PHY 554. Homework 2.

Handed: February 5
Return by: February 12
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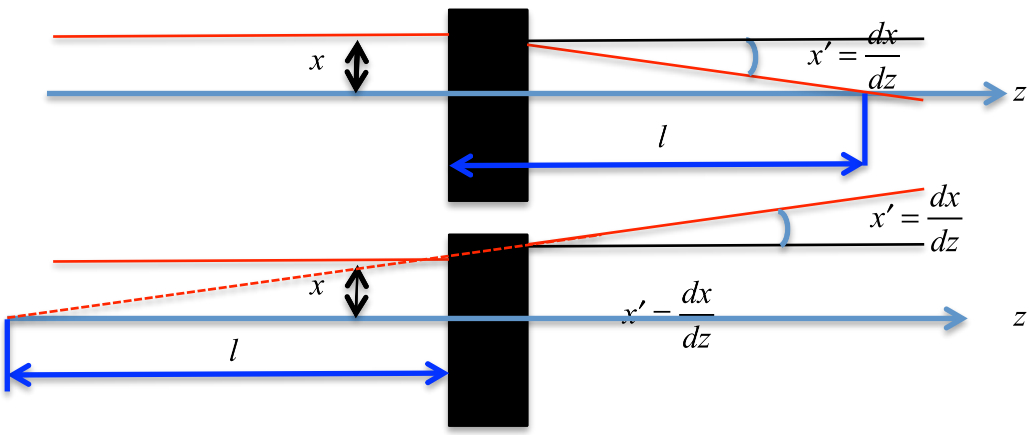
HW 1 (5 point): Let's first determine an effective focal length, F, of the of a paraxial (e.g. small angles!) focusing object (a black-box) as ratio between a parallel displacement of trajectory at its entrance to corresponding change of the angle at its exit (see figure below):

$$F = -\frac{x}{x'}; x' \equiv \frac{dx}{dz}$$

see figure below for



For completeness, the distance from the entrance to the object to the trajectory crossing the axis, l, in general is not equal to the focal length. In beam optics this is frequently, but not correctly, referred as astigmatism – in contrast, the astigmatism is defined as dependence of the focal strengths on the direction of propagation of the ray (particle).
Let consider a doublet of two thin lenses: a focusing (F) and defocusing (D) lenses with equal but opposite in sign focal length F with center separated by distance L as in Fig. 1.

