Modulator Simulations with Quadrupoles Focusing

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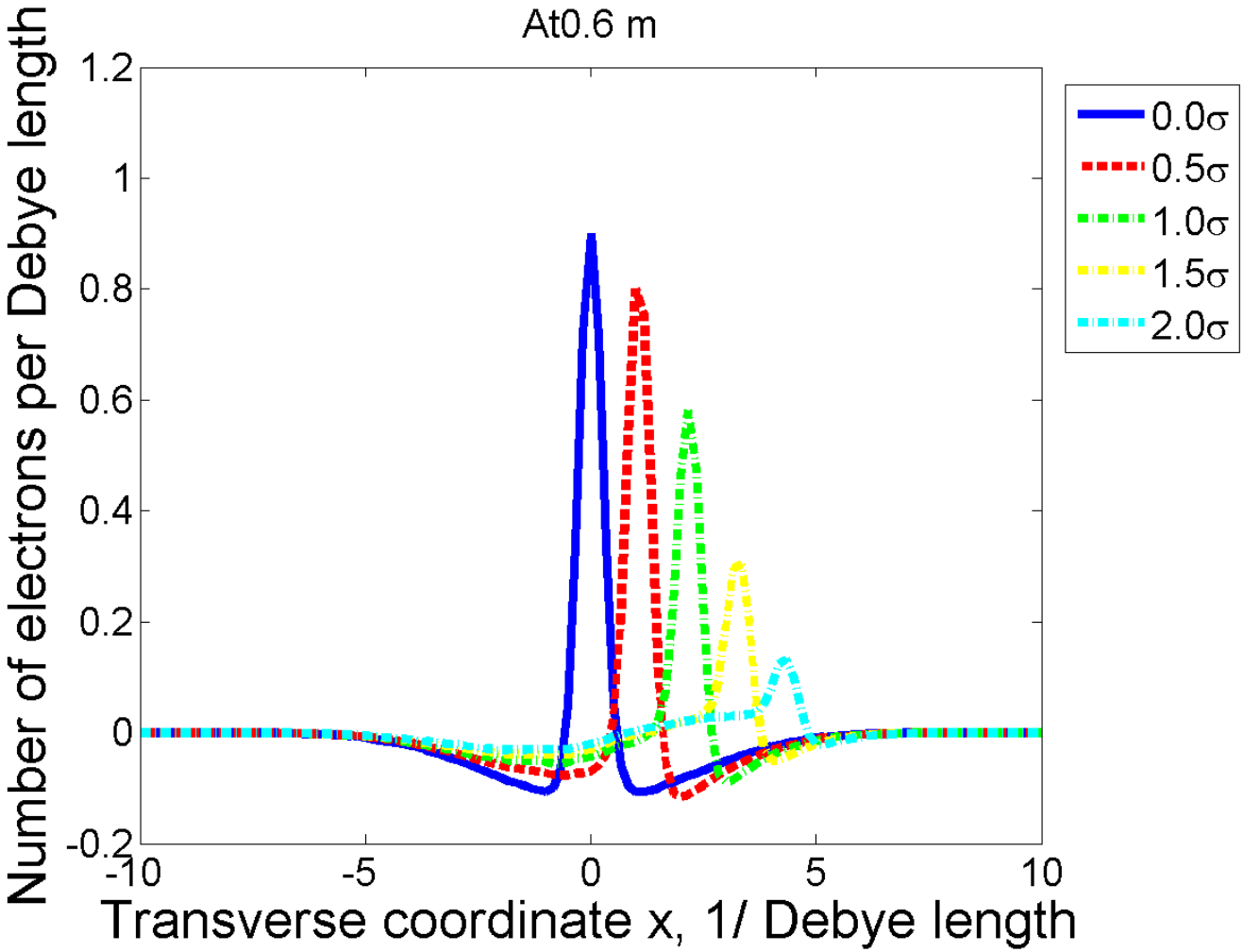
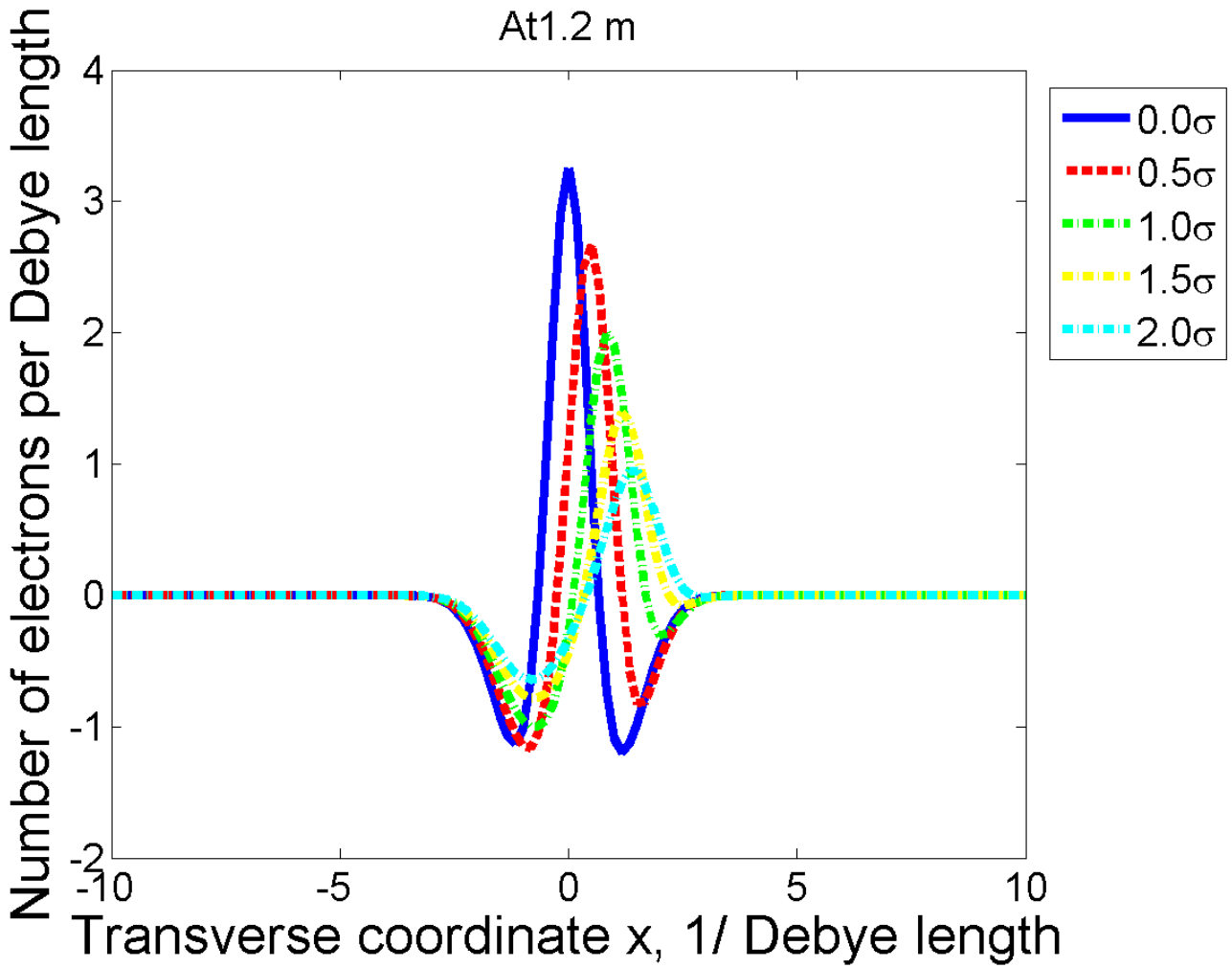
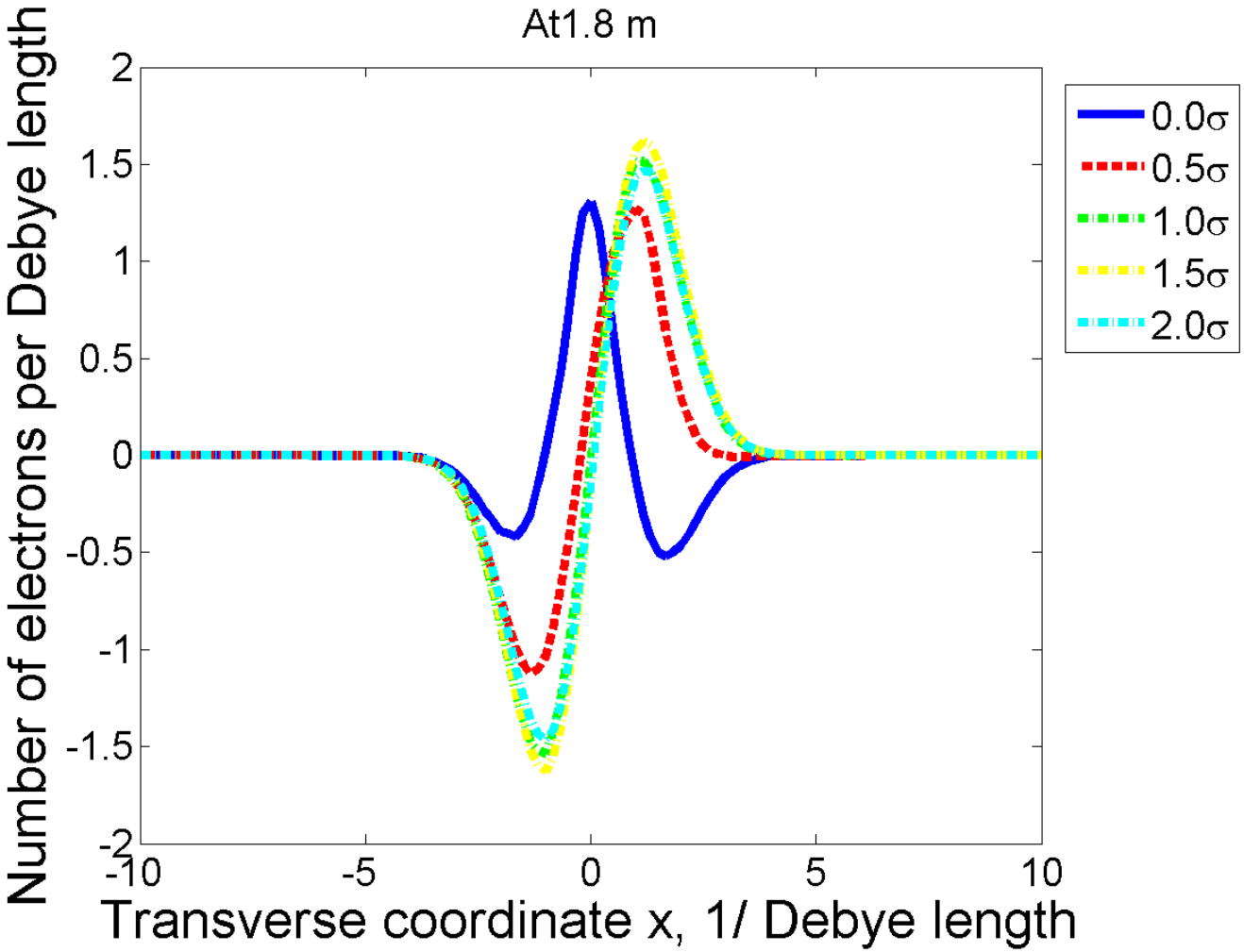
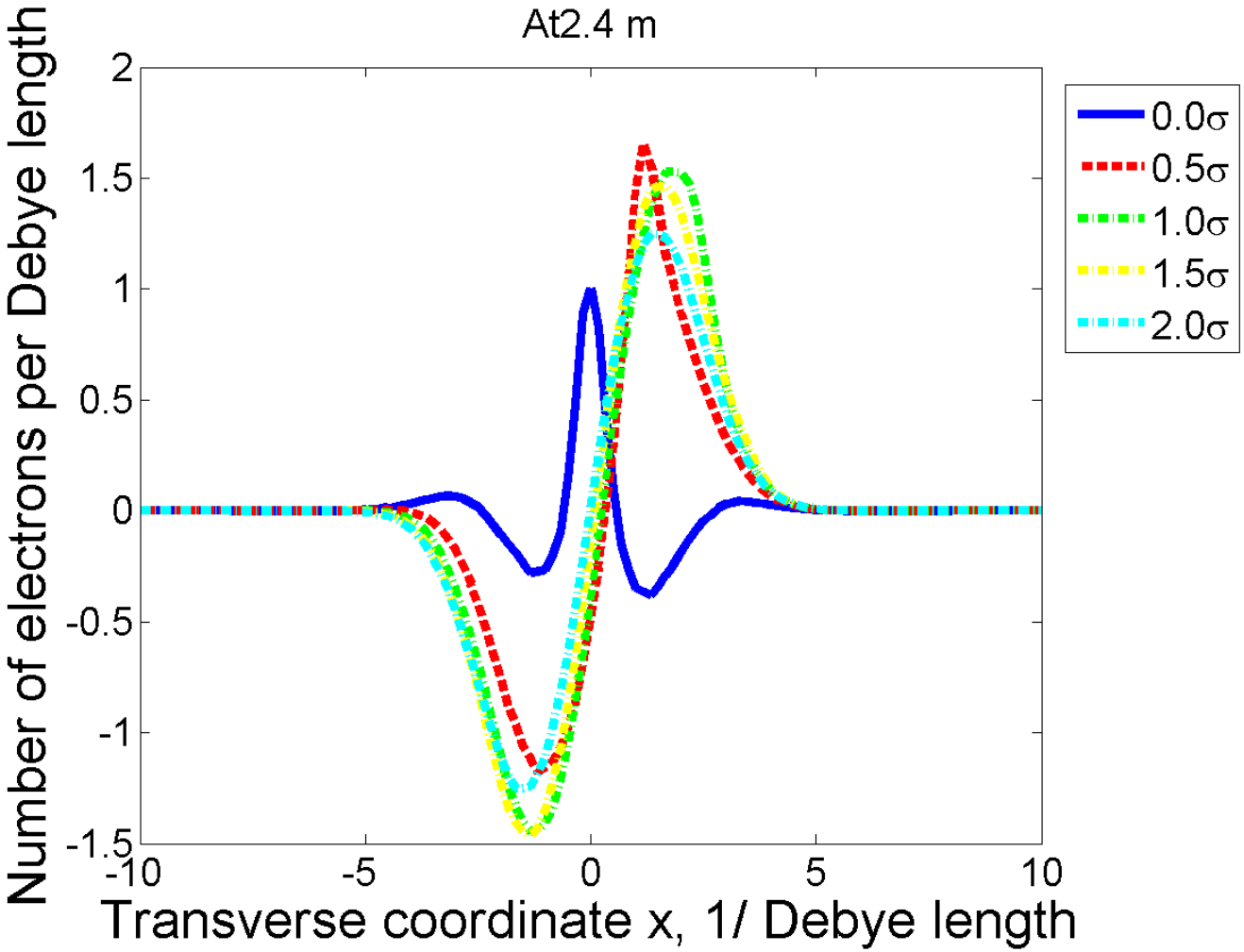
Notations

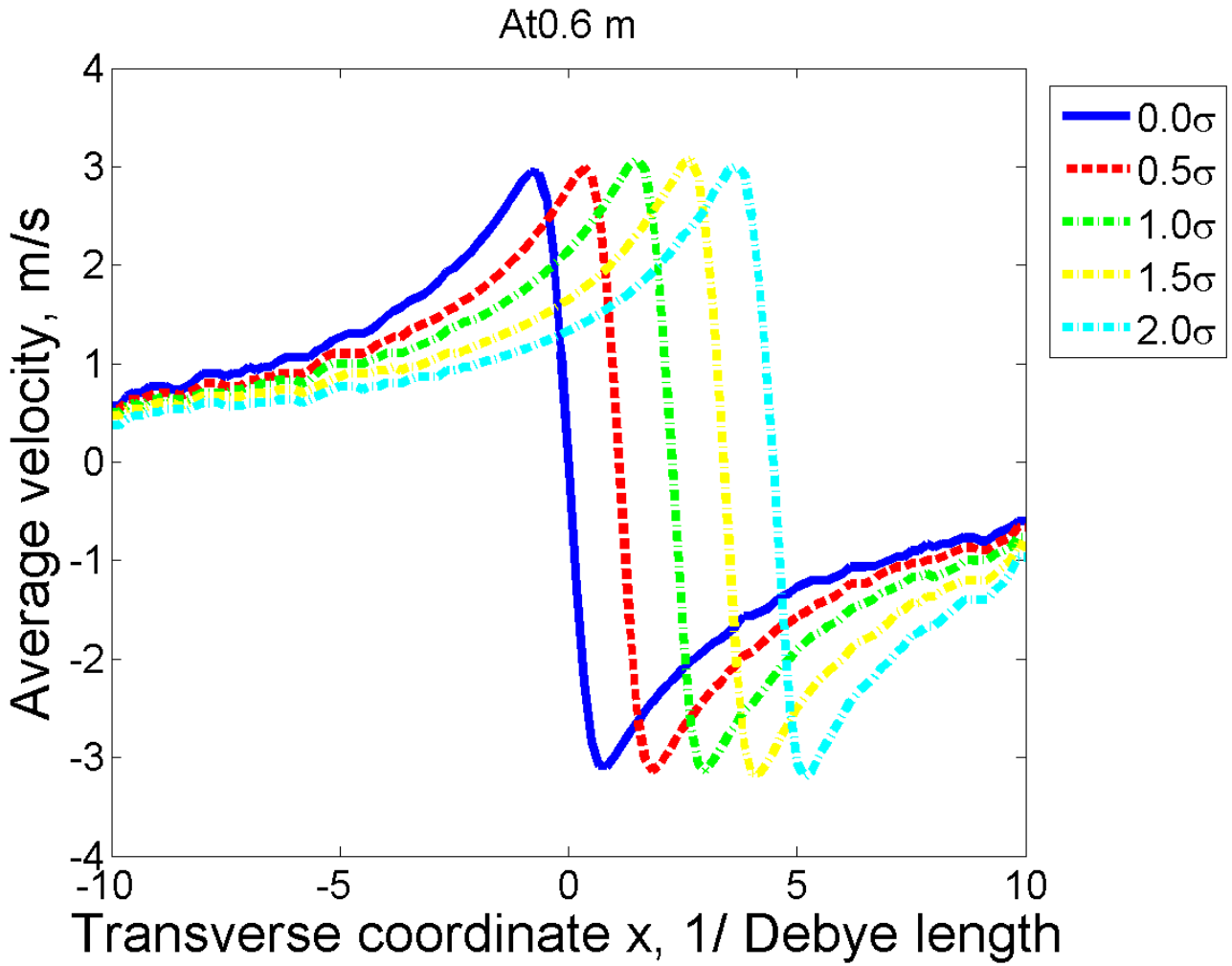
- nz : Longitudinal density distribution along z axis
- vz : Longitudinal velocity distribution along z axis
- nx : Transverse density distribution along x axis
- vx : Transverse velocity distribution along x axis
- ny : Transverse density distribution along y axis
- vy : Transverse velocity distribution along y axis

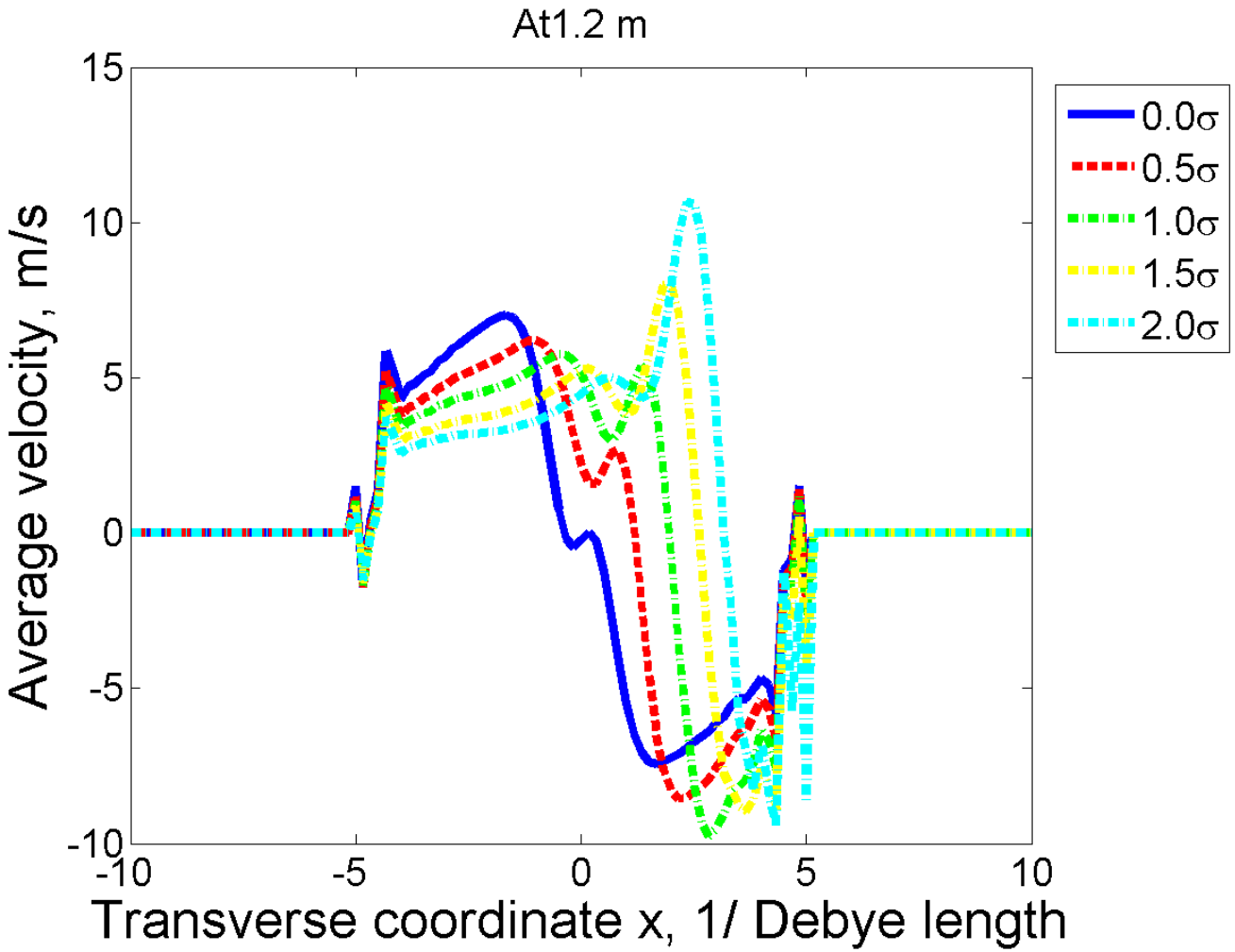

At0.6 m
Number of electrons per Debye length
Transverse coordinate x, 1/ Debye length
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-10
-5
0
5
10
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

