PHY564 | Advanced Accelerator Physics

Lecture #27:
Advanced Accelerator Concepts

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Outline:

- Motivation for new accelerator concepts
- Cold normal-conducting (NC) accelerating structures
- Photonic Band Gap (PBG) Accelerators
- Dielectric Wakefield Accelerators
- Laser Wakefield Plasma Accelerators

References:

- This lecture is an adaptation of the material taught by V. Litvinenko in PHY564'20 "Advanced Accelerator Physics"

- For more information, please refer to the following literature:

[1] V. Shiltsev, F. Zimmermann, ***Modern and future colliders***, DOI: https://doi.org/10.1103/RevModPhys.93.015006
[2] M. Nasr, et al., ***Experimental demonstration of particle acceleration with normal conducting accelerating structure at cryogenic temperature***, DOI: https://doi.org/10.1103/PhysRevAccelBeams.24.093201
[3] C. Jing, ***Dielectric wakefield accelerators***, DOI: https://doi.org/10.1142/S1793626816300061
[4] E. I. Smirnova, ***Photonic Band Gap Structures for Accelerator Applications,*** DOI: https://doi.org/10.1063/1.1524893
[5] E. Esarey, et al., ***Physics of laser-driven plasma-based electron accelerators***, DOI: https://doi.org/10.1103/RevModPhys.81.1229
[6] C. Joshi, ***Laser-Driven Plasma Accelerators Operating in the Self-Guided, Blowout Regime***, DOI: https://doi.org/10.1109/TPS.2017.2769455

Colliders: where are we at and what is next?

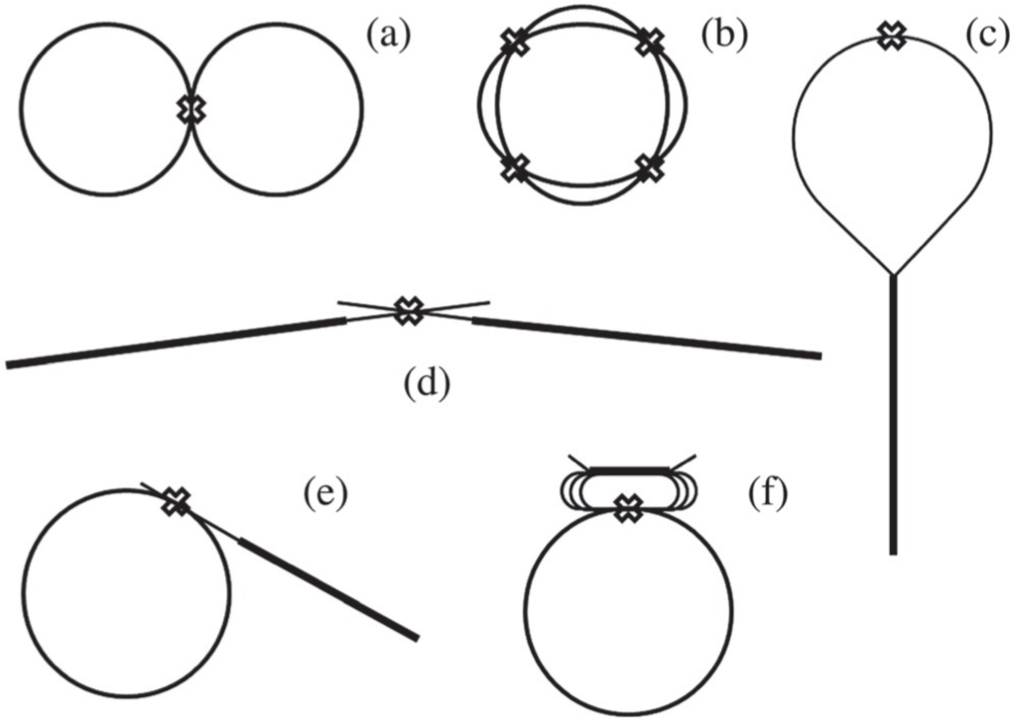
For the head-on collision of two particles of masses m_1 and m_2 with energies E_1 and E_2 colliding at a crossing angle θ_c:

$$E_{\text{c.m.e.}} = \left(2E_1E_2 + (m_1^2 + m_2^2)c^4 + 2\cos\theta_c\sqrt{E_1^2 - m_1^2c^4}\sqrt{E_2^2 - m_2^2c^4}\right)^{1/2}$$

Example:

Fixed target collision of 7,000 GeV protons with a stationary target produces $E_{\text{c.m.e.}} \approx 120$ GeV.

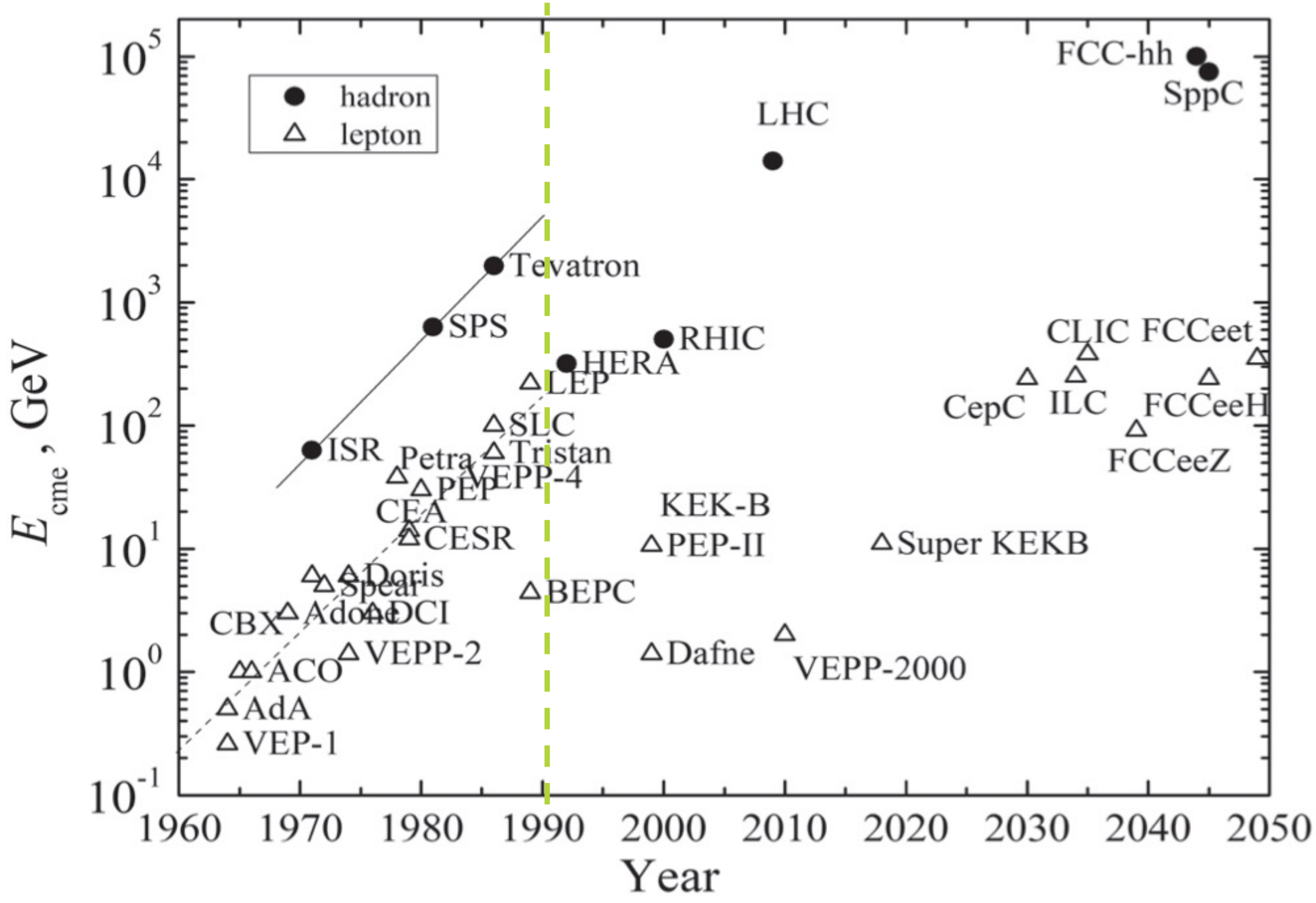
Proton-proton head-on collision of two 7,000 GeV beams results in $E_{\text{c.m.e.}} \approx 14{,}000$ GeV.



(a) & (b) storage ring colliders
(c) & (d) linac-based colliders
(e) linac-ring scheme
(f) ring + Energy Recovery Linac (ERL)

Livingston Chart: what's next?

- Until the early 1990s, the $E_{c.m.e.}$ increased on average by a factor of 10 every decade. **The exponential progress in energies then has stopped**.



Since then, the paths of different colliders have diverged:

- hadron colliders continued the quest for record high energies in particle reactions and the LHC was built at CERN
- highly productive e^+e^- colliders called *particle factories* focused on precise exploration of rare phenomena at much lower energies.

Luminosity

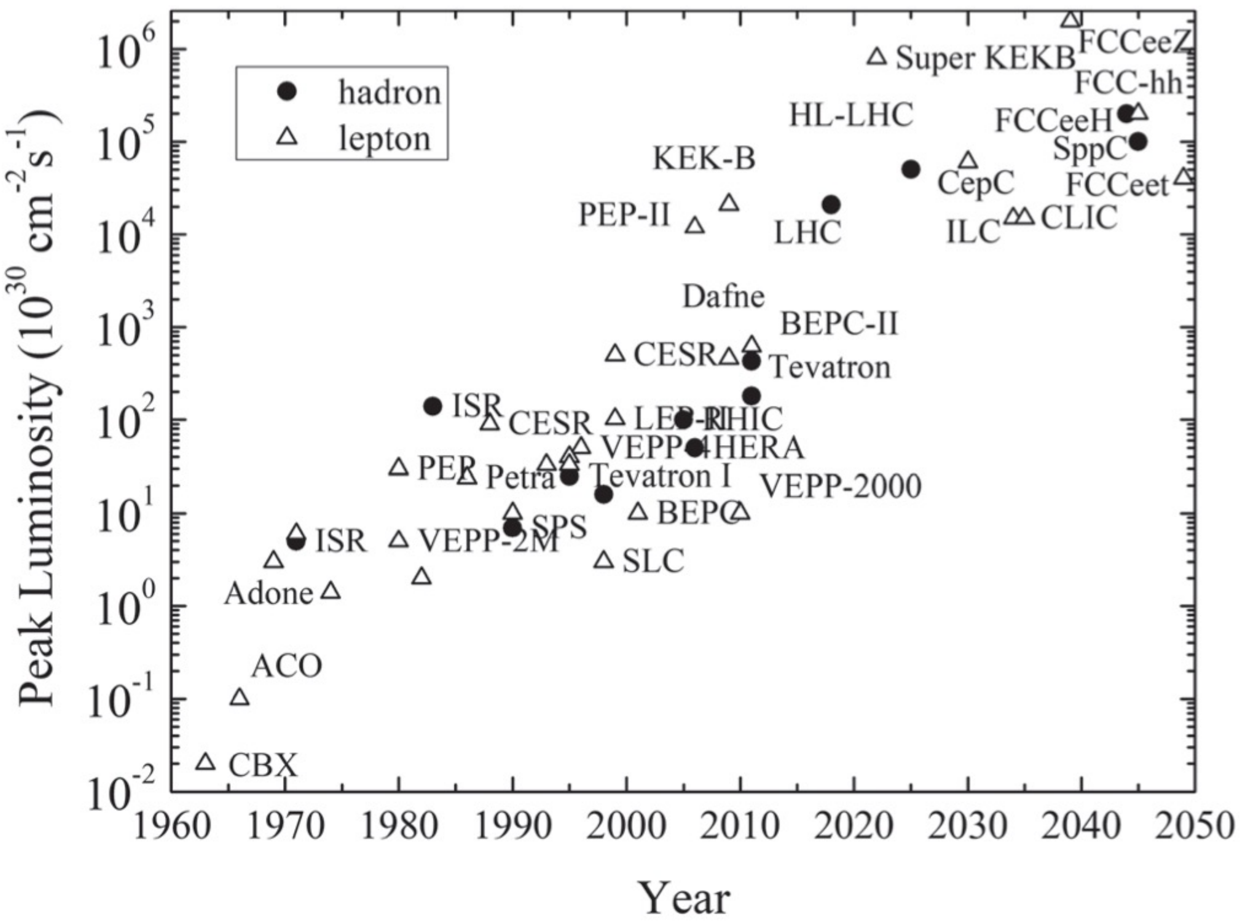
- The exploration of rare particle-physics phenomena requires **not only** an appropriately **high energy but also** a sufficiently **large number of detectable reactions**.

- **The number of events of interest** N_{exp} is given by the following product of the cross section of the reaction under study σ_{exp} and the time integral over the instantaneous luminosity L

$$N_{\text{exp}} = \sigma_{\text{exp}} \int \mathcal{L}(t)dt.$$

- *Luminosity* for two bunches containing N_1 and N_2 particles and colliding head on with frequency f_{coll},

$$\mathcal{L} = f_{\text{coll}} \frac{N_1 N_2}{4\pi\sigma_x^* \sigma_y^*},$$



To achieve high luminosity:

- maximize the population and number of bunches, either producing these narrowly or focusing them tightly and colliding them at high frequencies at dedicated locations where products of their reactions can be registered by detectors.

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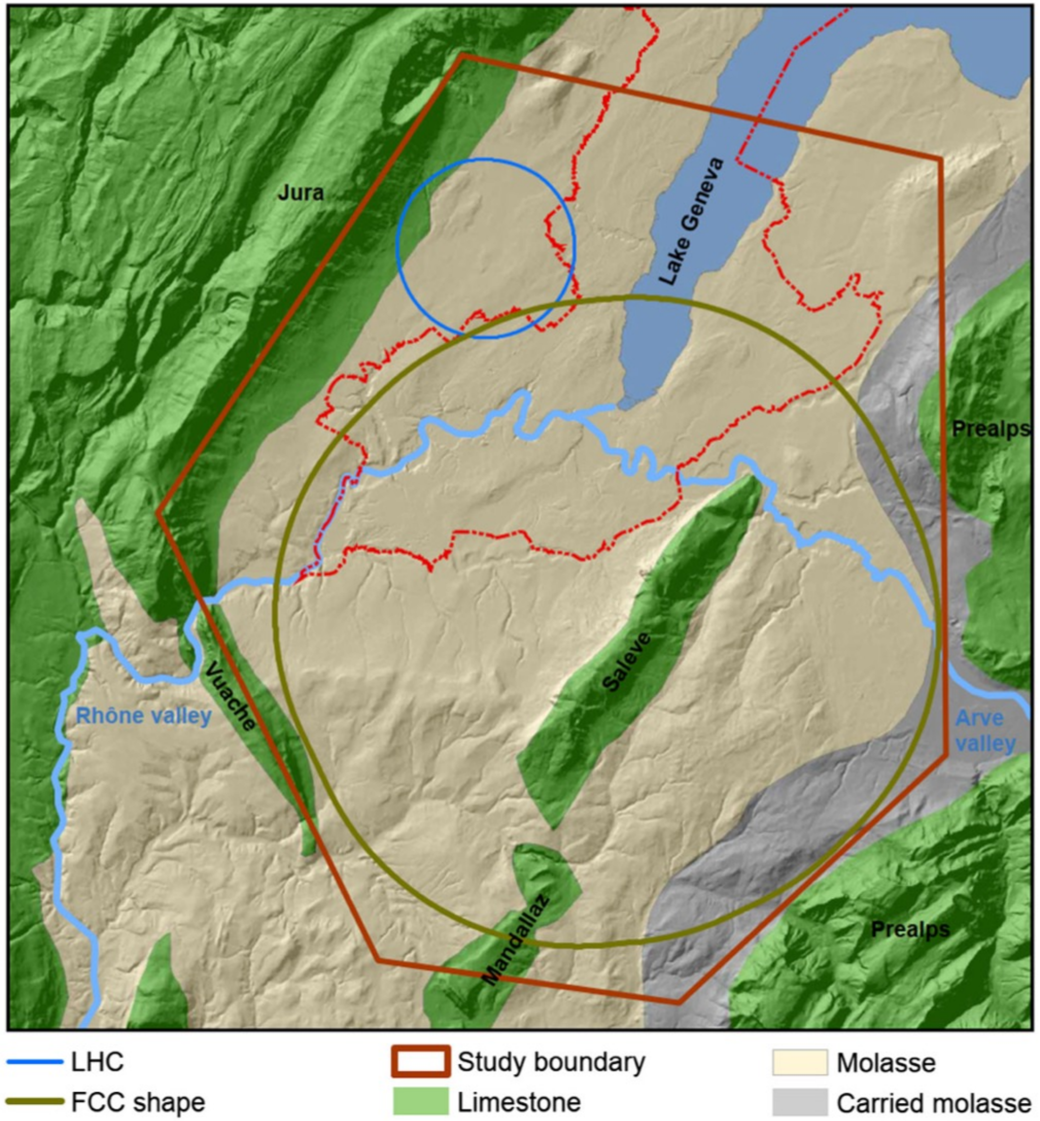
Circular colliders

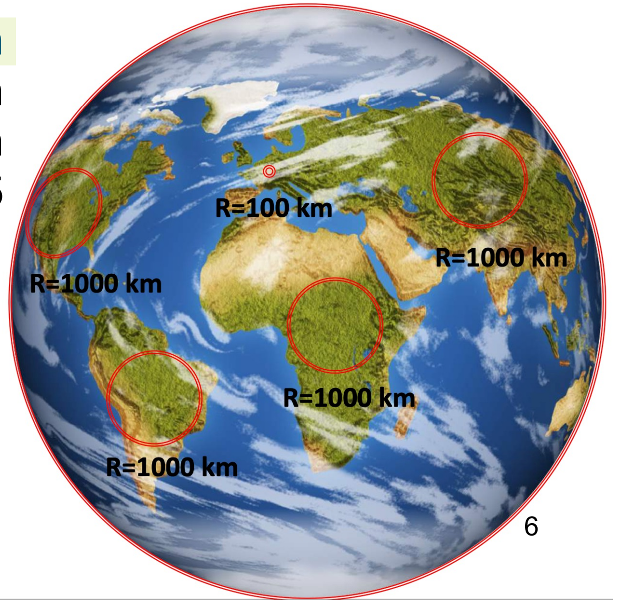


→ **Electron-position (e^+e^-) colliders** stopped at $E_{c.m.e.}$ of 209 GeV (104.5 GeV per beam, LEP, CERN, C = 27 km).
  → ***losses for synchrotron radiation became comparable with energy of the beam***

→ **Large Hadron Collider** (pp @ 2 x 7 TeV) occupies LEP tunnel with two 27 km rings powered superconducting 7.7 T magnets.
  → ***cost of such facility is in 10s of $B***

→ Physics community is discussing building **Future Circular Collider** (FCC) with circumference of 100 km
  → extends $E_{c.m.e.}$ to 350 GeV for e^+e^- collision: would need ***100 MW of RF power to compensate for synchrotron radiation*** losses
  → extends $E_{c.m.e.}$ to 100 TeV for pp collisions: would require ***superconducting magnets with field of 16 T***

→ **In the 1950s Fermi thought of an Earth-encircling "Globaltron"** with a circumference of C = 40 000 km and energy reach of 5000 TeV (5 PeV).



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Linear colliders

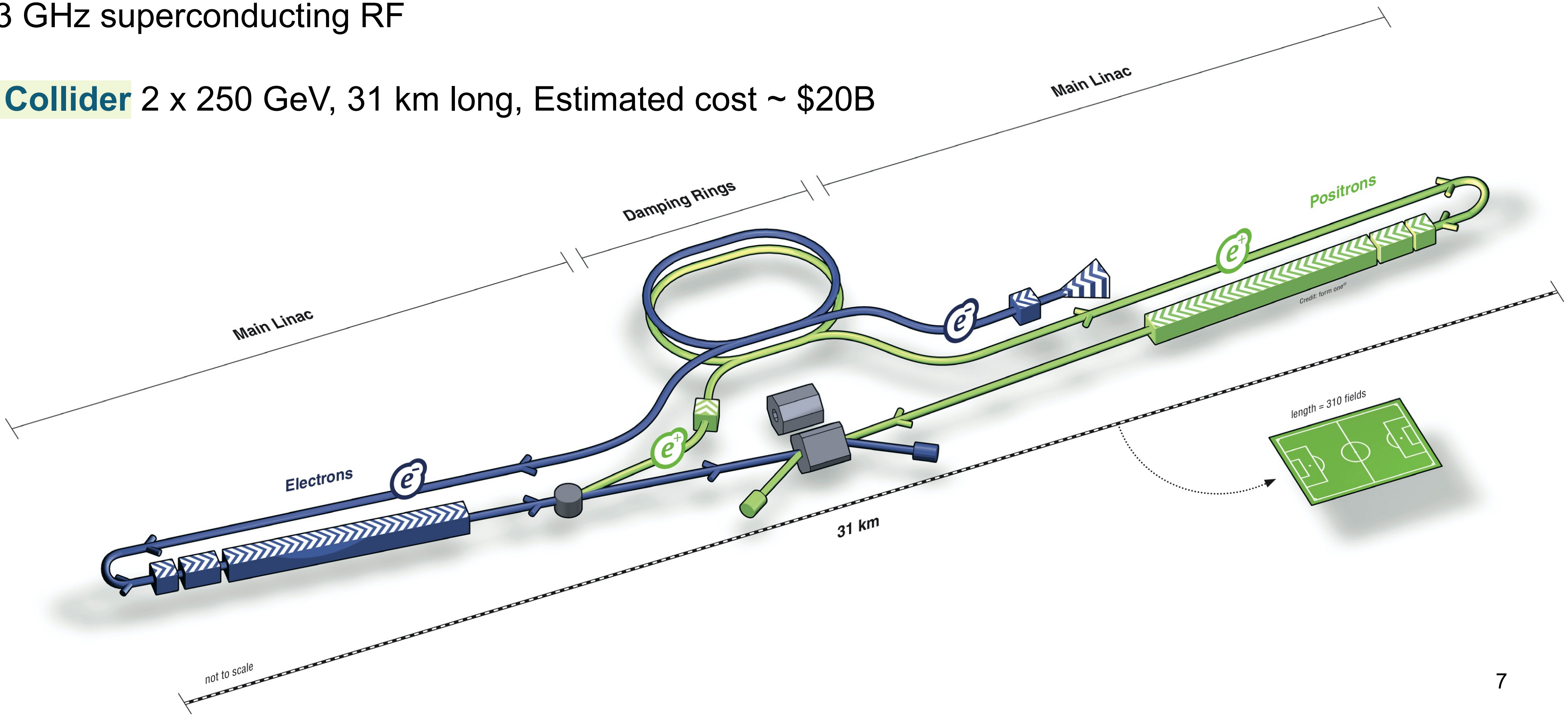
- **Synchrotron radiation is no longer a problem**
  - ***SR losses do not scale so aggressively with beam energy***

$$\Delta E_{\mathrm{SR}} = \frac{2e^4}{3m^2c^3}\int \frac{\left(\vec{E}+\vec{\beta}\times\vec{B}\right)^2-\left(\vec{\beta}\cdot\vec{E}\right)^2}{1-\vec{\beta}^2}dt,\ \vec{\beta}=\frac{\vec{v}}{c}$$

$$\vec{B}=0,\ \vec{E}\parallel\vec{\beta}\Rightarrow\Delta E_{\mathrm{SR}}=\frac{2e^4}{3m^2c^3}\int\vec{E}^2dt\approx\frac{2}{3}\frac{e^2}{L_{\mathrm{acc}}}\gamma^2$$

- However, standard **RF accelerators are limited in accelerating gradients**
  - 100 MV/m in 12 GHz normal-conducting (NC) RF cavities
  - 31.5 MV/m in 1.3 GHz superconducting RF

- **International Linear Collider** 2 x 250 GeV, 31 km long, Estimated cost ~ $20B



New accelerator concepts are required!

