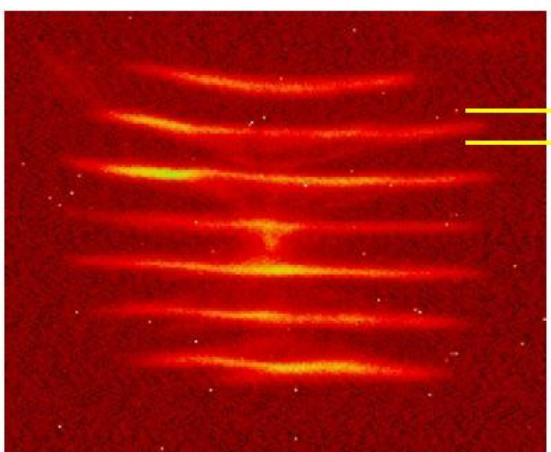


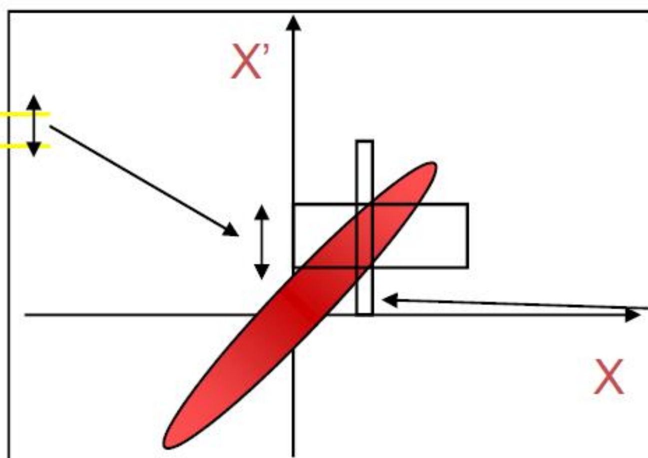



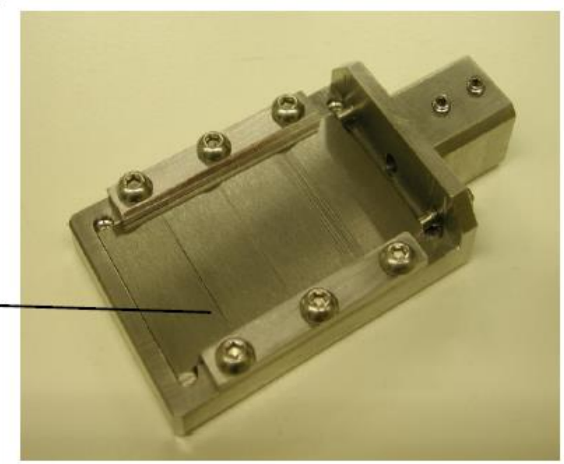

From width and position of slit image mean beam angle and divergence of slice at position *u* is readily computed.
By moving slit across the beam complete distribution in *x, x'* space is reconstructed.
Conditions for good resolution: *v >>s*

# Phase space reconstruction at SPARC (LNF)





# Computation work #3

The quadrupole scan technique is a standard technique used in accelerator facilities to measure the transverse emittance. It is based on the fact that the squared rms beam radius ($x_{rms}^2$) is proportional to the quadrupole "strength" or inverse focal-length f squared, so

$$x_{rms}^2 = < x^2 > = A\left(\frac{1}{f^2}\right) - 2AB\left(\frac{1}{f}\right) + (C + AB^2) \quad (1)$$

where A, B, C are constants and $f$ is the focal length defined as

$$\frac{1}{f} = \kappa l, \quad (2)$$

here κ is the magnet focusing strength in units of 1 over length squared and $l$ is the effective length of the magnet.

The emittance can be estimated according to

$$\varepsilon = \frac{\sqrt{AC}}{d^2} \quad (3)$$

where $d$ is the distance from the magnet you scan to the point you calculate the beam rms radius.

**OVERVIEW OF THE EXERCISE**

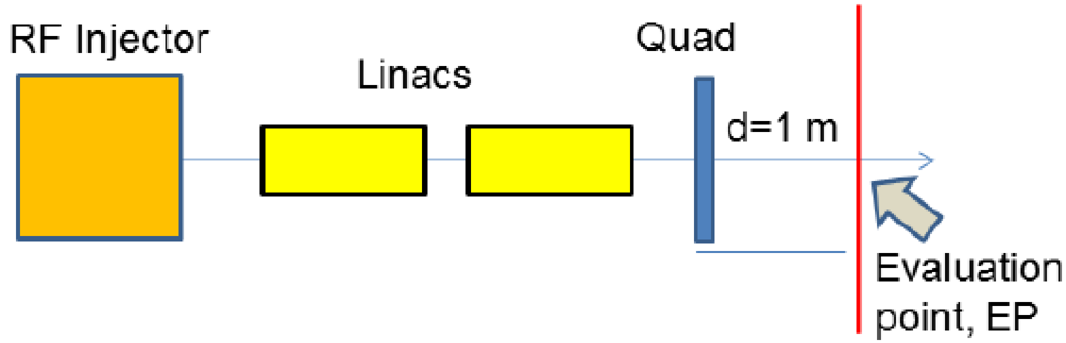



Figure 1: Simulation set-up

In the simulation you will vary the strength of the magnet and record the size of the beam on point EP. You will do it for several values of κ. Then, using excel you will fit the result into a polynomial fit and estimate the emittance from Eq. (3). You will compare the result with the emittance you obtain directly from ASTRA. In the exercise you will calculate the emittance on x axis, so you need to calculate only the x-beam size.

**CAUTION:** ASTRA is giving you the normalized emittance. Quad scan is giving you the unormalized emittance. You need to do the conversion through the following relation:

$$\varepsilon_{x,norm} = \beta\gamma\varepsilon_{x,un} \tag{4}$$

here beta and gamma are the usual relativistic factors.