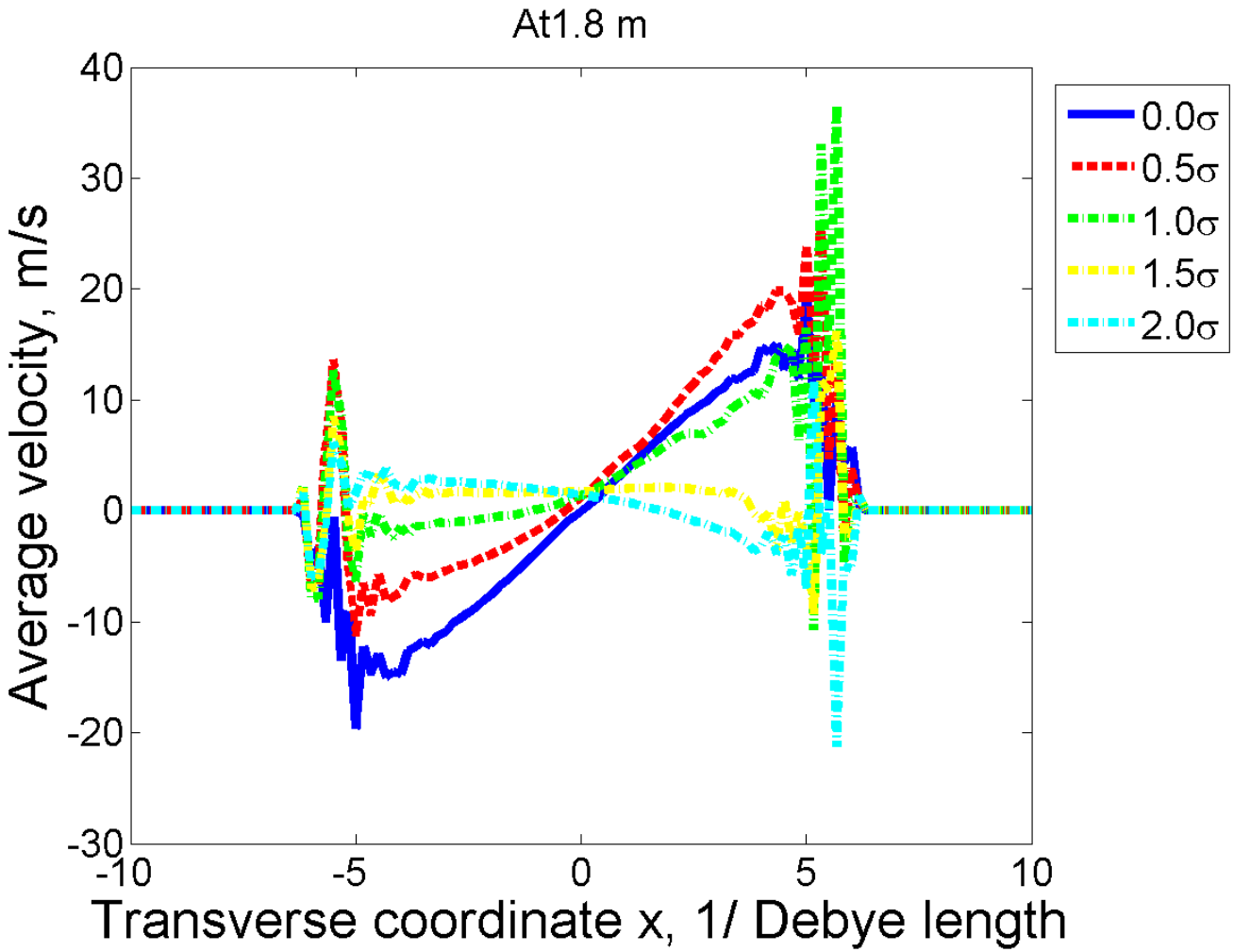

At1.2 m
Number of electrons per Debye length
Transverse coordinate x, 1/ Debye length
4
3
2
1
0
-1
-2
-10
-5
0
5
10
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

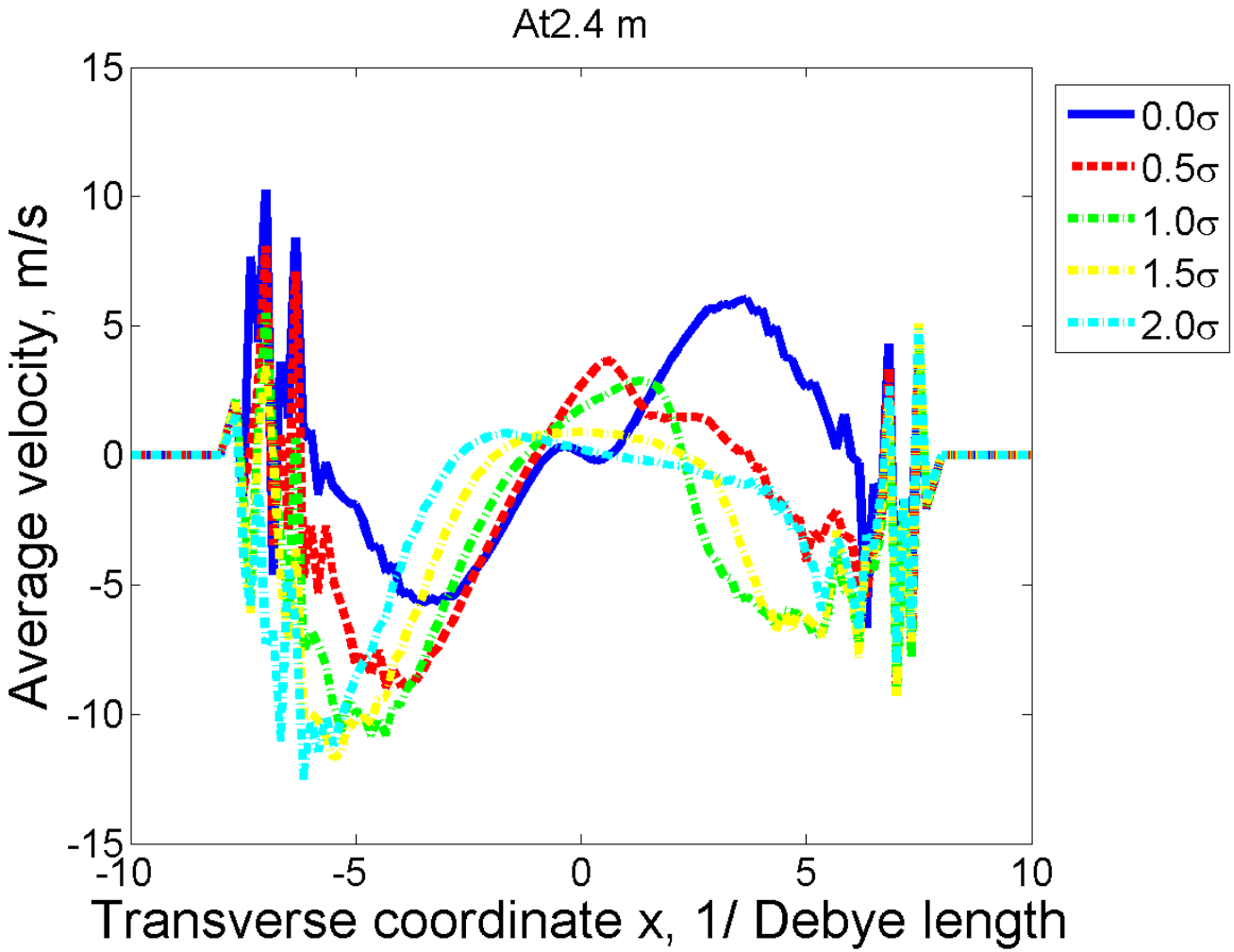

At1.8 m
Number of electrons per Debye length
Transverse coordinate x, 1/ Debye length
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-10
-5
0
5
10

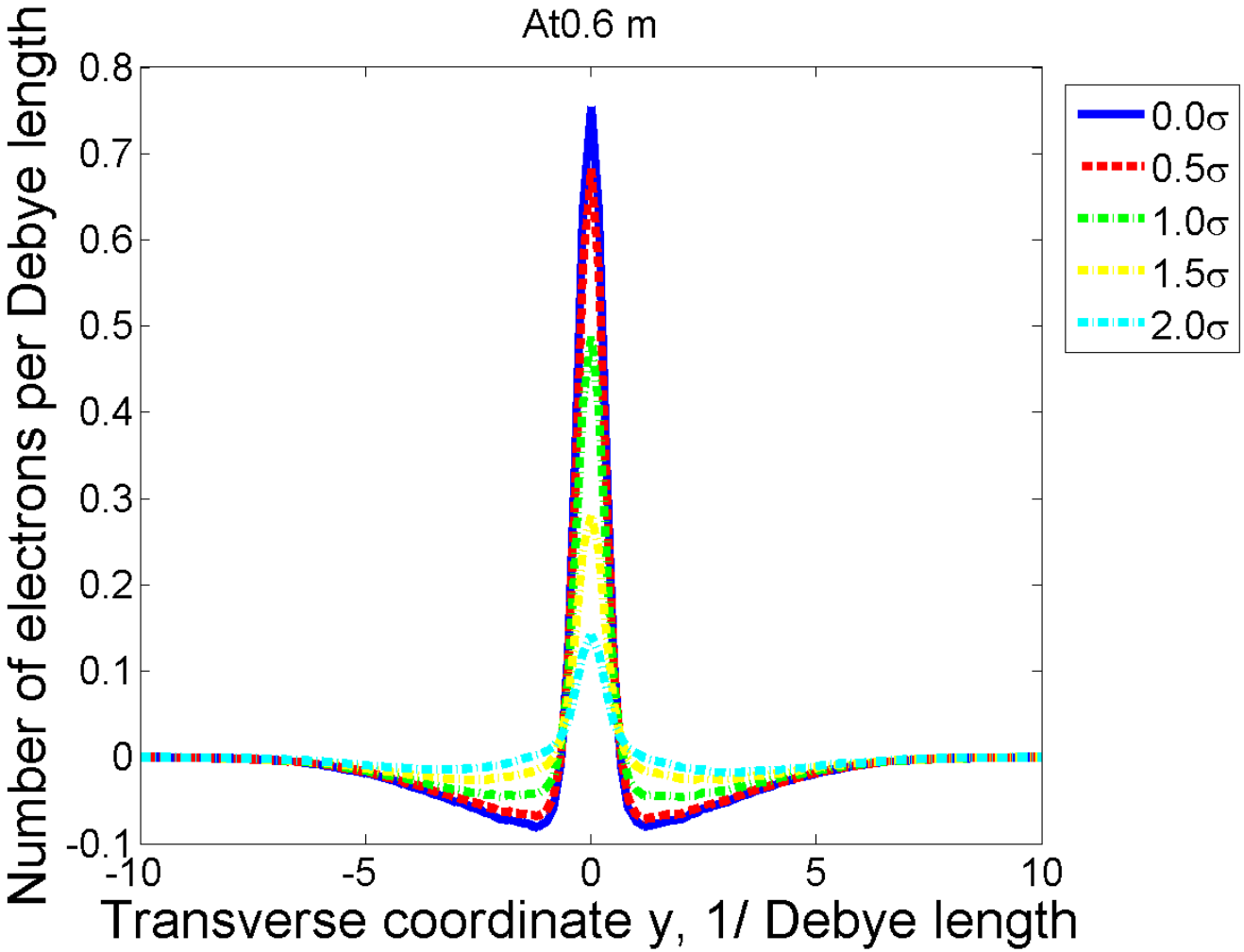

At2.4 m
Number of electrons per Debye length
Transverse coordinate x, 1/ Debye length
2
1.5
1
0.5
0
-0.5
-1
-1.5
-10
-5
0
5
10
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

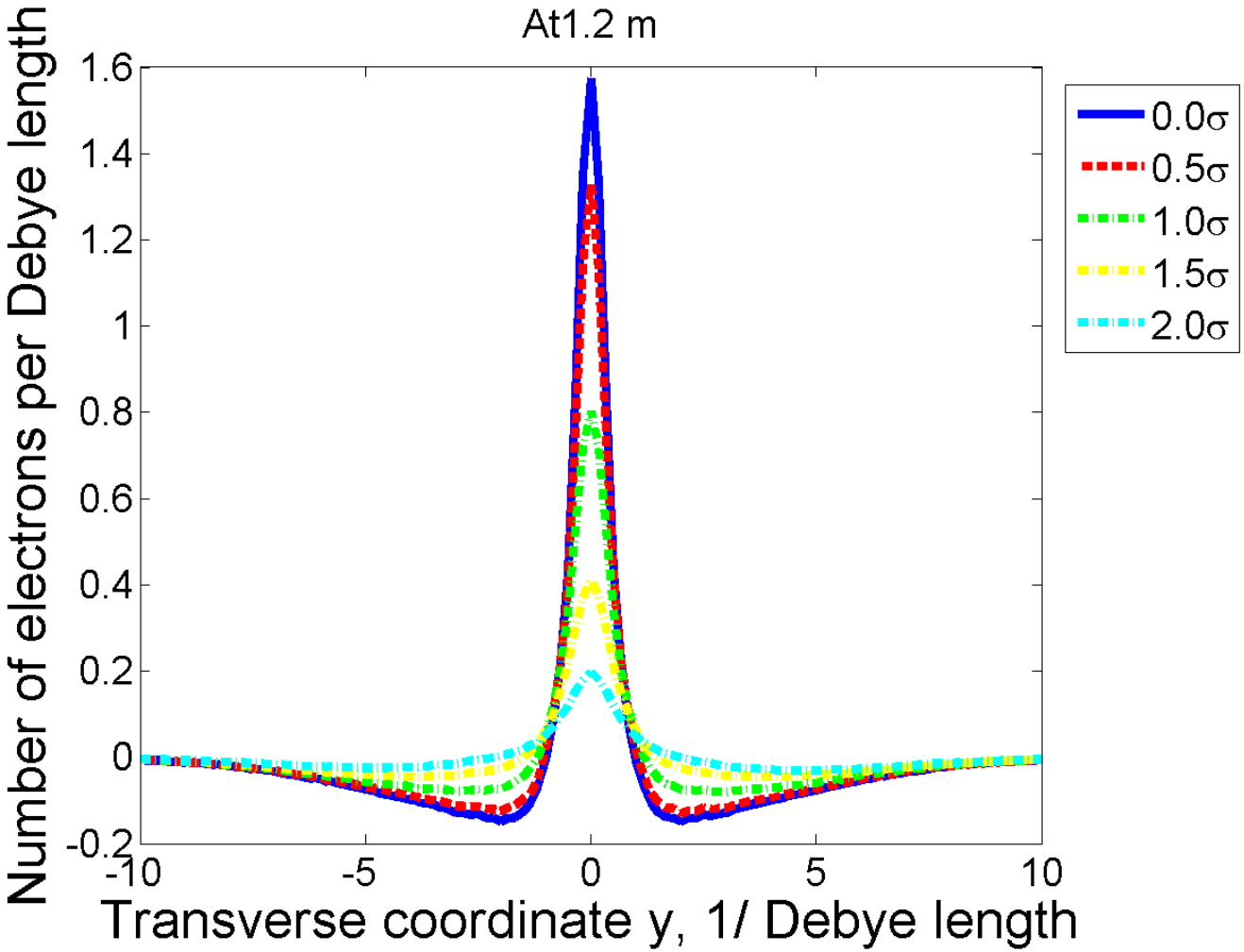

At0.6 m
Average velocity, m/s
Transverse coordinate x, 1/ Debye length
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ
4
3
2
1
0
-1
-2
-3
-4
-10
-5
0
5
10

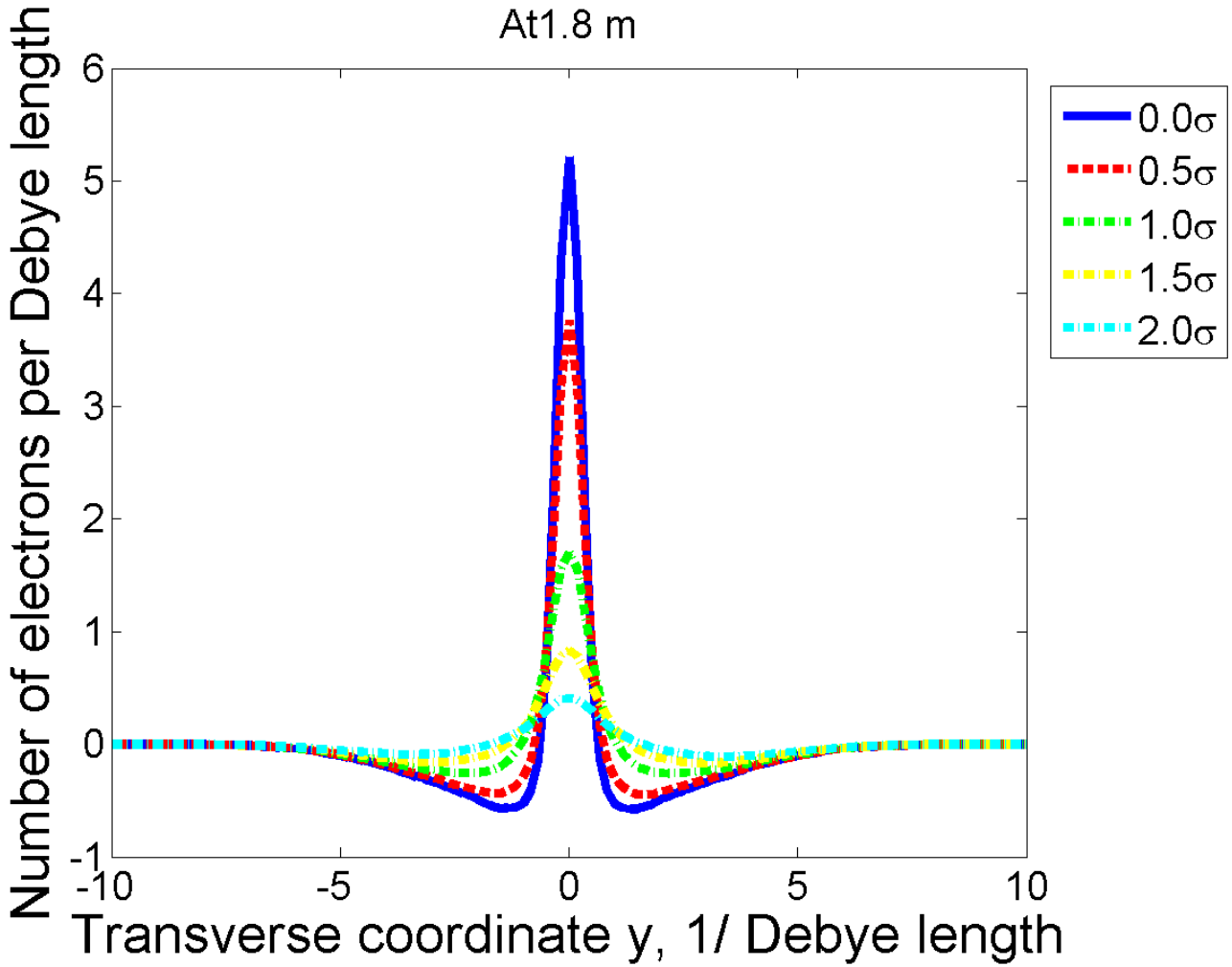

At1.2 m
Average velocity, m/s
Transverse coordinate x, 1/ Debye length
15
10
5
0
-5
-10
-10
-5
0
5
10
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