- **“Cold” normal-conducting RF**
- **Plasma accelerators:**
  - Laser-driven plasma accelerators
  - Beam-driven plasma accelerators
- **Dielectric wakefield accelerators**
- **Dielectric laser accelerators**
- **Photonic gap accelerators**

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“Cold” normal-conducting RF

Cool copper: normal conducting (NC) structures operating at cryogenic temperatures

Improving the NC structure performance:
Approach#1: choose a better material

- A **limiting factor** in the operation of high-gradient accelerating structures is the **vacuum arcs on the surface** that cause the **RF breakdowns** which can perturb the acceleration.

- **Breakdown** rates are largely correlated to the **peak electric and magnetic fields on the surface**.

- The breakdowns are **triggered by crystal defects** due to the cyclic fatigue which happens over many cycles. These defects cause surface protrusions seen by the electric field and then cause the breakdown events.

- Minimize the breakdown rates in high-gradient accelerators by **building accelerator structures from harder materials to reduce the cyclic fatigue** on the surface, and thus lower breakdown rates compared to the ones built from softer materials.

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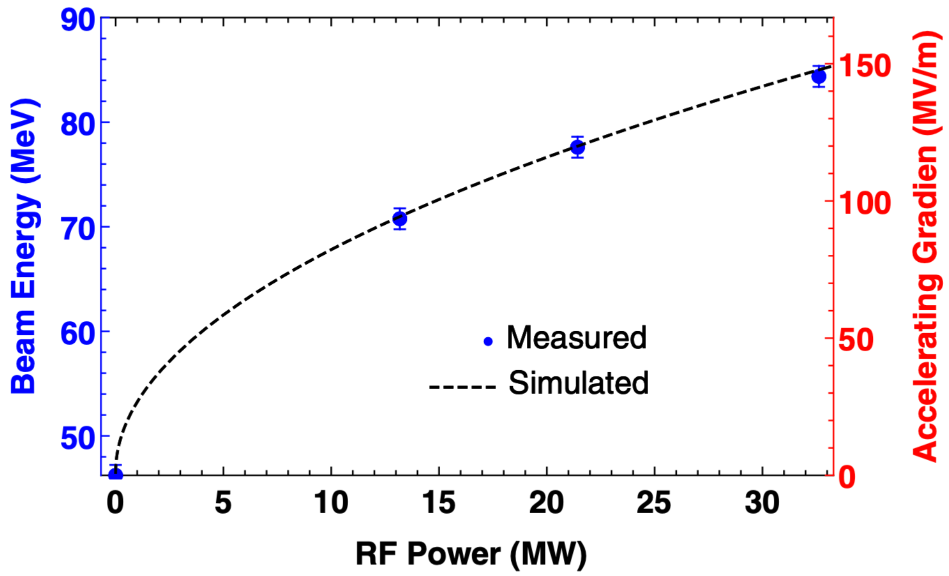
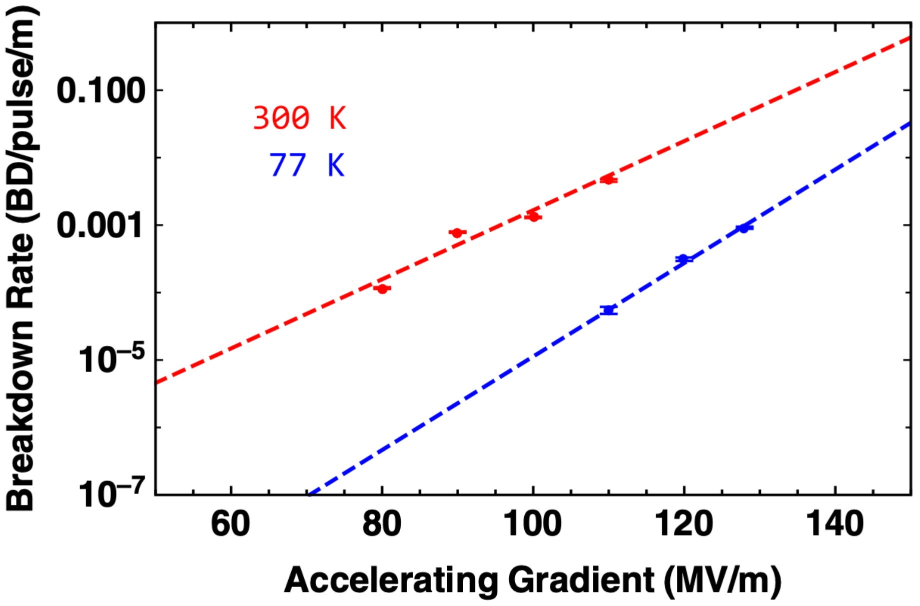
Cool copper: normal conducting (NC) structures operating at cryogenic temperatures

Improving the NC structure performance:
Approach#2: operate at cryogenic temperatures

- Operation at cryogenic temperatures **reduces the surface resistance**.
- This reduction **increases the shunt impedance and internal quality factor** of the accelerating cavities, leading to **increased rf-to-beam efficiency and beam loading capabilities**.
- The increased rf-to-beam efficiency can **increase the overall system efficiency** using optimized cryogenic systems.
- Low-temperature operation **increases hardness reducing the surface deformation and the breakdown rates** compared to room-temperature operation for the same accelerating gradient.





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Photonic Band Gap Accelerators

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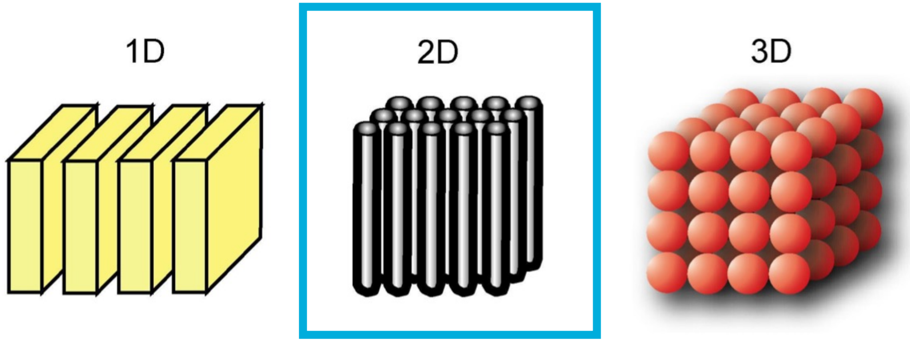
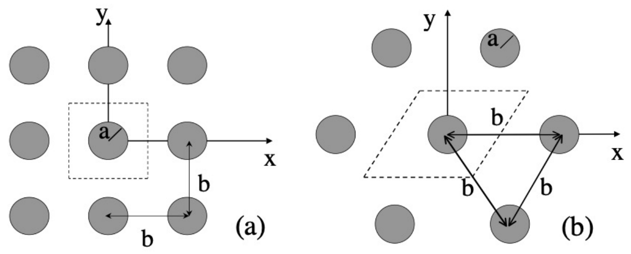
Photonic Band Gap (PBG)

- **A photonic band gap (PBG) structure is a 1-, 2- or 3-dimensional periodic metallic and/or dielectric system, which acts like a filter, reflecting RF fields in some frequency range and allowing RF fields at other frequencies to transmit through.**

- The range of frequencies which do not propagate through the photonic crystal is called a band gap.





- Metal PBG structures can be employed at X and Ku-band accelerators to suppress wakefields.

- Dielectric PBG structures are attractive at THz frequencies for construction of high gradient laser-driven accelerators.

- The presence of a global photonic band gap in a photonic crystal allows us to construct a **PBG resonator**, which can be formed by the **defect (missing rod) in a periodic lattice**. The mode, which has a frequency in a global photonic band gap, will not be able to propagate through the bulk of the PBG structure and will be localized around the defect.

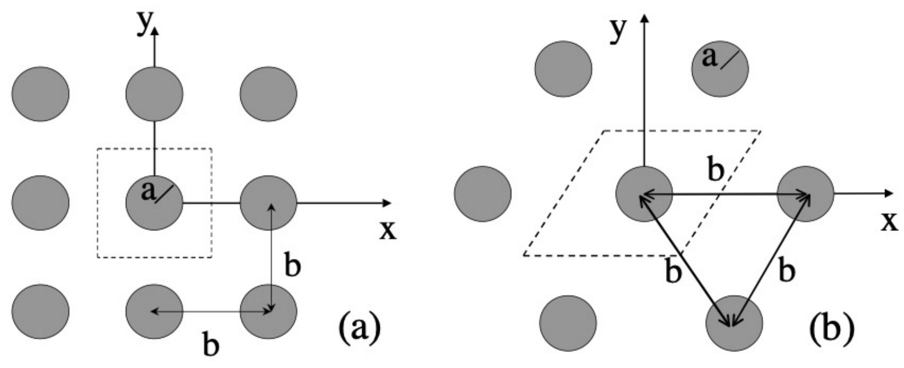
Photonic Band Gap (PBG)

The electromagnetic field in PBG structures is described with a set of Maxwell's equations:

$$\nabla \times \vec{E} = -i\mu_0\omega\vec{H},$$
$$\nabla \times \vec{H} = i\varepsilon\omega\vec{E},$$
$$\nabla \cdot (\varepsilon\vec{E}) = 0,$$
$$\nabla \cdot \vec{H} = 0,$$

In addition, each component of the field $\psi = E_i, H_i$ must satisfy the Floquet theorem $\psi(\vec{x}_\perp + \vec{T}) = \psi(\vec{x}_\perp)e^{i\vec{k}\cdot\vec{T}_{m,n}}$ where $\vec{k} = k_x\hat{e}_x + k_y\hat{e}_y + k_z\hat{e}_z$ is an arbitrary wave vector and. $\vec{T}_{m,n}$ Is a vector-period of the lattice.

$$\vec{T}_{m,n} = \begin{cases} mb\hat{e}_x + nb\hat{e}_y & \text{square lattice} \\ (m + n/2)\, b\hat{e}_x + \frac{\sqrt{3}}{2}nb\hat{e}_y & \text{triangular lattice} \end{cases}$$



For 2D lattice of metal rods, the set of Maxwell's equations can be simplified by splitting the possible modes into sets of the transverse electric (TE) and the transverse magnetic TM waves. In this case the Helmholtz equation is valid for $\psi = H_\parallel$ (TE modes) and $\psi = E_\parallel$ (TM modes).

$$\nabla_\perp\psi = \left(k_\parallel^2 - \frac{\omega^2}{c^2}\right)\psi = -\kappa^2\psi$$

The set of Maxwell's equations or the Helmholtz equation together with periodic boundary condition defines the eigenvalue problem of finding the propagating frequency as a function of the wave number.

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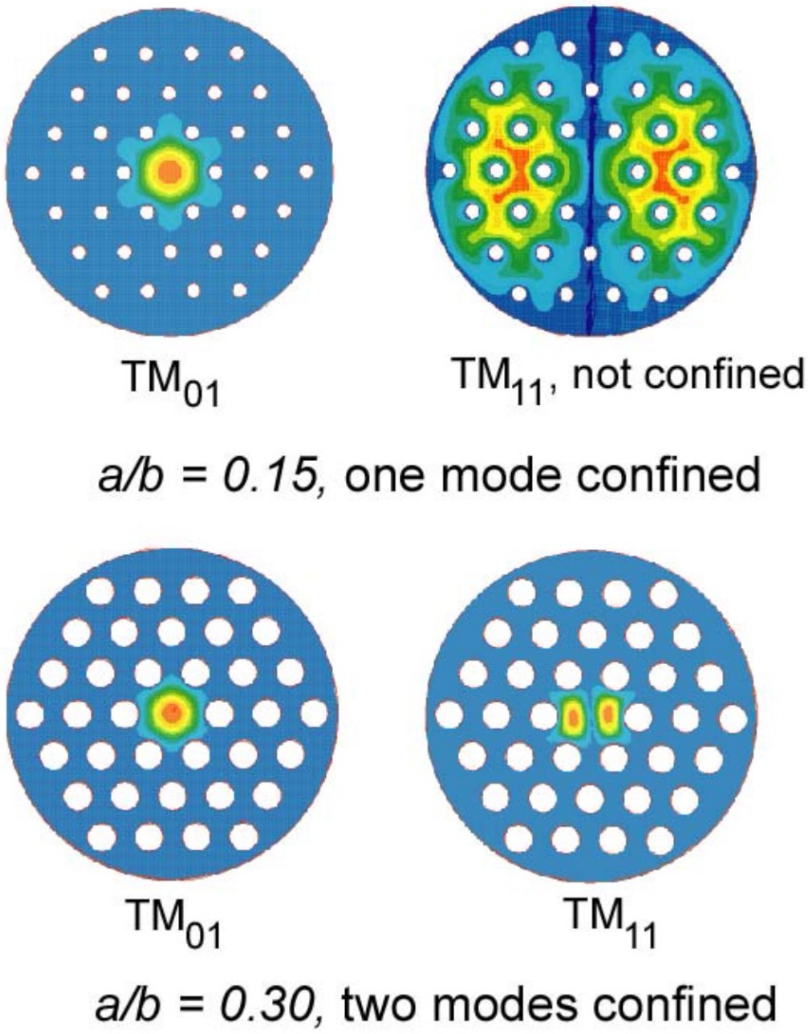
Modes in PBG resonator

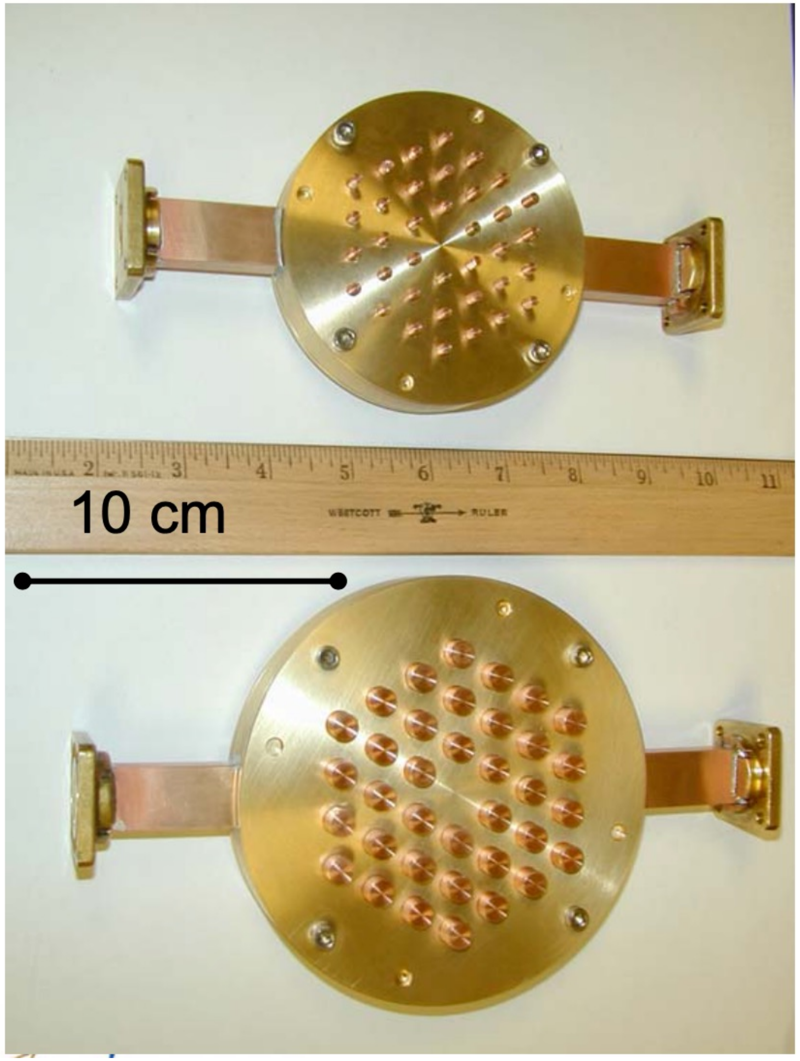
Consider, for example, a triangular lattice of metal rods with *a/b =0.15* and a defect formed by a missing rod.

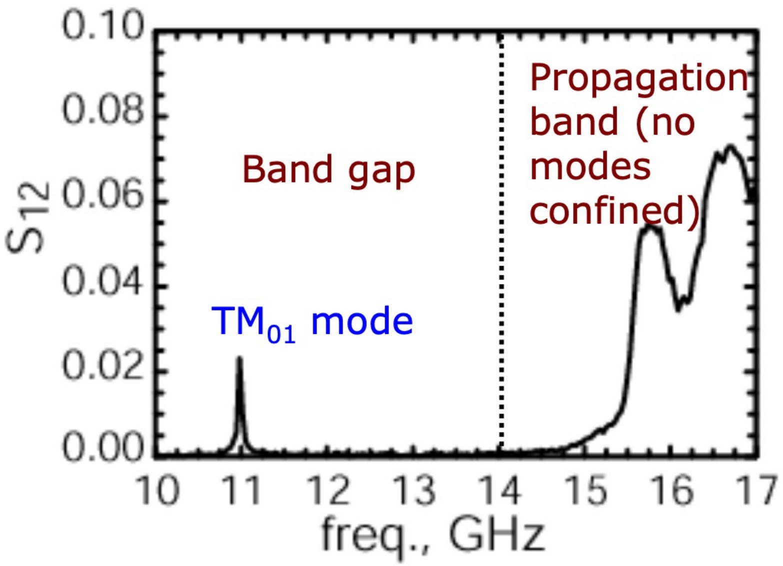


- The frequency ω lies inside the band gap and the wave of this frequency can not propagate through the PBG structure and will be confined around the defect.
- The PBG structure with a defect then acts like a resonator.
- The electric field pattern in the mode closely resembles the field pattern in a TM_{010} mode of a cylindrical pillbox cavity.

First PBG tests

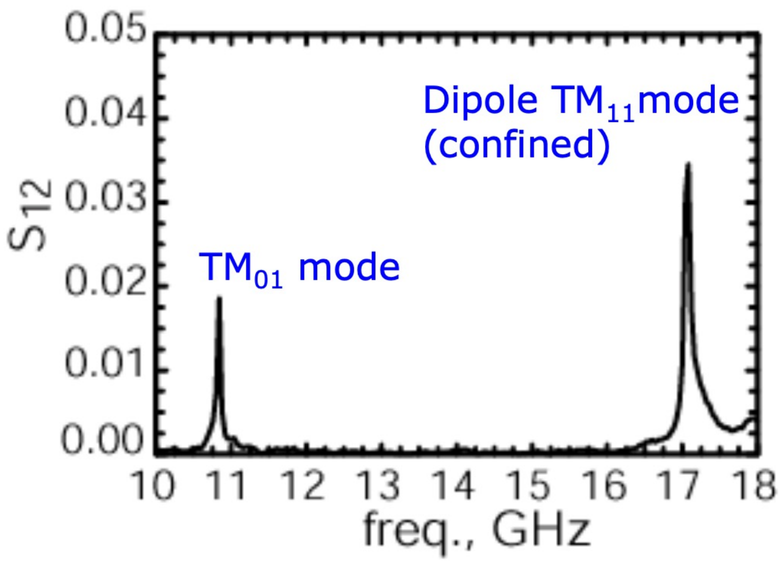
MIT 11 GHz PBG resonator





a/b=0.15

No confined wakefield modes
Good for accelerators.

