Options for Coherent electron Cooling

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Outline

- Options for Coherent electron Cooling
- What are advantages of various schemes
- What can be tested?
- How to evaluate cooling
- Conclusions
What is Coherent electron Cooling

- Short answer – stochastic cooling of hadron beams with bandwidth at optical wave frequencies: 1 – 1000 THz
- Longer answer
Litvinenko, Derbenev, PRL 2008

High gain FEL amplifier

Ratner, PRL 2013

Multi-Chicane Microbunching amplifier

Litvinenko, Wang, Kayran, Jing, Ma, 2017

Plasma-Cascade Microbunching amplifier

Litvinenko, Cool 13

Hybrid laser-beam amplifier
Advantages and Disadvantages

- The best studied and fully explored scheme
- Experimentally demonstrated both as instability and amplifier
- 3D FEL theory and simulation are very advanced
- Can operate at relatively low electron beam peak currents
- Allows – in principle – economic option without separating electron and hadron beams

CeC with High gain FEL amplifier

- When compared with micro-bunching amplifier, it has relatively lower bandwidth ~ few % of the FEL frequency
- FEL saturates at lower gain than micro-bunching amplifier
- Semi-periodic structure of the modulation limits the range where cooling occurs
Advantages and Disadvantages

- Very broad band amplifier
- Micro-bunching instability was experimentally demonstrated
- Can operate at significant gain without saturation and can be extended to LHC energies
- Ratner’s original scheme is - in principle - insensitive to longitudinal space charge effects in the electron beam

- Micro-bunching amplifier was not demonstrated
- Less studied – especially numerically in 3D - than other CeC schemes
- Requires electron beam with low energy spread
- Definitely require separation of electron and hadron beams

Multi-Chicane Microbunching amplifier
**Advantages and Disadvantages**

- Very broad band amplifier, can operate at significant gain without saturation
- Plasma-cascade micro-bunching instability was experimentally demonstrated
- Has good theoretical model and is extensively studied in 3D numerical simulations
- Cool hadrons with all energy deviation (no anti-cooling)
- Does not require (full) separation of electron and hadron beams

- Micro-bunching amplifier was not demonstrated
- Requires better quality electron beam than FEL amplifier
- Can operate for medium hadron energies (up to hundreds of GeV, such as US EIC), but cannot be extended to LHC energies
- Less studied than FEL-based CeC

**Plasma-Cascade Microbunching amplifier**

![Diagram of Plasma-Cascade Microbunching amplifier](image-url)
Advantages and Disadvantages

- Relatively broad-band amplifier
- Take advantage of laser technology
- Does not require high peak current electron beam
- Take advantage of flexibility provided by high K-wigglers to adjust wavelength of radiation to that of the laser-amplifier
- Has some synergy with optical-stochastic cooling

- Not studied in details
- Definitely require separation of electron and hadron beams
- Would require rather significant delay of the hadron beam – may require \( R_{56} \) reduction scheme for hadrons
- Semi-periodic structure of the modulation limits the energy range where cooling occurs
What can be tested experimentally?

Cooling test requires serious modification of the RHIC lattice & superconducting magnets +$20-$30M
OR
Building new CeC system at another hadron storage ring

RHIC Runs 20-22

Cooling test requires serious modification of the RHIC lattice & superconducting magnets +$20-$30M
OR
Building new CeC system at another hadron storage ring
Changing CeC amplifier from FEL to PCA

Small gap in FEL wigglers is not compatible with low energy RHIC operations of the Beam Energy Scan (BES-II) program

- Mechanical design new of the CeC system is completed. We used SBU NSF “Center for Accelerator Science and Education” grant to procure new hardware
- We procured and commissioned new laser system with controllable pulse structure
- All new vacuum chambers with beam diagnostics are built and installed
- All supports are built and installed
- All solenoids are designed, manufactured, delivered and undergo magnetic measurements
- Assembly of the plasma-cascade based CeC can be completed during this year’s RHIC shut-down
How to evaluate CeC: the original recipe

