Home Work PHY 554 #9. Due March 11, 2020

HW 1 (2 points): Calculate relations between three dimensionless infinitesimal parameters:

$$\frac{dE}{E} \equiv \frac{d\gamma}{\gamma}; \frac{dp}{p} \equiv \frac{d(\beta\gamma)}{\beta\gamma}; \frac{d\text{v}}{\text{v}} \equiv \frac{d\beta}{\beta}$$

where E is energy, p is momentum and v is velocity of a particle. Hint: use relativistic relations between β and γ.

HW 2 (5 points): In class we introduced the map of longitudinal motion in a storage ring

$$\delta_{n+1} = \delta_n + \frac{eV_{rf}}{\beta^2 E_o}(\sin\phi_n - \sin\phi_s); \qquad (1)$$
$$\phi_{n+1} = \phi_n + 2\pi h\eta \cdot \delta_{n+1},$$

1. For small oscillation variations of the RF phase about the synchronous phase

$$\varphi = \phi - \phi_s; |\varphi| << 1$$

linearize the map (1) by keeping only first order on φ and find one turn transport matrix M for longitudinal motion:

$$\begin{pmatrix} \varphi \\ \delta \end{pmatrix}_{n+1} = M \begin{pmatrix} \varphi \\ \delta \end{pmatrix}_n$$

2. Using Courant-Snyder parametrization we used for transverse motion find value of $\cos\mu_s, \beta_s, \alpha_s$ in parametric form (e.g. using $\sin\mu_s = \sqrt{1-\cos^2\mu_s}$, $\mu_s = 2\pi Q_s = \cos^{-1}(\cos\mu_s)$).

3. Assuming that μ_s<<1, find analytical expression for synchrotron tune and compare it with that we found in Lecture 12.

HW 3 (3 points): (4 points) For our example in lecture 12, find the synchrotron tunes for 100 GeV and 15 GeV protons in a storage ring for the following parameters (similar to RHIC collider at BNL):

RF voltage, V= 500 kV

Depending on the sing of the slip facor the synonymous phase is zero or 180 degrees,

$$\phi_s = 0, \pi$$

Harmonic number, h=360

Compaction factor, $\alpha_c = 0.002$