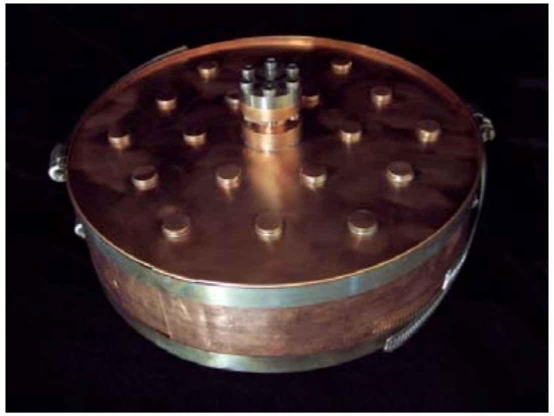


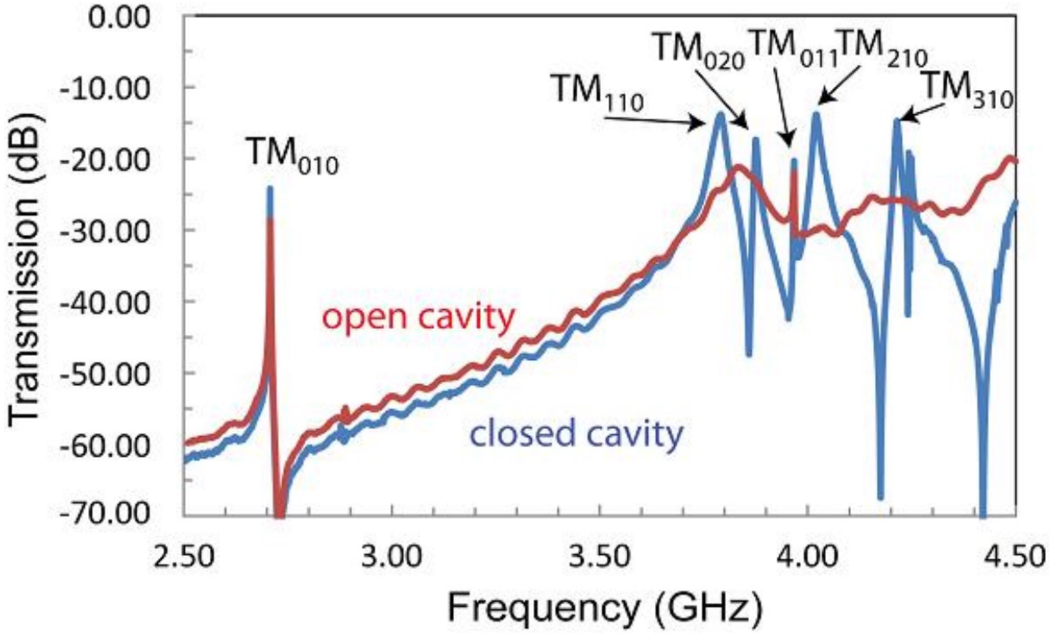
a/b=0.30

Confined TM_{11} mode.
Bad for accelerators.

First PBG tests

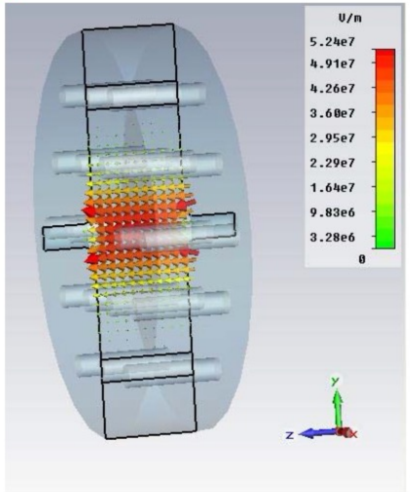
2.7 GHz open-wall PBG resonator



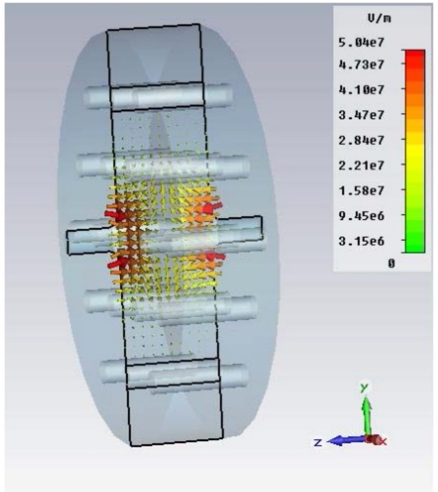


- HOMs were measured with two probes inserted off-axis.
- Confirmed that HOMs radiate out of the open structure except for TM_{011} mode.

TM_{010} mode, 2.708 GHz



TM_{011} mode, 3.970 GHz



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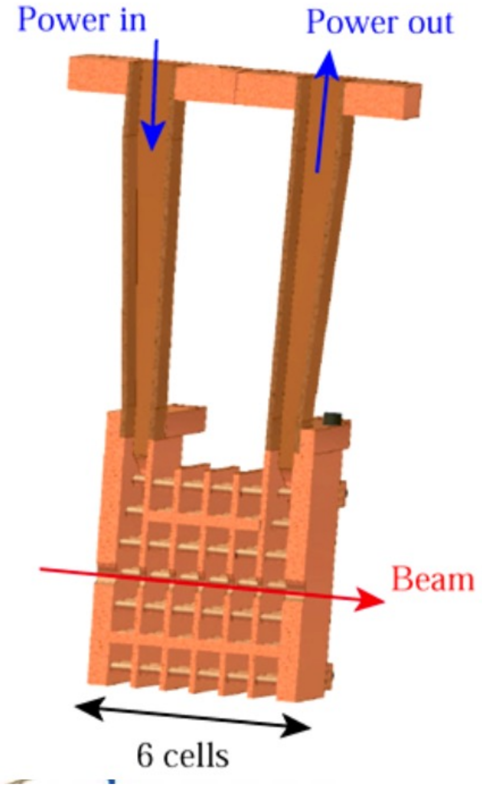
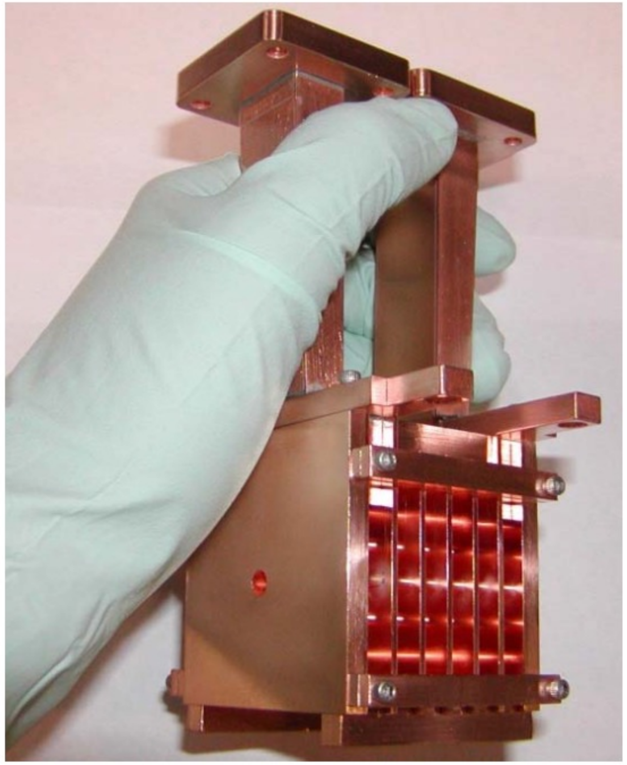
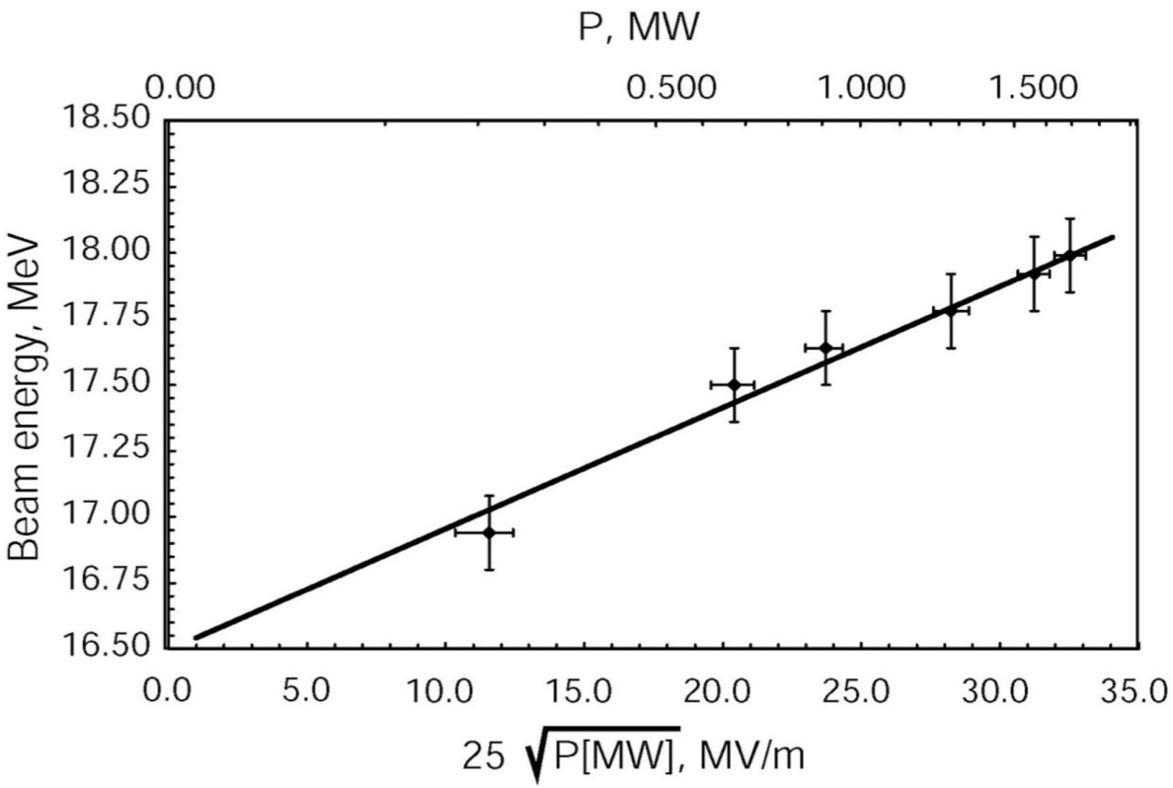
First PBG accelerator

MIT 17 GHz accelerator

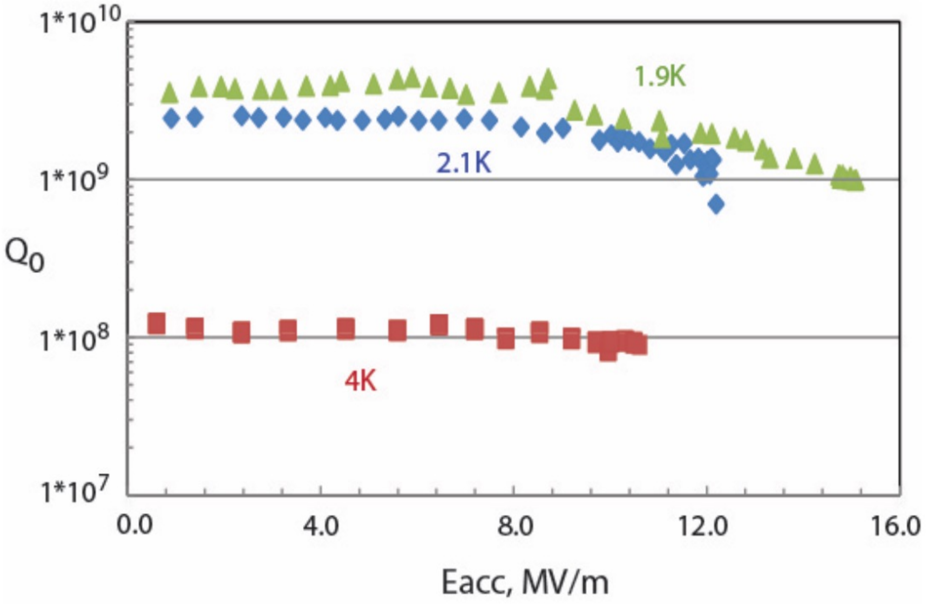




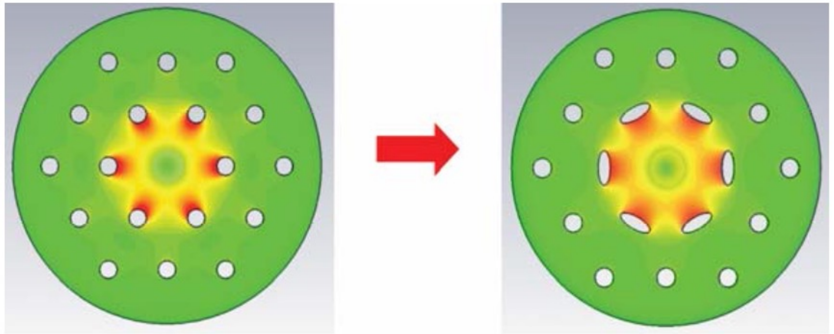


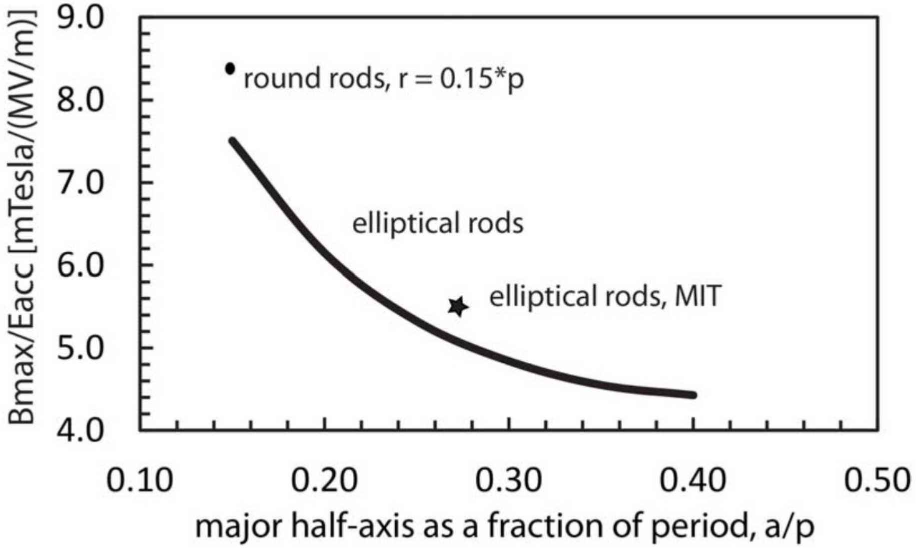
- Beam energy increased with power as $P^{1/2}$, as expected.
- First successful PBG accelerator demonstration.

SRF PBG cavities



- Changing the shape of the rods in a PBG structure to elliptical reduces surface magnetic fields and improves high gradient performance.
- Shape of the elliptical rods must be optimized.





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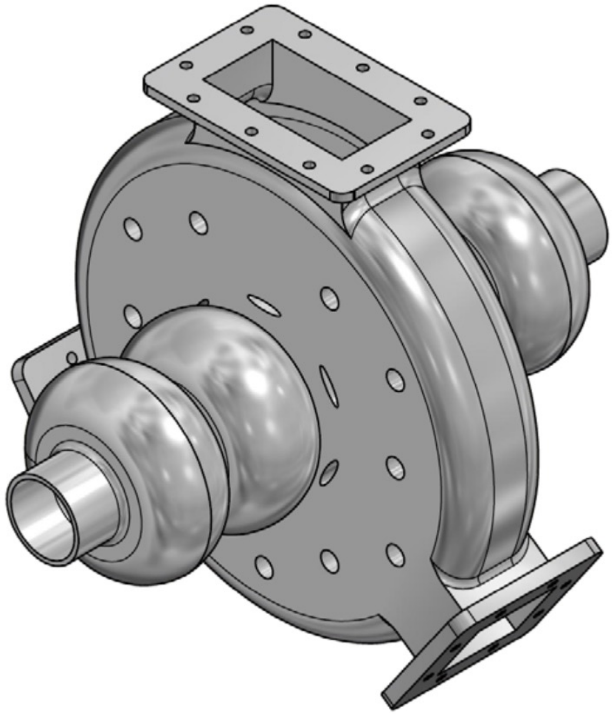
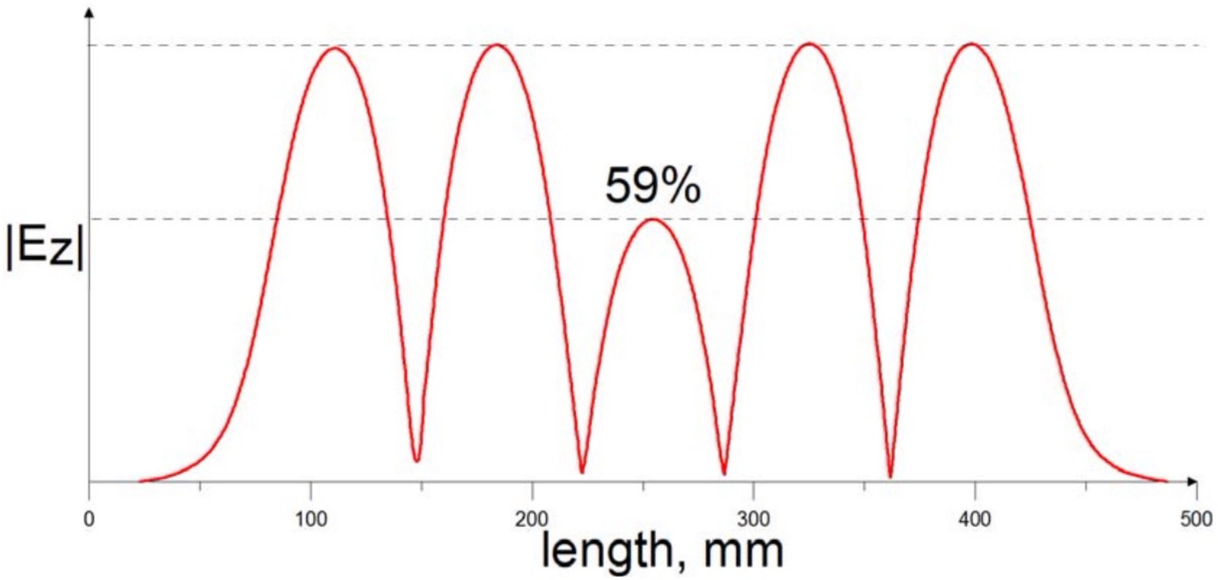
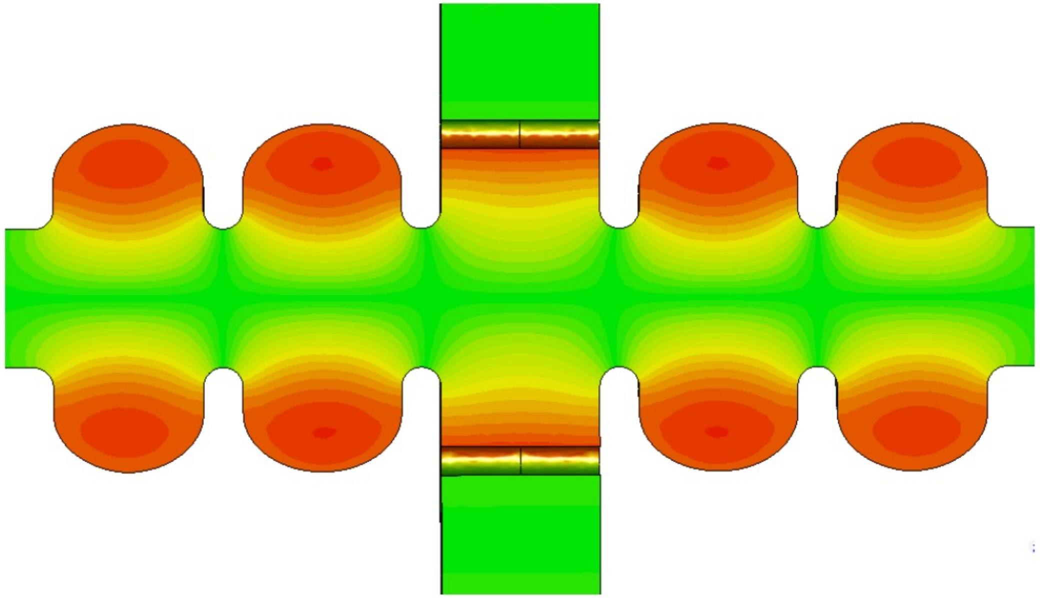
SRF PBG cavities

A 5-cell structure was designed with a PBG coupler cell in the middle.







- The peak magnetic field on the niobium surfaces in PBG cell can set the operational limits for the whole structure.
- Thus the cavity is tuned such that the peak magnetic fields are equal in all cells. This reduces the on-axis electric fields in the PBG cell.

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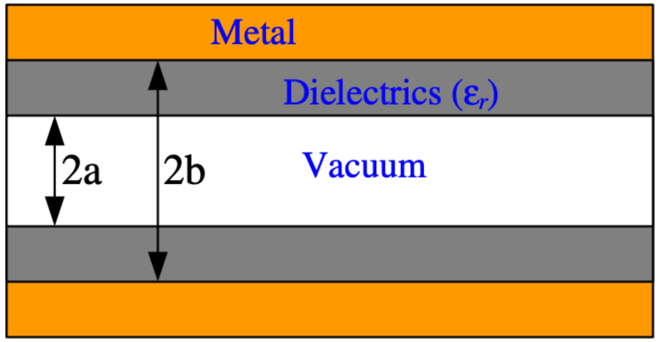
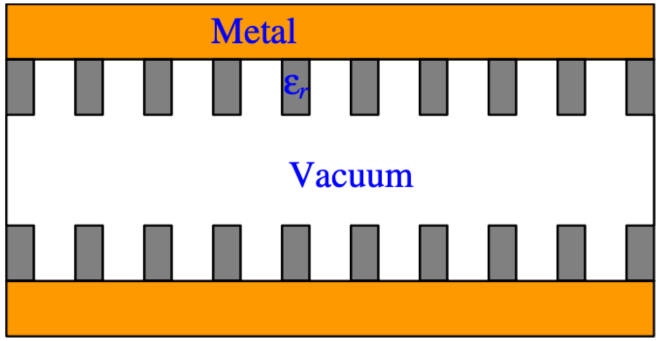
Dielectric Wakefield Accelerators

Dielectric Wakefield Accelerators (DWA) or Dielectric-Loaded Accelerators (DLA)

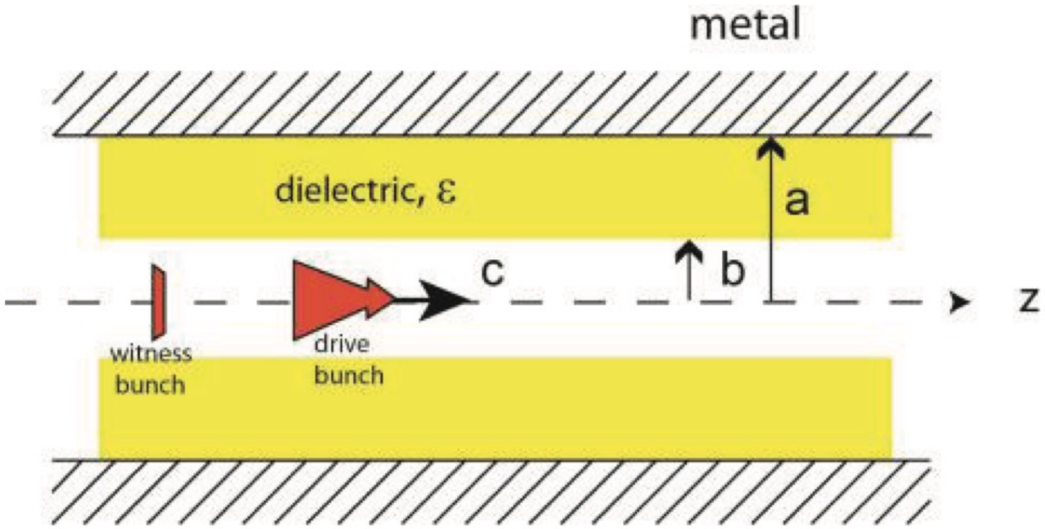
Dielectric-lined circular waveguide



Dielectric-disk loaded circular waveguide



- Dielectric wakefield accelerators are typically formed by one or several **coaxial dielectric layers surrounded by metal**.
- The **dielectric constant and the inner radius of the dielectric tube are chosen to adjust the phase velocity** of the fundamental monopole mode (TM_{01}) **to approximately the speed of light** so that the mode is effectively excited by the beam passing in the central vacuum channel.
- The fields generated by a leading, **high-charge drive bunch is used to accelerate a trailing, low-charge bunch**.
- Wakefields in dielectric structures may reach gradients on the order of 10 GV/m with 100 MV/m demonstrated in multiple experiments.



