Contents

Chapter 1

Betatron

As the betatron is part of the proposed projects for this course, this chapter will be limited to a brief introduction of its origins and principles.

The definition of the betatron requires introducing three ingredients [3,3.1,4]:

- induction acceleration on a circular, fixed radius orbit,

- the adiabatic theorem: the magnetic field increase is slow over a revolution around the ring, thus orbital motion and focusing behave, at each instant, as it would be constant (from the viewpoint of the equations of motion: time-varying corrective terms are neglected),

- simultaneous axial and radial focusing.

Noteworthy, betatron and cyclotron have something in common, history-wise: both technologies were used to demonstrate phase stability [4.1,4.2,4.3], after it had been conceived [5,6]

The theory and its application to writing a computer program to simulate a betatron accelerator, and testing it on an example, are left to the project.

1.1 Introduction

The idea of induction acceleration in a purely magnetic device, first known as the "ray-transformer" as the electron beam holds the role of the secondary winding, later denominated betatron (from the "beta-rays", high energy electrons), goes back to the early 1920s [1]. It would then be subject to studies and various attempts of operation and would eventually be made a functioning device in 1940 [3,4] (Fig. 1.1), with an history to come of a source of high energy electron beams for particle physics and a source of X- and γ-rays equivalent to grams of radium.

The betatron allows acceleration of electrons from very low to ultra-relativistic energies, a domain where the cyclotron finds its limits due to the rapid loss of isochronism (classical cyclotron) or prohibitive magnetic field $B(R) \propto \gamma$ (isochronous cyclotron). It would eventually be developed to produce electron beams in the MeV range, up to 300 MeV for particle physics, a limit arising from magnet size and synchrotron radiation. It has rapidly been outperformed by the electron synchrotron, following the advent of "synchronous acceleration" in the mid-1940s.

No RF voltage gap is required in the betatron, betatron acceleration is not resonant, the accelerating electric field is inductive and stems from the varying magnetic field, it is tangential to the reference circular orbit as a consequence of the toroidal current conductor symmetry (Fig. 1.3). The constant orbital radius over the acceleration cycle (under the "betatron condition", Sec. 1.3) was a specificity of the betatron, it would later be a major characteristic of the synchrotron. The theory of particle motion stability about a reference orbit was developed in that context [4], "betatron motion", "betatron oscillations" have become integral part of accelerator jargon.

The betatron concept does not present an interest for ions: at low energy, $v \ll c$, an ion would only get little energy increase over the short duration of a betatron pulse. On the other hand large rigidity $BR = p/q$ means large magnet size (proton BR is for instance 2.4 Tm at 250 MeV, 5.7 Tm at 1 GeV, R respectively 1.6 m, 3.8 m for $B_{max} = 1.5$ T), whereas magnet core volume increase as R^3 in corellation with return flux.

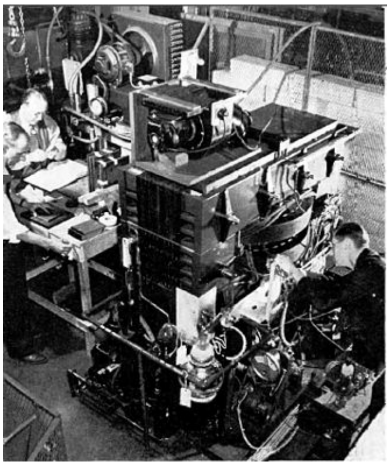

Fig. 1.1 Kerst (right) at work facing a 4-ton magnet betatron, University of Illinois, 1940 [2].

Fig. 1.2 A compact modern betatron X-ray source (yellow, left) and its power supply and control units [5]. The window is visible tho the front with an alignment laser on its left.

Betatrons have given way to linear accelerators in medical and X-ray e-beam applications. They are produced nowadays essentially as light (portable) compact X-ray sources for material analysis, a few MeV energy range (Fig. 1.2). Note though, betatron acceleration found extention to strong focusing FFAG accelerator technique [6], as a path to compact sources of high power electron beams in 100s of keV energy range [7].

1.2 Project

The betatron is proposed as a project during this course.

