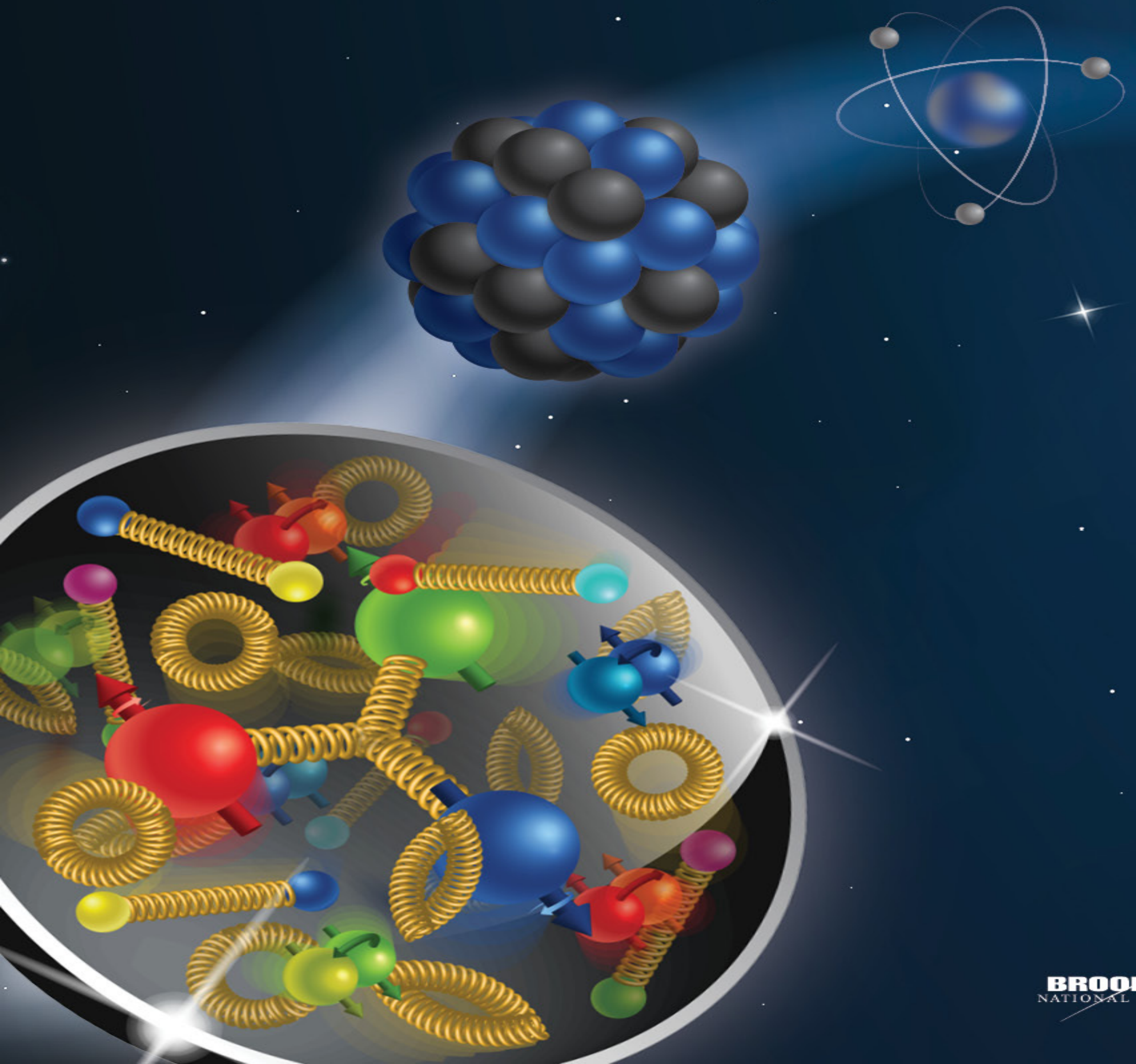

High Power RF Engineering -Waveguide (1)

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Note

- Speed of light in vacuum $c = 299\ 792\ 458\ m/s$
- Vacuum electric permittivity $\varepsilon_0 = 8.854187 \times 10^{-12}\ F/m$
- Vacuum magnetic permittivity $\mu_0 = 1.256637 \times 10^{-6}\ N/A^2$
- $\varepsilon_0 \mu_0 = 1/c^2$
- Impedance of free space $\eta = \sqrt{\mu_0/\varepsilon_0} = 376.730\ 313\ \Omega$

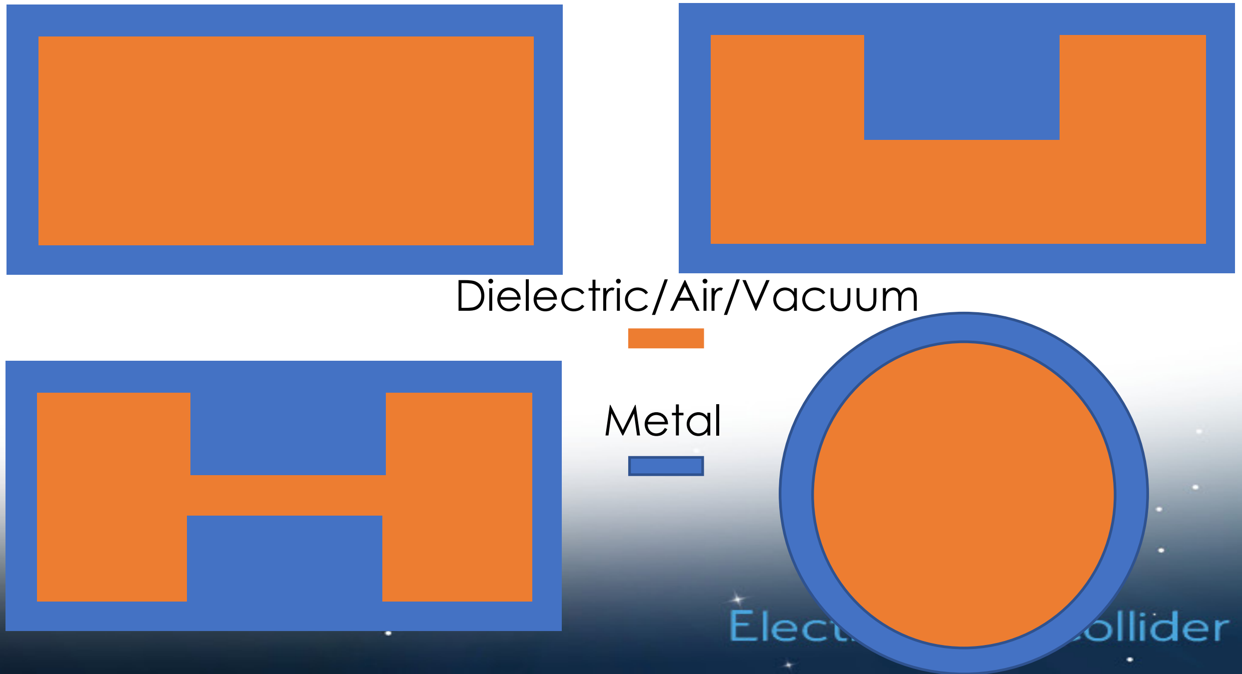
Waveguide

- Structure that guides the RF wave.
- Tube with metal wall, and gas-filled/vacuum (hollow) or filled with dielectric material.
- RF injected into this tube is regulated by the wall and is progressed down the waveguide.
- It follows Maxwell equations with boundary conditions.