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Dielectric–lined circular waveguide

The general solution using decomposition of the field in terms of $e^{in\phi}$, where ϕ is the azimuthal coordinate

$$E_z = \begin{cases} B_1 J_n(k_1 r)\cos(n\phi)e^{j(\omega t-\beta z)}, & 0 \le r < a, \\ B_2\left[J_n(k_2 r) - \frac{J_n(k_2 b)}{Y_n(k_2 b)}Y_n(k_2 r)\right] \\ \quad \cos(n\phi)e^{j(\omega t-\beta z)}, & a \le r \le b, \end{cases}$$

$$E_\phi = \begin{cases} \left[\frac{j\omega\mu_0}{k_1}A_1 J_n'(k_1 r) + \frac{j\beta n}{k_1^2 r}B_1 J_n(k_1 r)\right] \\ \quad \sin(n\phi)e^{j(\omega t-\beta z)}, \quad 0 \le r < a, \\ \left[\frac{j\omega\mu_0}{k_2}A_2 G_{nn}'(k_2 r) + \frac{j\beta n}{k_2^2 r}B_2 F_{nn}(k_2 r)\right] \\ \quad \sin(n\phi)e^{j(\omega t-\beta z)}, \quad a < r \le b, \end{cases}$$

$$H_\phi = \begin{cases} \left[-\frac{j\omega\varepsilon_0}{k_1}B_1 J_n'(k_1 r) - \frac{j\beta n}{k_1^2 r}A_1 J_n(k_1 r)\right] \\ \quad \cos(n\phi)e^{j(\omega t-\beta z)}, \quad 0 \le r < a, \\ \left[-\frac{j\omega\varepsilon_r\varepsilon_0}{k_2}B_2 F_{nn}'(k_2 r) - \frac{j\beta n}{k_2^2 r}A_2 G_{nn}(k_2 r)\right] \\ \quad \cos(n\phi)e^{j(\omega t-\beta z)}, \quad a < r \le b, \end{cases}$$

$$H_z = \begin{cases} A_1 J_n(k_1 r)\sin(n\phi)e^{j(\omega t-\beta z)}, & 0 \le r < a, \\ A_2\left[J_n(k_2 r) - \frac{J_n'(k_2 b)}{Y_n'(k_2 b)}Y_n(k_2 r)\right] \\ \quad \sin(n\phi)e^{j(\omega t-\beta z)}, & a \le r \le b, \end{cases}$$

$$E_r = \begin{cases} \left[-\frac{j\omega\mu_0 n}{k_1^2 r}A_1 J_n(k_1 r) - \frac{j\beta}{k_1}B_1 J_n'(k_1 r)\right] \\ \quad \cos(n\phi)e^{j(\omega t-\beta z)}, \quad 0 \le r < a, \\ \left[-\frac{j\omega\mu_0 n}{k_2^2 r}A_2 G_{nn}(k_2 r) - \frac{j\beta}{k_2}B_2 F_{nn}'(k_2 r)\right] \\ \quad \cos(n\phi)e^{j(\omega t-\beta z)}, \quad a < r \le b, \end{cases}$$

$$H_r = \begin{cases} \left[-\frac{j\omega\varepsilon_0 n}{k_1^2 r}B_1 J_n(k_1 r) - \frac{j\beta}{k_1}A_1 J_n'(k_1 r)\right] \\ \quad \sin(n\phi)e^{j(\omega t-\beta z)}, \quad 0 \le r < a, \\ \left[-\frac{j\omega\varepsilon_r\varepsilon_0 n}{k_2^2 r}B_2 F_{nn}(k_2 r) - \frac{j\beta}{k_2}A_2 G_{nn}'(k_2 r)\right] \\ \quad \sin(n\phi)e^{j(\omega t-\beta z)}, \quad a < r \le b. \end{cases}$$

B_1, B_2, A_1 and A_2 are the field amplitudes in region I (vacuum) and region II (dielectric); v_p is the phase velocity of the wave propagating inside the tube, k_1 and k_2 are the cutoff wave numbers in each region and is the wave propagation constant.

$$k_1 = \omega\sqrt{\frac{1}{c^2} - \frac{1}{v_p^2}},$$

$$k_2 = \omega\sqrt{\frac{\varepsilon_r}{c^2} - \frac{1}{v_p^2}},$$

$$\beta^2 = k_0^2 - k_1^2 = \varepsilon_r k_0^2 - k_2^2,$$

$$k_0^2 = \omega^2\mu_0\varepsilon_0.$$

$$F_{nn}(k_2 r) = J_n(k_2 r) - \frac{J_n(k_2 b)}{Y_n(k_2 b)}Y_n(k_2 r),$$

$$G_{nn}(k_2 r) = J_n(k_2 r) - \frac{J_n'(k_2 b)}{Y_n'(k_2 b)}Y_n(k_2 r),$$

$$F_{nn}'(k_2 r) = J_n'(k_2 r) - \frac{J_n(k_2 b)}{Y_n(k_2 b)}Y_n'(k_2 r),$$

$$G_{nn}'(k_2 r) = J_n'(k_2 r) - \frac{J_n'(k_2 b)}{Y_n'(k_2 b)}Y_n'(k_2 r).$$

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Dielectric–lined circular waveguide

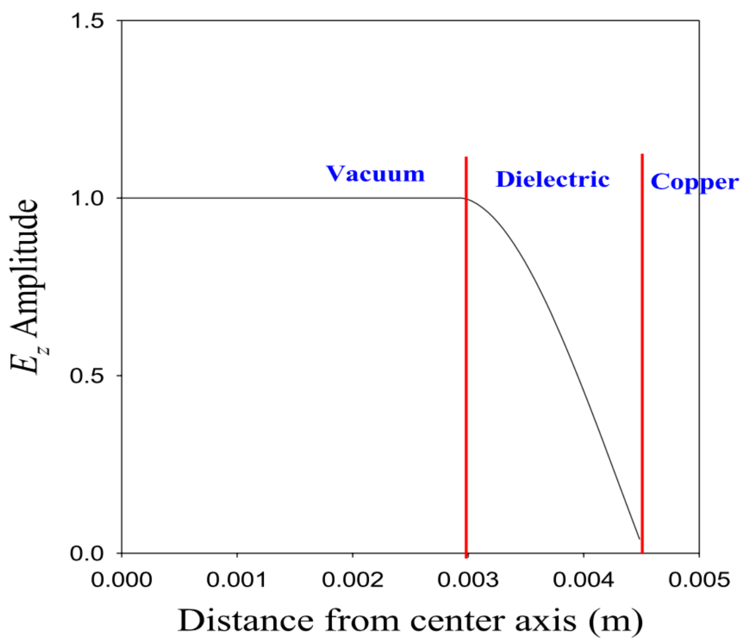
Using the boundary condition: at $r = a$ $\quad E_z, E_\phi, H_\phi, H_z$ are continuous.

For particle acceleration we use TM_{01}

$$E_z = \begin{cases} B_1 J_0(k_1 r) e^{j(\omega t - \beta z)}, & 0 \leq r < a, \\ B_2 F_{00}(k_2 r) e^{j(\omega t - \beta z)}, & a \leq r \leq b, \end{cases}$$

$$E_r = \begin{cases} -\frac{j\beta}{k_1} B_1 J_0'(k_1 r) e^{j(\omega t - \beta z)}, & 0 \leq r < a, \\ -\frac{j\beta}{k_2} B_2 F_{00}'(k_2 r) e^{j(\omega t - \beta z)}, & a < r \leq b, \end{cases}$$

$$H_\phi = \begin{cases} -\frac{j\omega\varepsilon_0}{k_1} B_1 J_0'(k_1 r) e^{j(\omega t - \beta z)}, & 0 \leq r < a, \\ -\frac{j\omega\varepsilon_r\varepsilon_0}{k_2} B_2 F_{00}'(k_2 r) e^{j(\omega t - \beta z)}, & a < r \leq b. \end{cases}$$



Important observations:

- When phase velocity of the TM_{01} wave is equal to the speed of light, the **longitudinal electric field is independent of the radial coordinate in the vacuum region.**
- This implies via the Panofsky-Wenzel theorem that there are no focusing/defocusing forces for a relativistic particle traveling inside the vacuum region.
- This is critical for emittance preservation!

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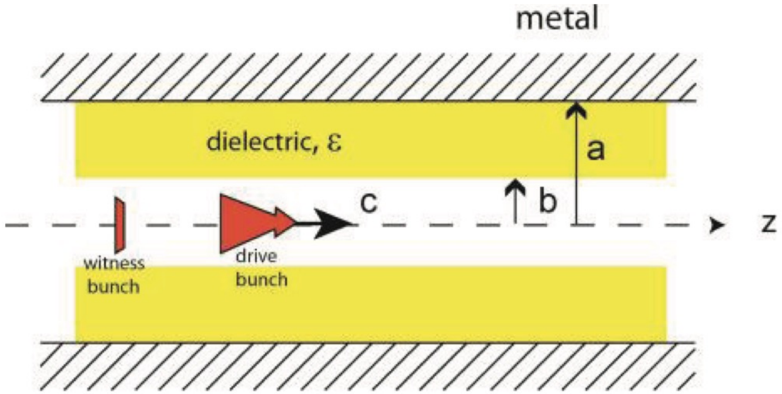
Dielectric–lined circular waveguide

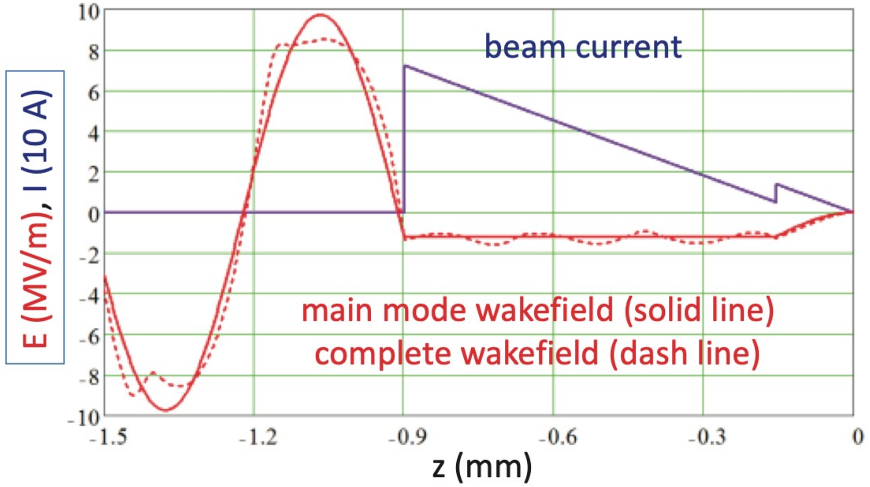
DWA are characterized by transformer ratio (TR) – ratio of the accelerating field acting on the witness bunch to the maximum deceleration field inside of the drive bunch.



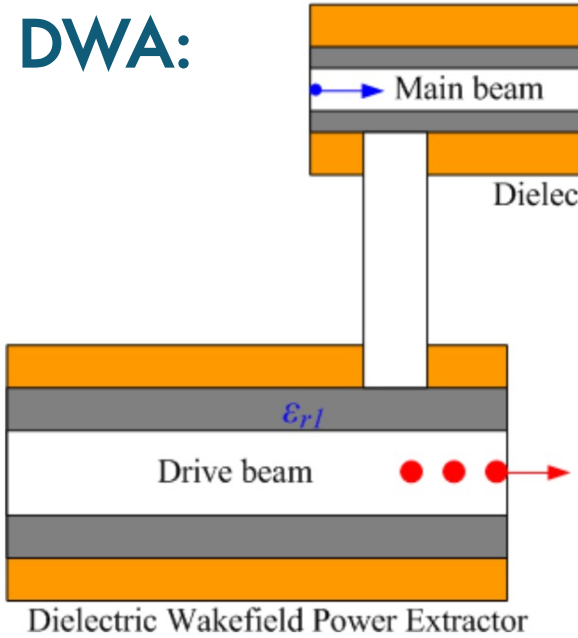
One can enhance TR by choosing the profile of the drive bunch:

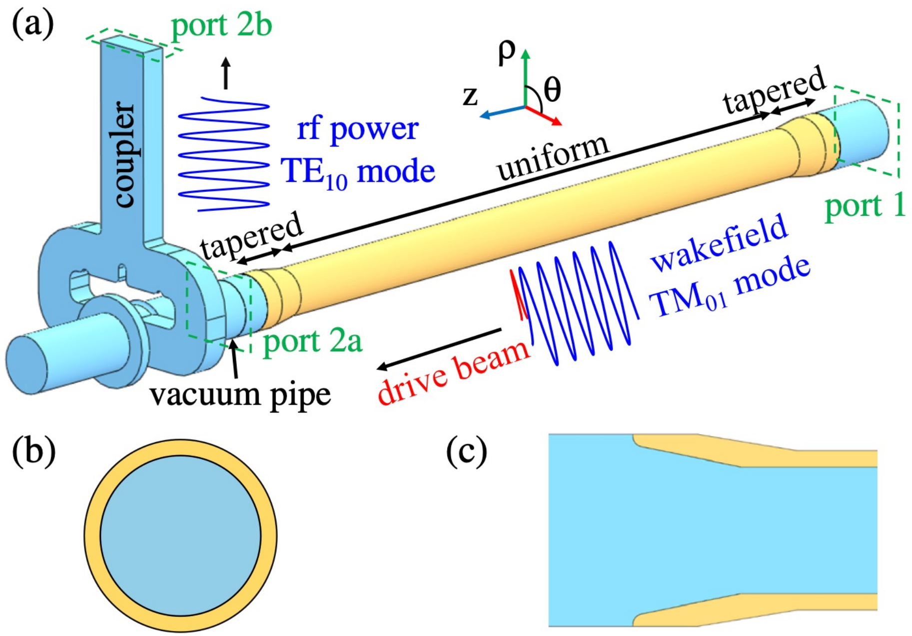
$$I(t) = \begin{cases} I_0 f t, \ 0 \le t < \frac{1}{4f} \\ I_0 \left(f t - \frac{1}{2\pi} \right), \ \frac{1}{4f} \le t < T \end{cases}$$

f - frequency of a single mode accelerator, T total bunch duration.



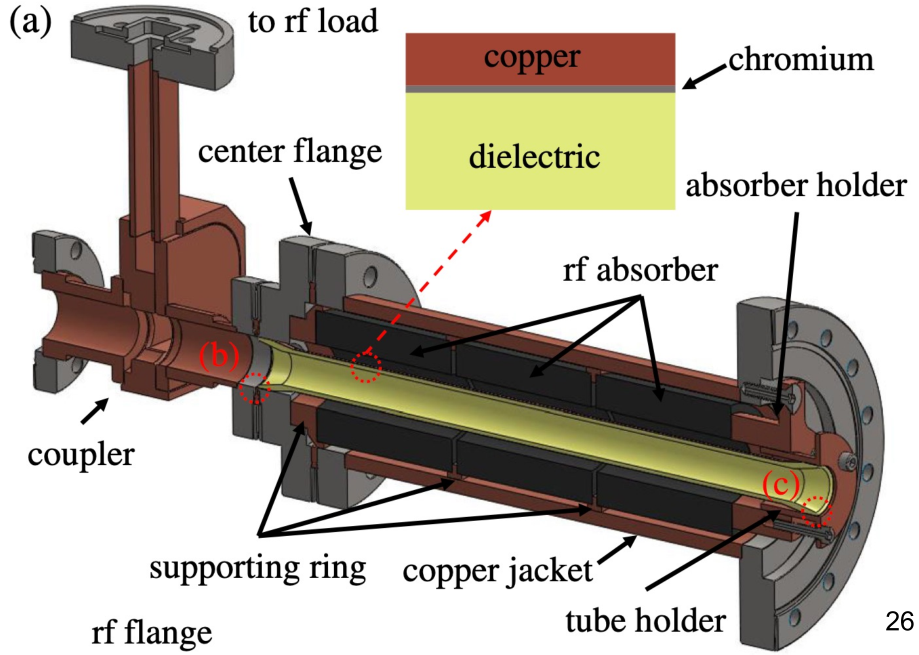
Two-beam DWA:





- The drive beam energy is extracted into an RF pulse, which is then used to accelerate the main beam in a separate accelerating structure.

- In the present room temperature, high gradient accelerator designs, a gradient of ~100 MV/m with a pulse length in the range of 200–400ns is usually the limit of normal operations. If the pulse length is reduced to ~20ns, a 200–300MV/m operational accelerating gradient may be achievable.



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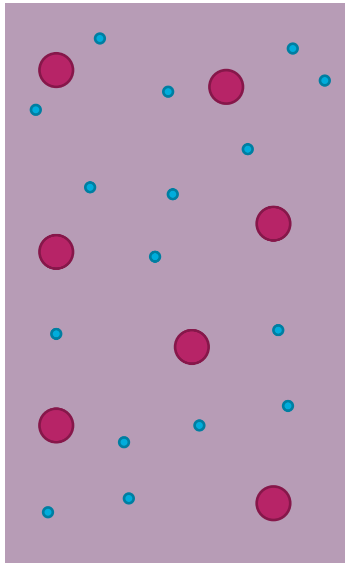
Plasma Wakefield Accelerators

Plasma accelerators: basics

Plasma – collection of electrons and ions that are not bound to each other

How does plasma respond to an external disturbance (EM field or particle beam)?

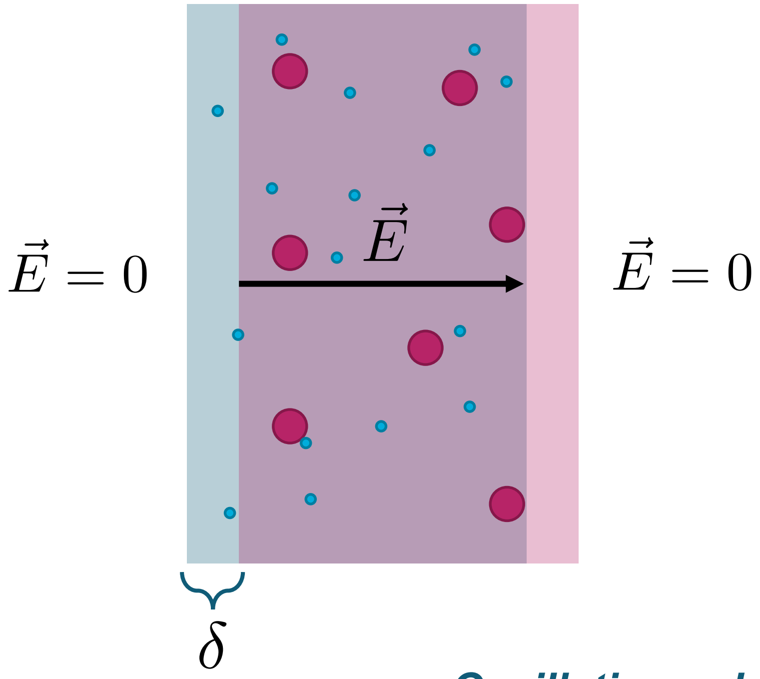


Plasma accelerators: basics

Plasma – collection of electrons and ions that are not bound to each other

How does plasma respond to an external disturbance (EM field or particle beam)?