Free Electron Lasers and High-energy Electron Cooling,

• Linear response of electron beam on perturbations – no saturation, superposition principle

\[
\delta \tilde{E}_h = Z e \cdot \tilde{G}_{Eh} (\vec{r}_h, \gamma_h, t_h, t_h); \delta \tilde{B}_h = Z e \cdot \tilde{G}_{Bh} (\vec{r}_h, \gamma_h, t_h, t_h);
\]
\[
\delta \tilde{E}_e = -e \cdot \tilde{G}_{Ee} (\vec{r}_e, \gamma_e, t_e, t_e); \delta \tilde{B}_e = -e \cdot \tilde{G}_{Be} (\vec{r}_e, \gamma_e, t_e, t_e);
\]
\[
\tilde{E} = Z e \cdot \sum_h \tilde{G}_{Eh} (\vec{r}_h, \gamma_h, t_h, t_h) - e \cdot \sum_e \tilde{G}_{Ee} (\vec{r}_e, \gamma_e, t_e, t_e);
\]
\[
\tilde{B} = Z e \cdot \sum_h \tilde{G}_{Bh} (\vec{r}_h, \gamma_h, t_h, t_h) - e \cdot \sum_e \tilde{G}_{Be} (\vec{r}_e, \gamma_e, t_e, t_e)
\]

• Evaluation of hadron distribution function using Fokker-Plank equation with both damping and diffusion terms

• Cooling transversely using coupling with longitudinal degrees of freedom
How to evaluate CeC: the original recipe

Free Electron Lasers and High-energy Electron Cooling,

- Linear response of electron beam on perturbations – no saturation, superposition principle

\[ \delta E_i = eZ \int \vec{E} \cdot d\vec{r}_i; \quad \delta \vec{p}_i = eZ \int \left( \vec{E} + \frac{\vec{p}_i \times \vec{B}}{\gamma m} \right) \cdot dt; \quad X^T = (x, P_x, y, P_y, z, P_z); \]

\[ \delta E_i = (eZ)^2 \cdot g_{Eh}(X_i, t_i) - Ze^2 \cdot g_{Ee}(X_i, t_i); \]

\[ g_{Eh}(X_i, t_i) = \sum_h \int \vec{G}_{Eh}(\vec{r}_i, \vec{r}_h, \gamma_h, t_i, t_h) \cdot d\vec{r}_i; \quad g_{Ee} = \int \sum_e \vec{G}_{Ee}(\vec{r}_i, \vec{r}_e, \gamma_e, t_i, t_e) \cdot d\vec{r}_i; \]

\[ \delta \vec{p}_i = (eZ)^2 \cdot \vec{g}_{ph}(X_i, t_i) - Ze^2 \cdot \vec{g}_{pe}(X_i, t_i); \]

\[ \vec{g}_{ph}(X_i, t_i) = \sum_h \int \left( \vec{G}_{Eh} + \frac{\vec{p}_i \times \vec{G}_{Bh}}{\gamma_i m} \right) \cdot dt_i; \quad \vec{g}_{pe} = \sum_e \int \left( \vec{G}_{Ee} + \frac{\vec{p}_i \times \vec{G}_{Be}}{\gamma_i m} \right) \cdot dt_i; \]

- Evaluation of hadron distribution function using Fokker-Plank equation with both damping and diffusion terms

- Cooling transversely using coupling with longitudinal degrees of freedom
How to evaluate CeC: the original recipe