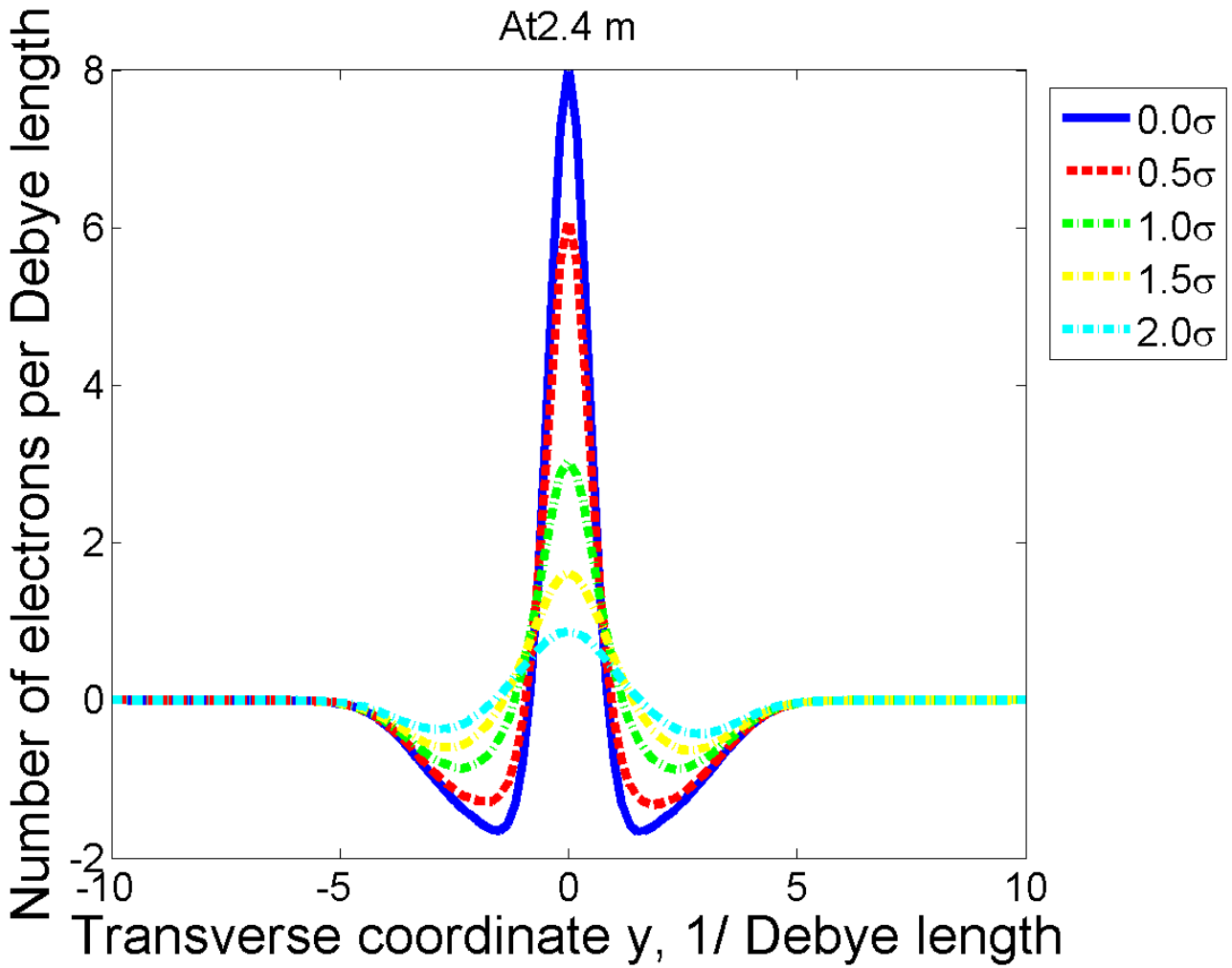

At1.8 m
Average velocity, m/s
Transverse coordinate x, 1/ Debye length
40
30
20
10
0
-10
-20
-30
-10
-5
0
5
10
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

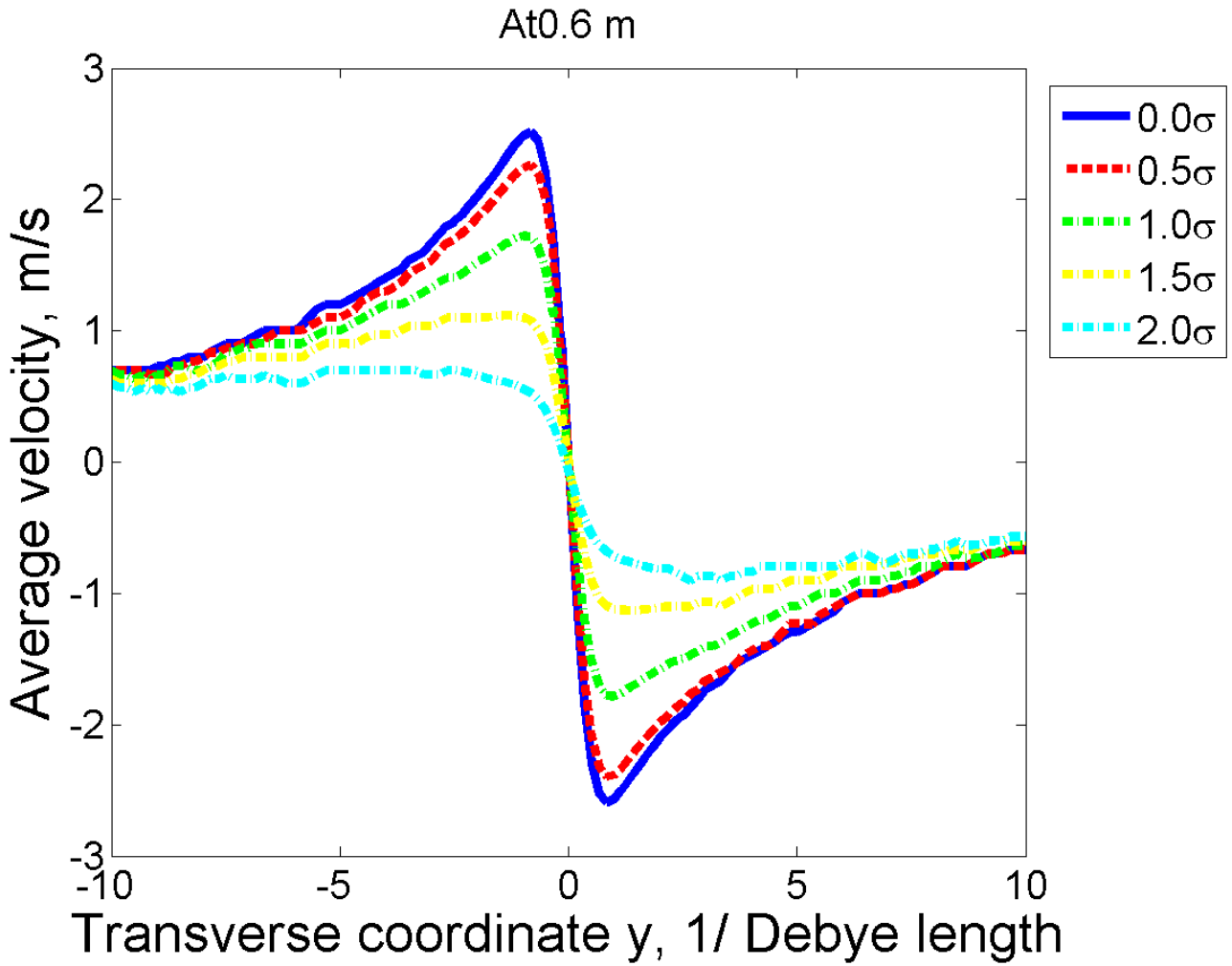

At2.4 m
Average velocity, m/s
Transverse coordinate x, 1/ Debye length
15
10
5
0
-5
-10
-15
-10
-5
0
5
10
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

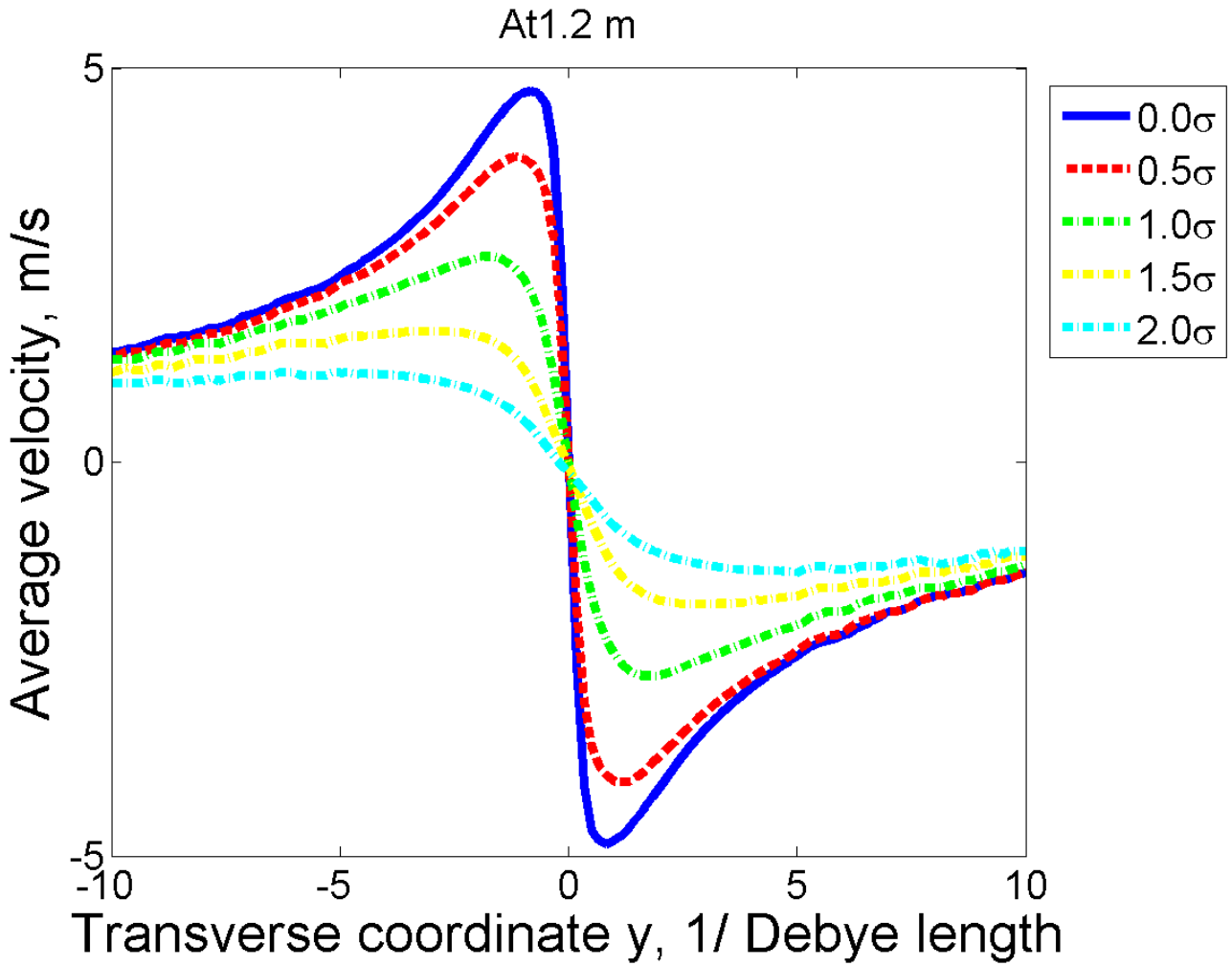

At0.6 m
Number of electrons per Debye length
Transverse coordinate y, 1/ Debye length
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-10
-5
0
5
10
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

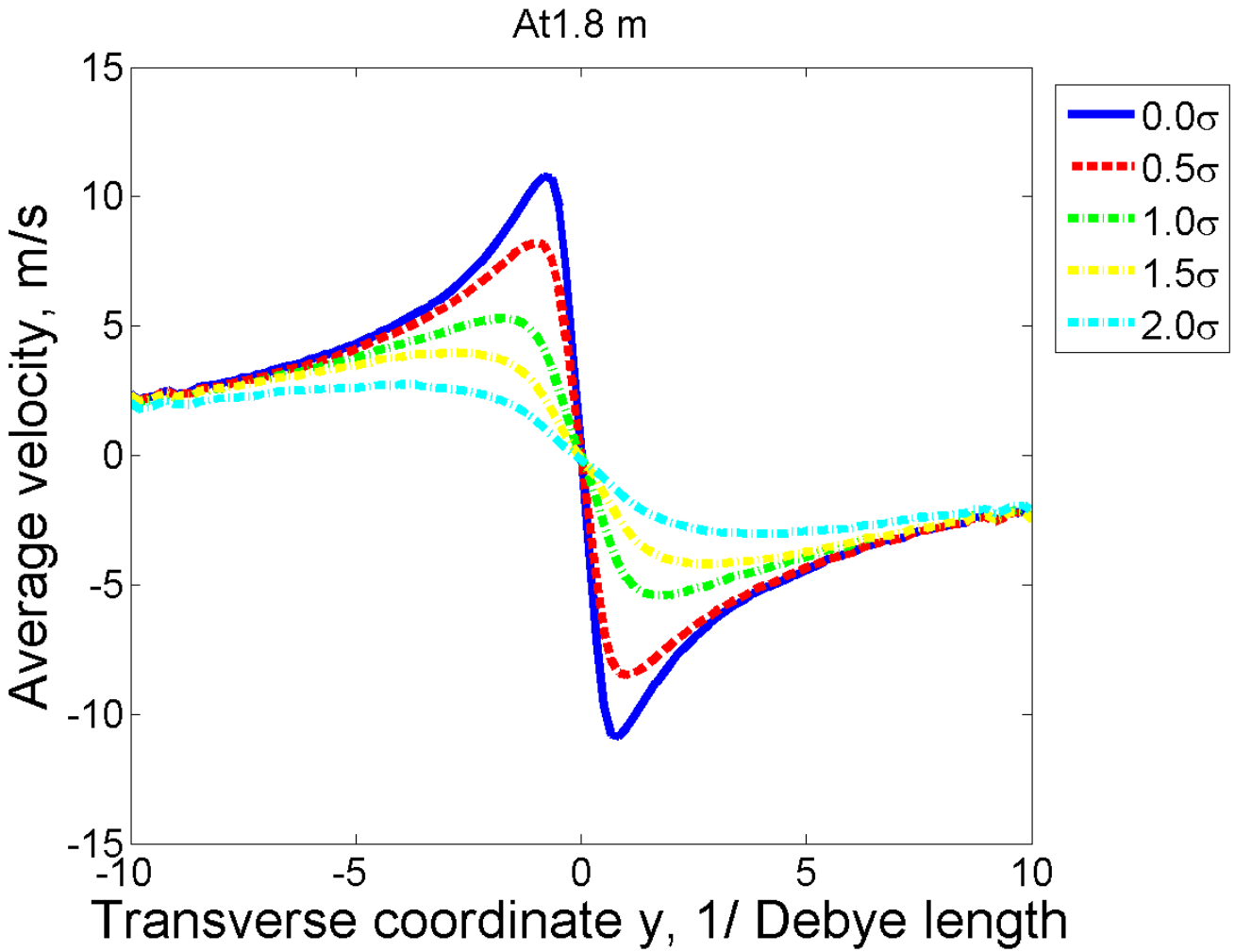

At1.2 m
Number of electrons per Debye length
Transverse coordinate y, 1/ Debye length
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-10
-5
0
5
10
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

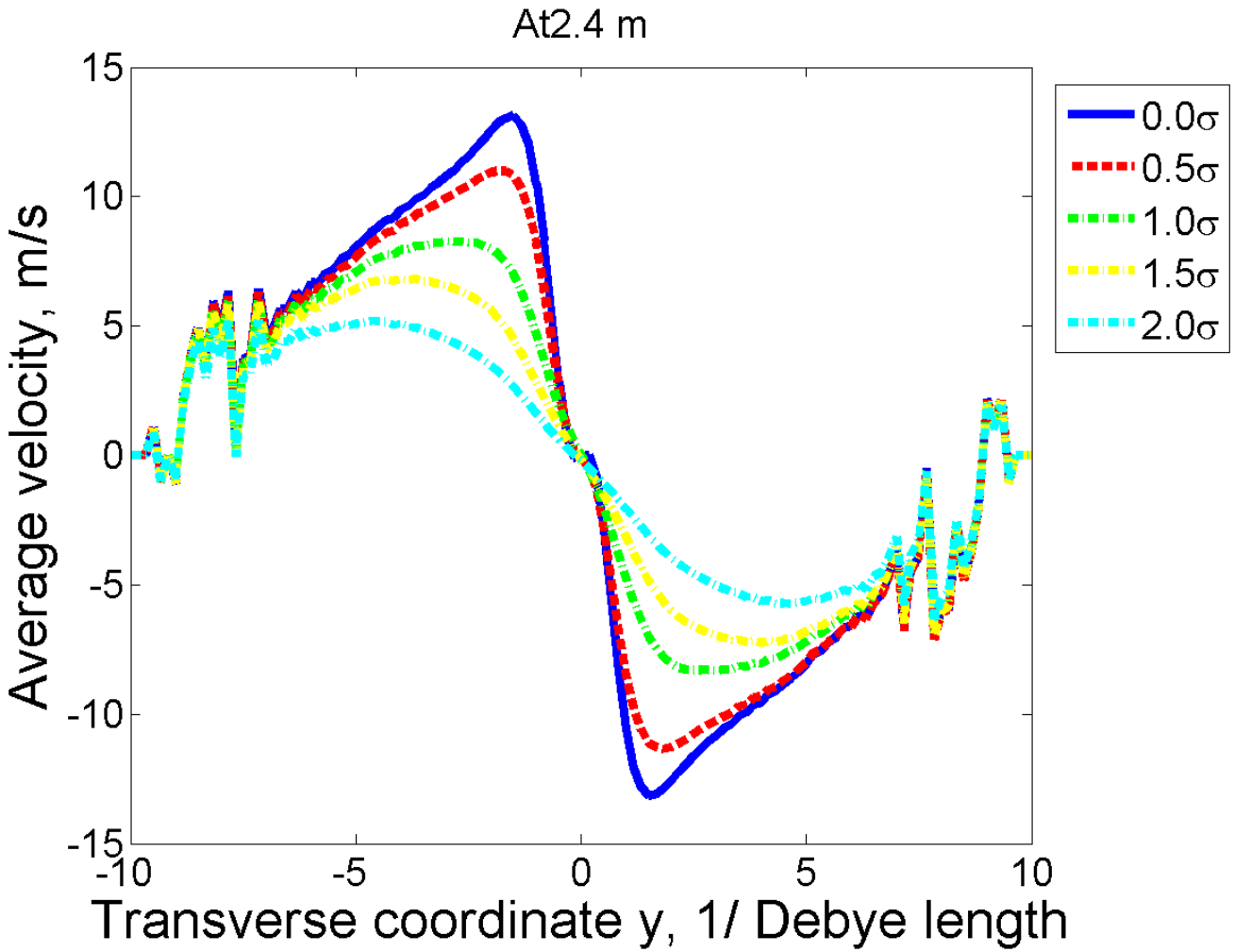

At1.8 m
Number of electrons per Debye length
Transverse coordinate y, 1/ Debye length
6
5
4
3
2
1
0
-1
-10
-5
0
5
10
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

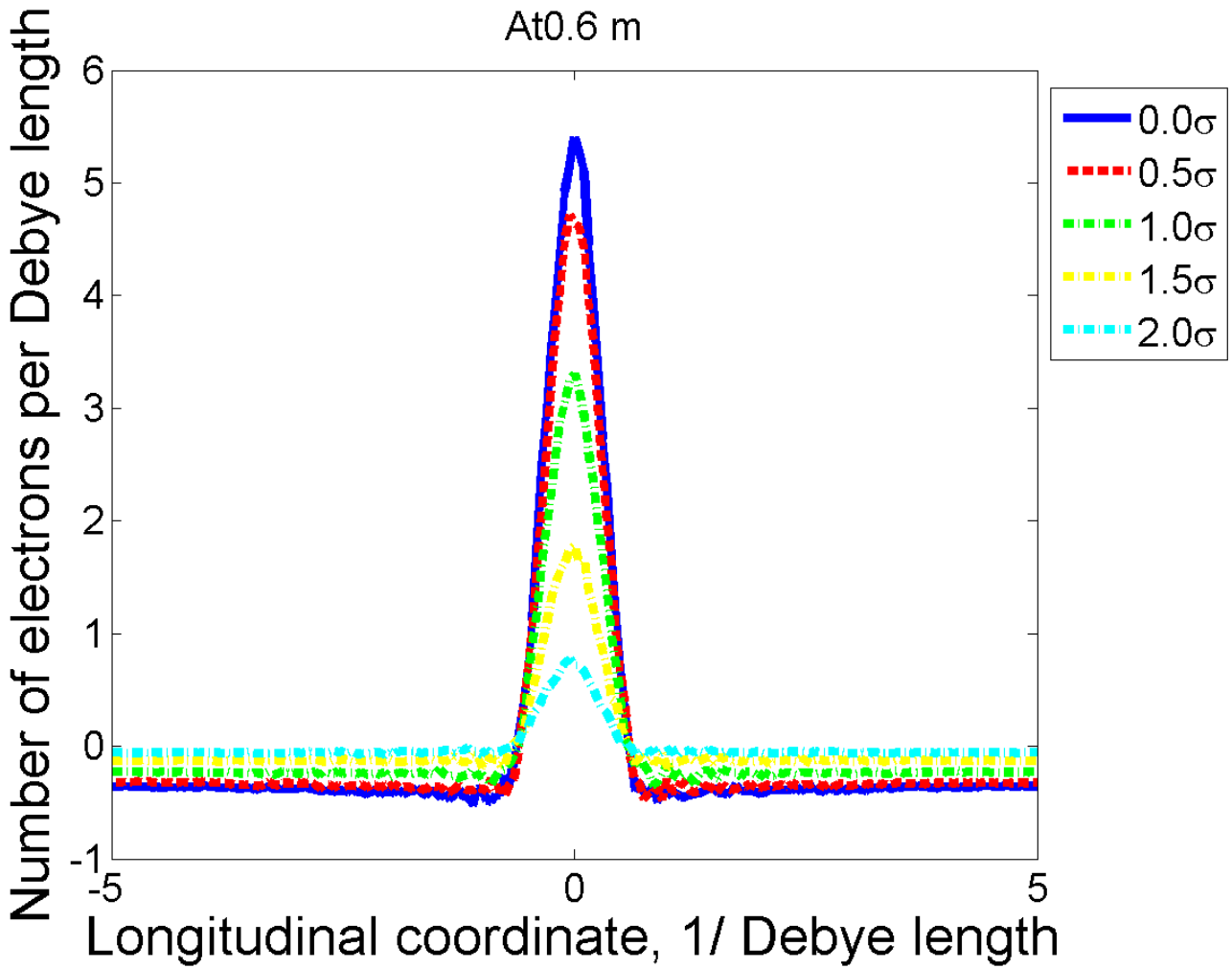

At2.4 m
Number of electrons per Debye length
Transverse coordinate y, 1/ Debye length
8
6
4
2
0
-2
-10
-5
0
5
10
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