Consider a quasi-neutral plasma slab in which an electron layer is displaced from its initial position by a distance δ:



Using Gauss Law: $\oint \vec{E} d\vec{A} = 4\pi Q = 4\pi \oint \sigma dA \Rightarrow E = 4\pi\sigma = 4\pi e n_0 \delta$

Equation of motion for electrons: $m_e \ddot{x}_e = -eE = -4\pi e^2 n_0 \delta$

Equation of motion for ions: $M_i \ddot{x}_i = eZ_i E = 4\pi Z_i e^2 n_0 \delta$

Respective frequencies:

$$\omega_{pe}^2 = \frac{4\pi e^2 n_0}{m_e}$$

$$\omega_{pi}^2 = \frac{4\pi Z_i e^2 n_0}{M_i}$$

Oscillating solution: $x_e = a_e \cos(\omega_p t);\ x_i = a_i \cos(\omega_p t) \to \omega_p^4 = \omega_p^2 \left(\omega_{pe}^2 + \omega_{pi}^2\right)$

$$\omega_p^2 = \omega_{pe}^2 + \omega_{pi}^2 \to \frac{a_i}{a_e} = -\frac{\omega_{pi}^2}{\omega_{pe}^2} = -\frac{Z_i m_e}{M_i} = \frac{Z_i m_e}{A_i M_p} \ll 1$$

Plasma frequency: $\omega_p \approx \sqrt{\frac{4\pi n_0 e^2}{m_e}} \equiv c\sqrt{4\pi n_0 r_e} \approx 5.6 \times 10^4\ [\mathrm{Hz}]\ \sqrt{n_0[\mathrm{cm}^{-3}]}$

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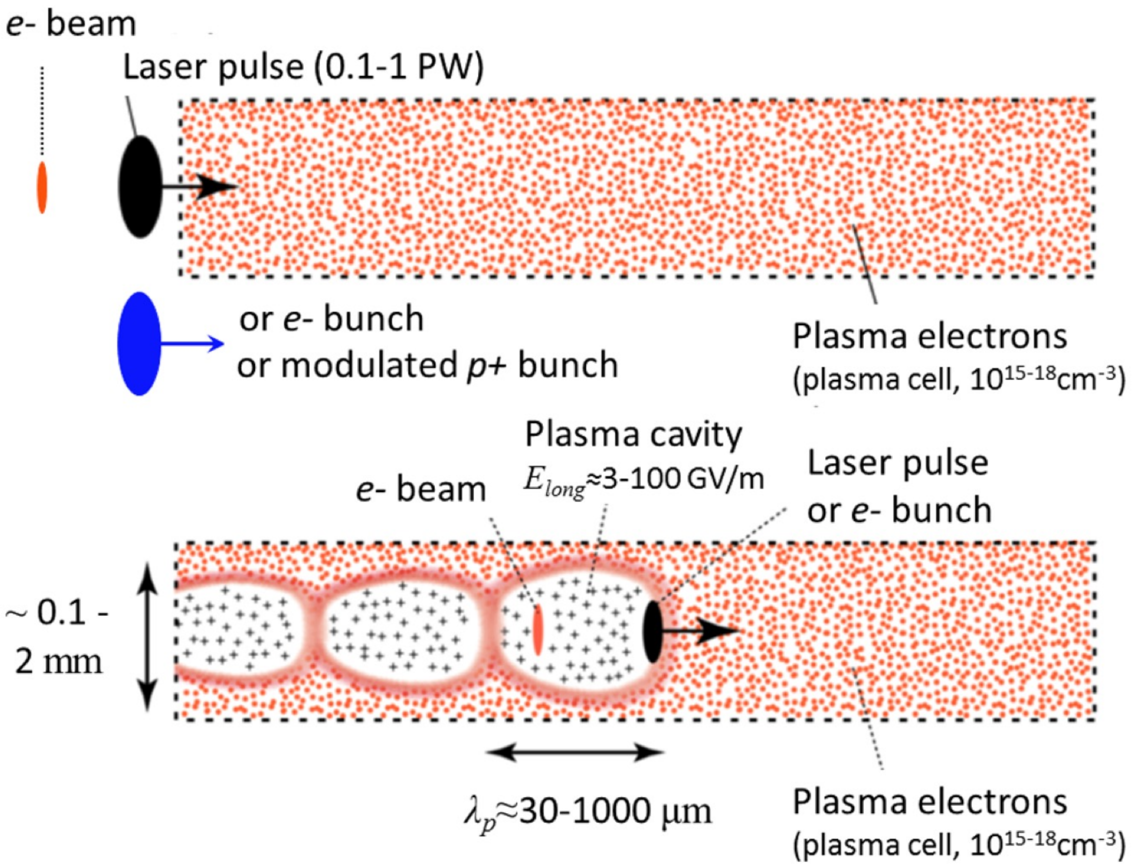
Plasma accelerators: basics



1. The laser **pulse enters the plasma and transversely accelerates plasma electrons** (ponderomotive force as transverse driving force).
2. The plasma ions move a negligible amount and a **positively charged ion channel is formed** along the laser path.
3. After the passage of the laser pulse, the **plasma electrons rush back** in, attracted by the transverse restoring force of the positively charged ion channel, **pass the center of the ion channel, rush back out and are attracted back** by the ion channel.
4. A space-charge-driven oscillation is formed, leaving **alternating regions of negative and positive net charge with strong induced longitudinal fields** behind the laser pulse (plasma wakefields).
5. If a **short test bunch** of charged particles, e.g., electrons, is **placed behind** the laser pulse at a proper distance, then it **will be accelerated** with high gradient.

Ionized plasmas can sustain waves with electric fields on the order of the nonrelativistic wave breaking field

$$E_0 = cm_e\omega_p/e \rightarrow E_0[\mathrm{V/cm}] \simeq 0.96 n_0^{1/2}[\mathrm{cm}^{-3}]$$

For example, $n_0 = 10^{18}\mathrm{cm}^{-3}$ results in $E_0 \simeq 100\mathrm{GV/m}$

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LWFA: linear regime

- Consider a laser pulse propagating in z-direction.
- General plasma oscillations driven by an intense beam can be described using Maxwell's equations in the Lorentz gauge and the equation of motion for an arbitrary number of laminar fluids:

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)\begin{bmatrix}\vec{A}\\ \phi\end{bmatrix} = 4\pi\begin{bmatrix}\vec{J}/c\\ \rho\end{bmatrix}$$

$$\frac{1}{c}\frac{\partial \phi}{\partial t} - \nabla\cdot\vec{A} = 0$$

$$\left(\frac{\partial}{\partial t} + \vec{v}_i\cdot\vec{\nabla}\right)\vec{P}_i = q_i\left[\vec{E} + \frac{\vec{v}_i}{c}\times\vec{B}\right]$$

$\rho,\ \vec{J}$ satisfy the charge conservation equation

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot\vec{J} = 0$$

One can assume the following approximations:

1. Upstream of the driver fluid can be treated as cold.
2. Quasistatic/frozen field: driver evolves on a time scale much longer than plasma response.

- Let us use $(\xi = z - v_\phi t, x, y; s = z)$, where

$\xi = z - v_\phi t$ is the distance from the front of the driver

$s = z$ is the distance the driver has propagated into the plasma

For a a fixed driver shape, the wake can be calculated. The wake only changes if the driver shape changes. The driver shape assumed to be changing slowly.

Quasistatic approximation: $\partial_\mathrm{s} \ll \partial_\xi$

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LWFA: linear regime

- For LWFA there are 3 time scales:

    1. high frequency laser
    2. plasma wakefield oscillation
    3. laser envelope evolution

- When calculating wakefield only the smooth motion is relevant—can **average out the high frequency of the laser**.

- LWFA theory utilizes normalized units

Quantity	Conversion
Time	$t' = t \times \omega_p$
Frequency	$\omega' = \omega/\omega_p$
Position	$\mathbf{x}' = \frac{\omega_p}{c}\mathbf{x}$
Momenta	$\mathbf{u}' = \frac{\mathbf{p}}{m_{sp}c} = \frac{\gamma\mathbf{v}}{c} = \frac{\mathbf{u}}{c}$
Electric field	$\mathbf{E}' = e\frac{c/\omega_p}{m_e c^2}\mathbf{E}$
Magnetic field	$\mathbf{B}' = e\frac{c/\omega_p}{m_e c^2}\mathbf{B}$

Then Maxwell's equations become:

$$-\nabla_\perp^2 \begin{bmatrix} \vec{A} \\ \phi \end{bmatrix} = \begin{bmatrix} \vec{J} \\ \rho \end{bmatrix}$$

$$\vec{\nabla}_\perp \cdot \vec{A}_\perp = -\frac{\partial \psi}{\partial \xi}$$

$$\psi = \phi - A_z, \;\; \vec{\nabla}_\perp = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y}, \;\; \vec{A}_\perp = \hat{x}A_x + \hat{y}A_y$$

Using the suggested approximations, we obtain:

$$E_z = \frac{\partial \psi}{\partial \xi} \qquad \vec{E}_\perp = -\nabla_\perp \phi - \frac{\partial \vec{A}_\perp}{\partial \xi}$$

$$B_z = (\vec{\nabla}_\perp \times \vec{A}_\perp) \cdot \hat{z} \qquad \vec{B}_\perp = \vec{\nabla}_\perp \times (A_z \hat{z}) + \vec{\nabla}_z \times \vec{A}_\perp$$

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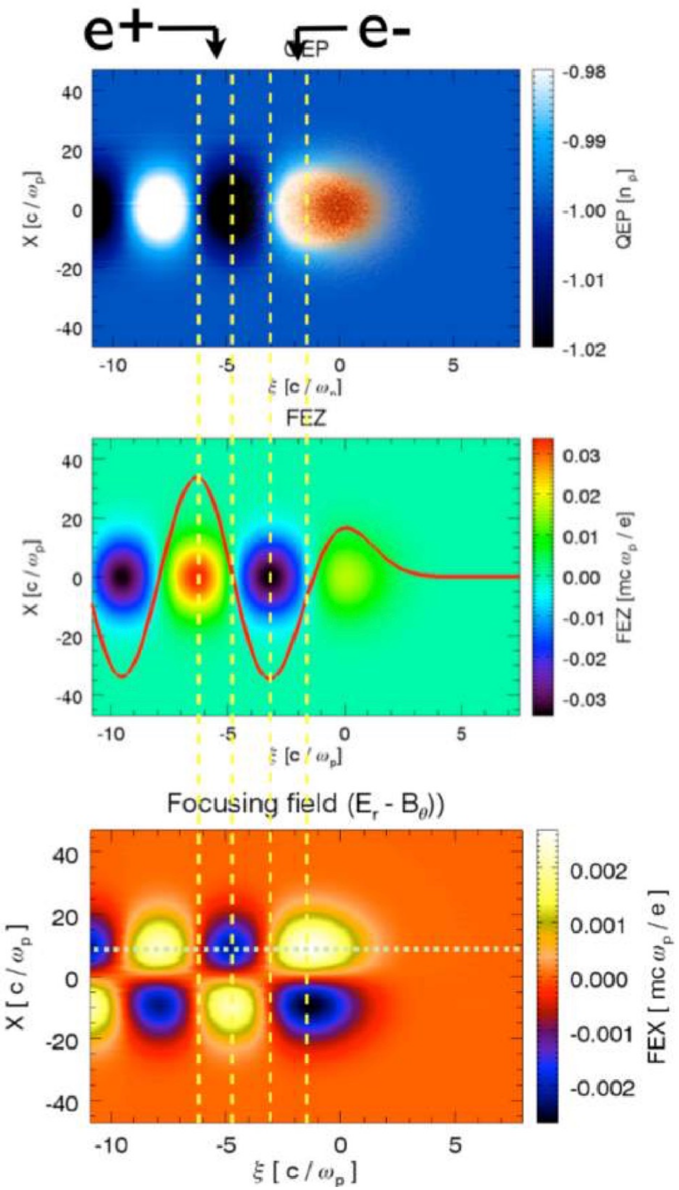
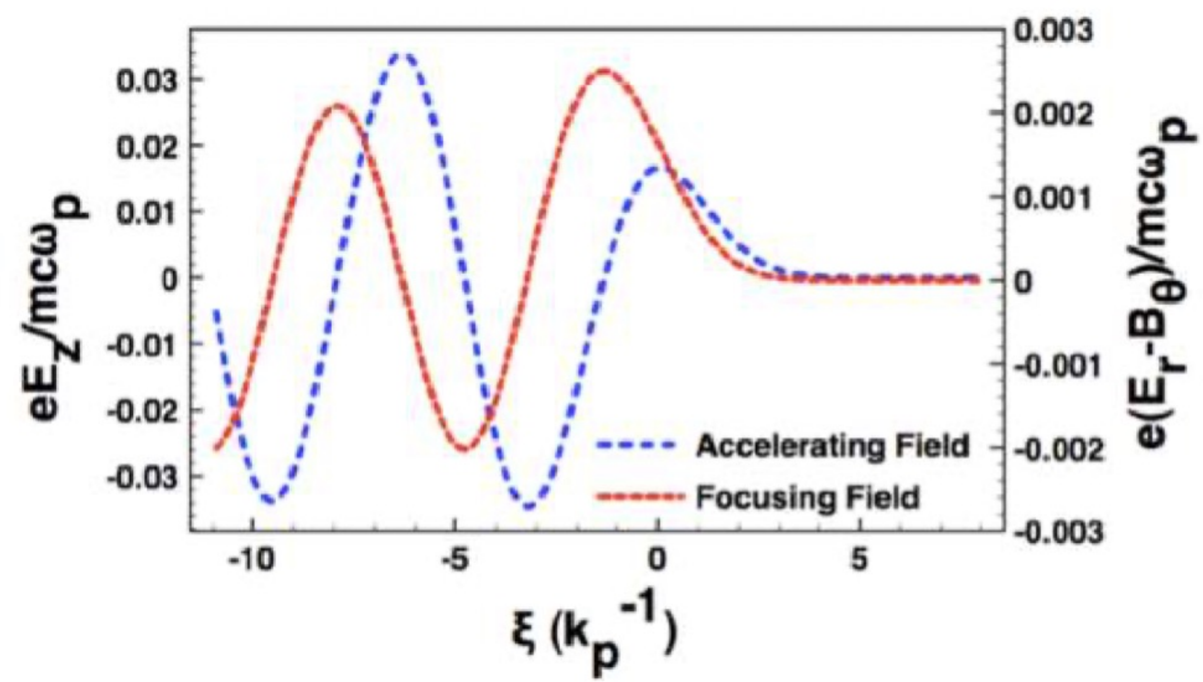
LWFA: linear regime



- **Fields can be calculated exactly** using Green's functions.
- The accelerating and focusing fields are $\pi/2$ out of phase.
- Only half of the accelerating phase can be used.
- Formalism for electrons and positrons is the same, just requires the shift of the accelerating and focusing phase.



Keep in mind:

Dephasing

- Consider an electron beam accelerated along z-axis by a linear wave $E_{\mathrm{z}} = E_{\mathrm{max}} \sin[\omega_{\mathrm{p}}(z/v_{\mathrm{p}} - t)]$
- As the **electron** is accelerated, its velocity will increase and **approach the speed of light** $v_{\mathrm{z}} \to c$
- If the phase velocity of the plasma wave is constant with $v_{\mathrm{p}} < c$, the **electrons** will eventually **outrun the plasma wave** and **move into a decelerating phase**.
- The **dephasing length** L_{d} is defined as the **length the electron must travel before it phase slips by one-half of a period** with respect to the plasma wave

$$L_{\mathrm{d}} = \frac{\lambda_{\mathrm{p}}}{2\left(1 - \frac{v_{\mathrm{p}}}{c}\right)} \simeq \gamma_{\mathrm{p}}^2 \lambda_{\mathrm{p}}$$

- The maximum energy gain after a dephasing length:

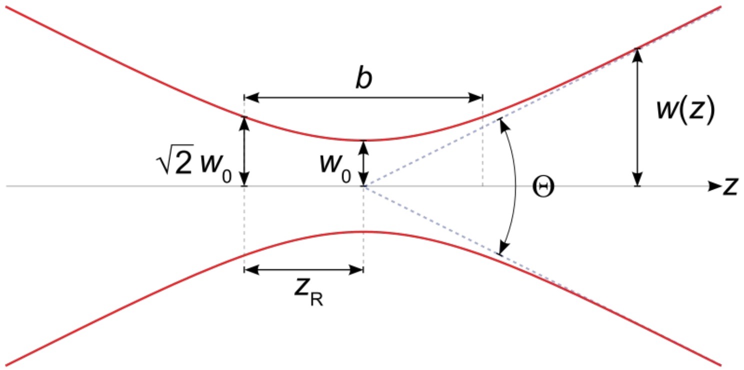
$$W_{\mathrm{max}} \simeq eE_{\mathrm{max}} L_{\mathrm{d}} \simeq 2\pi\gamma_{\mathrm{p}}^2 \left(\frac{E_{\mathrm{max}}}{E_0}\right) m_{\mathrm{e}} c^2$$

- *Possible solution:* need to increase plasma wavelength – adjust plasma density!

Keep in mind:

Diffraction

- **Rayleigh length** is the distance along the propagation direction of a beam from the waist to the place where the area of the cross section is doubled
- For a Gaussian beam with wave number $k = 2\pi/\lambda$, the Rayleigh length is given by: $z_\mathrm{R} = \dfrac{\pi w_0^2}{\lambda}$
- Beam envelope can be described as follows: $w(z) = w_0 \sqrt{1 + \left(\dfrac{z}{z_\mathrm{R}}\right)^2}$
- Diffraction of the laser is determined by the Rayleigh length
- *Possible solution:* the laser needs to be guided



Basic idea of self-guiding:

- When passing through the plasma, the laser expels electrons changing the plasma density.
- Change in the density causes change in the index of refraction (increase along the axis).
- Gradient of the index of refraction causes the phase velocity of the wave front to increase with the radial position.
- Wave front curves inward resulting in focusing.

Keep in mind:

Depletion

- **Loss of of laser power during the propagation**

- As the laser pulse gives energy to the wake, it is depleted and eventually no longer able to excite a wake.

- The distance over which the energy depletion occurs is called the pump depletion length L_{pd}

Dephasing: since the electron velocity is higher than phase velocity of a wake, electrons outrun and move into a decelerating mode.

Diffraction: determined by the Rayleigh length

Depletion: loss of laser power during the propagation

Breakdown of linear theory:

When the driver gets strong enough, the linear wave structure is no longer sustained!

For ***laser-driven*** wakes:

- Usually characterized by a normalized vector potential $a_0 = \frac{eA_0}{mc^2} = 0.85 \times 10^{-9} \sqrt{I[\mathrm{W/cm^2}]\lambda^2[\mu\mathrm{m}]}$
- Linear theory works when $a_0 \ll 1$ and $c\tau \sim \lambda_\mathrm{p}$

For ***beam-driven*** wakes:

- Usually characterized by a normalized charge per unit length $\Lambda = \frac{n_\mathrm{b}}{n_\mathrm{p}} k_\mathrm{p}^2 \sigma_\mathrm{r}^2$
- Linear theory works when $\Lambda < 1$ and $\frac{n_\mathrm{b}}{n_\mathrm{p}} < 10$ for electron beams.

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Breakdown of linear theory

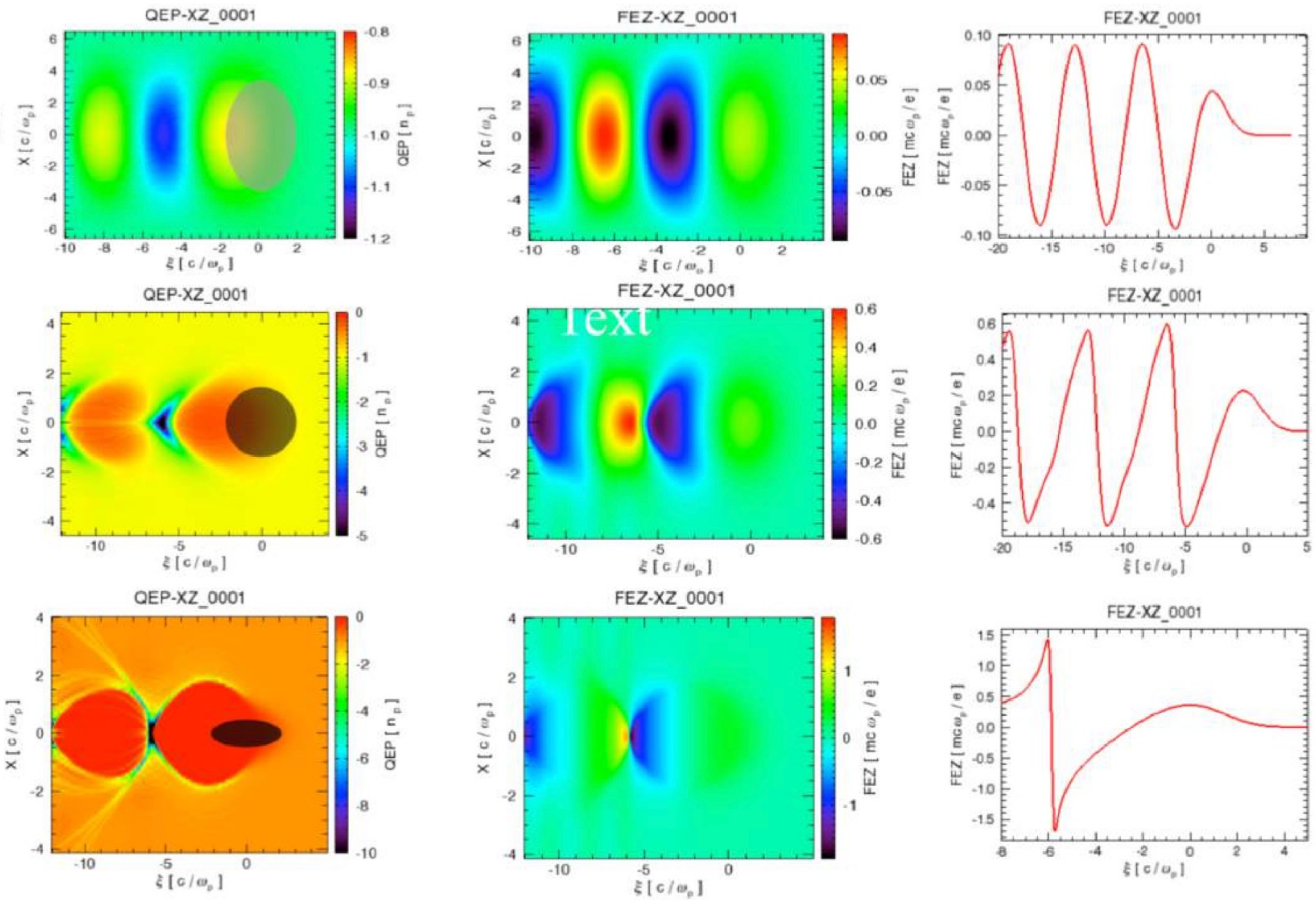
$k_p R_b \approx 2\sqrt{\Lambda}$ for $k_p \sigma_z \approx 1$

ne psi Ez

$k_p\sigma_r = 2.8$

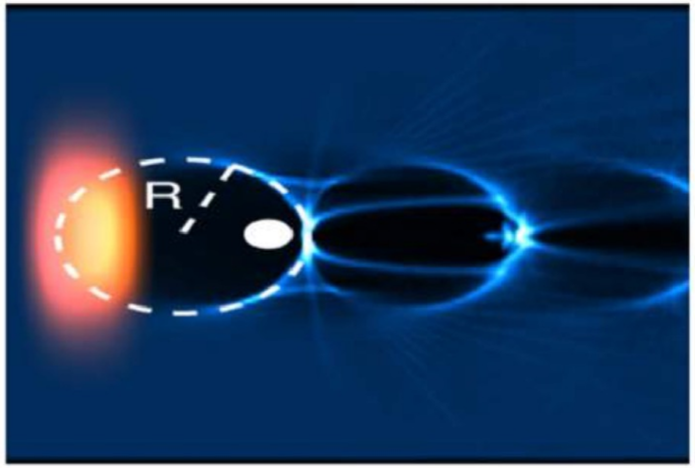
$k_p\sigma_r = 1.0$

$k_p\sigma_r = 0.38$



Beam-driven "bubble"