The field computation and beam dynamics simulation work consists in the fol-

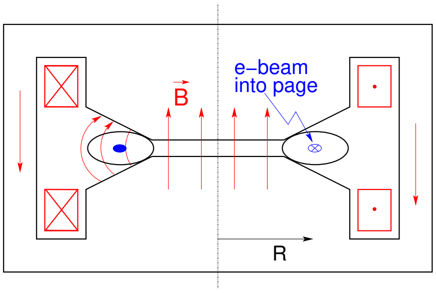

Fig. 1.3 Betatron ring, principle. The current in the coils (left graph, red) is pulsed (sine-like wave in 100s kHz range) and produces the time-varying magnetic field at the origin of the accelerating azimuthal electric field. The ion bunch centroid follows a fixed orbit of radius R. Curved B-field lines in the vacuum chamber region confine the beam in the vicinity of the orbit. The vacuum chamber is made from non-conducting material (high resistance, or glass), or presents non-conducting gaps.

lowing:

- write a program (fortran or C) that simulates the magnetic and induction electric fields in the useful 3D region in the gap of a betatron magnet,
- install it in zgoubi,
- validate by appropriate numerical simulations, modeling an existing a (past or present) betatron,
- confront to theory [3,4]
- produce start-to-end bunch tracking simulations: betatron motion, “damping” of the betatron oscillations, etc.

1.3 Principle

In virtue of the research project, the discussion of the principles is limited to a brief outline aimed as a guidance. Note that, (i) axial and radial focusing has been studied in the Cyclotron chapter, the same principles of weak radial index applies for the betatron, (ii) focusing concurrent with slowly rising field is further discussed in the Synchrotron chapters.

The rising magnetic field ensures the guiding of the bunch on a circular fixed-radius orbit, it increases in proportion of the change in momentum of the particles. Transverse focusing (axial and radial confinement of the bunch particles in the vicinity of the fixed-orbit), is ensured by the shape of the magnetic pole in the region of the reference orbit. The geometry is sketched in Fig. 1.3.

In the absence of any scalar electric potential, the electric field is purely inductive and satisfies $\tilde{\mathrm{E}} = -\frac{\partial\tilde{\mathrm{A}}}{\partial \mathrm{t}}$, with $\tilde{\mathrm{A}}$ the magnetic vector potential.

Given the current symmetries, the latter satisfies $\mathrm{A_r} = \mathrm{A_y} = 0$, $\tilde{\mathrm{A}} \equiv \tilde{\mathrm{A}}_\theta \tilde{\mathrm{u}}$, $\tilde{\mathrm{u}}$ a unitary vector tangent to the circular orbit. $\tilde{\mathrm{B}} = \tilde{\mathrm{curl}}\tilde{\mathrm{A}}$ thus yields $B_r = -\frac{\partial A}{\partial y}$, $\mathrm{B}_\theta = 0$, $\mathrm{B_y} = \frac{1}{\mathrm{r}}\frac{\partial \mathrm{rA}}{\partial \mathrm{r}}$.

The magnetic flux across the surface encompassed by the orbit, radius R, is

$$\phi = \int_0^R B_y \times 2\pi r dr = 2\pi \int_0^R \frac{1}{r}\frac{\partial rA}{\partial r} r dr = 2\pi RA$$

Introduce an average field value $\overline{\mathrm{B}}_\mathrm{y} = \phi/\pi\mathrm{R}^2$, thus $\mathrm{A} = \mathrm{r}\overline{\mathrm{B}}_\mathrm{y}/2$ and $2\pi\mathrm{R}\mathrm{E}_\theta = -2\pi\mathrm{R}\frac{\mathrm{dA}}{\mathrm{dt}} = -\pi\mathrm{R}^2\frac{\mathrm{d}\overline{\mathrm{B}}_\mathrm{y}}{\mathrm{dt}} = -\frac{\mathrm{d}\phi}{\mathrm{dt}}$. The acceleration satisfies $\frac{\mathrm{dp}}{\mathrm{dt}} = \mathrm{eE}_\theta = \frac{-\mathrm{eR}}{2}\frac{\mathrm{d}\overline{\mathrm{B}}_\mathrm{y}}{\mathrm{dt}}$ whereas on the other hand $\frac{\mathrm{dp}}{\mathrm{dt}} = -\mathrm{eR}\frac{\mathrm{dB}_0}{\mathrm{dt}}$: the change in the magnetic flux through the orbit is twice that of the orbital field. By integration,

$$\mathrm{B}_0 = \frac{1}{2}\overline{\mathrm{B}}_\mathrm{y} + \mathrm{constant}, \quad \mathrm{the}\text{“betatron condition”}$$

Note that this property of constant radius under the betatron condition holds independently of the velocity, relativistic or not.

By contrast with the characteristic bunched beam of the resonant acceleration, the beam is longitudinally non-confined in the betatron, spread around the ring.