Types of RF Waveguide

- There are many kinds of structures that can transmit RF power. We covered coaxial line, stripline, microstrip and twin wires previously. Now we focus on two types of waveguides: rectangular waveguide (and single-ridged, double-ridged) and circular waveguide (and elliptical).

Cross section of:
Rectangular (Top-left)
Single ridged (Top-right)
Double ridged (Bottom-left)
Circular (Bottom-right)



Maxwell Equations

$$\nabla \cdot \boldsymbol{D} = \rho$$

$$\nabla \cdot \boldsymbol{B} = 0$$

$$\boxed{\begin{aligned} &\nabla \times \boldsymbol{E} = -\partial B/\partial t \\ &\nabla \times \boldsymbol{H} = J + \partial D/\partial t \end{aligned}}$$

With material property

$$\boldsymbol{D} = \varepsilon \boldsymbol{E}$$

$$\boldsymbol{B} = \mu \boldsymbol{H}$$

$$\boldsymbol{J} = \sigma(\boldsymbol{E} + \boldsymbol{E}_{ext})$$

and boundary conditions TBD

Maxwell Equations - Right Side

In the waveguide $\rho=0$ & $\mathbf{J}=0$

Fields are time dependent $e^{j\omega t}$, $\partial/\partial t \to j\omega$

$\partial\mathbf{H}/\partial t = j\omega\mathbf{H}$ & $\partial\mathbf{E}/\partial t = j\omega\mathbf{E}$

$-\partial\mathbf{B}/\partial t = -\mu\partial\mathbf{H}/\partial t = -\, j\omega\mu\mathbf{H}$ & $\partial\mathbf{D}/\partial t = -\varepsilon\partial\mathbf{E}/\partial t = j\omega\varepsilon\mathbf{E}$

We have $\nabla \times \mathbf{E} = -j\omega\mu\mathbf{H}$ & $\nabla \times \mathbf{H} = j\omega\varepsilon\mathbf{E}$

From now on time dependent is omitted, and we assume it is under vacuum.

Maxwell Equations - Left Side

Fields travelling along +z direction $e^{-j\beta z}$, $\partial/\partial z \rightarrow -j\beta$

$$\nabla \times \boldsymbol{E} = \begin{bmatrix} \boldsymbol{x} & \boldsymbol{y} & \boldsymbol{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{bmatrix} = \boldsymbol{x}\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right) + \boldsymbol{y}\left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right) + \boldsymbol{z}\left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)$$

with $\frac{\partial E_x}{\partial z} = -j\beta E_x$ and $\frac{\partial E_y}{\partial z} = -j\beta E_y$, the above becomes

$$\nabla \times \boldsymbol{E} = \boldsymbol{x}\left(\frac{\partial E_z}{\partial y} + j\beta E_y\right) + \boldsymbol{y}\left(-j\beta E_x - \frac{\partial E_z}{\partial x}\right) + \boldsymbol{z}\left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)$$

$\nabla \times \boldsymbol{H}$ can be expressed in a similar way

Maxwell Equations to Field Distribution

- Compare the **x**, ***y***, **z** components of ***E*** & ***H***
- Get E_x, E_y, H_x, H_y as a function of E_z & H_z
- *The reason is that EM waves are transverse wave and there is a chance that* $E_z = 0$ and/or $H_z = 0$

Field Distribution

$$\frac{\partial E_z}{\partial y} + j\beta E_y = \text{-}j\omega\mu H_x$$

$$-j\beta E_x - \frac{\partial E_z}{\partial x} = \text{-}j\omega\mu H_y$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = \text{-}j\omega\mu H_z$$

$$\frac{\partial H_z}{\partial y} + j\beta H_y = j\omega\varepsilon E_x$$

$$-j\beta H_x - \frac{\partial H_z}{\partial x} = j\omega\varepsilon E_y$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega\varepsilon E_z$$

$$k_c^2\ H_x = j\left(\omega\varepsilon\frac{\partial E_z}{\partial y} - \beta\frac{\partial H_z}{\partial x}\right)$$

$$k_c^2\ H_y = -j\left(\omega\varepsilon\frac{\partial E_z}{\partial x} + \beta\frac{\partial H_z}{\partial y}\right)$$

$$k_c^2\ E_x = -j\left(\beta\frac{\partial E_z}{\partial x} + \omega\mu\frac{\partial H_z}{\partial y}\right)$$

$$k_c^2\ E_y = j\left(-\beta\frac{\partial E_z}{\partial y} + \omega\mu\frac{\partial H_z}{\partial x}\right)$$

$$k_c^2 = k^2 - \beta^2\ \&\ k = \omega\sqrt{\mu\varepsilon}$$

Note: there are two equations that have not been used yet.

Cutoff Frequency

- $k_c = \frac{2\pi f_c}{c}$, f_c is the cutoff frequency
- $\beta^2 = k^2 - k_c^2$
- Frequency f below f_c
  - $\beta^2 < 0$, α appears as a non-perturbation term
  - Field decays in the waveguide and cannot propagate
  - It is also called "evanescent field"

Transverse Electric (TE)

- $E_z = 0$ & $H_z \neq 0$.
- Wave impedance

$Z_{TE} = E_x/H_y = -E_y/H_x = \omega\mu/\beta = k\eta/\beta$

$$H_x = \frac{-j\beta}{k_c^2}\frac{\partial H_z}{\partial x}$$

$$H_y = \frac{-j\beta}{k_c^2}\frac{\partial H_z}{\partial y}$$

$$E_x = \frac{-j\omega\mu}{k_c^2}\frac{\partial H_z}{\partial y}$$

$$E_y = \frac{j\omega\mu}{k_c^2}\frac{\partial H_z}{\partial x}$$

$$\frac{\partial H_y}{\partial x} = \frac{\partial H_x}{\partial y}$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu H_z$$