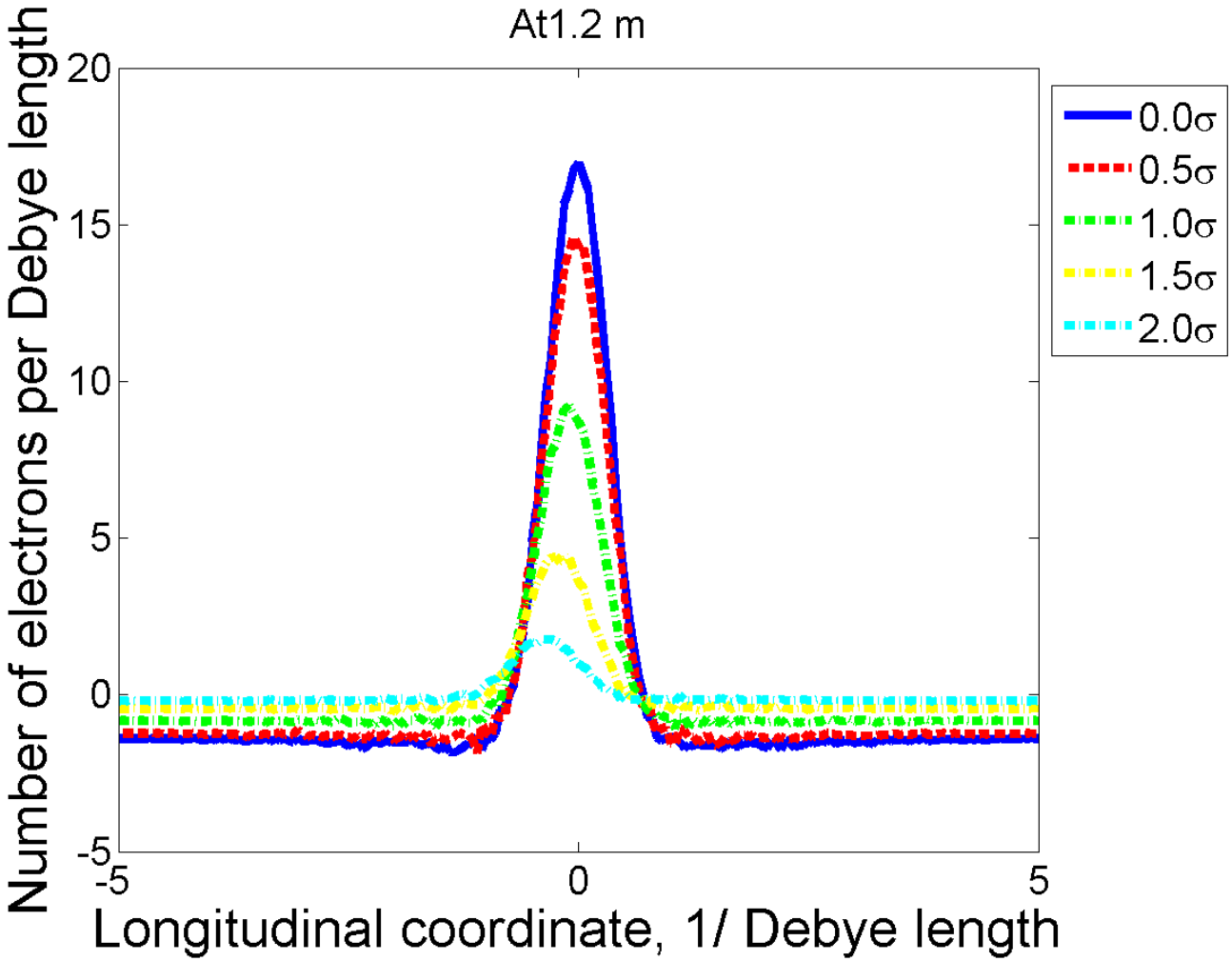

At0.6 m
Average velocity, m/s
Transverse coordinate y, 1/ Debye length
3
2
1
0
-1
-2
-3
-10
-5
0
5
10
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

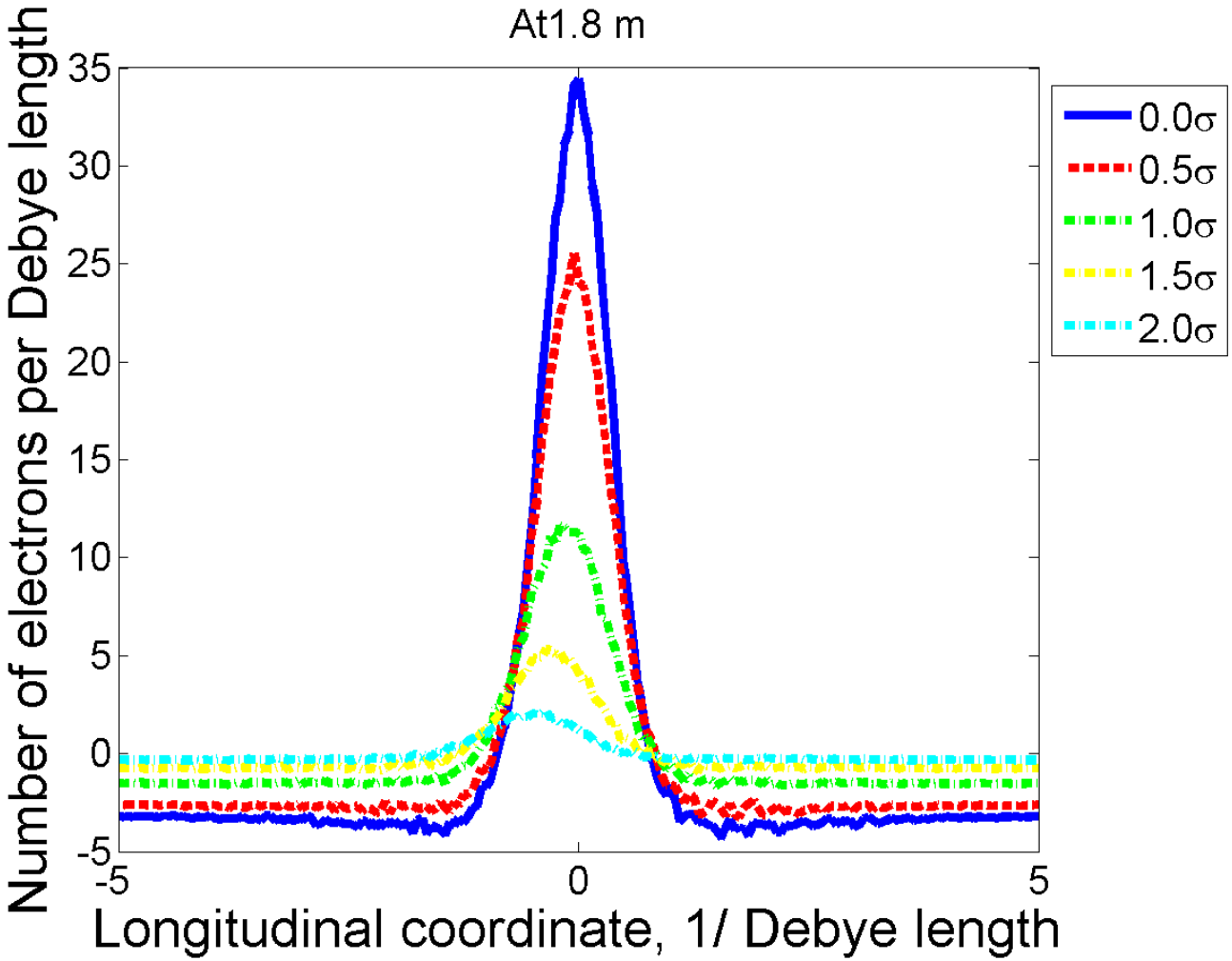

At1.2 m
Average velocity, m/s
Transverse coordinate y, 1/ Debye length
5
0
-5
-10
-5
0
5
10
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

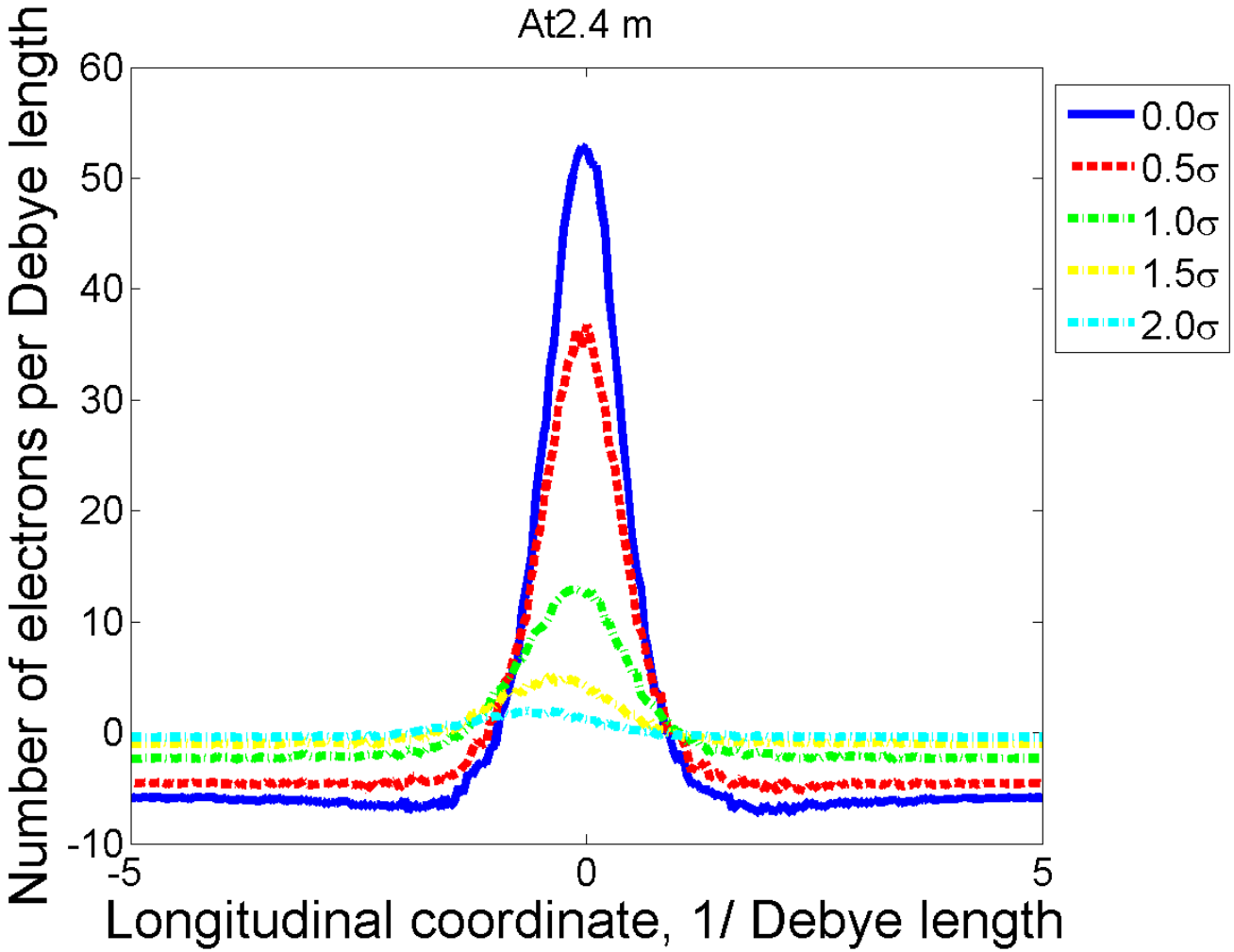

At1.8 m
Average velocity, m/s
Transverse coordinate y, 1/ Debye length
15
10
5
0
-5
-10
-15
-10
-5
0
5
10
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

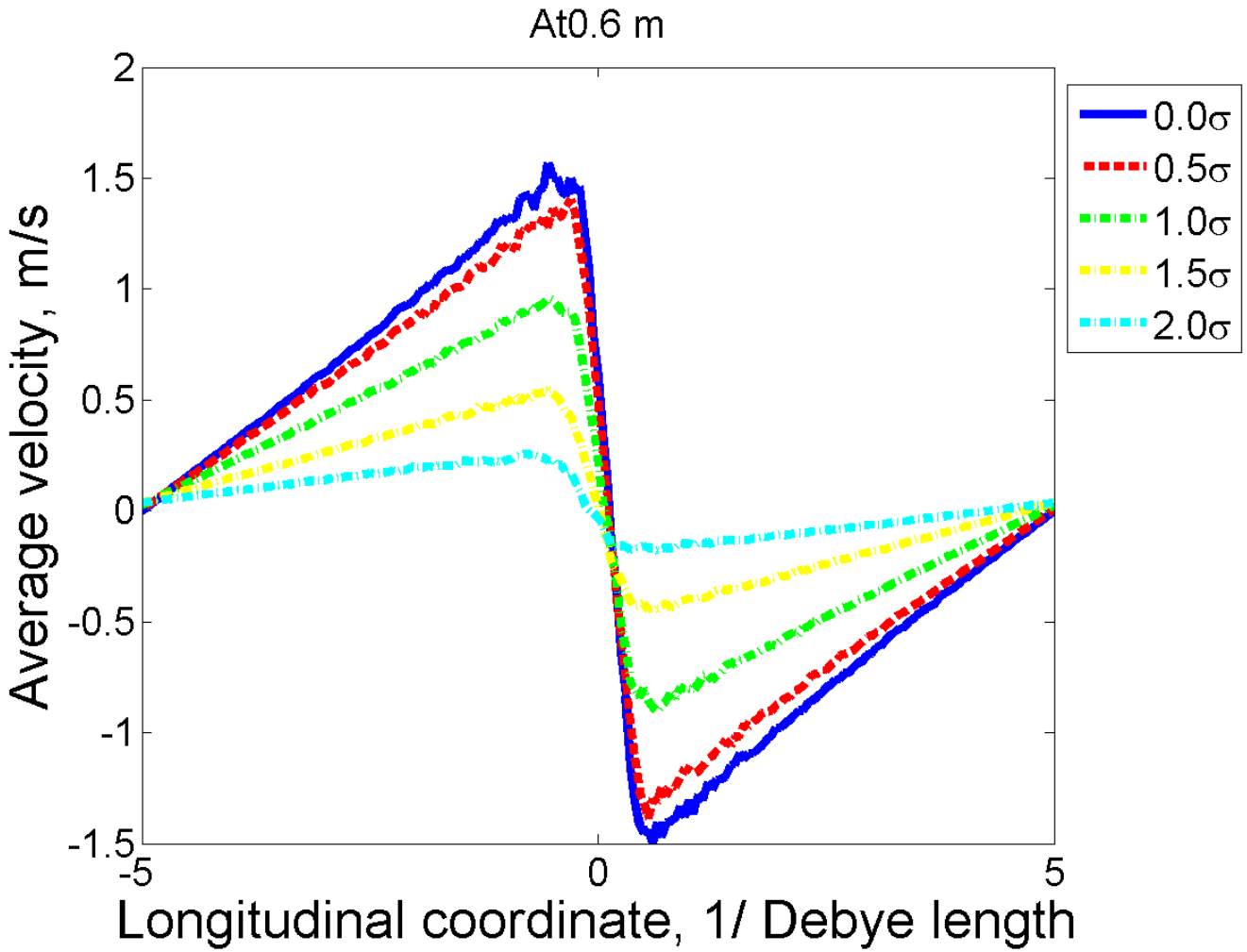

At2.4 m
Average velocity, m/s
Transverse coordinate y, 1/ Debye length
15
10
5
0
-5
-10
-15
-10
-5
0
5
10
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

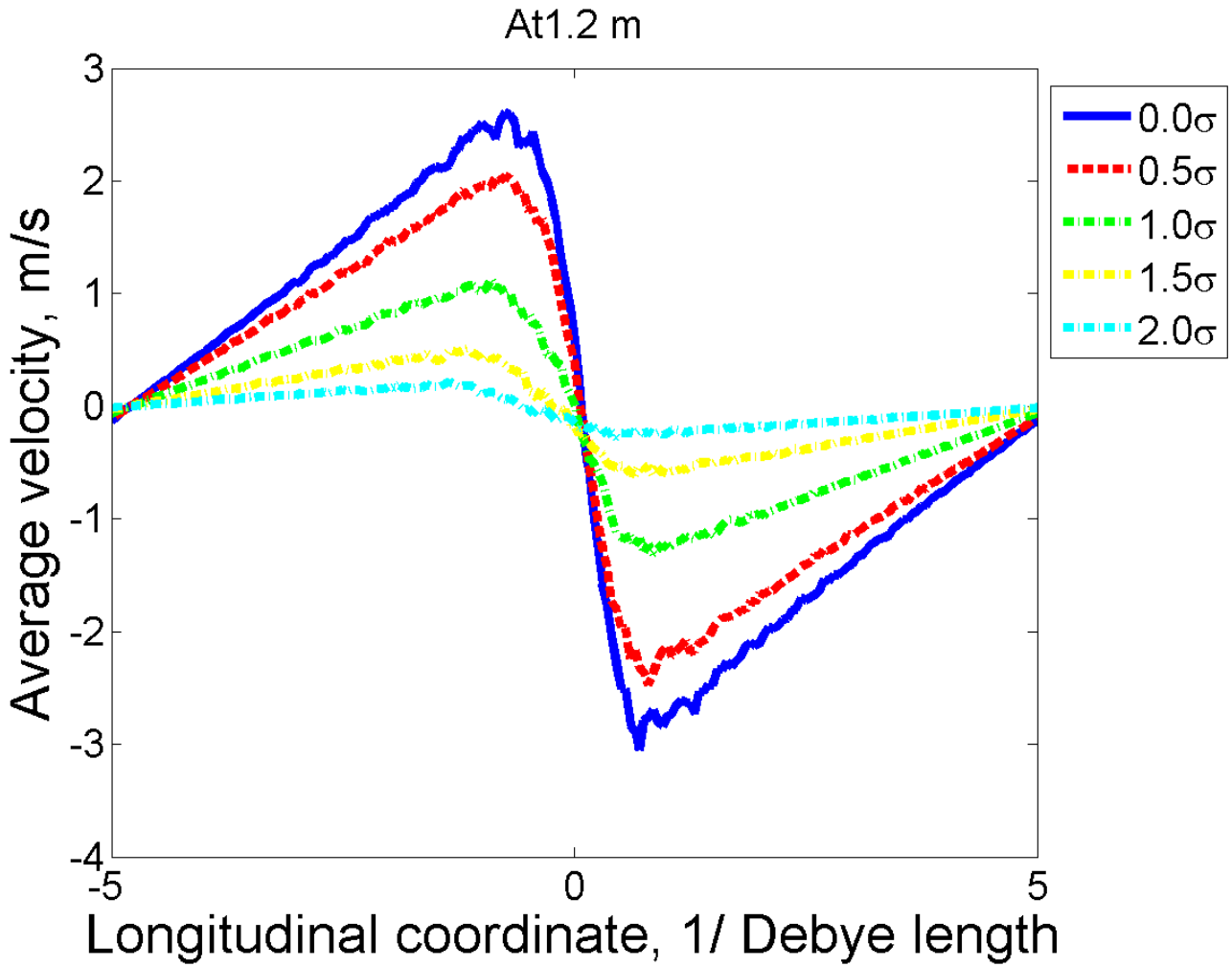

At0.6 m
Number of electrons per Debye length
Longitudinal coordinate, 1/ Debye length
6
5
4
3
2
1
0
-1
-5
0
5
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