The driving volage form is not a strong constraint, it can be a sine-like (Fig. 1.3), beam energy follows and the beam stays on the reference orbit. Its frequency (which is also the “repetition rate” of the acceleration cycle) is typically of several 100 Hz. As early as the first betatron, two beams were successively accelerated, both ways, on respectively the rising and falling phase of the driving voltage (so doubling the “repetition rate”, with the magnetic field pulsing in an interval $[-\mathrm{B}_\mathrm{max}, +\mathrm{B}_\mathrm{amx}]$), allowing the production of two X-ray beams, of opposing directions, on the internal target [3].

Other properties include:

- as the momentum increases, chromatic orbits converge towards the R-radius reference closed orbit, following (see Cyclotron chapter) $\Delta R = \eta_x(p'(t) - p(t))/p(t)$, with p(t) (resp. p'(t)) the momentum on the reference R-orbit (resp. on the chromatic $\mathrm{R}'(\mathrm{t}) = \mathrm{R} + \Delta\mathrm{R}(\mathrm{t})$ orbit),
- particles perform “betatron oscillations” around their R'(t)-radius instantaneous orbit, with frequencies $\omega_\mathrm{rev}\sqrt{1-\mathrm{n}}$ (horizontal) and $\omega_\mathrm{rev}\sqrt{\mathrm{n}}$ (vertical), the amplitude of these oscillations “damps” (decreases) under the effect of acceleration,
- both effects, orbital and focusing, combine at injection, allowing part of the injected beam to miss the injector on successive turns.

Adiabatic change of the field

The change in the magnetic field on the orbit is slow, thus the change in momentum upon the effect of the azimuthal induction electric field is weak.

To put numbers on it, take a turn duration $\mathrm{T}_\mathrm{rev} = 2\pi\mathrm{R}/\mathrm{c} \approx 10$ ns for a R=50 cm radius orbit, take a 1 kHz, 1 ms period drive voltage, a ratio of 10^5, meaning a change in $B = p/qR$ of $\approx 10^{-5}\mathrm{B}_\mathrm{max}$ at each turn.

Thus from one revolution to the other the orbital motion and the focusing are considered to behave at each instant as if B and p would be constant, corresponding time-varying corrective terms in the equations of motion are neglected. This allows a linear theory of betatron motion and chromatism.

• Exercise 1.3-1.
1/ Calculate typical values for the followng quantities:
- dB/dt,
- change in energy per turn,
- cycle time.
- maximum theoretical energy attainable.
- number of turns to 5 MeV.

Assume a maximum 1.5 T field, r_0=7.5 cm orbit radius, 1 keV injection energy.
2/ What is the location of minimum electric accelerating field?
3/ How can the beam be extracted? •

Bibliography

[1] A. Sessler, E. Wilson, A Century of Particle Accelerators. World Scientific (2006).
[2] https://en.wikipedia.org/wiki/Betatron.
[3] D.W. Kerst, Phys. Rev. 58, 841 (1940).
[3.1] D.W. Kerst, The Acceleration of Electrons by Magnetic Induction, Phys. Rev. 60, 47-53 (1941).
[4] D.W. Kerst and R. Serber, Electronic orbits in the induction accelerator, Phys. Rev. 60, 53-58 (1941).
[4.1] First demonstration of phase stability, this was using an x-ray betatron, by F. Goward, Woolwich Arsenal Research Lab., UK, in "Engines of Discovery", A.Sessler, E.Wilson, World Sci (2007), 56-57.
[4.2] Demonstration of phase stability using Berkeley 37-inch and 184-inch cyclotrons: D. Bohm and L. Foldy, Theory of the Synchrocyclotron, Phys. Rev. 60, 47-53 (1941).
[4.3] W. M. Brobeck et al., Initial performance of the 184-inch Cyclotron of the University of California, Phys. Rev. 71, 449-450 (1947).

[5] V. Veksler, A new method of acceleration of relativistic particles, J. of Phys. USSR 9 153-158 (1945), Translation L. Bell in [2].

[6] E.M. McMillan, The Synchrotron, Phys. Rev. 68 143-144 (1945).

[5] ADVANCED INSPECTION SYSTEMS. JME Portable 6 MeV. X-RAY BETATRON. Microprocessor model: PXB-6 M. Jun 15, 2010.
[6] C. L Hammer et al., Slow Extraction from FFAG Accelerators, accelconf.web.cern.ch/AccelConf/p67/PDF/PAC1967_0702.PDF.
[7] Takashi Baba et al., DEVELOPMENT OF FFAG ELECTRON ACCELERATOR, THPP001, Proceedings of EPAC08, Genoa, Italy. accelconf.web.cern.ch/AccelConf/e08/papers/thpp001.pdf.