Transverse Magnetic (TM)

- $E_z \neq 0$ & $H_z = 0$.
- Wave impedance

$Z_{TM} = E_x/H_y = -E_y/H_x = \beta/\omega\varepsilon = \beta\eta/k$

$$H_x = \frac{j\omega\varepsilon}{k_c^2}\frac{\partial E_z}{\partial y}$$

$$H_y = \frac{-j\omega\varepsilon}{k_c^2}\frac{\partial E_z}{\partial x}$$

$$E_x = \frac{-j\beta}{k_c^2}\frac{\partial E_z}{\partial x}$$

$$E_y = \frac{-j\beta}{k_c^2}\frac{\partial E_z}{\partial y}$$

$$\frac{\partial E_y}{\partial x} = \frac{\partial E_x}{\partial y}$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega\varepsilon E_z$$

Transverse Electromagnetic (TEM)

- $E_z = 0$ & $H_z = 0$.
- $k_c = 0,\ k = \beta$
- Wave impedance

$Z_{TEM} = E_x/H_y = -E_y/H_x = \eta$

$$\sqrt{\varepsilon}E_y = -\sqrt{\mu}H_x$$

$$\sqrt{\varepsilon}E_x = \sqrt{\mu}H_y$$

$$\frac{\partial E_y}{\partial x} = \frac{\partial E_x}{\partial y}$$

$$\frac{\partial H_y}{\partial x} = \frac{\partial H_x}{\partial y}$$

Field pattern is frequency independent

Transverse Electromagnetic (TEM)

- *TEM* mode cannot exist in a waveguide with only one conductor (rectangular or circular waveguide).
- Within the hollow space defined by one conductor: no longitudinal magnetic component → magnetic field must close its loop transversely → it will produce longitudinal electric field, contradict to no longitudinal electric component in TEM.
- It can exist in those with two conductors, for example coaxial line, stripline or microstrip.

Guide wavelength, phase and group velocities

- $V_{ph} = \frac{\omega}{\beta} = f\lambda_{\text{guide}}$ & $k_c^2 = k^2 - \beta^2$ & $k = \omega\sqrt{\mu\varepsilon} = \frac{\omega}{c}$.
- $\beta = k$ for TEM, the guide wavelength λ_{guide} and phase velocity V_{ph} are the same as those in free space. Group velocity $V_g = \frac{\partial\omega}{\partial\beta} = c = V_{ph}$.
- $\beta < k$ for TE & TM, the guide wavelength λ_{guide} and phase velocity V_{ph} are larger than those in free space. Group velocity $V_g = \frac{\partial\omega}{\partial\beta} = c\sqrt{1 - (\frac{k_c}{k})^2}$ and phase velocity $V_{ph} = \frac{c}{\sqrt{1-(\frac{k_c}{k})^2}}$
- $V_{ph} \times V_g = c^2$ in coaxial lines for TE, TM and TEM, as well as in rectangular and circular waveguides (will show in following lectures).

Rectangular Waveguide

TE (1)

$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}$ = -jωμH_z & $E_x = \frac{-j\omega\mu}{k_c^2}\frac{\partial H_z}{\partial y}$ & $E_y = \frac{j\omega\mu}{k_c^2}\frac{\partial H_z}{\partial x}$

$\rightarrow \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k_c^2\right) H_z = 0$ two-dimensional Helmholtz equation

and then use "separation of variables" $H_z(x, y) = X(x)Y(y)e^{-j\beta z}$

$\frac{d^2X}{dx^2} + k_x^2 X = 0$ & $\frac{d^2Y}{dx^2} + k_y^2 Y = 0$ & $k_c^2 = k_x^2 + k_y^2$

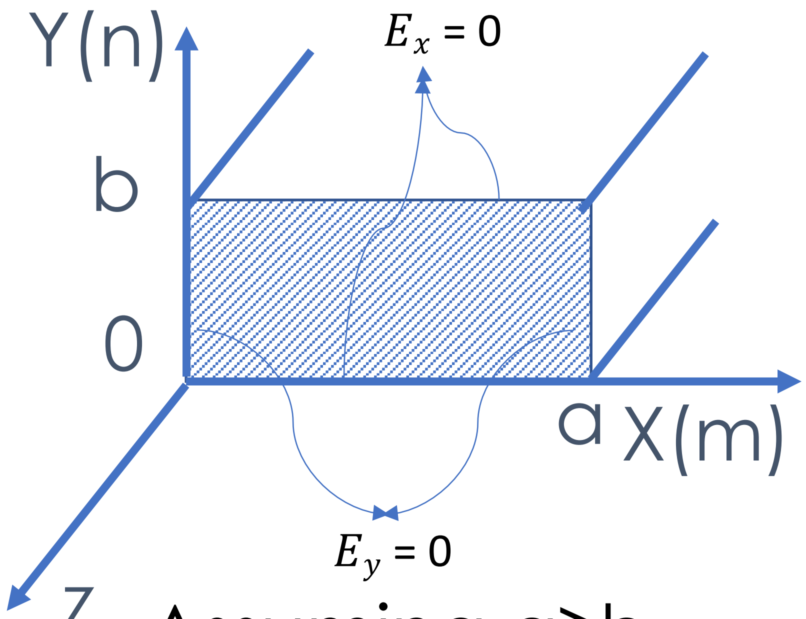
We get $H_z = (A\cos k_x x + B\sin k_x x)(C\cos k_y y + D\sin k_y y)$

and $E_x = -\frac{j\omega\mu}{k_c^2}(A\cos k_x x + B\sin k_x x)(-C\sin k_y y + D\cos k_y y)k_y$

$E_y = \frac{j\omega\mu}{k_c^2}(-A\sin k_x x + B\cos k_x x)k_x(C\cos k_y y + D\sin k_y y)$

We have lots of unknowns here: k_x k_y A B C D

TE (2)



Assuming a≥b

Boundary conditions:

E field should be perpendicular to the metal walls, thus

$$E_x|_{\mathrm{y=0,b}} = 0 \;\&\; E_y|_{\mathrm{x=0,a}} = 0$$

$$-\frac{j\omega\mu}{k_c^2}(Acosk_x x + Bsink_x x)Dk_y = 0$$

$$-\frac{j\omega\mu}{k_c^2}(Acosk_x x + Bsink_x x)\big(-Csink_y b + Dcosk_y b\big)k_y = 0$$