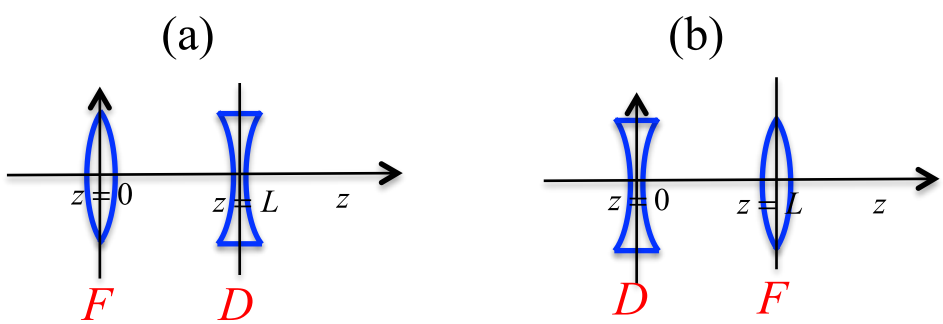


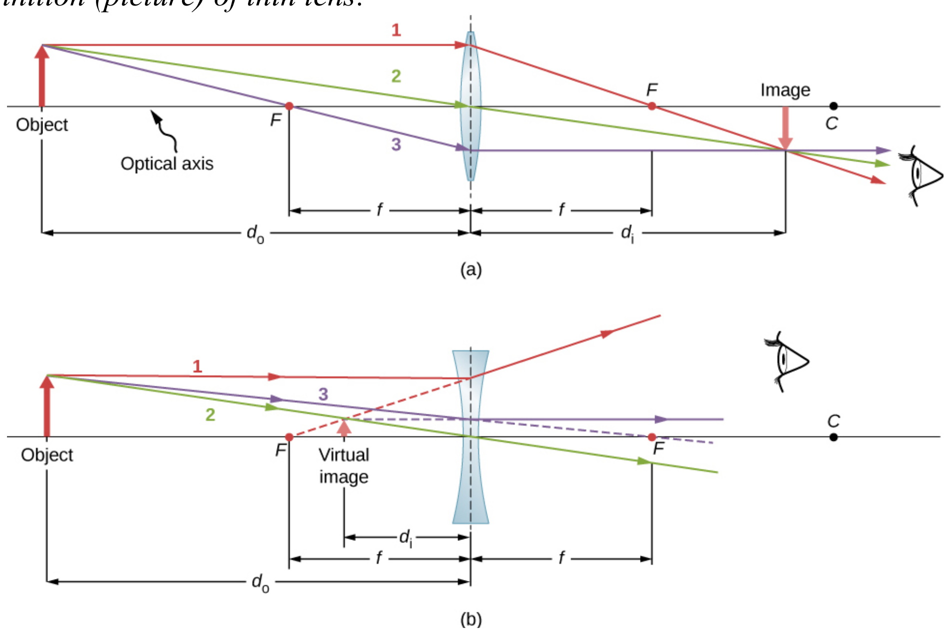
Fig.1. Two combinations of a doublet: *FD* and *DF*.

1. (3 points) Show through a calculation of the ray trajectory that the focal lengths of *FD* and *DF* doublets are equal and given by following expression:

$$F_{eff} = \frac{F^2}{L}$$

2. (2 points) Determine location of the ray crossing the axis and find their difference between *FD* and *DF* doublets – this indeed would be an astigmatism of doublet built from two quadruples.

P.S. Definition (picture) of thin lens:



HW 2 (2 points): Spectral brightness (sometimes called brilliance) of a light source is defined as

$$B = \frac{dN_{ph}}{dtd\Omega dA\left(d\lambda / \lambda\right)} = \frac{dN_{ph}}{dtd\Omega dA\left(d\omega / \omega\right)};$$

where $\frac{dN_{ph}}{dt}$ is the number of photons per second with the spectral bandwidth $d\omega / \omega$ radiated from an area dA into the solid angle $d\Omega$. The units used for brightness are expressed in photons per second

$$\left[B\right] = \frac{photons}{\sec \cdot mm^2 \cdot mrad^2 \left(10^{-3} d\lambda / \lambda\right)}$$

As an exercise, calculate spectral brightness of NdYAG laser with average power of 10 W, wavelength of λ=1.064 μm, Bandwidth of $\Delta\omega = 700\ GHz$ and with diffraction limited spot size and angular spread:

$$\Delta x \cdot \Delta \theta_x = \frac{\lambda}{4\pi}; \Delta y \cdot \Delta \theta_y = \frac{\lambda}{4\pi}.$$

HW 3 (3 points): In a fixed Cartesian coordinates for a trajectory with $\frac{dz}{dt} \neq 0$ of a particle moving in magnetic field $\vec{B} = \hat{x}B_x + \hat{y}B_y + \hat{z}B_z$ equation for its trajectory can be written in terms of z as independent variable:

$$\frac{d^2x}{dz^2} = \frac{e}{p}\sqrt{1+x'^2+y'^2}\left(y'B_z - (1+x'^2)B_y + x'y'B_x\right);$$

$$\frac{d^2y}{dz^2} = -\frac{e}{p}\sqrt{1+x'^2+y'^2}\left(x'B_z - (1+y'^2)B_x + x'y'B_y\right);$$

$$x' \equiv \frac{dx}{dz}; y' \equiv \frac{dy}{dz};$$

where *e* is the particle's charge and $p = \gamma m \mathrm{v}$ is its relativistic momentum.

Hint: consider constants of motion in a magnetic field.