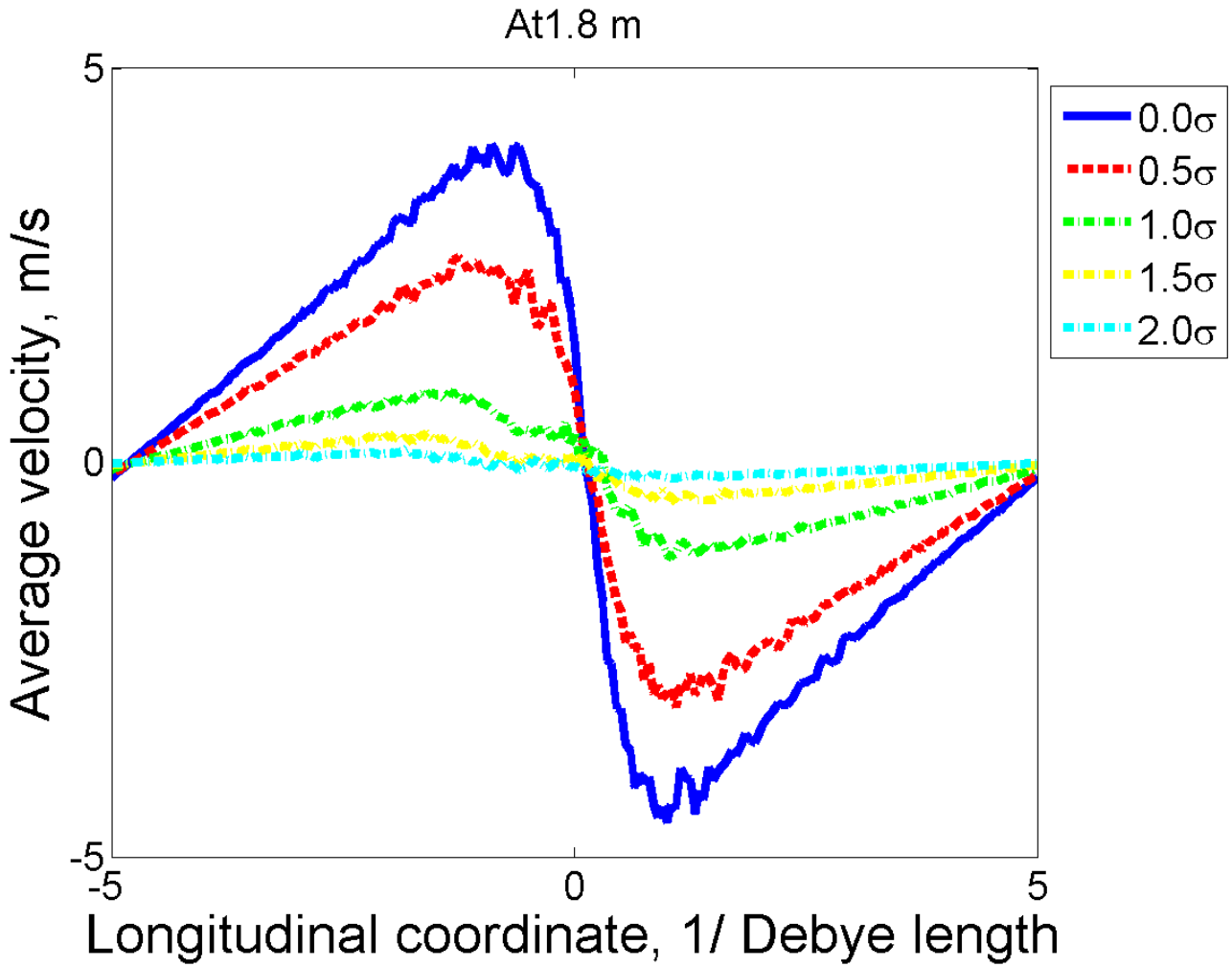

At1.2 m
Number of electrons per Debye length
Longitudinal coordinate, 1/ Debye length
20
15
10
5
0
-5
-5
0
5
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

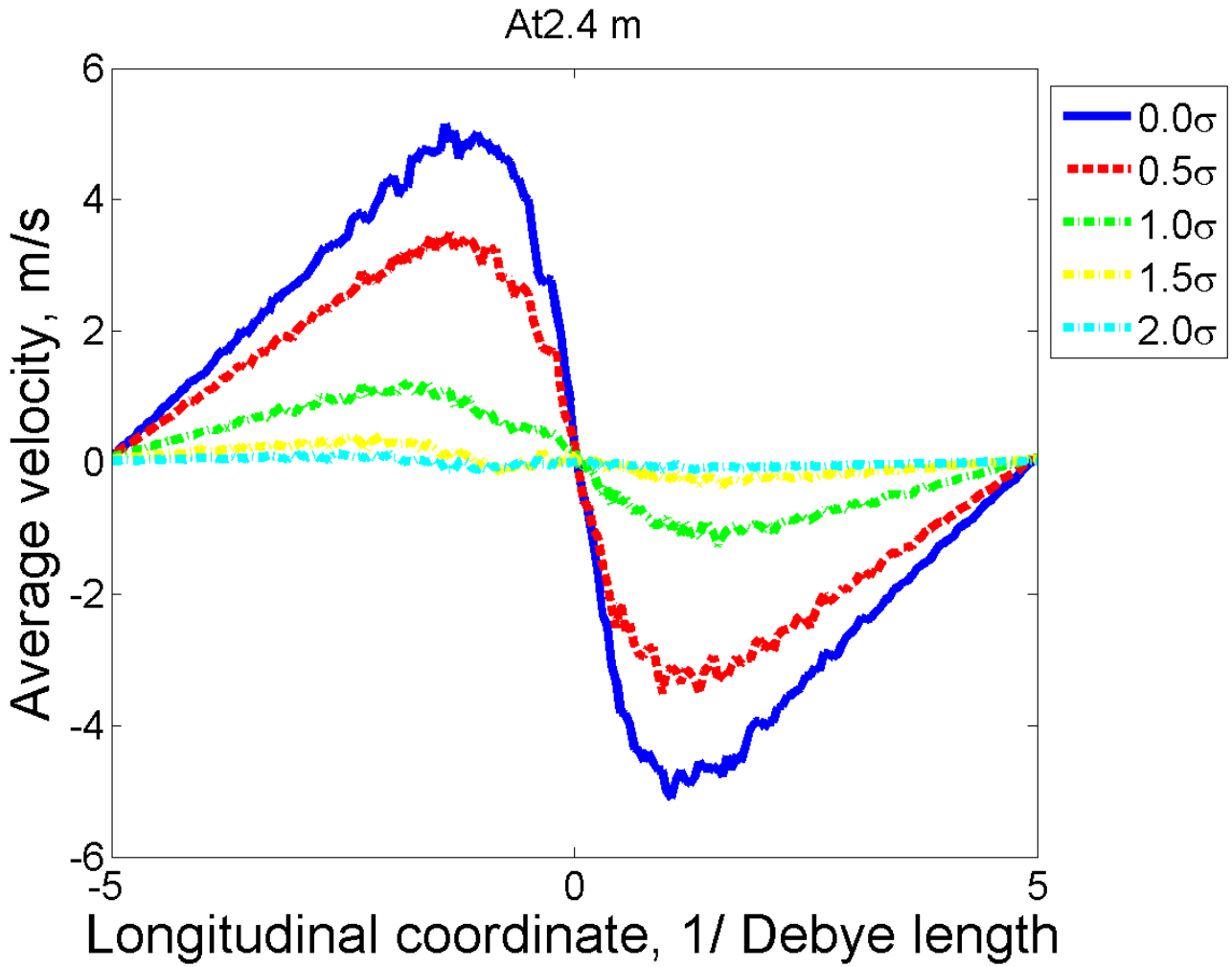

At1.8 m
Number of electrons per Debye length
Longitudinal coordinate, 1/ Debye length
35
30
25
20
15
10
5
0
-5
-5
0
5
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

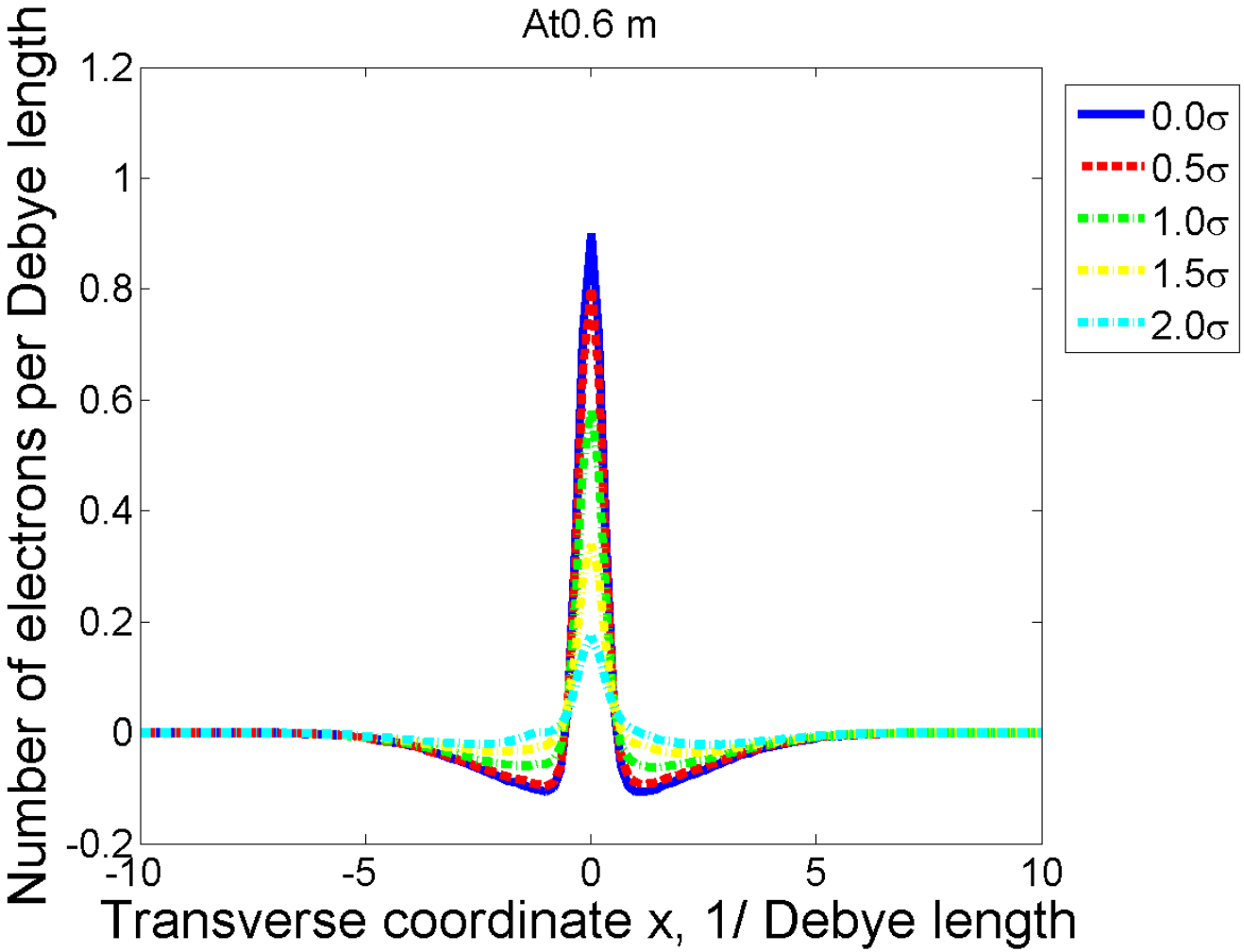

At2.4 m
Number of electrons per Debye length
Longitudinal coordinate, 1/ Debye length
60
50
40
30
20
10
0
-10
-5
0
5
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

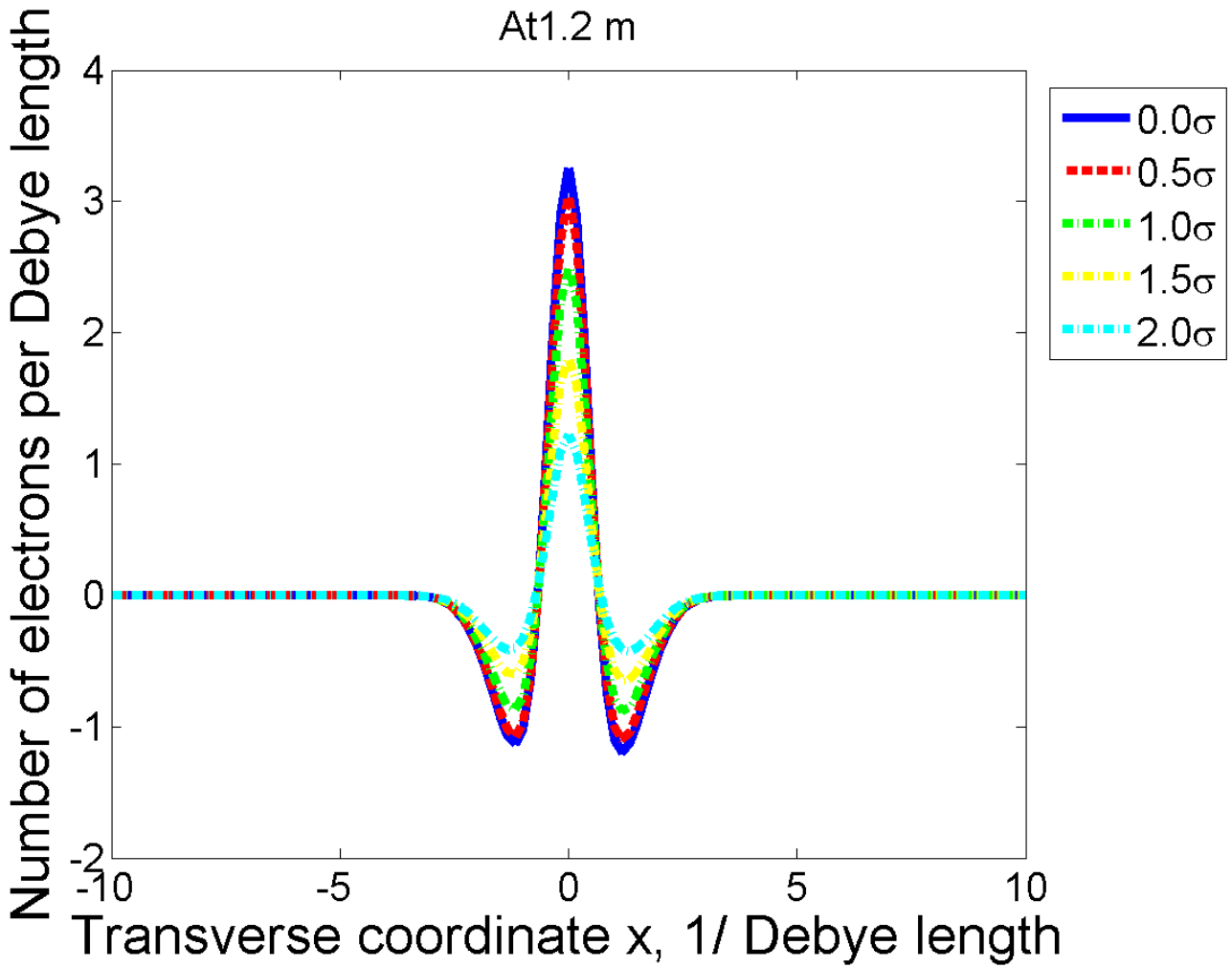

At0.6 m
Average velocity, m/s
Longitudinal coordinate, 1/ Debye length
2
1.5
1
0.5
0
-0.5
-1
-1.5
-5
0
5
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

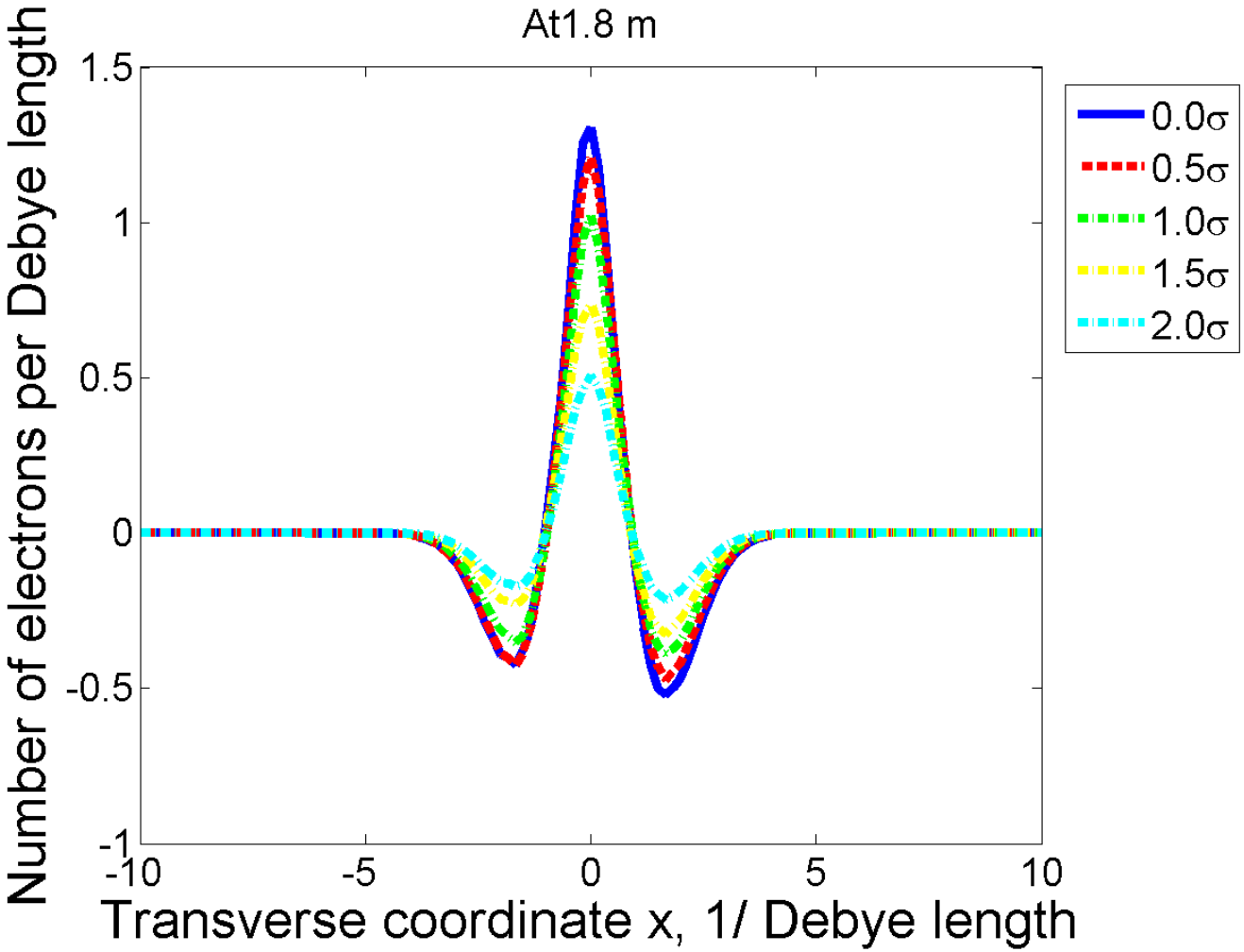

At1.2 m
Average velocity, m/s
Longitudinal coordinate, 1/ Debye length
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ
3
2
1
0
-1
-2
-3
-4
-5
0
5

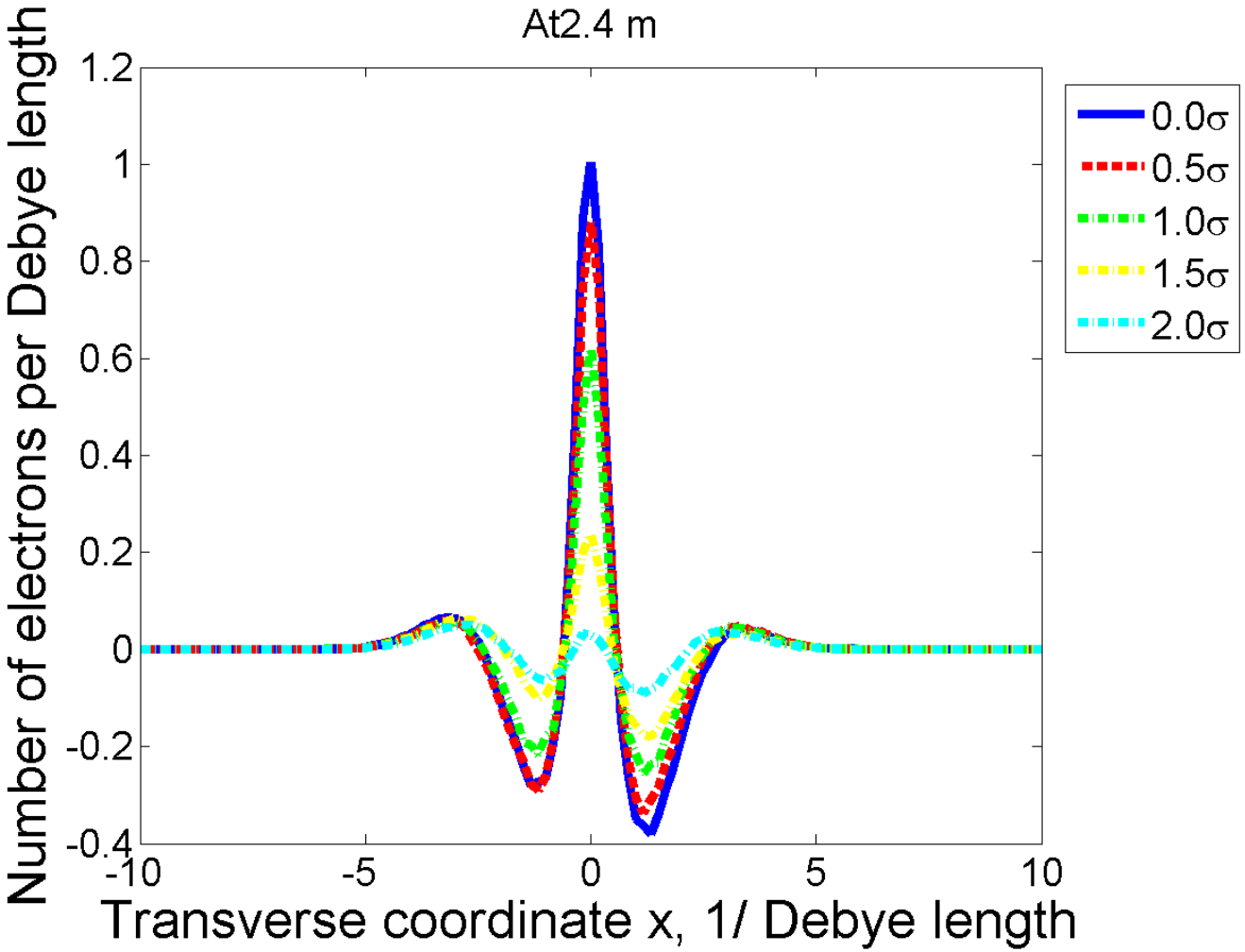

At1.8 m
Average velocity, m/s
Longitudinal coordinate, 1/ Debye length
5
0
-5
-5
0
5
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