So $D = 0$ & $sink_y b = 0$, we have $k_y = n\pi/b$ with $n = 0,1,2\ldots$

And similarly $B = 0$ & $k_x = m\pi/a$ with $m = 0,1,2\ldots$

So: $H_z = A_{mn}cos(\frac{m\pi x}{a})cos(\frac{n\pi y}{b})e^{-j\beta z}$

TE (3)

$$E_x = \frac{j\omega\mu n\pi}{k_c^2 b} A_{mn} cos(\frac{m\pi x}{a}) sin(\frac{n\pi y}{b}) e^{-j\beta z}$$

$$E_y = \frac{-j\omega\mu m\pi}{k_c^2 a} A_{mn} sin(\frac{m\pi x}{a}) cos(\frac{n\pi y}{b}) e^{-j\beta z}$$

$$E_z = 0$$

$$H_x = \frac{j\beta m\pi}{k_c^2 a} A_{mn} sin(\frac{m\pi x}{a}) cos(\frac{n\pi y}{b}) e^{-j\beta z}$$

$$H_y = \frac{j\beta n\pi}{k_c^2 b} A_{mn} cos(\frac{m\pi x}{a}) sin(\frac{n\pi y}{b}) e^{-j\beta z}$$

$$H_z = A_{mn} cos(\frac{m\pi x}{a}) cos(\frac{n\pi y}{b}) e^{-j\beta z}$$

A "full map" is derived. A_{mn} scales the field strength up and down.

TE (4)

Cutoff frequency $f_c = \frac{c}{2\pi}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$

For TE_{mn} mode

- There is no TE_{00}, all components will be 0 in this case. Note that $\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu H_z$ thus H_z is also 0.
- The mode with lowest cutoff frequency for *TE* modes is TE_{10}, with $f_{c\_TE_{10}} = \frac{c}{2a}$. It is also the mode with lowest cutoff frequency for all modes in the rectangular waveguide, called dominant mode.

TE_{10}

$$E_x = 0$$

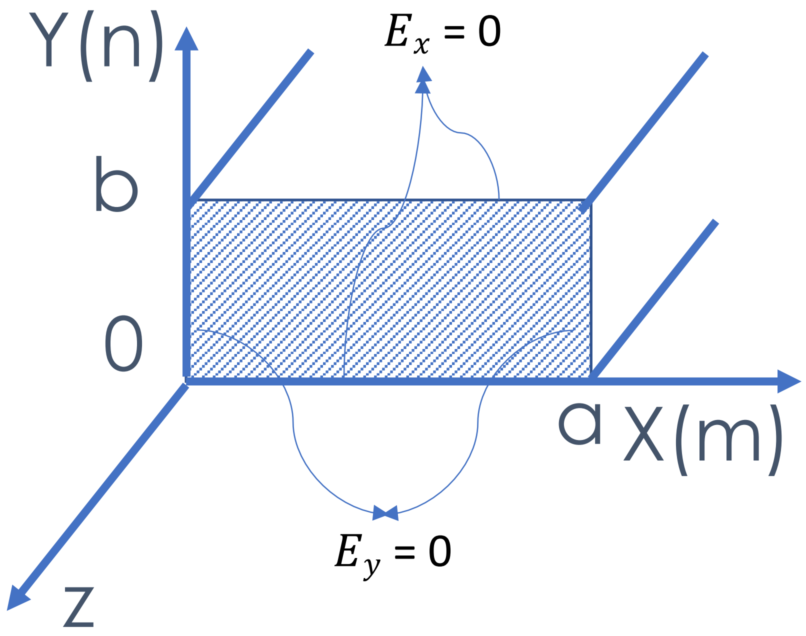
$$E_y = \frac{-j\omega\mu a}{\pi} A_{10} sin(\frac{\pi x}{a})\ e^{-j\beta z}$$

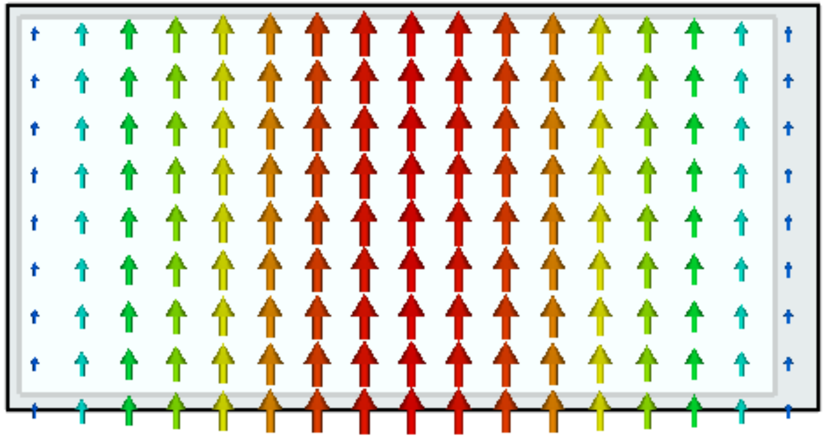
$$E_z = 0$$

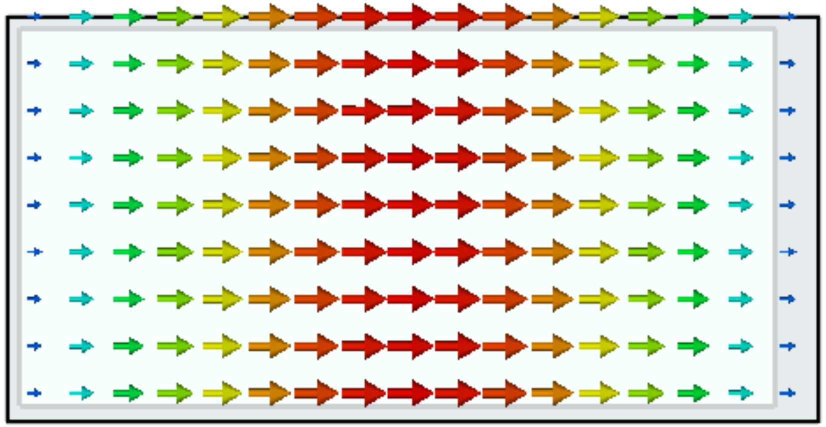
$$H_x = \frac{j\beta a}{\pi} A_{10} sin(\frac{\pi x}{a})\ e^{-j\beta z}$$

$$H_y = 0$$

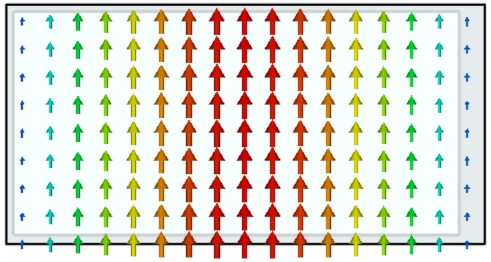
$$H_z = A_{10} cos(\frac{\pi x}{a})\ e^{-j\beta z}$$