Free Electron Lasers and High-energy Electron Cooling,

- Linear response of electron beam on perturbations – no saturation, superposition principle
- Evaluation of hadron distribution function using Fokker-Plank equation with both damping and diffusion terms

\[ \bar{f} = \langle \tilde{f} \rangle; \tilde{f} = \sum_h \delta \left( X - X_i (t) \right) \]

\[ \frac{\partial \bar{f}(X,s)}{\partial t} + \frac{\partial}{\partial X_i} \left[ \frac{dX_i(X,t)}{dt} \bar{f}(X,s) \right] - \frac{1}{2} \frac{\partial^2}{\partial X_i \partial X_k} \left[ D_{ik}(X,t) \bar{f}(X,t) \right] = 0 \]

\[ \left\langle \frac{dX_i(X,t)}{dt} \right\rangle = \frac{1}{\tau} \int (X_i - Z_i) \cdot W(Z,X|\tau,t) dZ = \frac{1}{T_o} \langle \delta X_i \rangle \]

\[ 2D_{ik}(X,t) = \frac{1}{2\tau} \int (X_i - Z_i)(X_k - Z_k) W(Z,X|\tau,t) dZ = \frac{1}{T_o} \langle \delta X_i \cdot \delta X_k \rangle \]

- Cooling transversely using coupling with longitudinal degrees of freedom
Transverse cooling: the original recipe

- Cooling transversely using coupling with longitudinal degrees of freedom by making energy kick depending on transverse motion (via $R_{51}$, $R_{52}$, $R_{53}$, $R_{54}$ or by displacing beam center in the kicker section)

$$X^T = [x, x', y, y', \tau, \delta]; \tau = -c( t - t_o); \delta = \frac{E - E_o}{\beta_o E_o};$$

$$\Delta E = F(X); \Delta \delta \equiv \Delta x_6 = \frac{F(X)}{\beta_o E_o} = \text{const} - \sum_{i=1}^{6} \zeta_i \cdot x_i;$$

$$\Delta X^T = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \Delta x_6 \end{bmatrix}; S = \begin{bmatrix} \sigma & 0 & 0 \\ 0 & \sigma & 0 \\ 0 & 0 & \sigma \end{bmatrix}; \sigma = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix};$$

$$X = \sum_{k=1}^{3} (a_k Y_k e^{i \psi_k} + a_k^* Y_k^* e^{-i \psi_k}); \quad Y_j S Y_k^* = -2i \delta_{jk}; Y_k^T = \begin{bmatrix} y_{1k} & y_{2k} & y_{3k} & y_{4k} & y_{5k} & y_{6k} \end{bmatrix};$$

$$\Delta a_k = \frac{i}{2} \Delta X^T S Y_k^* = \frac{i y^*_5}{2} \Delta x_6 e^{-i \psi_k} = -\frac{i y^*_5}{2} e^{-i \psi_k} \sum_{i=1}^{6} \zeta_i \cdot x_i; \quad x_i = \sum_{j=1}^{3} (a_j y_{ij} e^{i \psi_j} + a_j^* Y_k^* e^{-i \psi_k});$$

$$\Delta a_k = -\frac{i y^*_5}{2} \sum_{i=1}^{6} \zeta_i \sum_{j=1}^{3} a_j y_{ij} e^{i (\psi_j - \psi_k)} \rightarrow \langle \Delta a_k \rangle = -\zeta_k a_k \rightarrow a_k = a_k e^{-n \zeta_k}; \quad \sum_k \text{Re} \zeta_k = \text{Tr} D = \zeta_6 \equiv \zeta_\delta$$

$$\langle e^{i (\psi_j - \psi_k)} \rangle = \delta_{jk}; \langle e^{-i (\psi_j + \psi_k)} \rangle = 0; \quad \zeta_k = \frac{i}{2} \sum_{i=1}^{6} \zeta_i y_{5k}^* y_{ik}; \quad \text{Re} \zeta_k = \text{Im} \sum_{i=1}^{6} \zeta_i y_{5k}^* y_{ik}$$
Transverse cooling: the original recipe

- Cooling transversely using coupling with longitudinal degrees of freedom by making energy kick depending on transverse motion (via $R_{51}, R_{52}, R_{53}, R_{54}$ or by displacing beam center in the kicker section) – we can only redistribute cooling decrements between three eigen modes

$$\langle \Delta a_k \rangle = -\xi_k a_k \rightarrow a_k = a_{k0} e^{-n\xi_k}; \quad \xi_k = \frac{i}{2} \sum_{i=1}^{6} \zeta_i y_{5k} y_{ik}; \quad \sum_k \xi_k = TrD = \zeta_6 \equiv \zeta_\delta$$