Laser-driven "bubble"



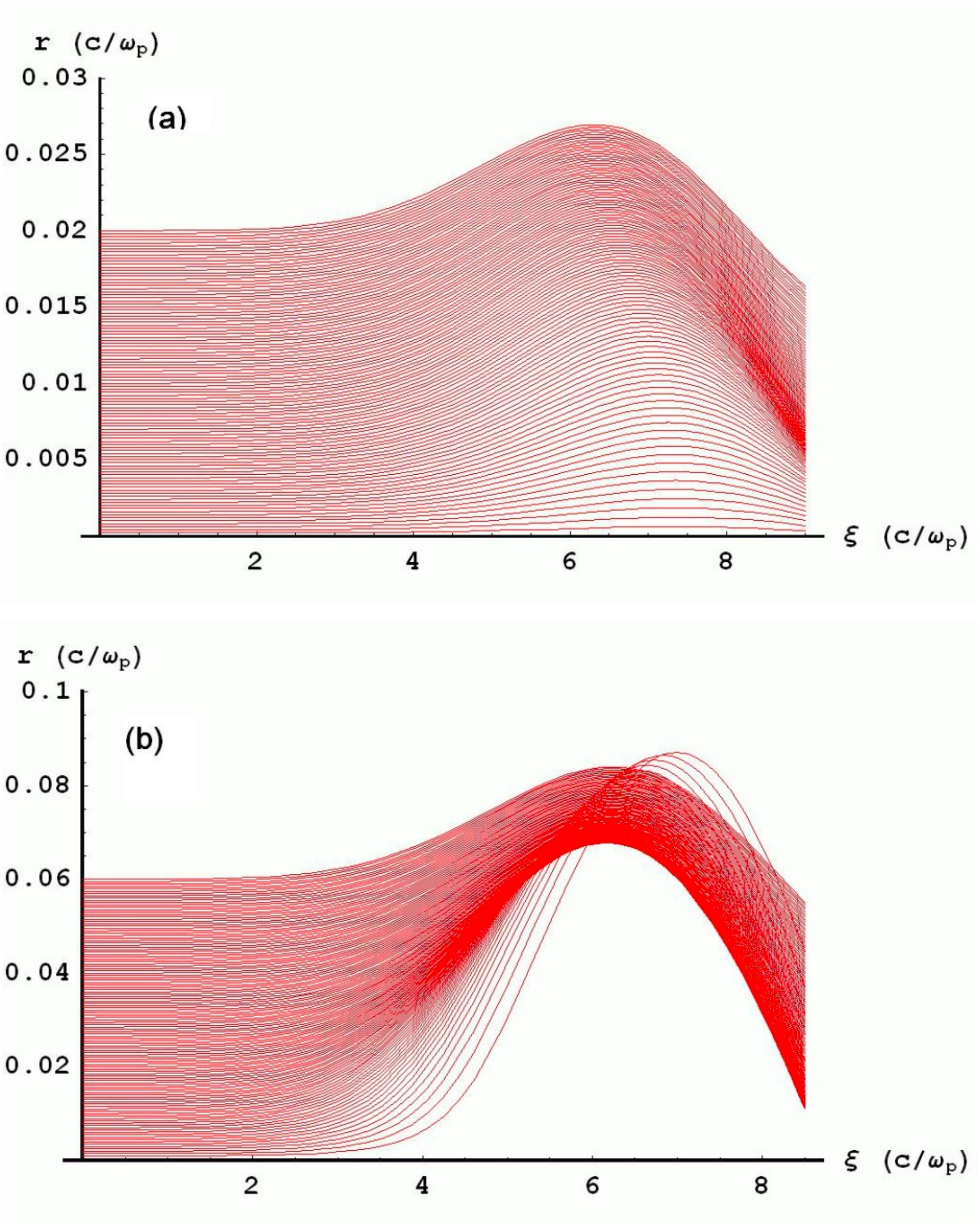
Very stable wake structure-–perfect for acceleration.

Trajectory crossing: necessary, but not sufficient to reach blowout regime

- The trajectory crossing condition defines the transition from the linear regime to the weakly blown out regime.

- Imagine electrons which begin at different radial positions. If no trajectory crossing occurs then each electron will always see other electrons at smaller radii and hence there is no region void of electrons.

- However, if the trajectory of an electron with a sufficiently small initial radius crosses that of another electron then for radii smaller than this it is possible for an ion column to form.



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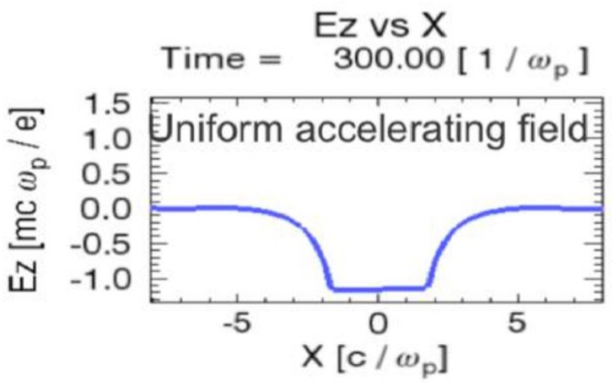
Strongly nonlinear regime

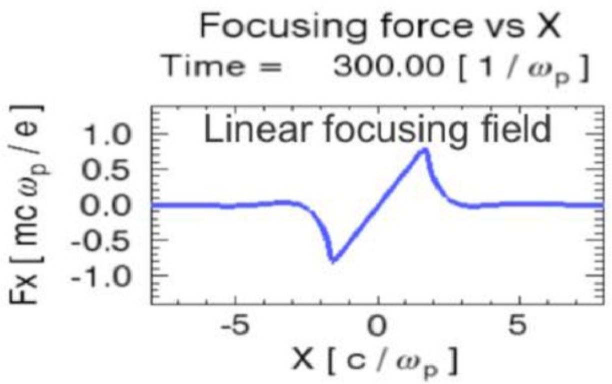
- The power needed to excite nonlinear wakes approaches or even exceeds critical power for relativistic self-focusing.

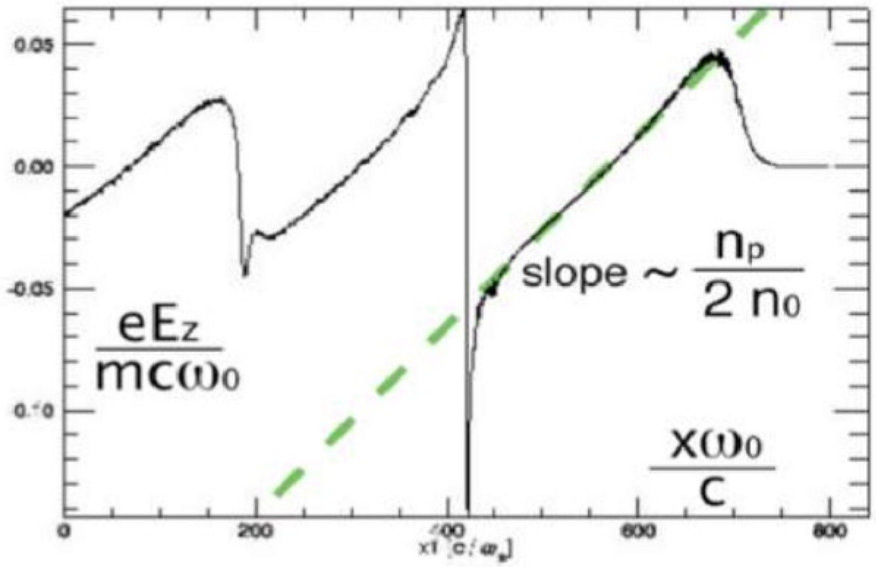
$$P_c(\mathrm{GW}) = 17.4(\omega_0^2/\omega_p^2)$$

- If the laser pulse is “matched,” (i.e., its spreading by diffraction is exactly balanced by relativistic self-focusing) and its power is above P_c, the laser pulse will continue to focus until the plasma electrons are completely expelled by the ponderomotive force of the laser, thus saturating the process.

- If the laser $a_0 \geq 2$ and $c\tau \sim \lambda_p/2$, matching occurs when $k_p w_0 \cong k_p R_m = k_p R_b = 2\sqrt{a_0}$







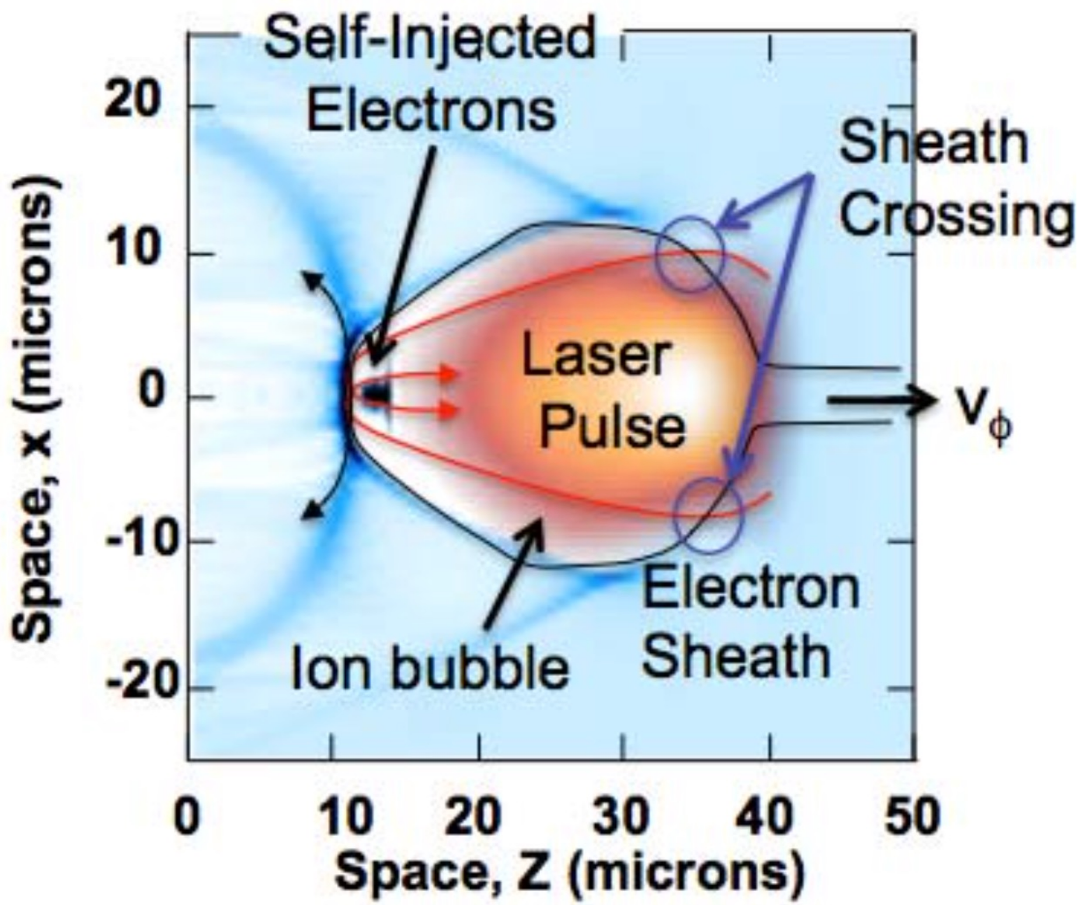
$$\frac{eE_z}{mc\omega_p} = \frac{r_b}{2}\frac{dr_b}{d\xi} \approx \frac{1}{2}\xi$$

$$\frac{eE_M}{mc\omega_p} \approx \frac{1}{2}k_p R_b \approx \sqrt{\Lambda}$$

Injection Methods: Where do particles come from?

- **Self Injection**—charge is injected into the bubble due to the fluctuations of the bubble sheath.
- **Colliding Pulse Injection**—two pulses collide head-on: the “driver” creates the wakefield, and the “injector” injects electrons.
- **Ionization Injection**—mixture of high-ionization-threshold (HIT) and low-ionization threshold (LIT) gases is used. Electrons from HIT are ionized and injected within the bubble driver in the LIT gas
- **External Injection**–using a beam produced by a conventional accelerator to inject into an LWFA bubble

Self-Injection:



Electron Sheath (black lines):

- plasma electrons stream toward the laser pulse, are then blown out by it, and form part of a negatively charged sheath around the ion cavity.
- These electrons cross the axis of the laser pulse, overshoot, and form the wake.

Self-injected electrons (red lines):

- If the laser pulse is intense enough $a_0>4$, some electrons that started out farther from the laser axis can cross the sheath and gain enough energy from the accelerating field to be injected and trapped inside the potential well of the wake.
- If they can gain sufficient energy from the wakefield to move at the phase velocity v_φ of the wake as they reach the near the $E_{z,max}$, they will be trapped and further accelerated by the wakefield.

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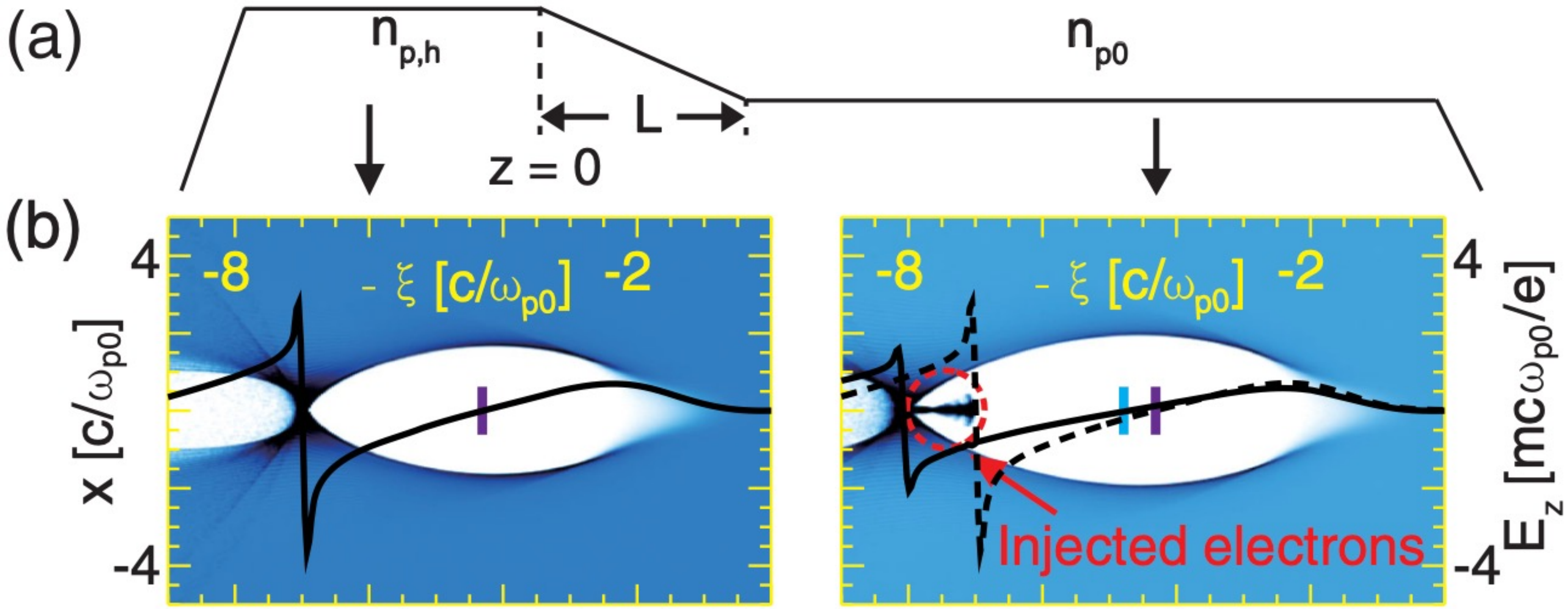
Downramp Injection:

- A short driver pulse is sent through an underdense plasma with a sharp downward density transition marking the boundary between a dense upstream region (I) and a less dense downstream region (II).
- When the beam passes a sharp downward plasma density transition, the wavelength of the plasma wave changes rapidly.
- The plasma electrons that originate just inside region II spend much of their oscillation in region I before returning to near their initial position in z, advanced in wave phase compared to the nominal (uniform plasma) region II oscillation.
- The faster oscillation of the electron in region I allows the electron entering region II to remain in an accelerating phase.

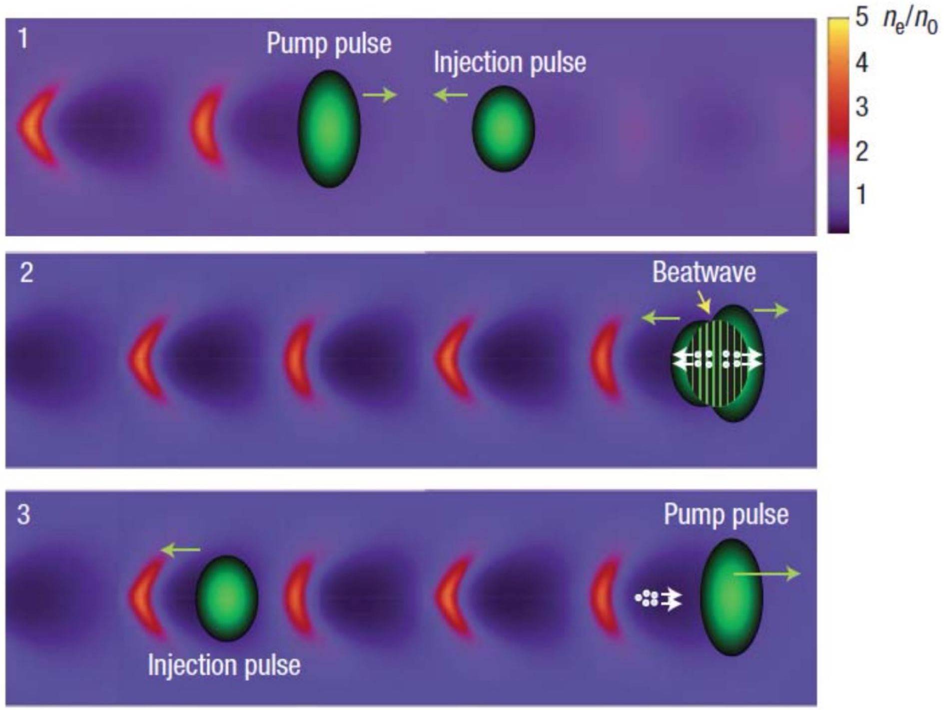


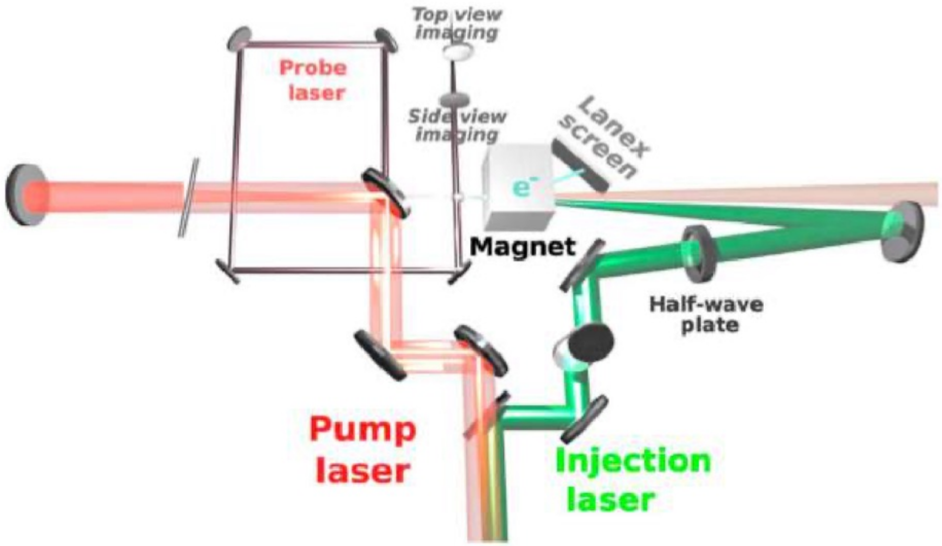
Colliding Pulse Injection:

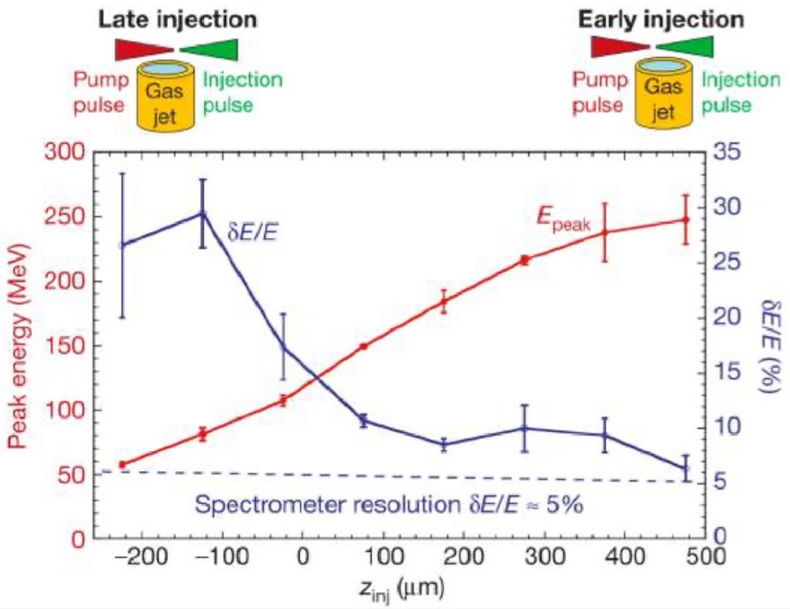
1. **Pump laser produces a wake**, but the wake is not intense enough to trap plasma electrons.