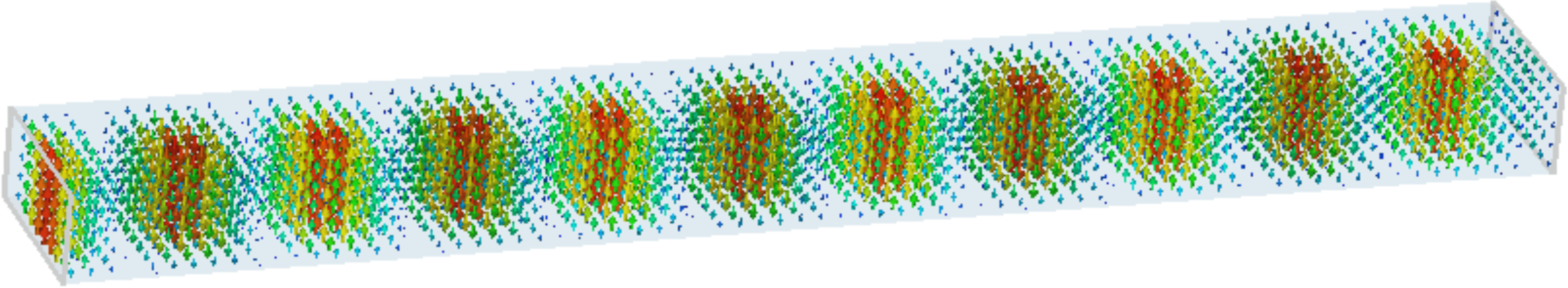


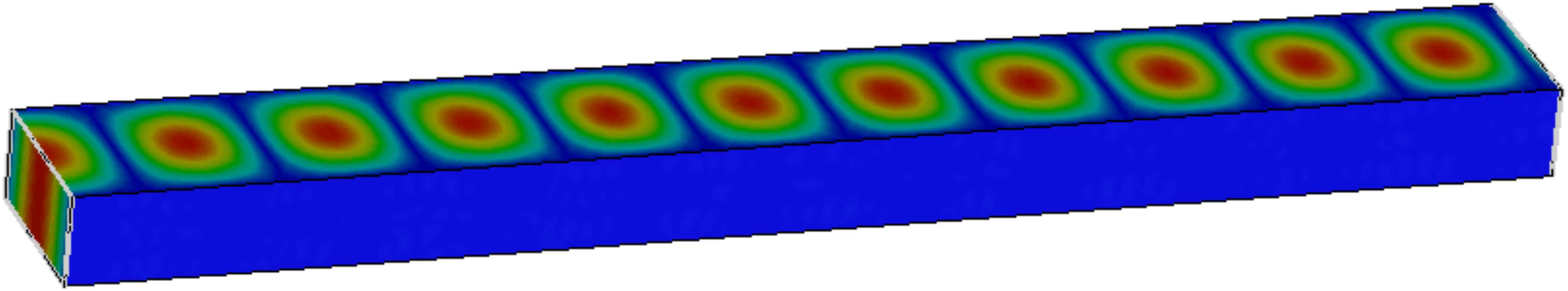




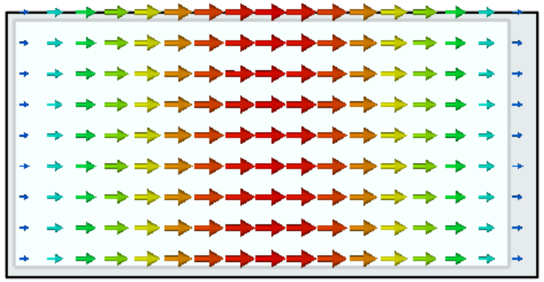
TE_{10} E

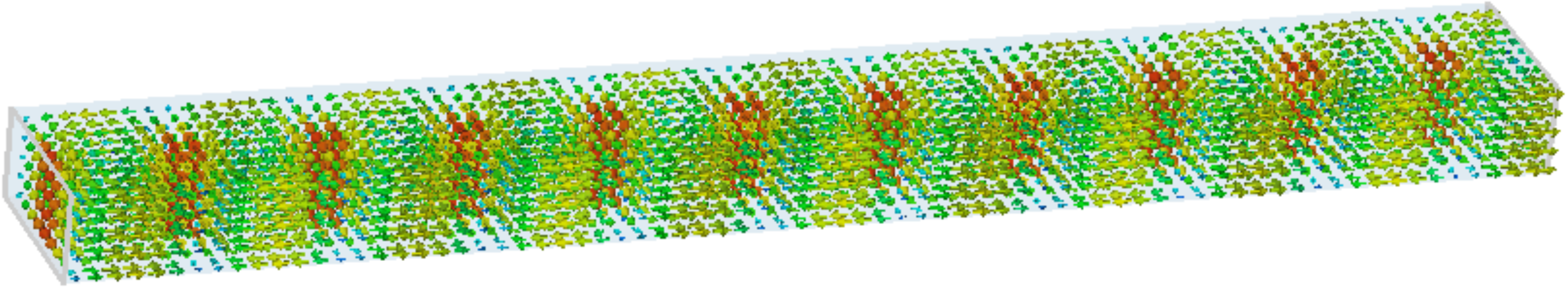


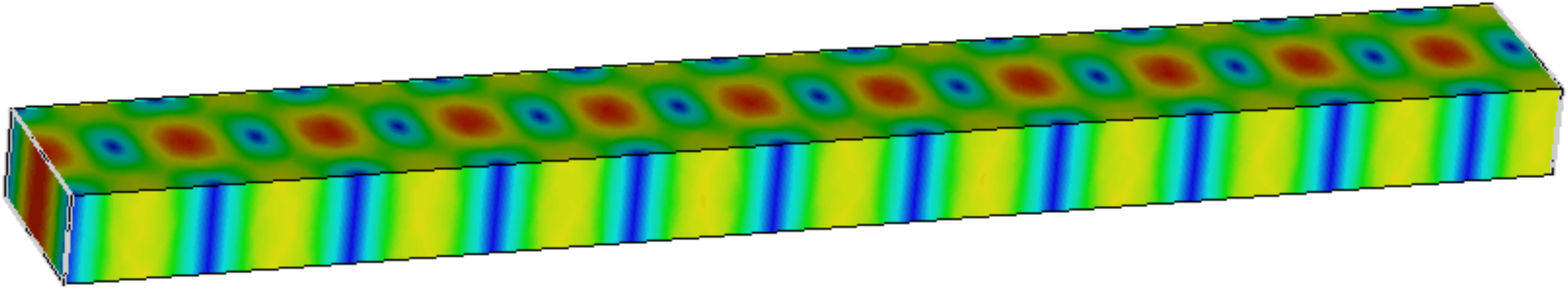




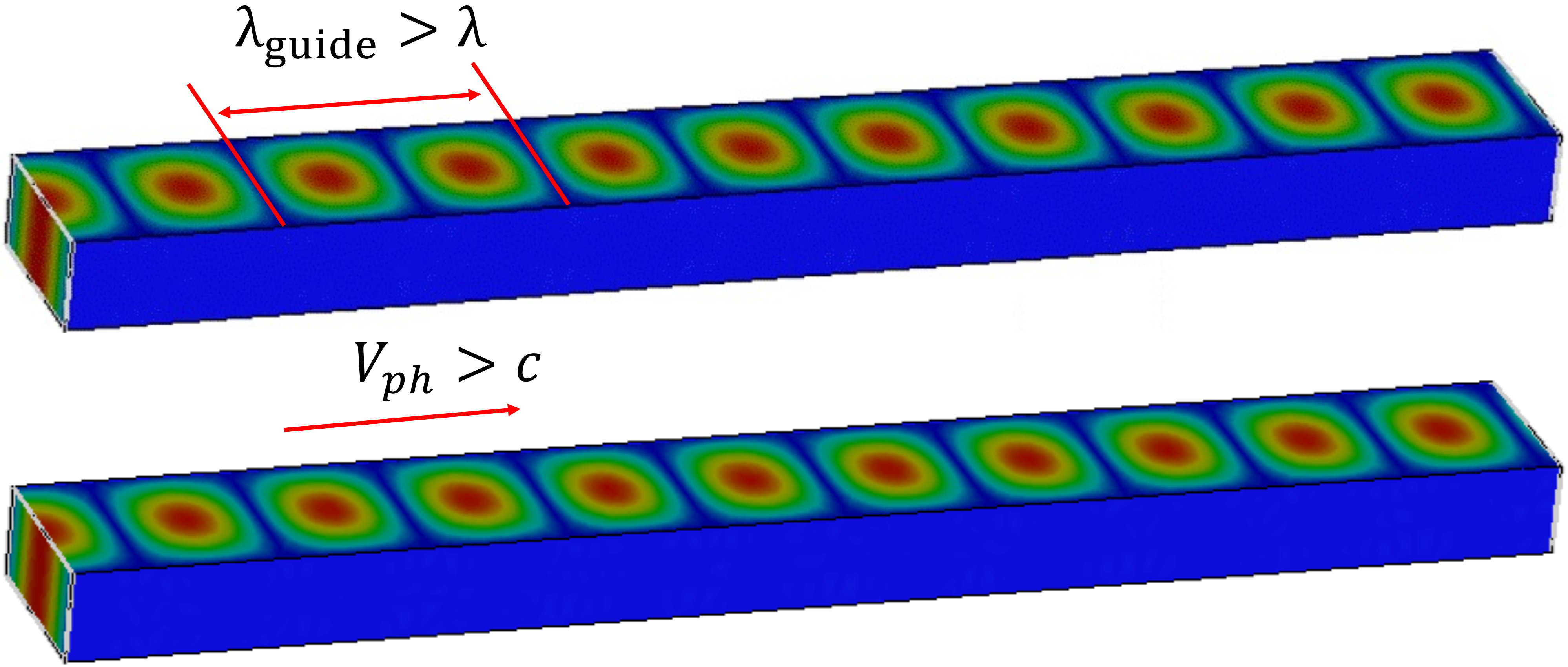
TE_{10} H







Guide wavelength, phase and group velocities



Energy propagates with group velocity V_g, which equals to c^2/V_{ph}

TE_{10} Peak E Field

- $E_y = \frac{-j\omega\mu a}{\pi} A_{10} sin(\frac{\pi x}{a})\ e^{-j\beta z}$
- Peak E field at x=a/2 & phasor=0, $E_{pk} = \frac{\omega\mu a}{\pi} A_{10}$
- E_{pk} is a constant for a given material under certain conditions. A_{10} is inversely proportional to $\boldsymbol{a}$.

TE_{10} Power Flow

- Time-average Poynting vector in z direction: $\langle \boldsymbol{P} \rangle = \frac{1}{2} Re[\boldsymbol{E} \times \boldsymbol{H}^*] \cdot \boldsymbol{z} = \boldsymbol{z} \frac{\omega\mu a}{2\pi} \frac{\beta a}{\pi} A_{10}^2 sin^2\left(\frac{\pi x}{a}\right) = \frac{\beta}{2\omega\mu} sin^2\left(\frac{\pi x}{a}\right) E_{pk}^2$
- Integration: $\int_0^b \int_0^a \langle \boldsymbol{P} \rangle dxdy = \frac{ab}{4} \frac{\beta}{\omega\mu} E_{pk}^2$
- Example: WR975 with 9.75" x 4.875" cross section, cutoff frequency c/2a = 0.605GHz, for 0.75GHz application, $\frac{\beta}{\omega\mu} = 0.00157/\Omega$, ab/4 = 0.007666 m^2, peak E field is ~3MV/m the breakdown voltage of dry air at 1 atm. The maximum (pulsed) power flow is 108.3MW. For 1.12GHz application this number increases to 154.1MW.