For slow synchrotron oscillations ($Q_s << 1$)

$$Y_{5k} = -Y_{k\beta}^T SD$$

$$\xi_k = \frac{i}{2} (Y_{k\beta}^T SD) \sum_{i=1}^{6} \zeta_i y_{ik};$$

Hence, introduction dependencies on the components of transverse motion and non-zero dispersion in the kicker section allows to re-distribute cooling decrements.
Instead of conclusion

- There is a variety of amplifiers suitable for CeC
  - In addition to what we discussed now, Yaroslav Derbenev is proposing using coherent synchrotron radiation instability as CeC amplifier – one needs a specific schematic to understand how it fits into the CeC family

- Theoretical evaluation is typically limited to 1D, but 3D simulation are performed for two CeC schemes

- Two CeC options can be tested experimentally at RHIC – we are currently pursuing CeC with plasma-cascade amplifier

- The evaluation scheme that I presented only looking simple – evil is always in details

- Following presentations will give a much deeper view into physics and realities of CeC
Back-up
Sum of decrements theorem

Let’s consider an arbitrary linear \( s \)-dependent equation:

\[
\frac{dX}{ds} = D(s) \cdot X; \tag{SD-1}
\]

e.g. the overall motion is not necessary symplectic

\[
X(s) = R(s)X_o \rightarrow \frac{dR}{ds} = DR \rightarrow \frac{d}{ds} \det[R(s)] = \text{Trace}[D(s)] \cdot \det[R(s)] \tag{SD-2}
\]

\[
\det[R(s)] = \int_o^s \text{Trace}[D(\xi)]d\xi.
\]

Prove of the later is rather trivial

\[
R(s+ds) = (I + dsD(s^*) + ds^2O) \cdot R(s); \quad s^* \in \{s, s+ds\} \rightarrow \det[R(s+ds)] = \det(I + dsD(s^*) + ds^2O) \cdot \det[R(s)];
\]

\[
\det A = \sum_{i,j,k} e_{ijk} \cdot a_i a_j a_k \cdots; \det(I + dsD(s^*) + \epsilon^2O) = \sum_{i,j,k} e_{ijk} \cdot (\delta_{ii} + ds d_{i1}) (\delta_{jj} + ds d_{j1}) (\delta_{kk} + ds d_{k1}) \cdots O(\epsilon^2)
\]

\[
\sum_{i,j,k} e_{ijk} \cdot (\delta_{ii} + ds d_{i1}) (\delta_{jj} + ds d_{j1}) (\delta_{kk} + ds d_{k1}) \cdots = \prod_{i=1}^{2n} (\delta_{ii} + ds d_{i1}) + ds^2 \sum_{i\neq 1, j,k} e_{ijk} \cdot d_{i1} \sum_{i\neq 1, j,k} e_{ijk} \cdot (\delta_{ii} + ds d_{i1}) (\delta_{jj} + ds d_{j1}) (\delta_{kk} + ds d_{k1}) \cdots d_{i1} \cdots + \]

\[
+ ds^2 \sum_{i,j \neq 2, k} e_{ijk} \cdot \sum_{i,j \neq 2, k} e_{ijk} \cdot (\delta_{ii} + ds d_{i1}) d_{j2} (\delta_{jj} + ds d_{j1}) \cdots d_{j2} \cdots = \prod_{i=1}^{2n} (\delta_{ii} + ds d_{i1}) + O(ds^2);
\]

\[
\prod_{i=1}^{2n} (\delta_{ii} + ds d_{i1}) = 1 + ds \sum_{i=1}^{2n} d_{i1} + O(ds^2) = 1 + ds \cdot \text{Tr}[D(s^*)];
\]