2. **Beat wave generated by colliding the pump and injection pulses**. The excited short wavelength, low phase velocity plasma wave perturbs the normally passing trajectories of the **plasma electrons** so that they **are pushed longitudinally.**

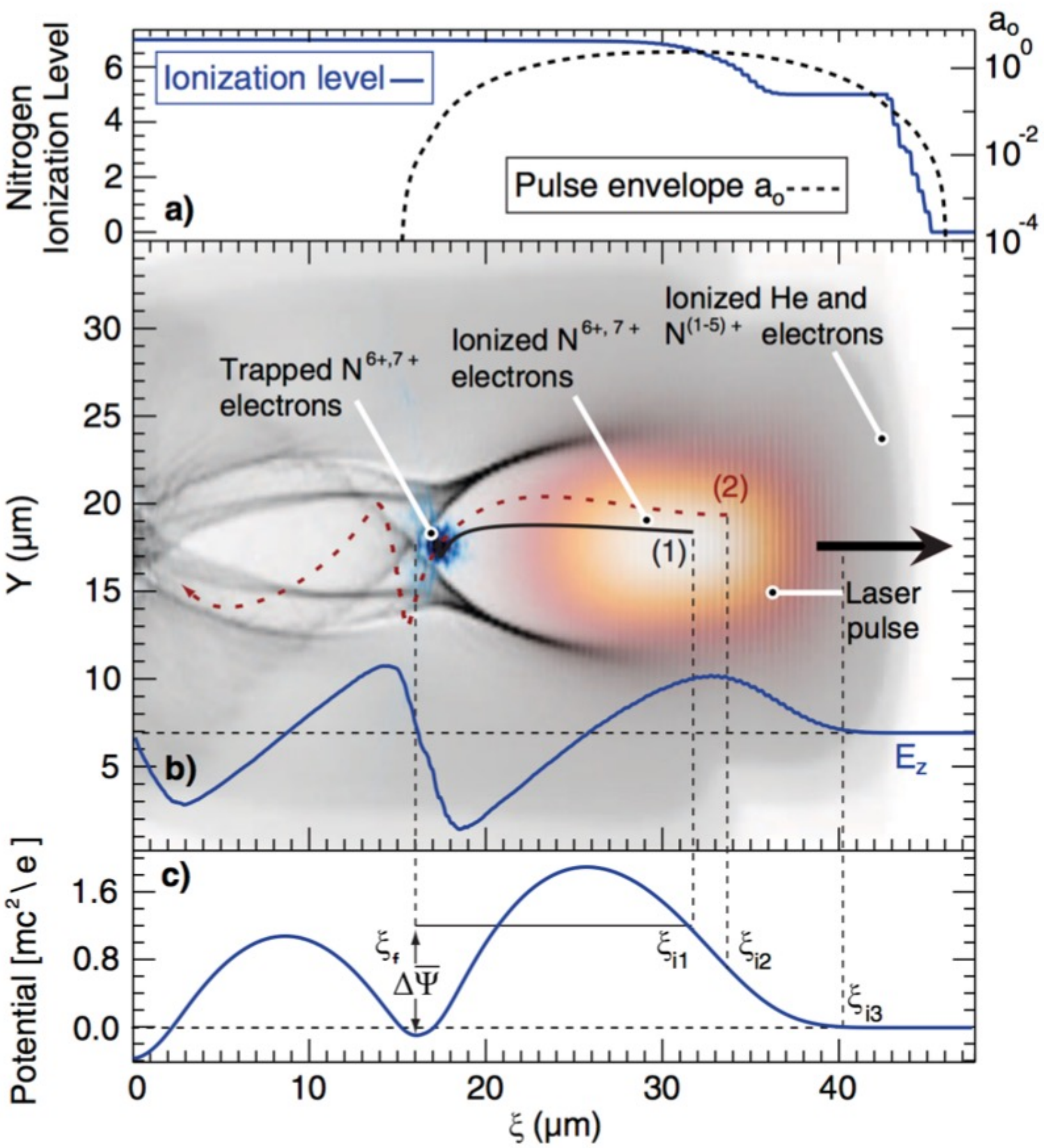
3. As the injection pulse passes, **some of these electrons can be injected into the first "bucket"** of the wake to be accelerated.







Ionization Injection:



- Use a **gas mixture that has multiple ionization states** (for example He+N). The mixture contains one (majority) species of atoms that is rather easily ionized by the rising edge of the laser pulse and form the wake. The second (minority) species of atoms has a large step in the ionization potential between two closed shells.
- The **leading edge of the laser** pulse fully **ionizes He and the outer 5 electrons of N**.
- The ponderomotive force pushes electrons out creating a wake.
- The inner-shell electrons, however, are not ionized until the laser intensity exceeds their ionization threshold near the peak of the pulse.
- Have the laser intensity profile and plasma density such that the 6th and 7th electrons of N are ionized by the E-field of the laser.
- The electrons produced later now see a larger wake potential difference. $\Delta\xi = \xi_i - \xi_f$
- If $\Delta\xi < -1$, electrons may be trapped in the cavity.

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Trojan Horse Injection:

Needs “two colors”:

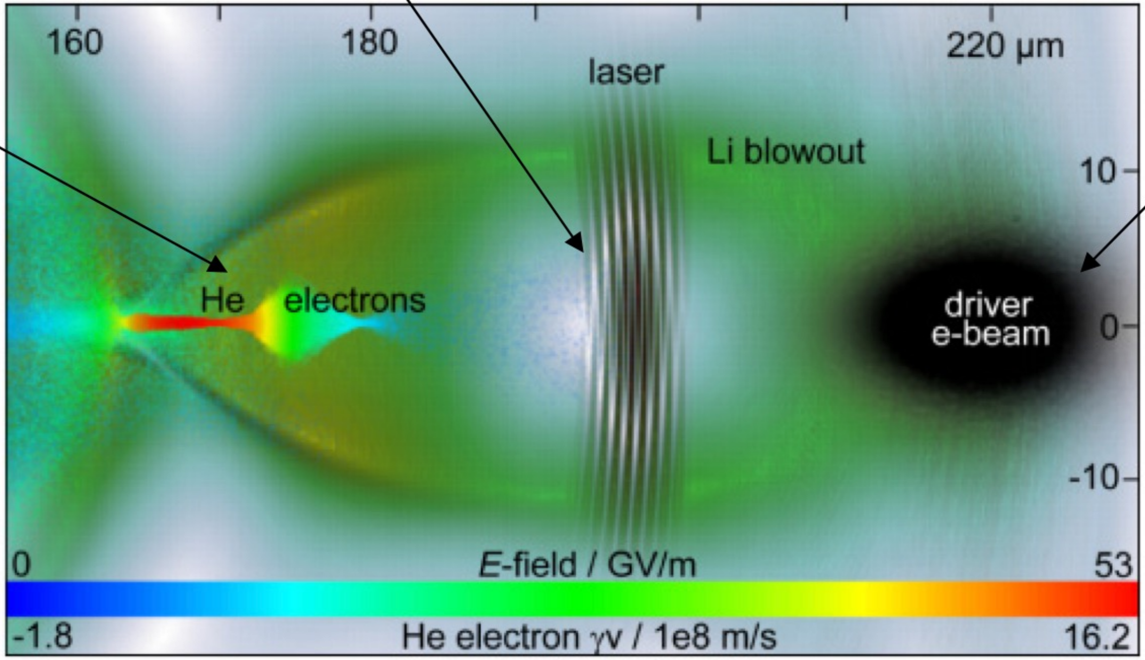
- Option #1: e-beam as a driver + laser as an injector
- Option #2: low intensity, high energy driver (e.g. CO_2) + high intensity, low energy injector (Ti:Sapphire)

Synchronized laser pulse tunnel ionizes in focus and releases ultracold electron population

Released electrons are rapidly accelerated and form bunch with ultralow emittance

Electron beam drives plasma wave over long distance

Key is to have two plasma components, one with low ionization threshold (LIT) and one with high ionization threshold (HIT)



Phys. Rev. Letters 108, 035001 (2012)

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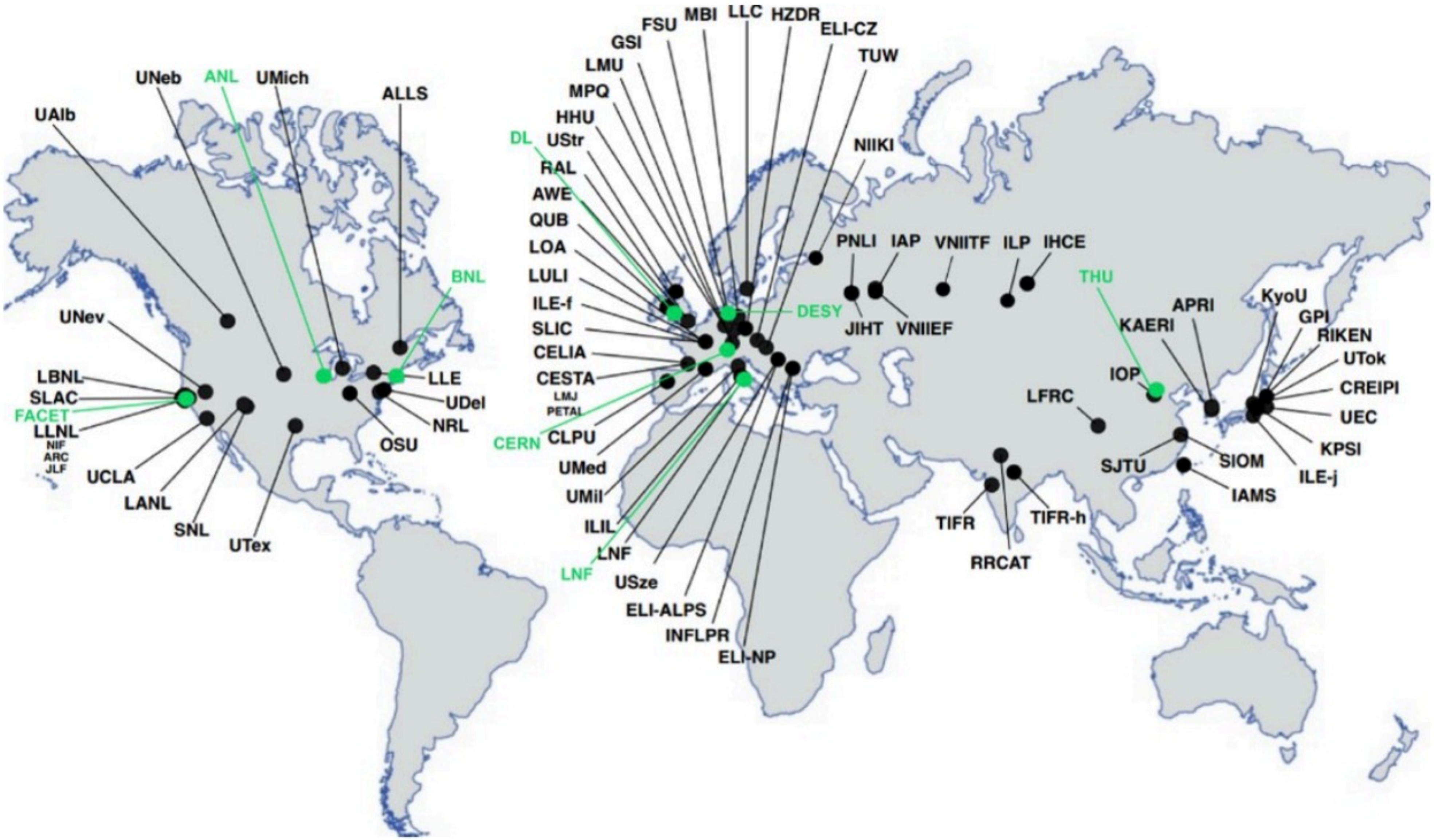
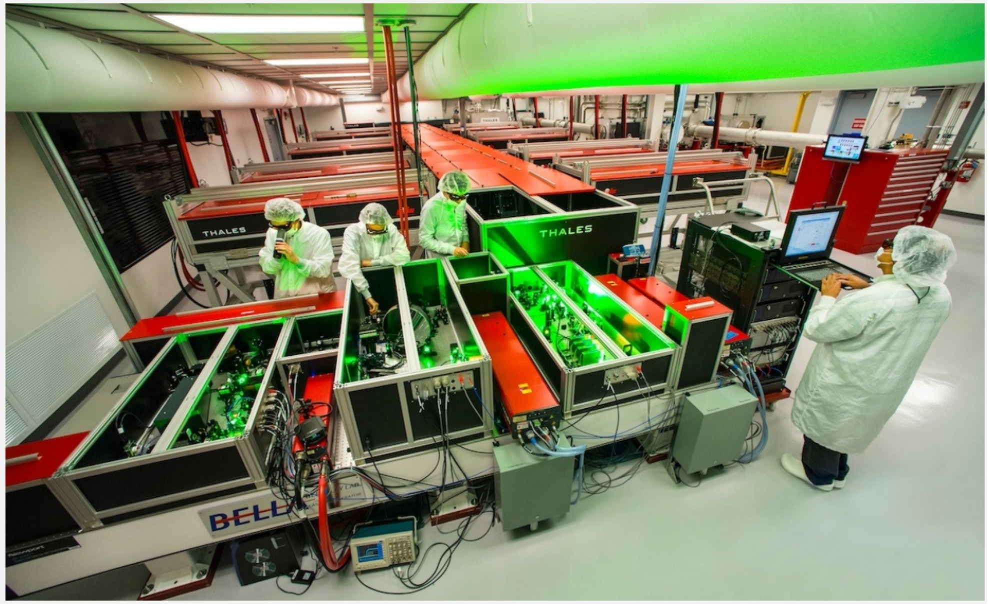
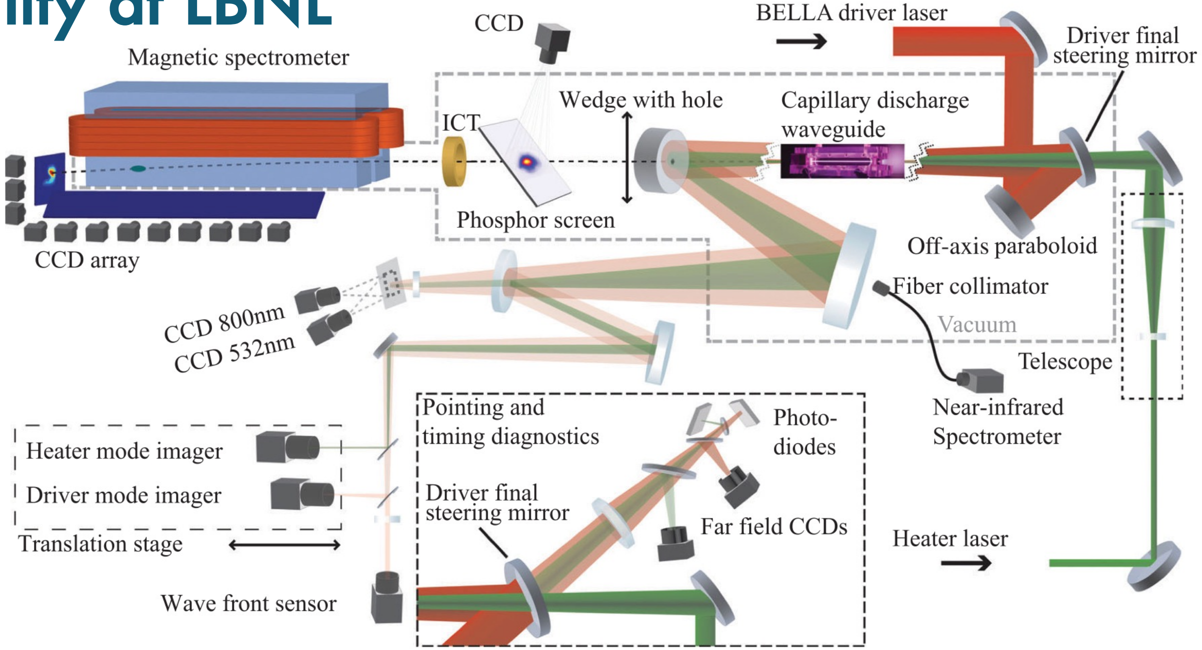
Current status, challenges & outlook

Laboratories around the world working on plasma wakefield accelerators:

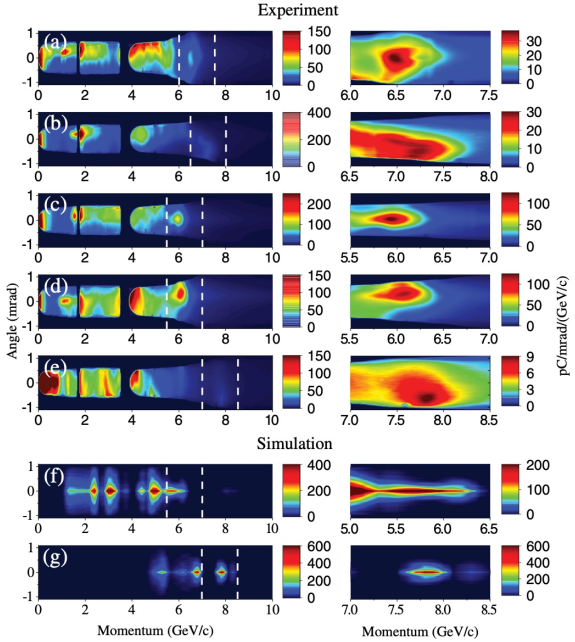


Recent Progress in LWFA: BELLA PW facility at LBNL



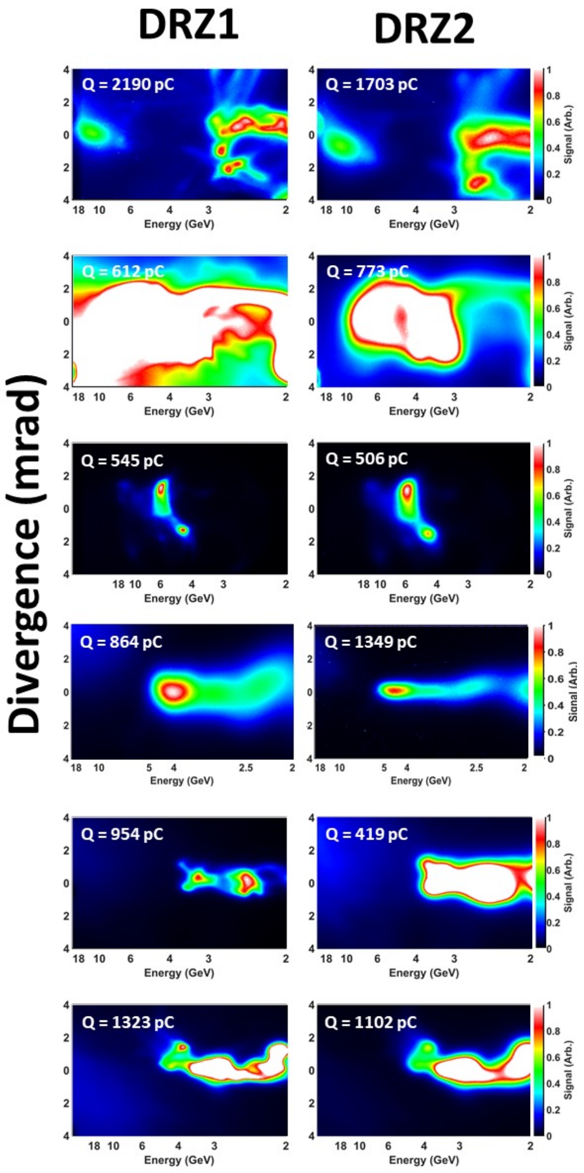


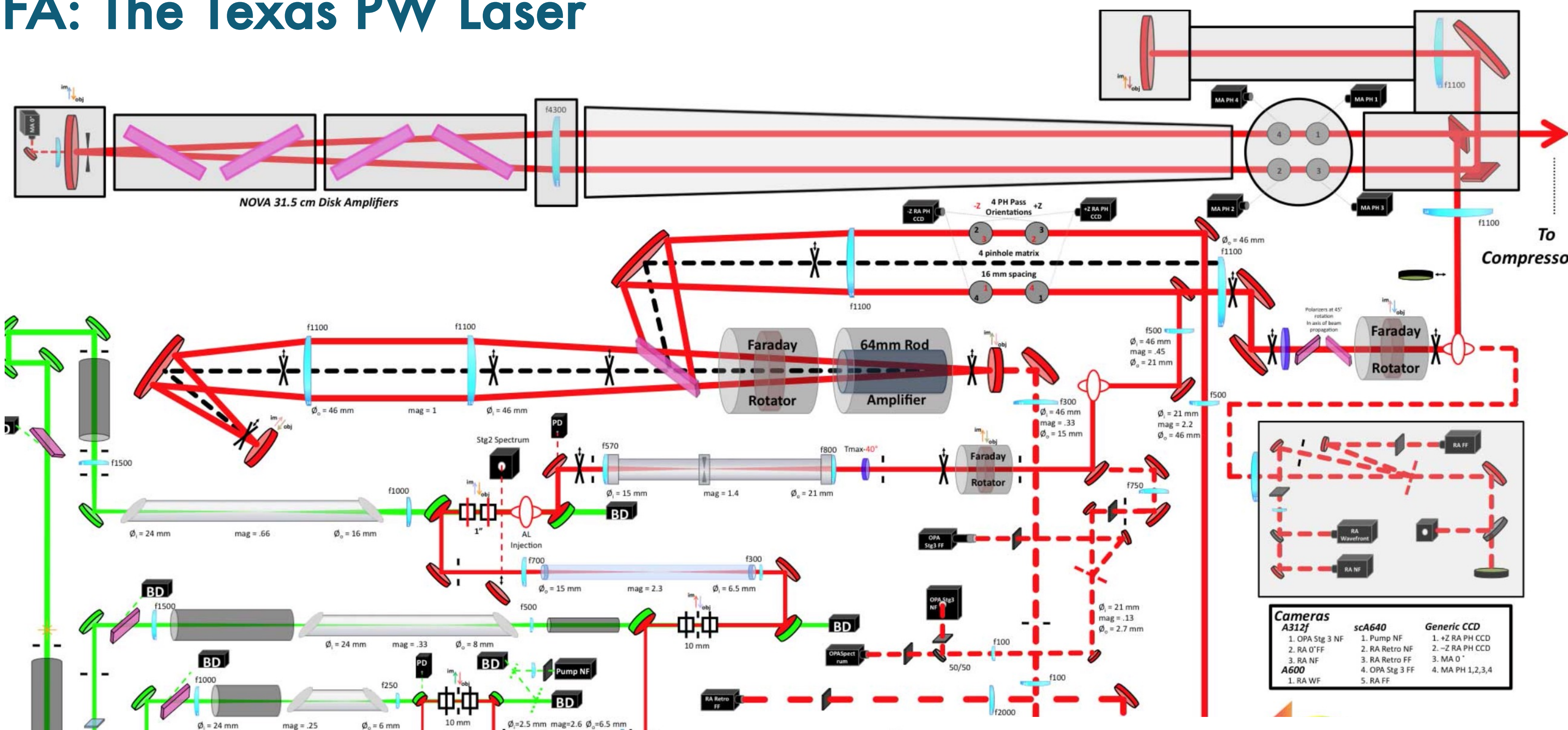
A laser-driven electron energy gain of about 8 GeV over 20 cm of plasma with density 3×10^{17} cm^{-3}.