TE_{10} wall loss

- RF loss per unit length: $P_{rf} = \frac{R_s}{2}\iint |H|^2 dS = \frac{R_s}{2}\iint ((\frac{\beta a}{\pi} A_{10} sin(\frac{\pi x}{a}))^2 + (A_{10} cos(\frac{\pi x}{a}))^2) dS$ on the walls.
- Use $E_{pk} = \frac{\omega\mu a}{\pi} A_{10}$, $P_{rf} = \frac{R_s}{2}(\frac{E_{pk}}{\omega\mu})^2 \iint ((\beta sin(\frac{\pi x}{a}))^2 + (\frac{\pi}{a} cos(\frac{\pi x}{a}))^2) dS$
- Walls: x=0&a, y from 0 to b, and y=0&b, x from 0 to a.
- $P_{rf} = \frac{R_s}{2}\left(\frac{E_{pk}}{\omega\mu}\right)^2 \left(2b\left(\frac{\pi}{a}\right)^2 + 2\int_0^a \left(\left(\beta \sin\left(\frac{\pi x}{a}\right)\right)^2 + \left(\frac{\pi}{a}\cos\left(\frac{\pi x}{a}\right)\right)^2\right)dx\right) = \frac{R_s}{2}\left(\frac{E_{pk}}{\omega\mu}\right)^2 \left(2b\left(\frac{\pi}{a}\right)^2 + a\beta^2 + a\left(\frac{\pi}{a}\right)^2\right)$
- with $\frac{\omega_c}{c} = \frac{\pi}{a}$ & $\beta^2 = k^2 - k_c^2 = \left(\frac{\omega}{c}\right)^2 - \left(\frac{\omega_c}{c}\right)^2$ $P_{rf} = \frac{R_s}{2}\left(\frac{E_{pk}}{\omega\mu}\right)^2 \left((a+2b)\left(\frac{\omega_c}{c}\right)^2 + a\left(\left(\frac{\omega}{c}\right)^2 - \left(\frac{\omega_c}{c}\right)^2\right)\right) = \frac{R_s}{2}\left(\frac{E_{pk}}{\omega\mu}\right)^2 \left(2b\left(\frac{\omega_c}{c}\right)^2 + a\left(\frac{\omega}{c}\right)^2\right) = \frac{R_s a E_{pk}^2}{2\eta^2}\left(1 + \frac{2b}{a}\left(\frac{\omega_c}{\omega}\right)^2\right)$

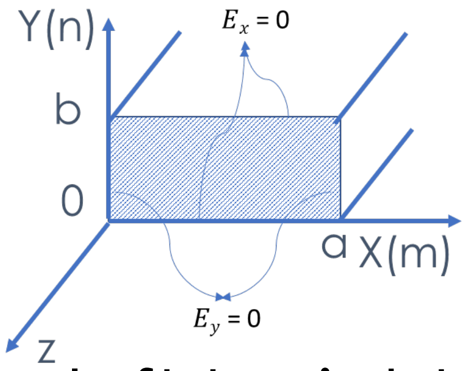
TE_{10} attenuation

- Attenuation (Np/m) equals to loss per unit length over 2 times the power flow:

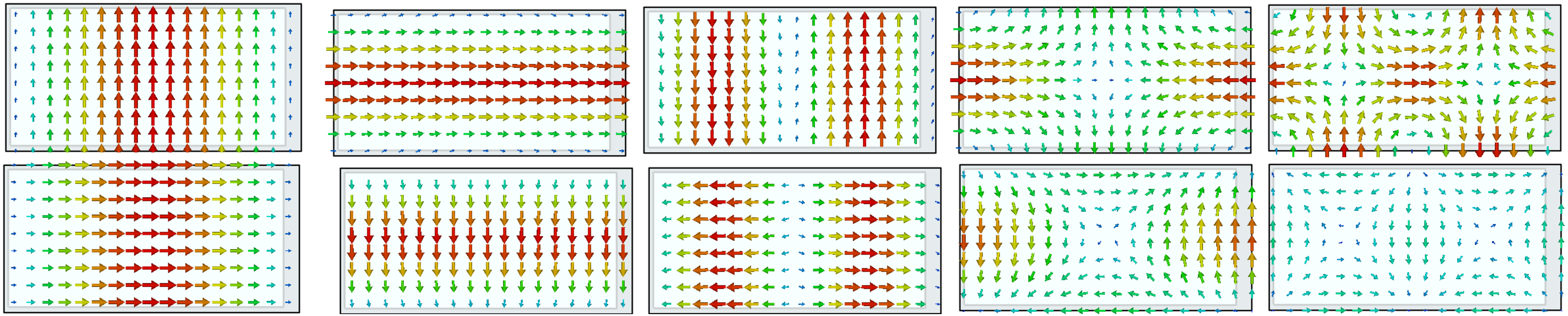
$$\alpha = \frac{\frac{R_S a E_{pk}^2}{2\eta^2}\left(1+\frac{2b}{a}\left(\frac{\omega_c}{\omega}\right)^2\right)}{2\frac{ab}{4}\frac{\beta}{\omega\mu}E_{pk}^2} = \frac{R_S}{\eta b}\frac{1+\frac{2b}{a}\left(\frac{\omega_c}{\omega}\right)^2}{\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}}$$

- R_s of Al @ 0.75GHz is 8.86mΩ and @1.12GHz is 10.82mΩ.
- Example: WR975, the attenuation from wall loss is roughly 0.00037~0.00055Np/m, corresponding to -0.32~-0.48dB/100m, smaller than the LMR1700 coax which is around -2dB/100m at 1GHz

Meaning of m & n



E (top) and H (bottom) of TE_{10} TE_{01} TE_{20} TE_{11} TE_{21}, from left to right.



m and n denote the change in x and y direction, with 0 no change, 1 one change, 2 two changes.

Here 1 change is half (or π) of the sin or cos, for sin it is zero to max and then to zero, for cos it is max to zero to min.

TM (1)

$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega\varepsilon E_z$ & $H_x = \frac{j\omega\mathcal{E}}{k_c^2}\frac{\partial E_z}{\partial y}$ & $H_y = \frac{-j\omega\mathcal{E}}{k_c^2}\frac{\partial E_z}{\partial x}$

$\rightarrow \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k_c^2\right) E_z = 0$

and then use " separation of variables" $E_z(x, y) = X(x)Y(y)e^{-j\beta z}$

$\frac{d^2X}{dx^2} + k_x^2 X = 0$ & $\frac{d^2Y}{dx^2} + k_y^2 Y = 0$ & $k_c^2 = k_x^2 + k_y^2$

We get $E_z = (A\cos k_x x + B\sin k_x x)(C\cos k_y y + D\sin k_y y)$

We have lots of unknowns here: k_x k_y A B C D

TM (2)

Boundary conditions:

E field should be perpendicular to the metal walls, thus

$E_z|_{\mathrm{x=0,a\ \&\ y=0,b}} = 0$

$(Acosk_x x + Bsink_x x)(Ccosk_y y + Dsink_y y)$

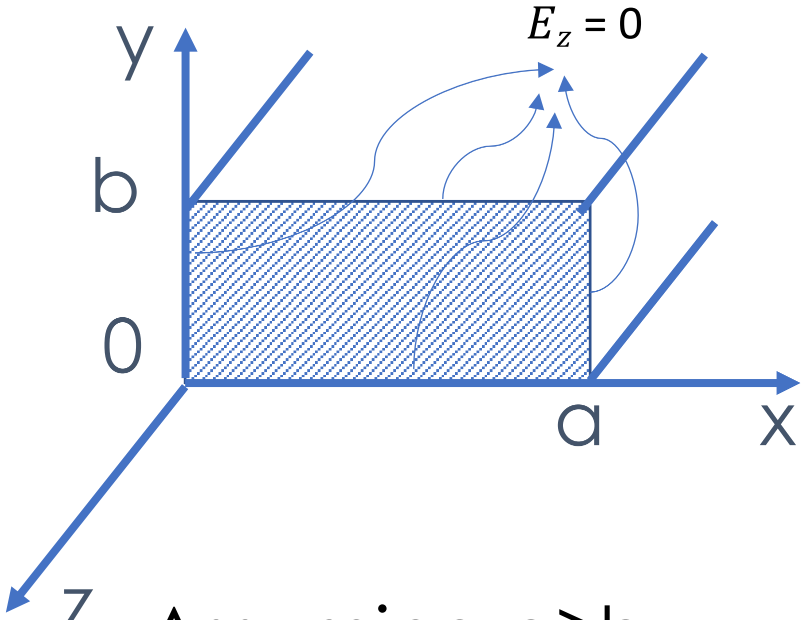
$A(Ccosk_y y + Dsink_y y) = 0$

$(Acosk_x a + Bsink_x a)(Ccosk_y y + Dsink_y y) = 0$

So $A = 0$ & $sink_x a = 0$, we have $k_x = m\pi/a$ with $m = 0,1,2\ldots$

And similarly $C = 0$ & $k_y = n\pi/b$ with $n = 0,1,2\ldots$

So: $E_z = B_{mn} sin(\frac{m\pi x}{a}) sin(\frac{n\pi y}{b}) e^{-j\beta z}$



Assuming a≥b

TM (3)

$$E_x = \frac{-j\beta m\pi}{k_c^2 a} B_{mn} cos(\frac{m\pi x}{a}) sin(\frac{n\pi y}{b}) e^{-j\beta z}$$

$$E_y = \frac{-j\beta n\pi}{k_c^2 b} B_{mn} sin(\frac{m\pi x}{a}) cos(\frac{n\pi y}{b}) e^{-j\beta z}$$

$$E_z = B_{mn} sin(\frac{m\pi x}{a}) sin(\frac{n\pi y}{b}) e^{-j\beta z}$$

$$H_x = \frac{j\omega\varepsilon n\pi}{k_c^2 b} B_{mn} sin(\frac{m\pi x}{a}) cos(\frac{n\pi y}{b}) e^{-j\beta z}$$

$$H_y = \frac{-j\omega\varepsilon m\pi}{k_c^2 a} B_{mn} cos(\frac{m\pi x}{a}) sin(\frac{n\pi y}{b}) e^{-j\beta z}$$

$$H_z = 0$$

A “full map” is derived. B_{mn} is related to the field strength.

TM (4)

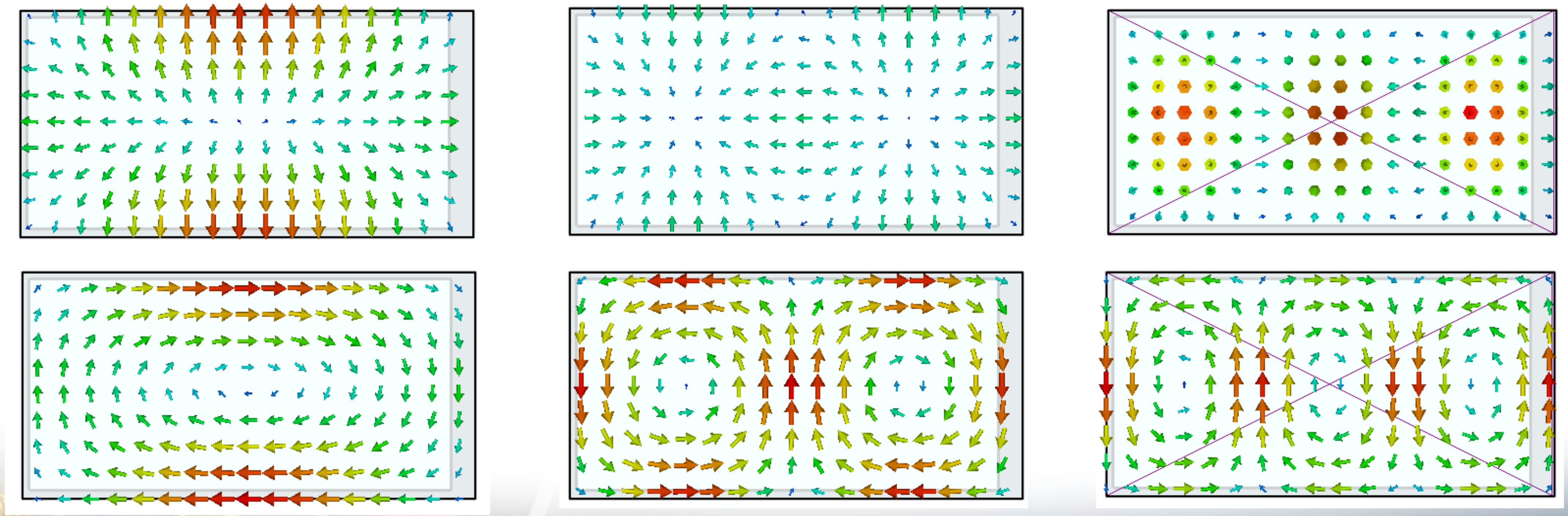
Cutoff frequency $f_c = \frac{c}{2\pi}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$

For TM_{mn} mode

- There is no TM_{00} TM_{01} TM_{10}… all components will be 0 in these cases. (No mode with m=0 or n=0 exists)
- The mode with lowest cutoff frequency for *TM* modes is TM_{11}, with $f_{c\_TM_{11}} = \frac{c}{2}\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}$, which is larger than $f_{c\_TE_{10}}$.

TM modes

E (top) and H (bottom) of TM_{11} TM_{21} TM_{31}, from left to right.



Application

- One basic application of rectangular waveguide is to transmit RF power by using dominant mode only (not overmoded), meaning the working frequency is between the lowest cutoff frequency and the 2nd lowest cutoff frequency.
- One of the commonly used series is named "WR" (stands for rectangular waveguides) following with a number, and this number divide by 100 is a in inch, with a:b=2:1.
- For example, WR-650 has a cross section of 6.5" x 3.25". Cutoff frequency of WR-650 is 0.908GHz for TE_{10}, 1.816GHz for TE_{20} & TE_{01} (Twice of that for TE_{10}, this is the reason of using 2:1), 2.030GHz for TE_{11} & TM_{11}. The recommended frequency band for WR-650 is 1.15 to 1.72GHz (125% and 189% of cutoff for TE_{10}).
- In special applications overmoded waveguides are used.