\[
\det[R(s+ds)] \equiv \det[R(s)] + d(\det[R(s)]) + O(ds^2) = \left(1 + ds \cdot \text{Tr}[D(s^*)] + O(ds^2)\right) \cdot \det[R(s)];
\]

\[
ds \rightarrow 0 \Rightarrow d(\det[R(s)]) = ds \cdot \text{Tr}[D(s)] \cdot \det[R(s)];
\]

\[
\frac{d}{ds} \det[R(s)] = \text{Tr}[D(s)] \cdot \det[R(s)]. \tag{SD-2}
\]
Status: more details in my afternoon talk

- Mechanical design of the new CeC system is completed.
- We procured and commissioned a new laser system with controllable pulse structure.
- All new vacuum with beam diagnostics are built; chambers are installed.
- All supports are built and installed.
- All solenoids are designed, manufactured, delivered, and undergo magnetic measurements.
- Assembly of the plasma-cascade based CeC can be completed during this year’s RHIC shut-down period.
Distribution of the decrements

\[ X = \frac{1}{2} \sum_{k=1}^{3} (a_k Y_k(s) e^{i\psi_k} + c.c.); \quad Y_j^* S Y_k = 2i \delta_{jk}; Y_j^T S Y_k = 0; \quad S = \begin{bmatrix} \sigma & 0 & 0 \\ 0 & \sigma & 0 \\ 0 & 0 & \sigma \end{bmatrix}; \sigma = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \]

\[ \delta X = -\xi \begin{bmatrix} 0 \\ 0 \\ 0 \\ \delta + k_x x \end{bmatrix} = -\xi K \cdot X = -\xi \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ k_x & 0 & 0 & 0 & 1 \end{bmatrix} \cdot \frac{1}{2} \sum_{k=1}^{3} (a_k Y_k(s) e^{i\psi_k} + c.c.); \]

\[ \delta a_k = -\xi e^{-i\psi_k} \frac{Y_k^* S K \cdot \sum_{j=1}^{3} (a_j Y_j(s) e^{i\psi_j} + c.c.)}{2i} \]

\[ \xi_k = \frac{\langle \delta a_k \rangle}{a_k} = -\xi \frac{Y_k^* S Y_k}{2i}; \quad 2 \sum_{k=1}^{3} \xi_k = \xi \cdot Tr(K) = \xi; \]

\[ \xi_k = \frac{\xi}{2i} \cdot Y_k^* \left( k_x Y_k^1 + Y_k^6 \right) \]

\[ X^T = \{ x, x', y, y', -c\tau, \delta \} \]
Distribution of the decrements

\[ Y_{k=1,2} = \begin{pmatrix} Y_{k1} \\ Y_{k2} \\ Y_{k3} \\ Y_{k4} \\ Y_{k5} \\ 0 \end{pmatrix} = \begin{pmatrix} Z_k \\ -Z_k^T SD \\ 0 \end{pmatrix}; Y_3 \equiv \frac{1}{\sqrt{\Omega}} \begin{pmatrix} D_x \\ D'_x \\ D_y \\ D'_y \\ i\Omega \\ 1 \end{pmatrix}; \]

\[ \xi_k = \frac{\xi}{2i} Y_{k}^{*5} (k_x Y_{k}^1 + Y_{k}^6) \]
\[ \xi_s = \frac{\xi}{2} (k_x D_x + 1); \]
\[ \xi_{k=1,2} = -\frac{\xi}{2i} (Z_k^{*T} SD) \cdot k_x Z_k^1 \]
\[ \xi_1 + \xi_2 = -k_x D_x \frac{\xi}{2} \]

Uncoupled case

\[ \xi_y = 0; \quad \text{Re} \xi_x = -\frac{\xi}{2} \cdot R_{52e} \frac{D_{xh}}{D_{zh}}; \quad \text{Re} \xi_s = \frac{\xi}{2} \left(1 - R_{52e} \frac{D_{xh}}{D_{zh}}\right) \]