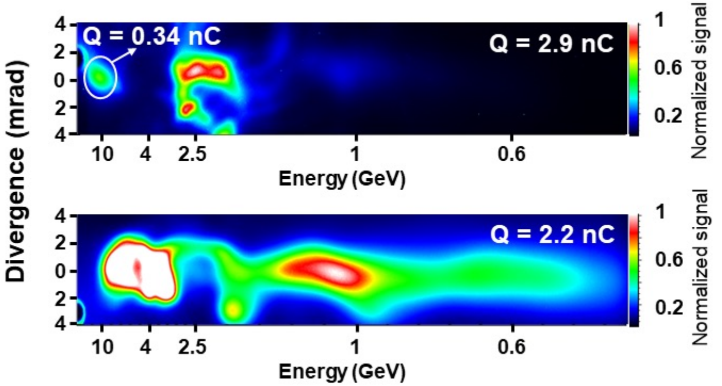


The facility has recently finished an upgrade of the BELLA PW laser system to allow a variety of experimental capabilities.

Recent Progress in LWFA: The Texas PW Laser







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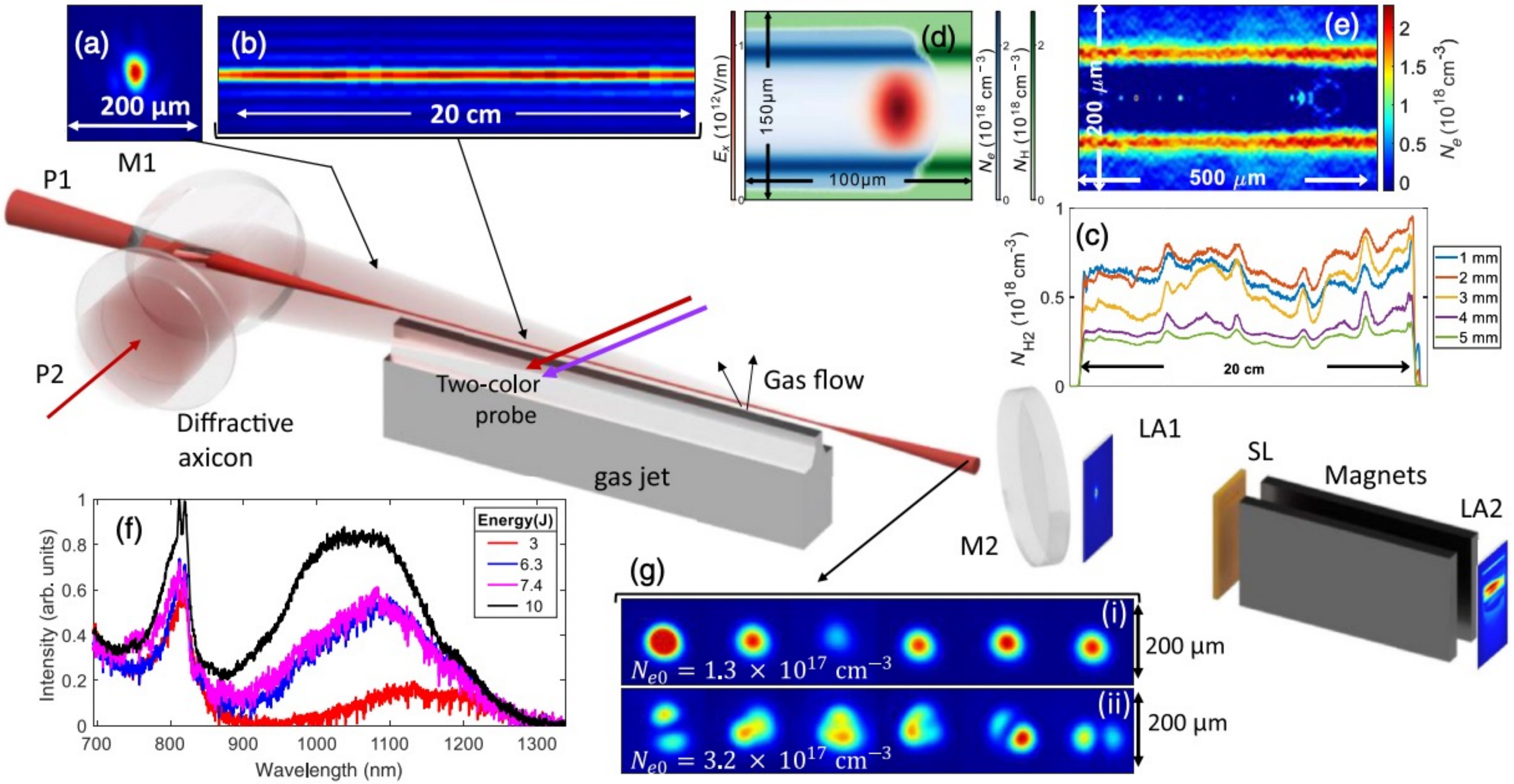
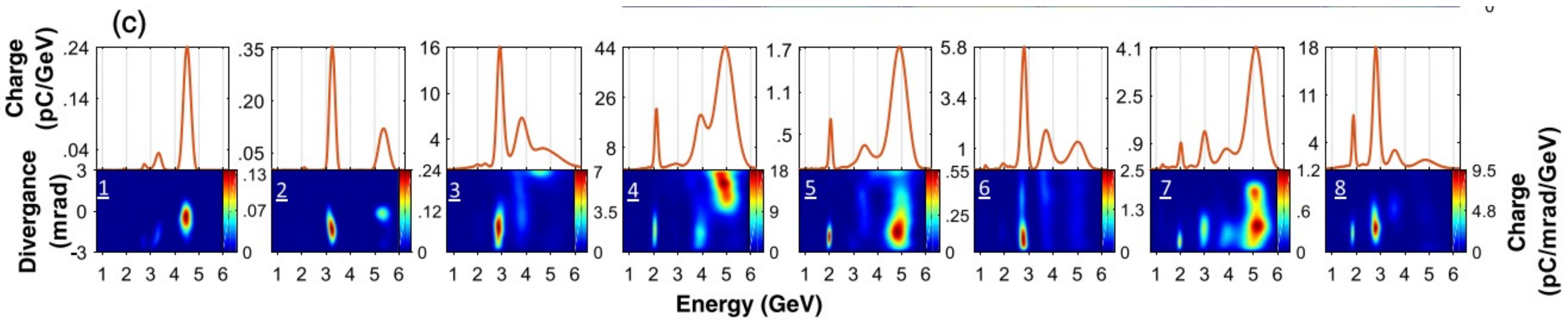
- ✓ Two shots out of 26 showed a spectrum extending beyond 10 GeV.
- ✓ Highest energy electron spectrum has the centroid at 10.4±0.6 GeV with 3.4 GeV RMS convolved energy spread, 0.9 mrad RMS divergence, and 340 pC charge.

Recent Progress in PWFA: University of Maryland

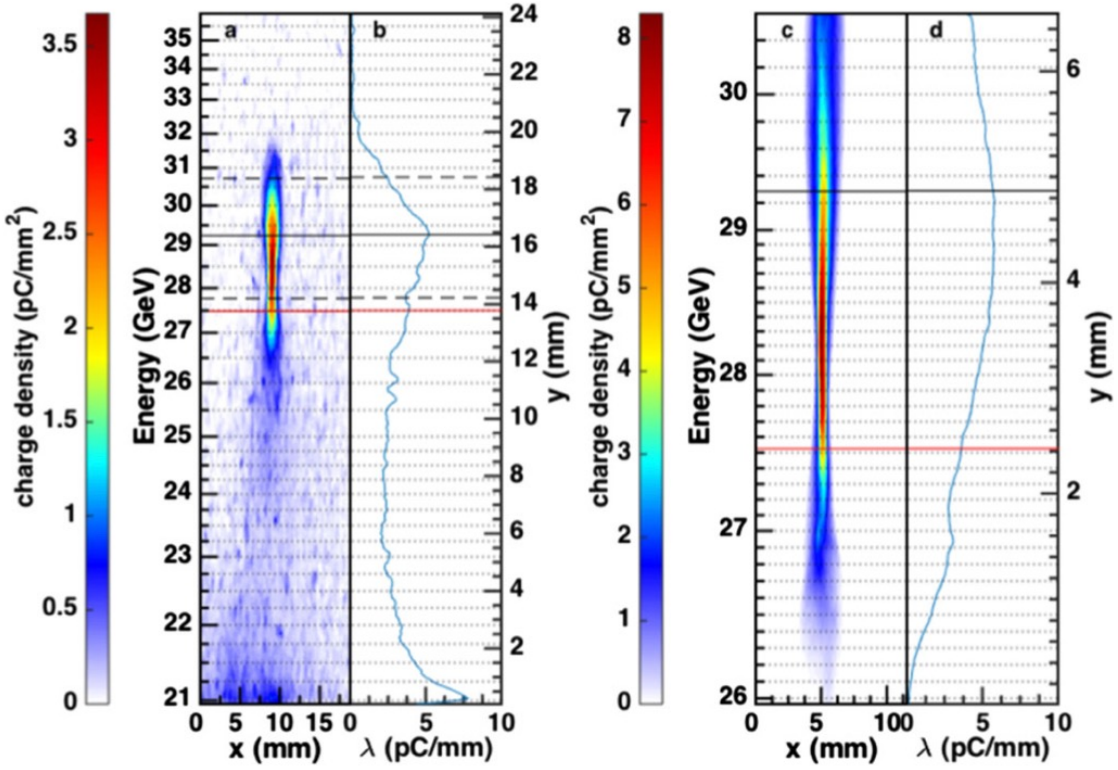
- Multi-GeV Electron Bunches from an All-Optical Laser Wakefield Accelerator.
- Accelerating gradient as high as 25 GeV/m.
- The guide was formed via self-waveguiding of <15 J, 45 fs pulses over 20 cm in a low-density hydrogen gas jet
- Accelerated electron bunches up to 5 GeV, relative energy width as narrow as ~15%, with divergence down to ~1 mrad and charge up to tens of pC.





Recent Progress in PWFA: FACET

Short electron bunches were used to boost the energy externally injected electron bunches by 9 GeV over 1.3 m $\sim10^{17}cm^{-3}$ plasma at the FACET facility in SLAC



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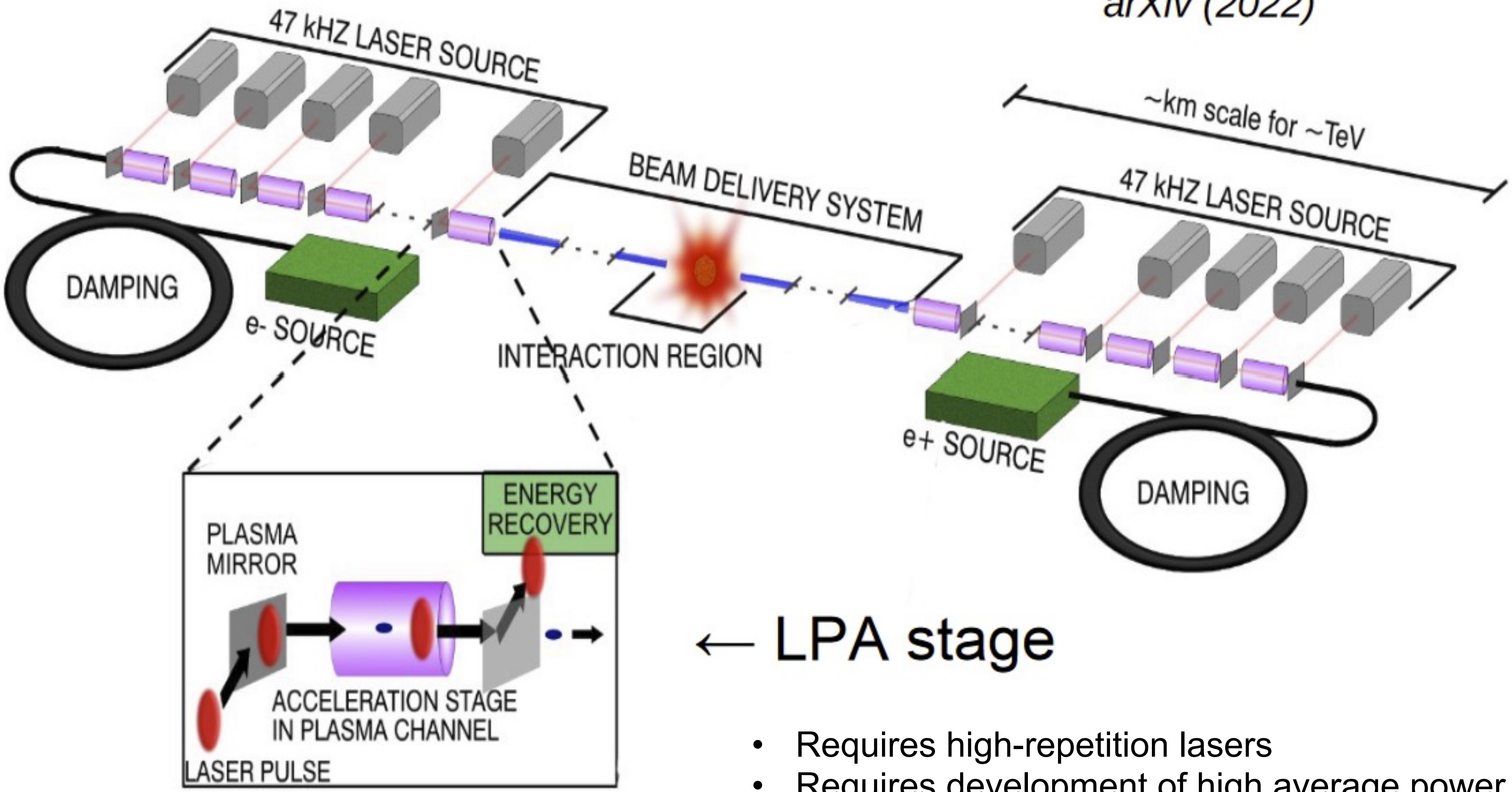
Ultimate goal: LPA collider

Benedetti et al., arXiv (2022)



← LPA stage

- Requires high-repetition lasers
- Requires development of high average power lasers

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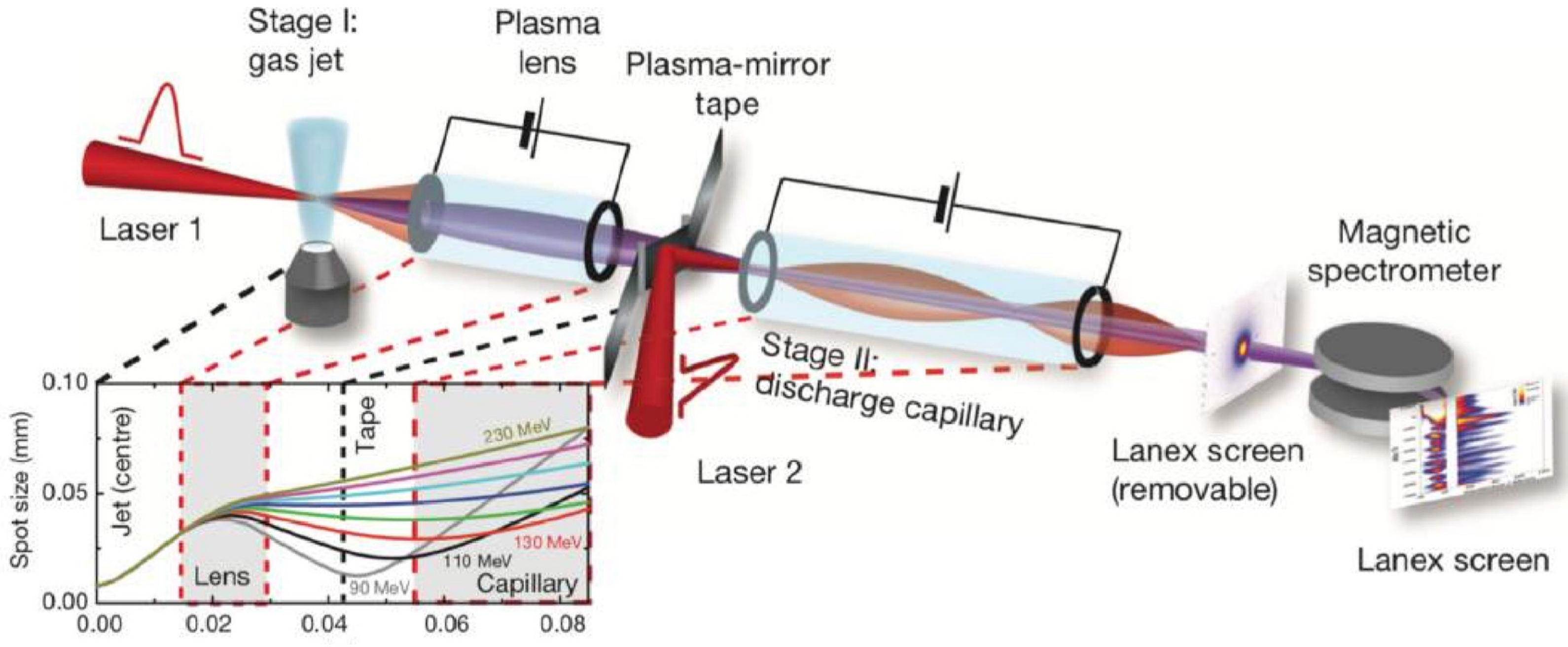
LWFA challenges

- Instabilities in accelerated beams and beam emittance control in scattering media
- Final focusing of e^+ and e^- bunches with significant energy spread acquired during acceleration
- Efficiency of staging [beam transfer and matching from one O(1m)-long plasma cell to another]. Strong transverse focusing gradients O(10MT/m) are generated inside the ion channel of plasma accelerators. Such focusing is equivalent to small beta functions $\beta_{x,y}$ in the range of a couple of centimeters to a few millimeters for high-energy beams accelerated in the 10^{14}-10^{17} cm^{-3} plasma.
- Working towards better repeatability from shot to shot and better beam quality.



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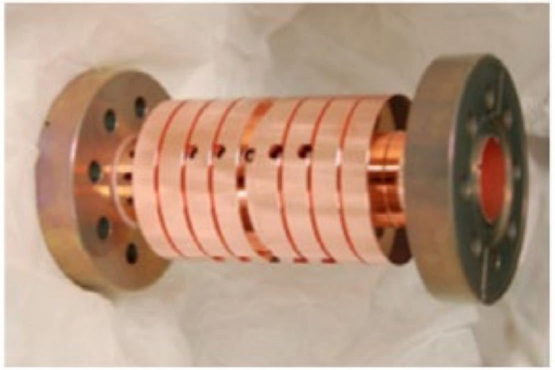
Conclusions

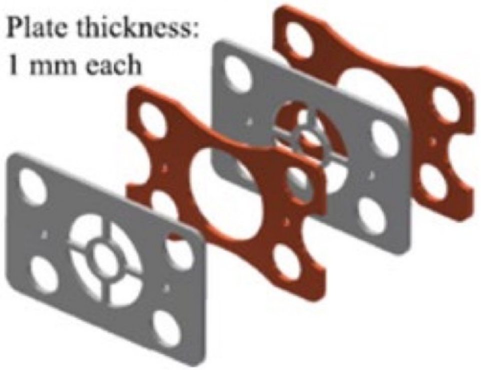
- A large variety of Advanced Accelerator techniques is currently under development.
- All acceleration techniques share common challenges: high-energy staging with independent drive beams, equal energy gain and good beam transmission; emittance preservation, high energy extraction efficiency, etc.

Metallic

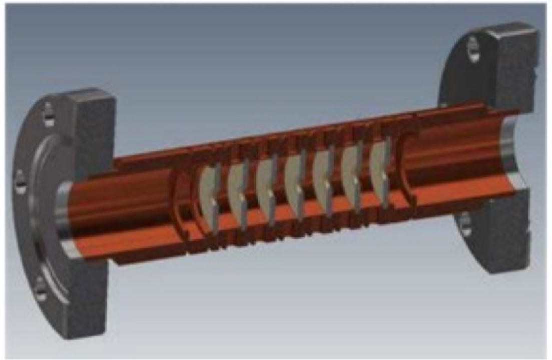


Dielectric

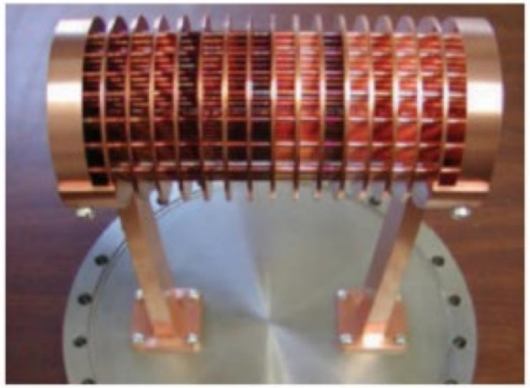
Metamaterial



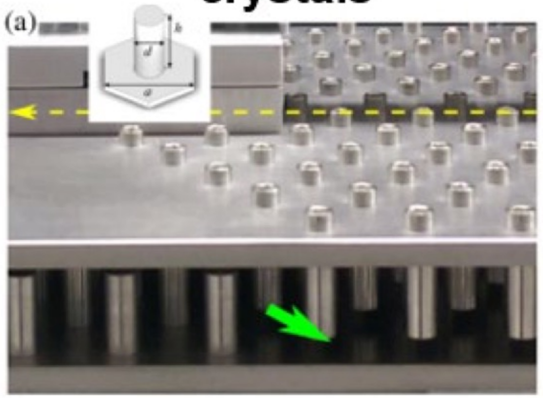
Dielectric disk structure



Photonic band-gap



Photonic topological crystals



The End