

PHY 554: RF Physics Midterm Review

Fundamentals of Accelerator Physics

Jun Ma

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Concepts and methods from Lectures 9, 10, and 16

RF acceleration: the essential picture

- A particle gains energy from the **longitudinal electric field**: $d\mathcal{E}/dt = q\mathbf{E} \cdot \mathbf{v}$.
- RF fields oscillate: acceleration depends on the phase at which the bunch traverses a cavity.
- For a uniform on-axis field over length d , a particle with speed $v = \beta c$ sees a reduced effective voltage:

$$V_{\text{acc}} = E_0 d T, \quad T = \frac{\sin(\omega d/2v)}{\omega d/2v}.$$

Check your understanding

Why is the energy gain smaller than qE_0d when transit time is significant?

Pillbox cavity and the TM_{010} mode

- Accelerating modes need $E_z \neq 0$ (TM modes).
- For an ideal cylindrical pillbox of radius a and length L :

$$E_z(r, t) = E_0 J_0\left(\frac{\chi_{01} r}{a}\right) \cos \omega t$$

$$H_\phi(r, t) = \frac{E_0}{Z_0} J_1\left(\frac{\chi_{01} r}{a}\right) \sin \omega t$$

Boundary condition

$E_z(a, t) = 0$ gives $J_0(\chi_{01}) = 0$.
 $\chi_{01} \simeq 2.405$ is the first positive zero of J_0 .

Important

The TM_{010} resonant frequency depends on radius, not length, for an ideal closed pillbox.

Resonant frequency and geometry

$$\omega_0 = \frac{\chi_{01} c}{a}, \quad f_{010} = \frac{\chi_{01} c}{2\pi a}$$

- Larger radius $a \Rightarrow$ lower resonant frequency.
- For TM_{010} , $k_z = 0$: no longitudinal field variation in the ideal pillbox.
- The cavity length L still affects **stored energy and wall loss**.

Concept check

If frequency is doubled while the cavity remains in TM_{010} , how must the radius change?

Electromagnetic energy in a resonant cavity

$$W_E(t) = \frac{\epsilon_0}{2} \int_V |\mathbf{E}(t)|^2 dV, \quad W_H(t) = \frac{\mu_0}{2} \int_V |\mathbf{H}(t)|^2 dV.$$

- In a lossless standing-wave mode, energy exchanges between electric and magnetic fields:

$$W_E(t) = W \cos^2 \omega t, \quad W_H(t) = W \sin^2 \omega t.$$

$$W = W_E + W_H = \text{constant}$$

How to calculate W

Choose an instant when $H = 0$, integrate the electric-field energy density over the *whole volume*; use $dV = 2\pi r L dr$ for an axisymmetric pillbox.

Stored energy: radial field distribution matters

- The on-axis peak field E_0 is *not* the field everywhere in the cavity.
- Use J_0^2 when integrating the electric energy:

$$W = \frac{\epsilon_0 E_0^2}{2} (2\pi L) \int_0^a J_0^2\left(\frac{\chi_{01} r}{a}\right) r dr.$$

$$W = \pi \epsilon_0 E_0^2 a^2 L \int_0^1 J_0^2(\chi_{01} x) x dx$$

Common pitfalls

Do not replace J_0 by 1 throughout the cavity. Do not add the *separate maxima* of electric and magnetic energy.

RF surface resistance and wall dissipation

- Real cavity walls dissipate power through the **tangential magnetic field**.
- For uniform surface resistance R_s , with H_t the *peak* RF field amplitude:

$$P_{\text{wall}} = \frac{R_s}{2} \oint_S |H_t|^2 dS$$

Cylindrical wall ($r = a$)

$dS = a d\phi dz$; H_t is constant over this surface for TM_{010} .

Two end plates ($z = 0, L$)

$dS = r dr d\phi$; integrate J_1^2 radially and count **both** plates.

Quality factor and SRF cavity performance

$$Q_0 = \frac{\omega_0 W}{P_{\text{wall}}}, \quad Q_0 = \frac{G}{R_s}$$

- Q_0 is the intrinsic (unloaded) quality factor; G is the geometry factor.
- Superconducting cavities have very small, but nonzero, RF surface resistance.
- Two contributions discussed in Lecture 16:

$$R_s(T, f) = R_{\text{BCS}}(T, f) + R_{\text{res}}.$$

Beyond the isolated cavity

External couplers introduce Q_{ext} ; for one port, $1/Q_L = 1/Q_0 + 1/Q_{\text{ext}}$.

Review checklist: can you explain each step?

- ➊ From the conducting-wall boundary condition, obtain the TM_{010} resonance condition.
- ➋ Write the spatial dependence of E_z and H_ϕ ; identify their phase difference.
- ➌ Set up the stored-energy integral with the correct cylindrical volume element.
- ➍ Set up wall-loss integrals on the side wall and *both* end plates.
- ➎ Explain the $1/2$ in time-averaged RF surface loss for peak field amplitudes.
- ➏ Relate W , P_{wall} , Q_0 , and R_s ; check units.

Understand the derivations, assumptions, and physical meaning—not just formulas.