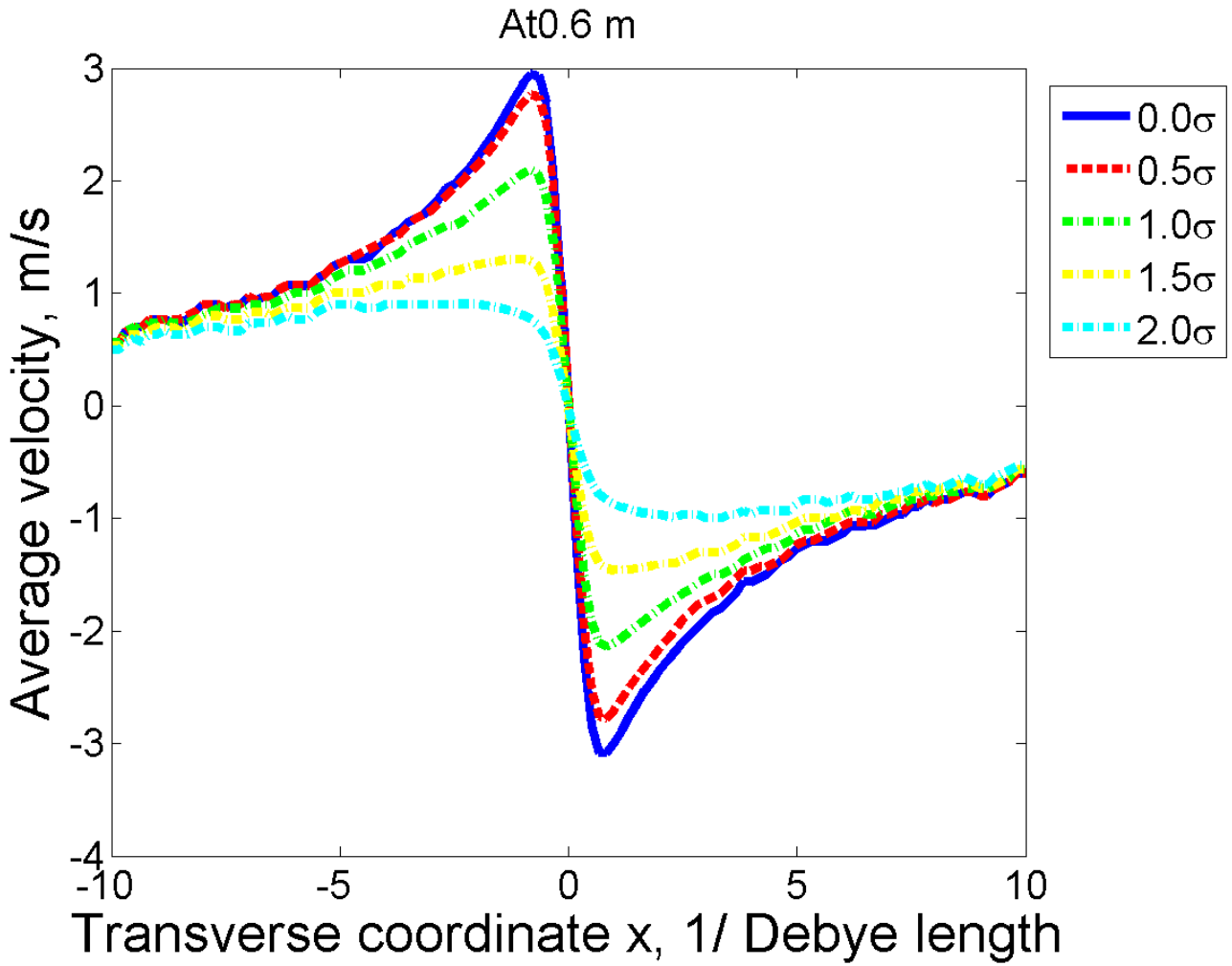

At2.4 m
Average velocity, m/s
Longitudinal coordinate, 1/ Debye length
6
4
2
0
-2
-4
-6
-5
0
5
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

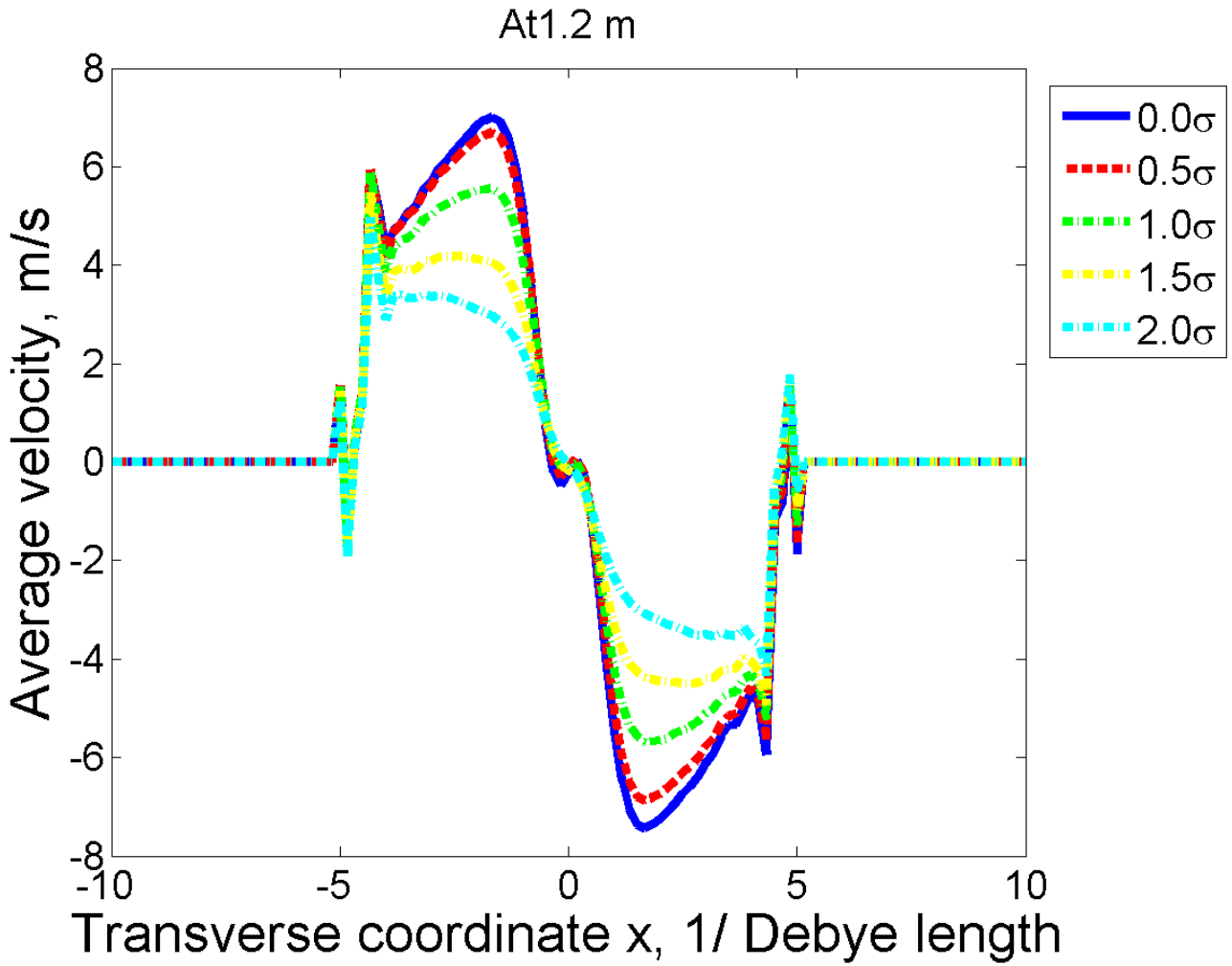

At0.6 m
Number of electrons per Debye length
Transverse coordinate x, 1/ Debye length
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-10
-5
0
5
10
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

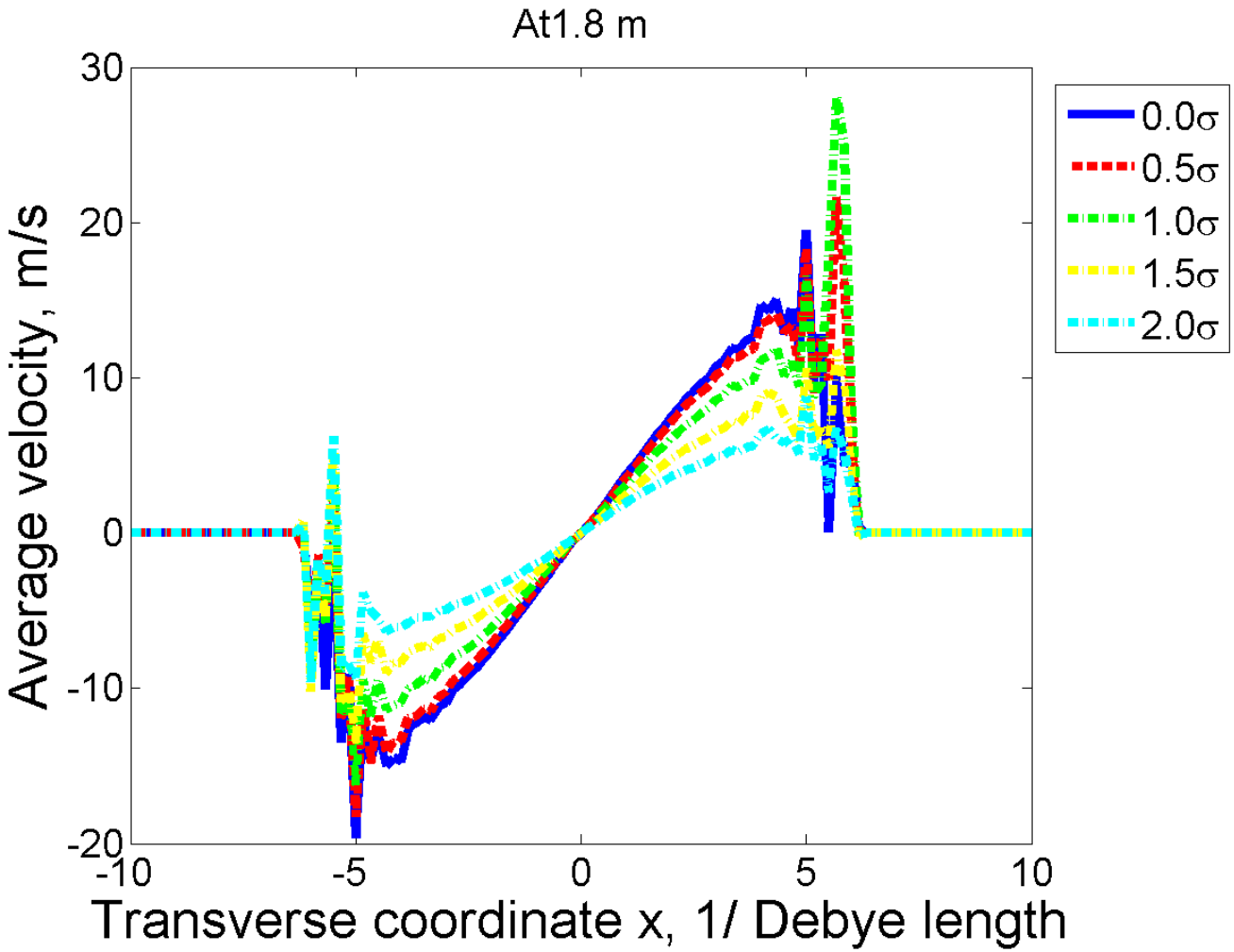

At1.2 m
Number of electrons per Debye length
Transverse coordinate x, 1/ Debye length
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ
4
3
2
1
0
-1
-2
-10
-5
0
5
10

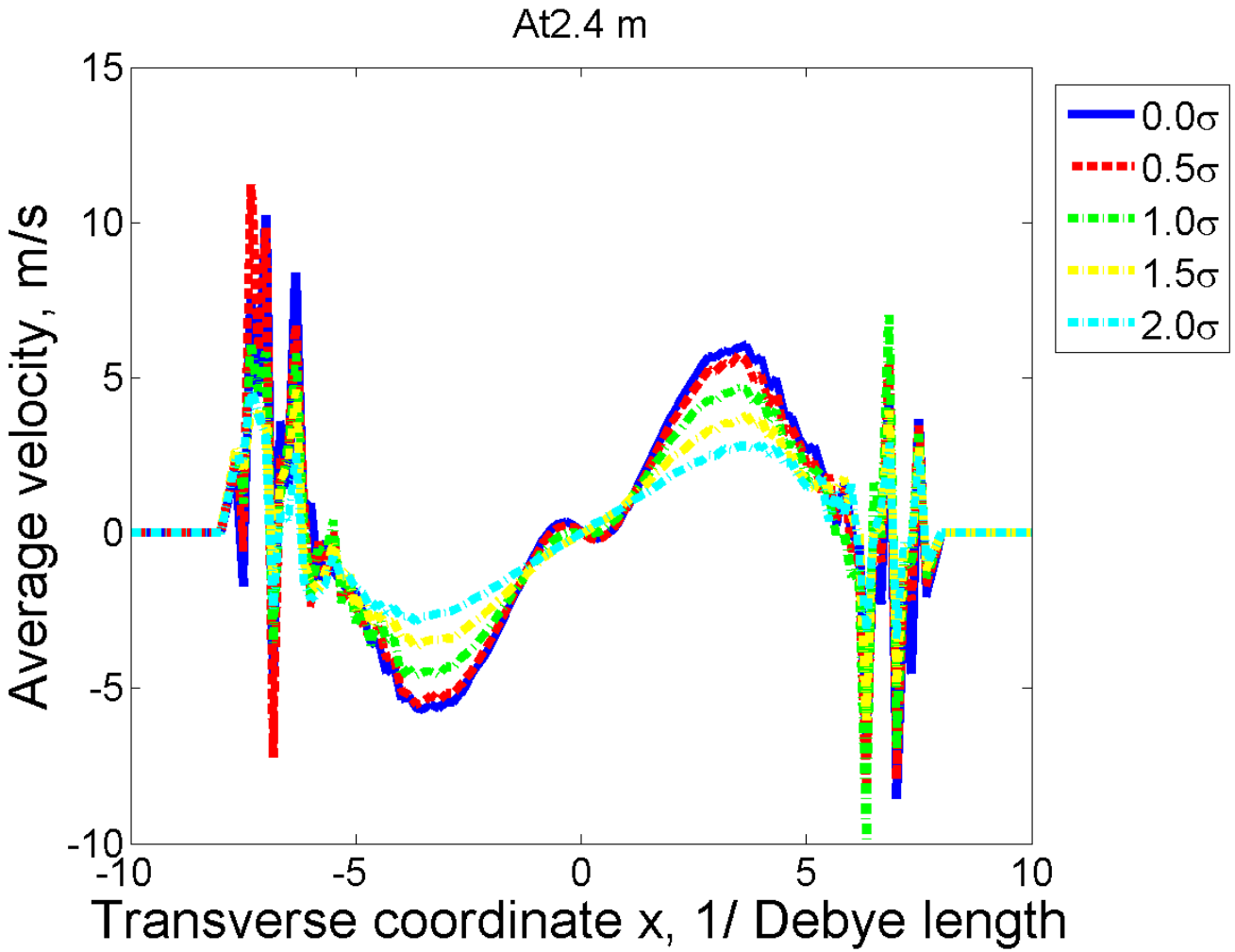

At1.8 m
Number of electrons per Debye length
Transverse coordinate x, 1/ Debye length
1.5
1
0.5
0
-0.5
-1
-10
-5
0
5
10
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

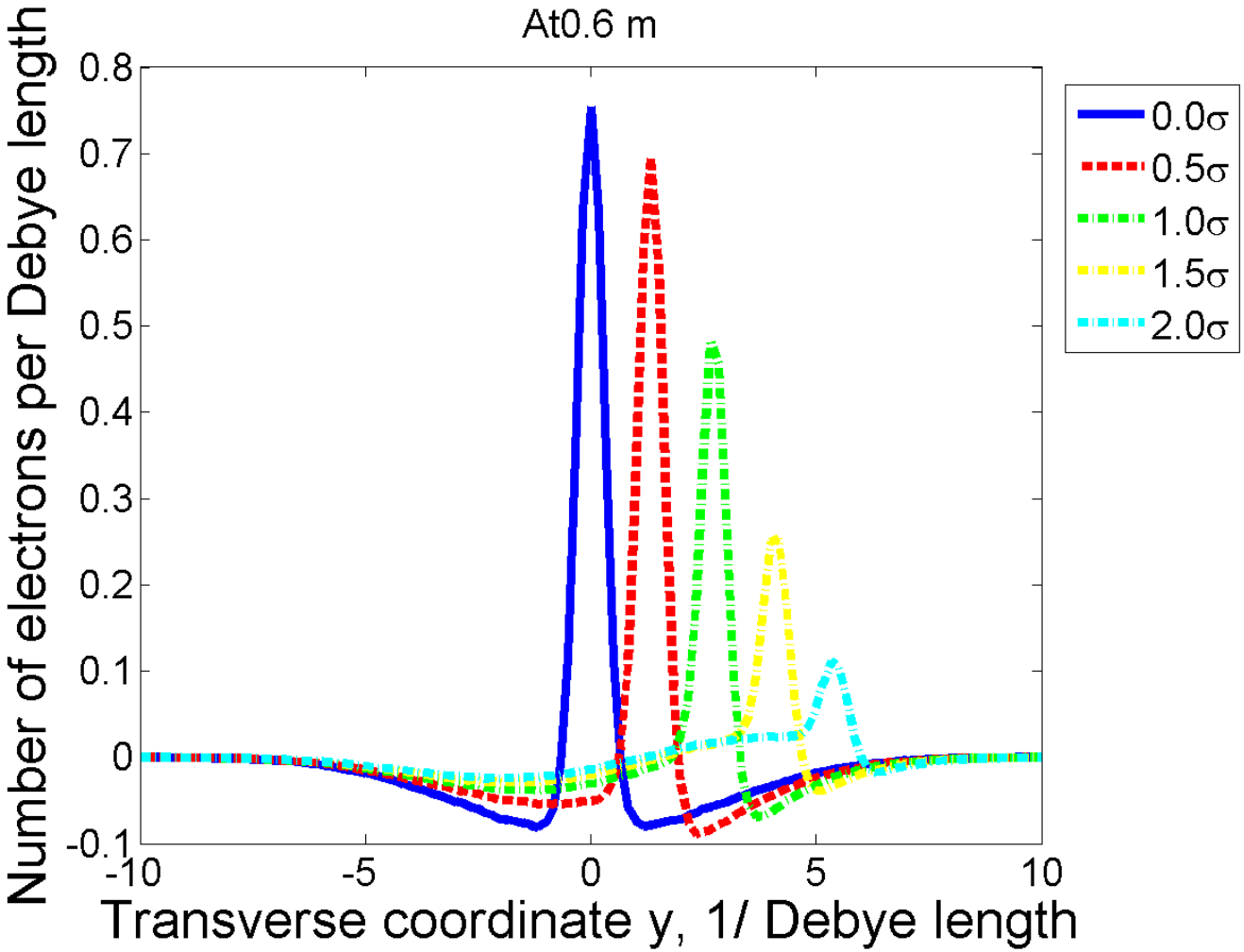

At2.4 m
Number of electrons per Debye length
Transverse coordinate x, 1/ Debye length
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-10
-5
0
5
10
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

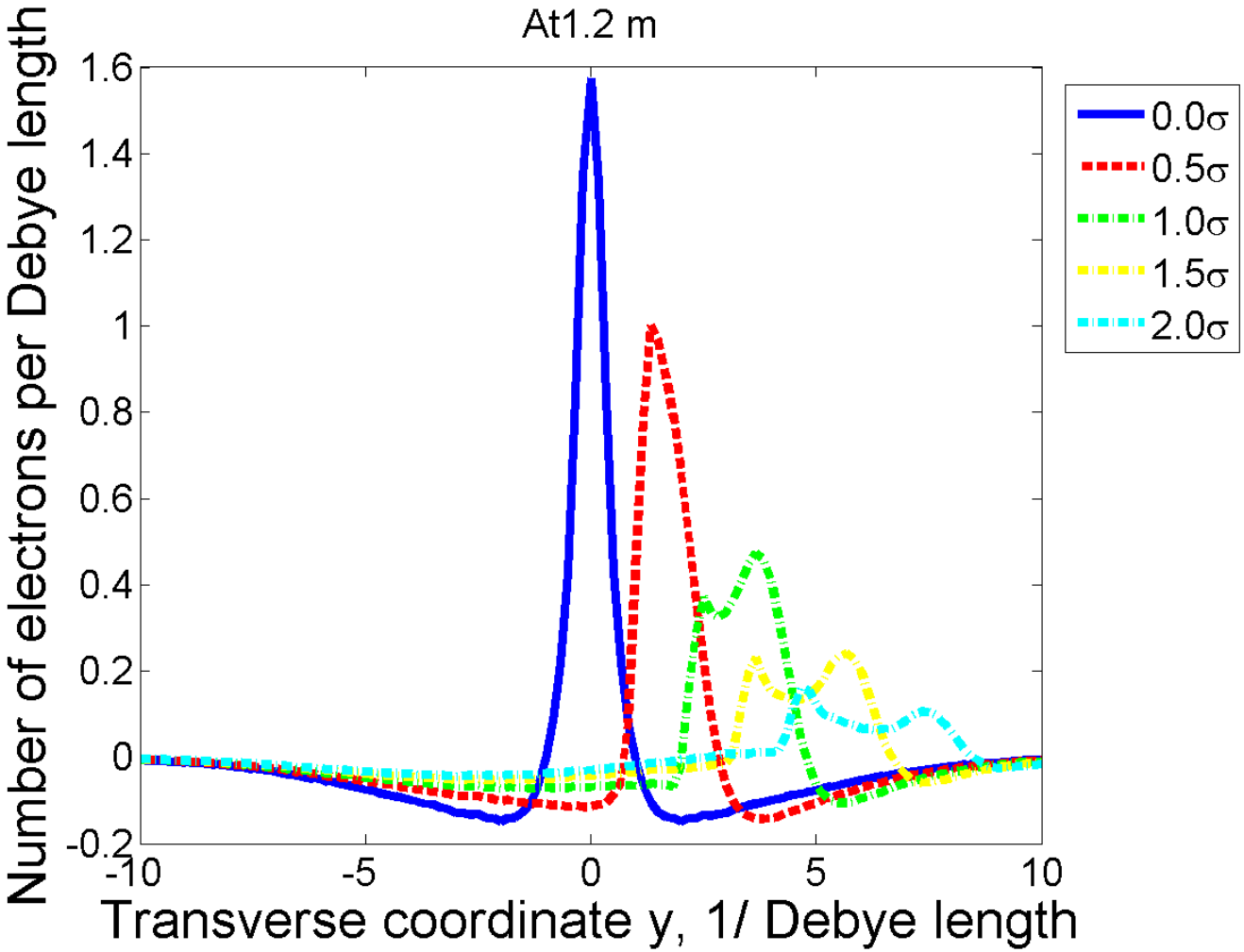

At0.6 m
Average velocity, m/s
Transverse coordinate x, 1/ Debye length
3
2
1
0
-1
-2
-3
-4
-10
-5
0
5
10
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

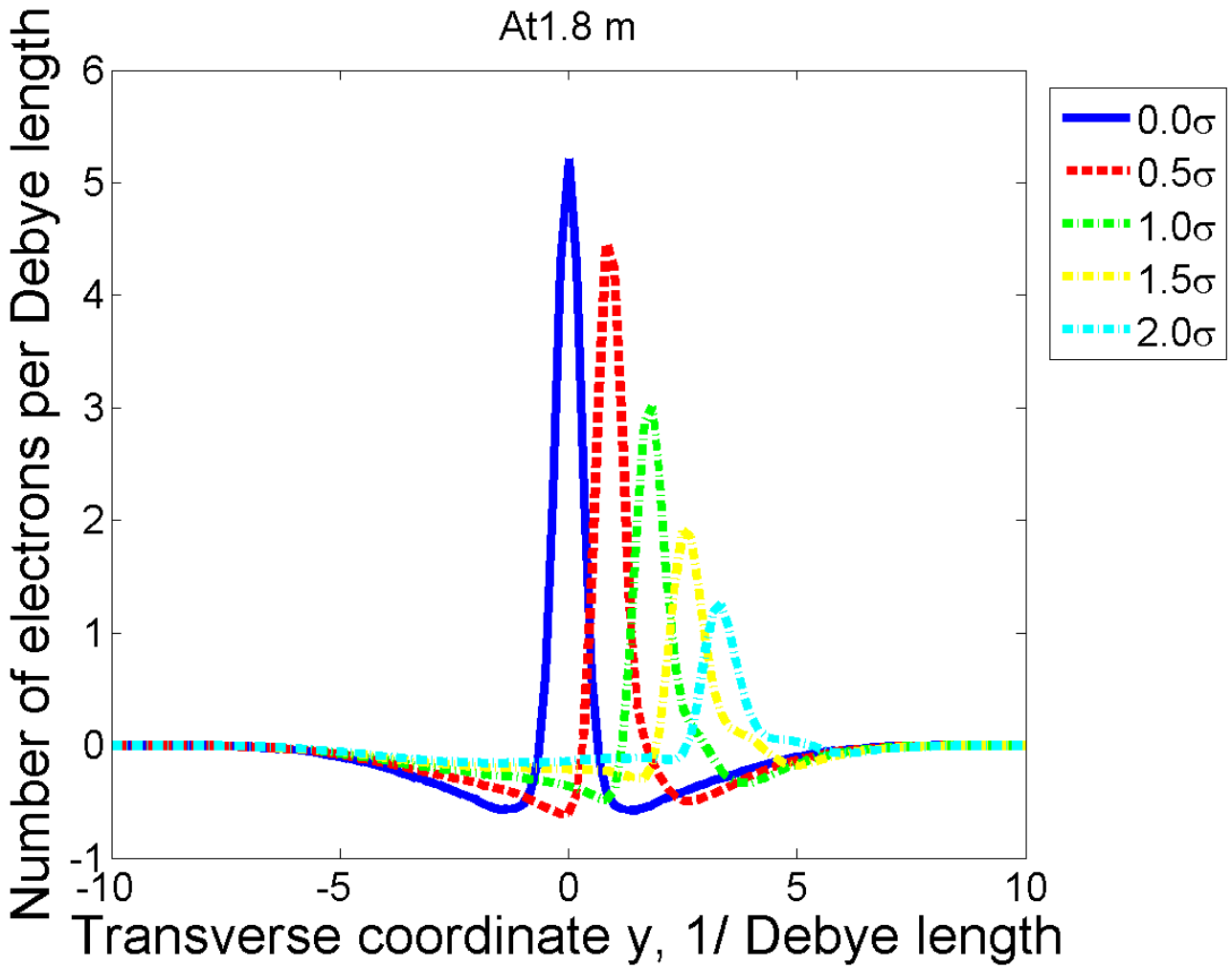

At1.2 m
Average velocity, m/s
Transverse coordinate x, 1/ Debye length
8
6
4
2
0
-2
-4
-6
-8
-10
-5
0
5
10
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

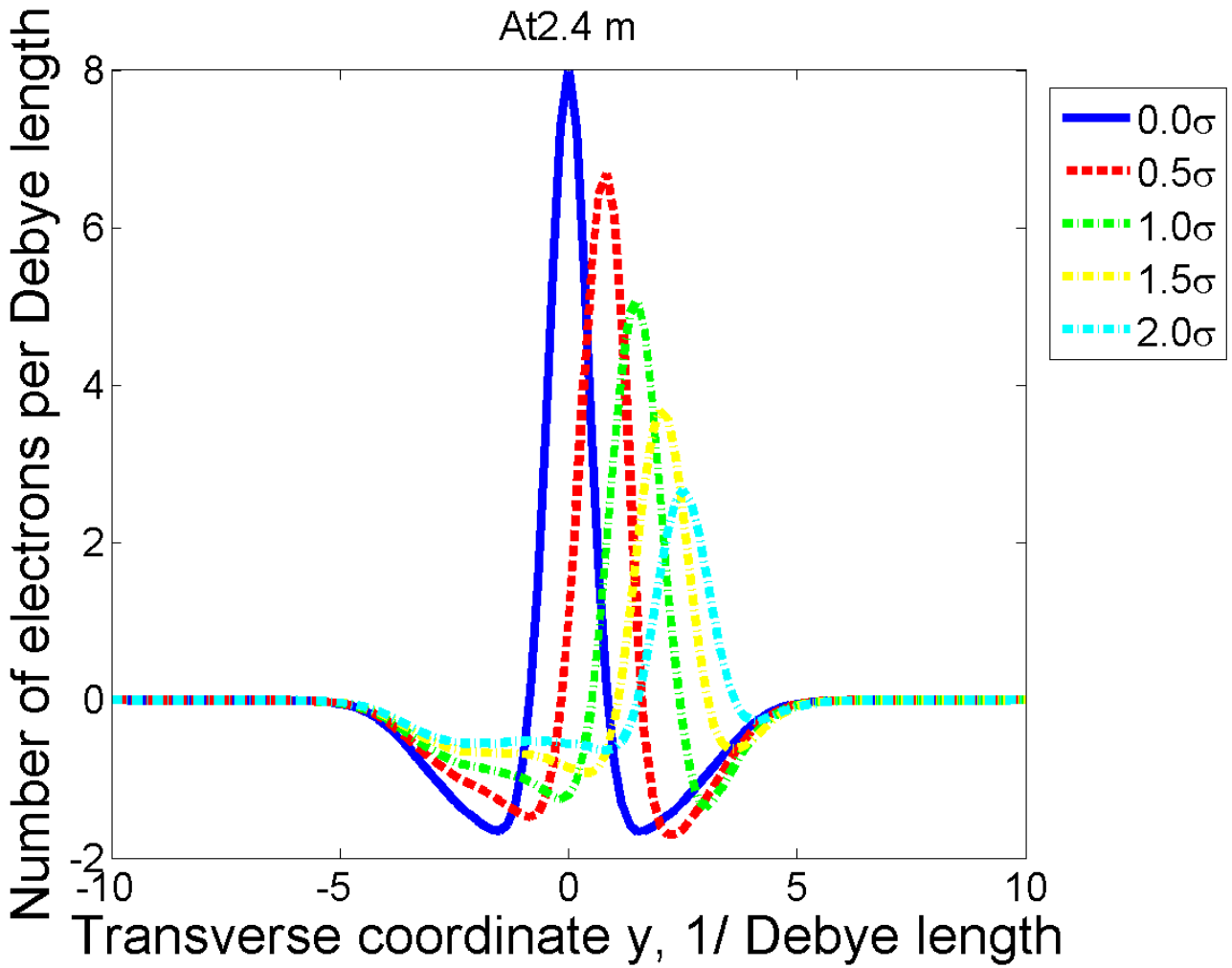

At1.8 m
Average velocity, m/s
Transverse coordinate x, 1/ Debye length
30
20
10
0
-10
-20
-10
-5
0
5
10
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

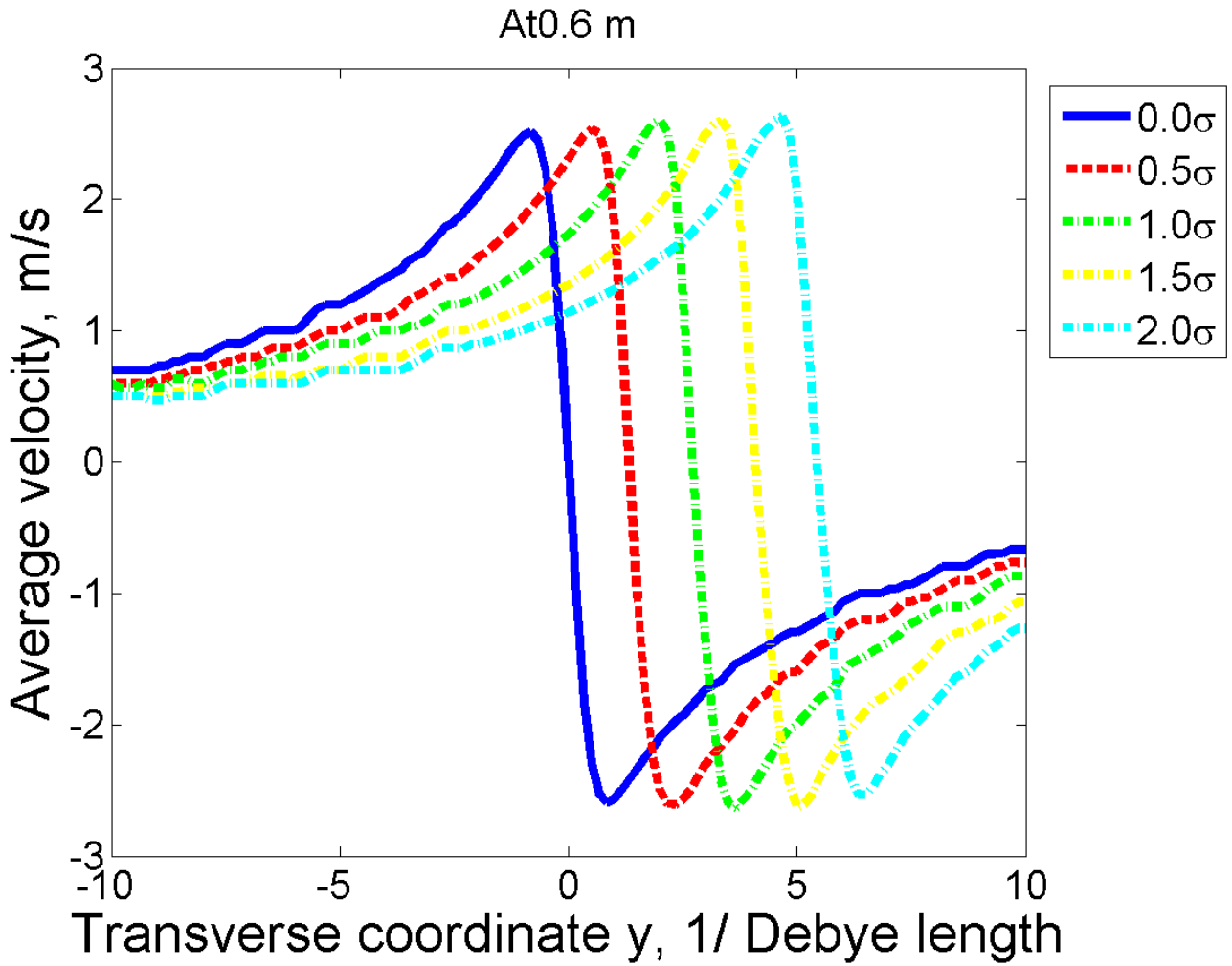

At2.4 m
Average velocity, m/s
Transverse coordinate x, 1/ Debye length
15
10
5
0
-5
-10
-10
-5
0
5
10
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

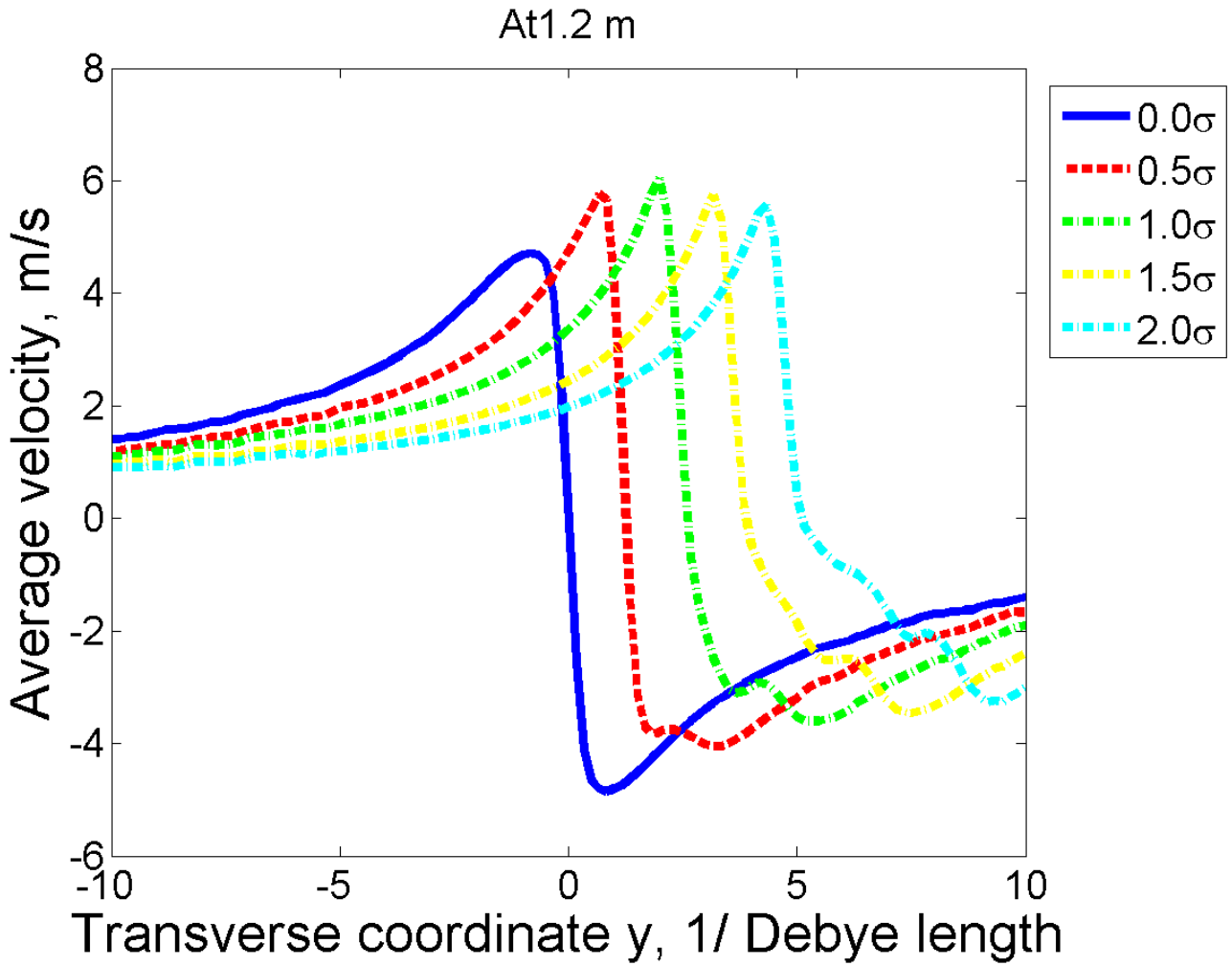

At0.6 m
Number of electrons per Debye length
Transverse coordinate y, 1/ Debye length
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-10
-5
0
5
10
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

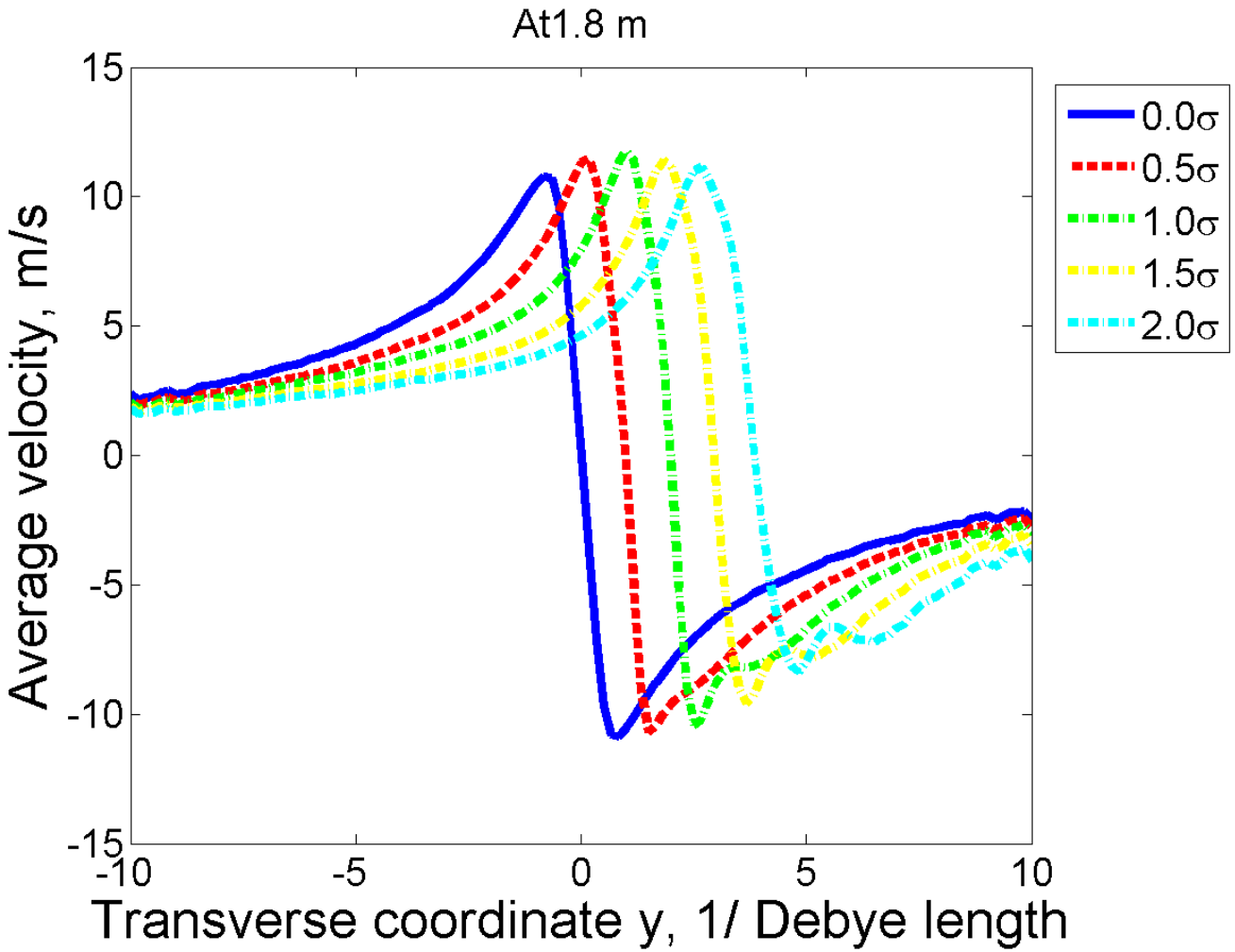

At1.2 m
Number of electrons per Debye length
Transverse coordinate y, 1/ Debye length
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-10
-5
0
5
10
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

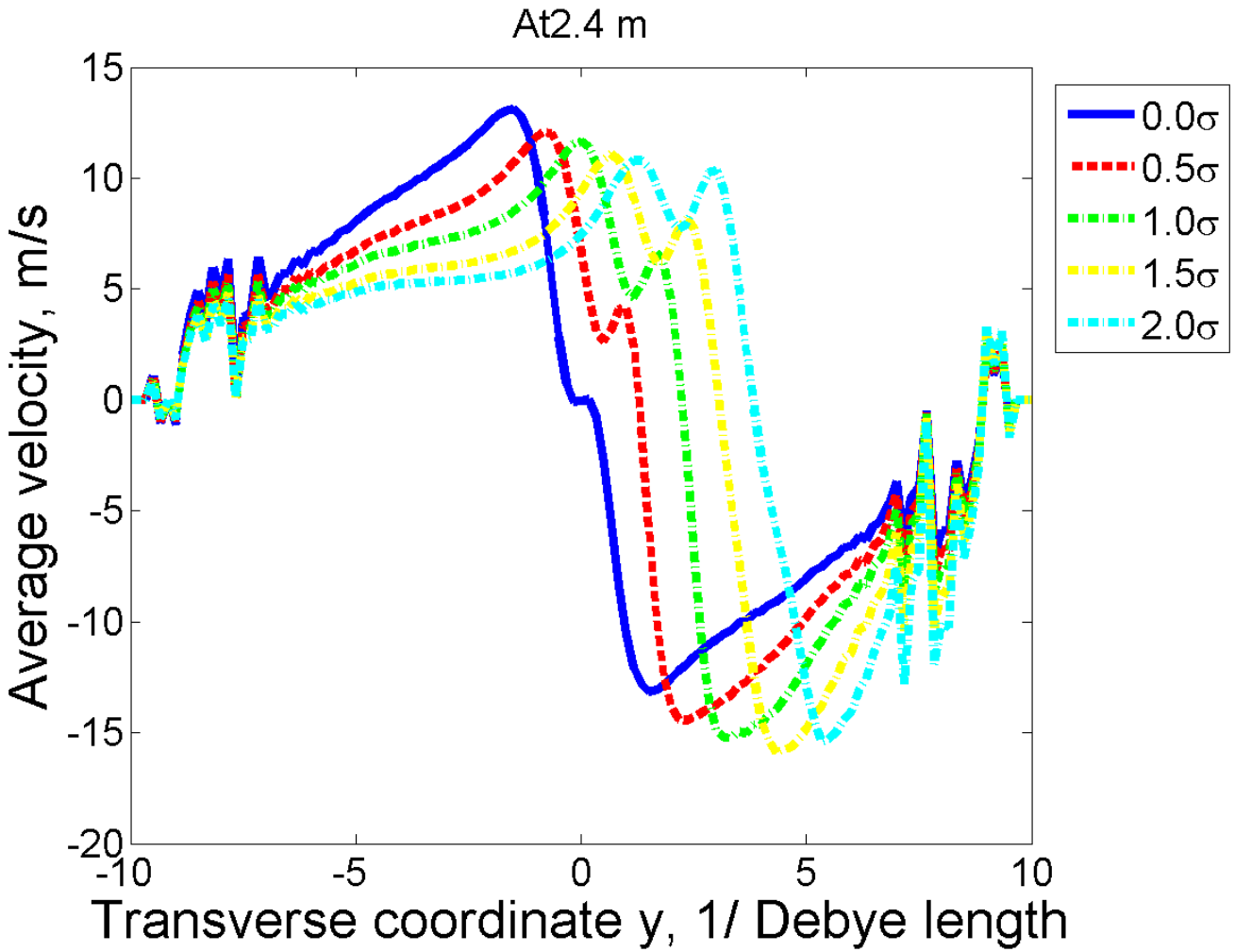

At1.8 m
Number of electrons per Debye length
Transverse coordinate y, 1/ Debye length
6
5
4
3
2
1
0
-1
-10
-5
0
5
10
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

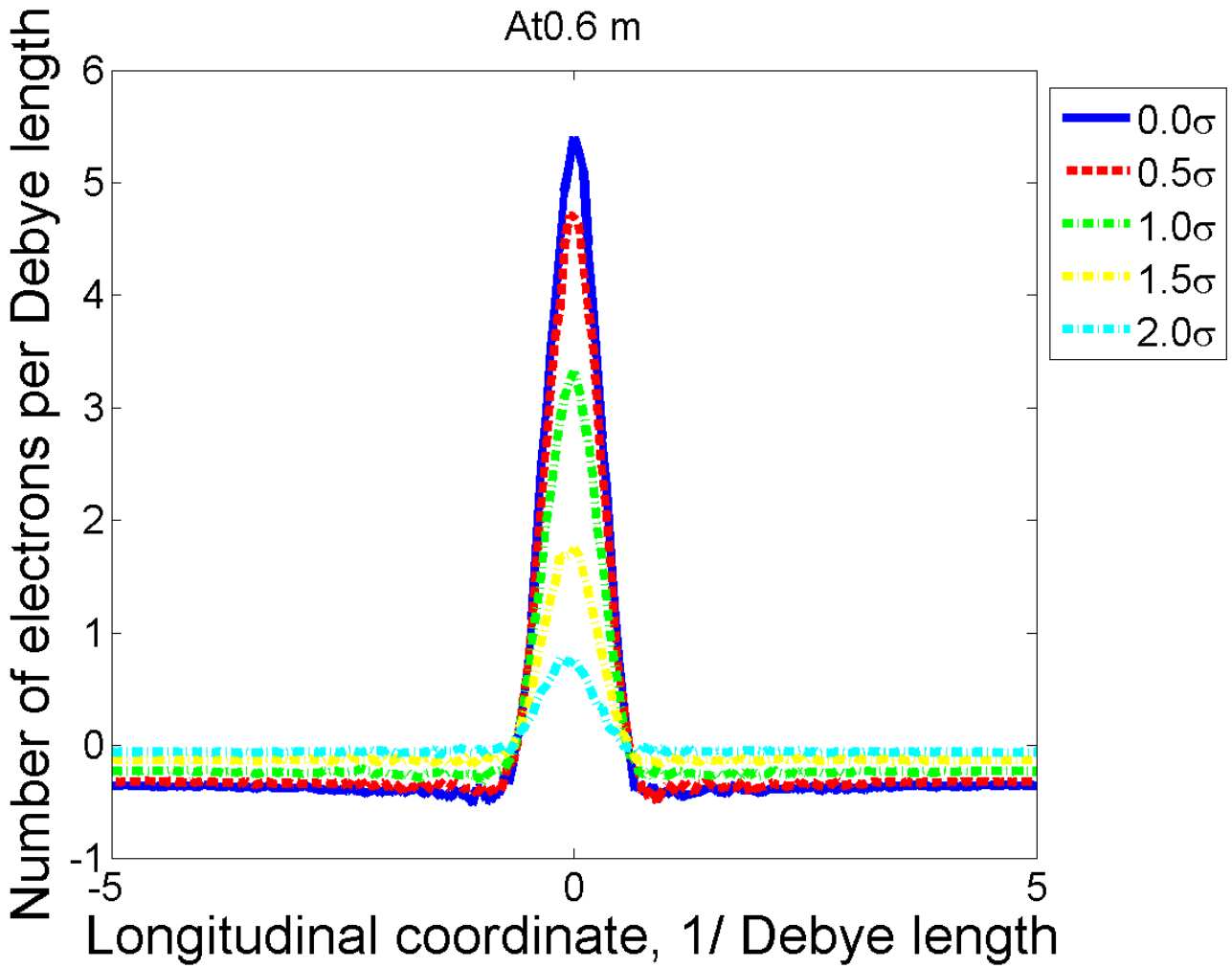

At2.4 m
Number of electrons per Debye length
Transverse coordinate y, 1/ Debye length
8
6
4
2
0
-2
-10
-5
0
5
10
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

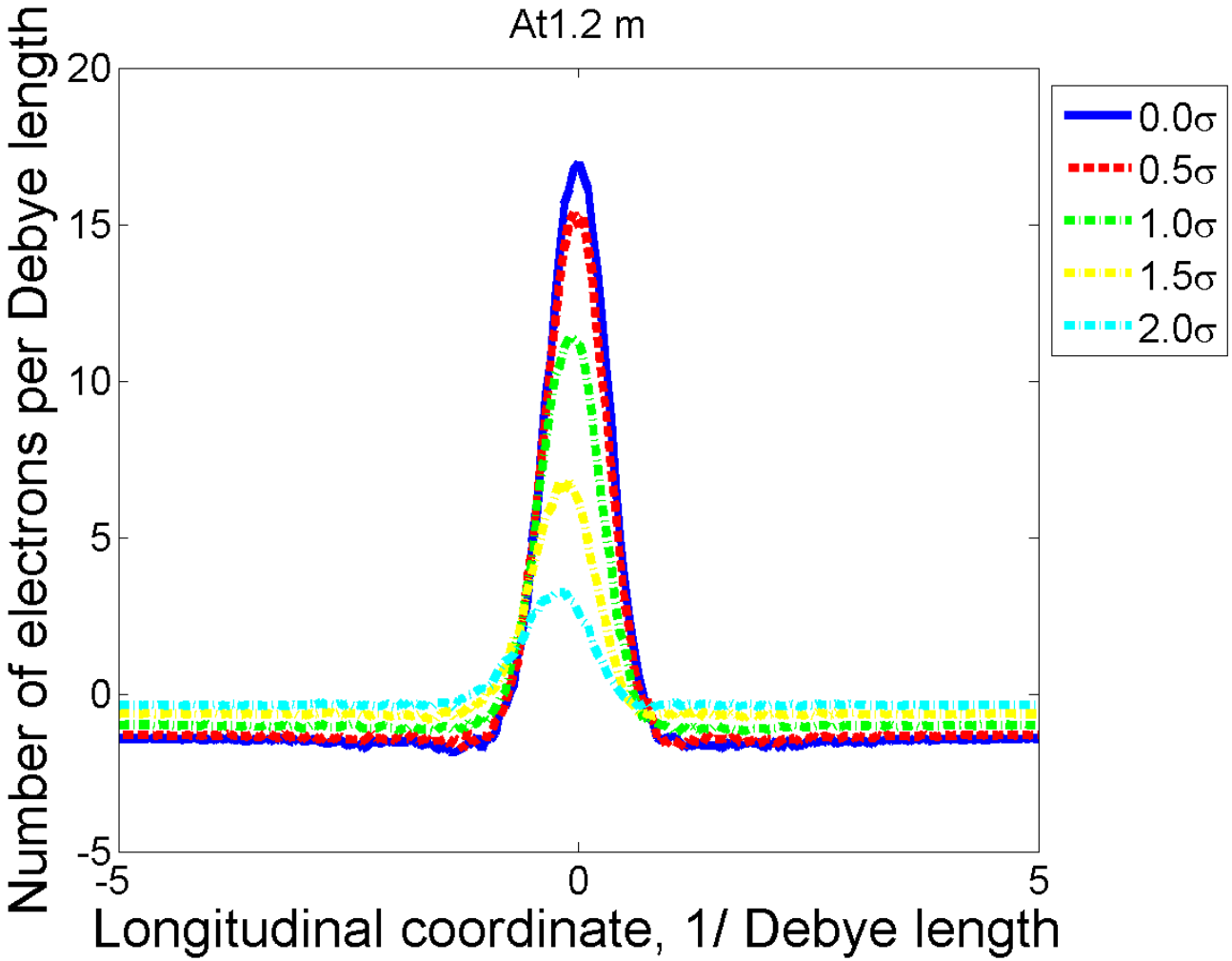

At0.6 m
Average velocity, m/s
Transverse coordinate y, 1/ Debye length
3
2
1
0
-1
-2
-3
-10
-5
0
5
10
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

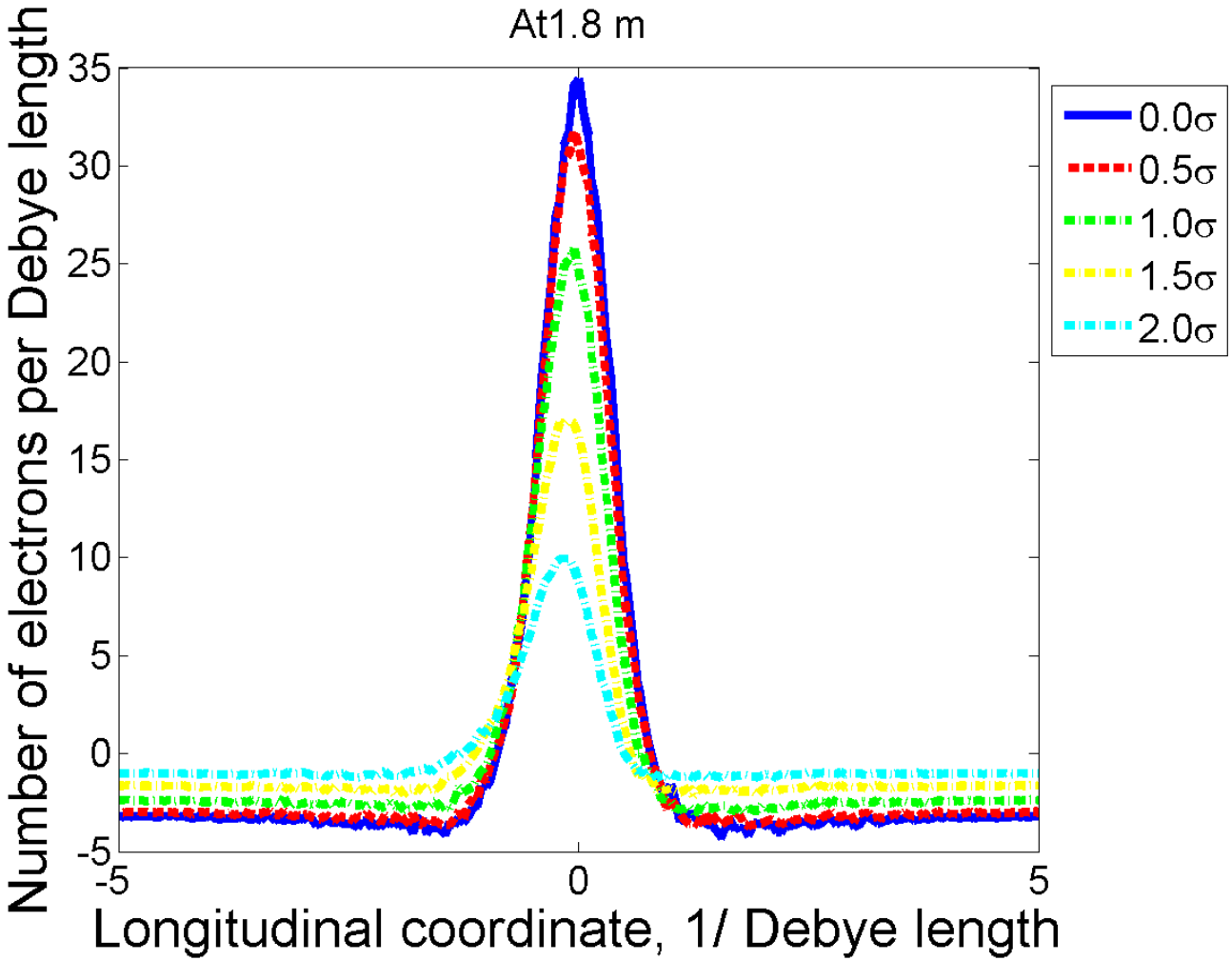

At1.2 m
Average velocity, m/s
Transverse coordinate y, 1/ Debye length
8
6
4
2
0
-2
-4
-6
-10
-5
0
5
10
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

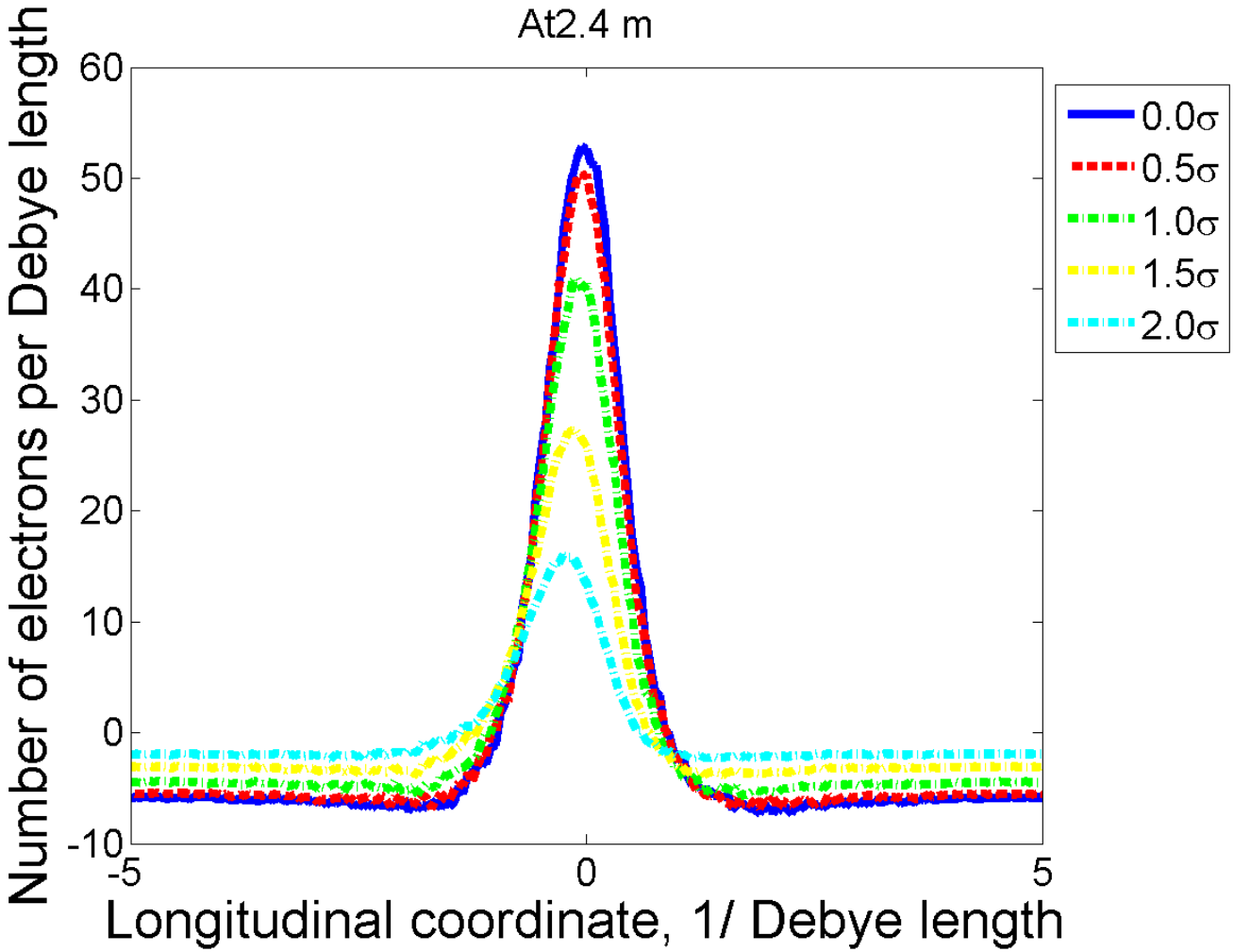

At1.8 m
Average velocity, m/s
Transverse coordinate y, 1/ Debye length
15
10
5
0
-5
-10
-15
-10
-5
0
5
10
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

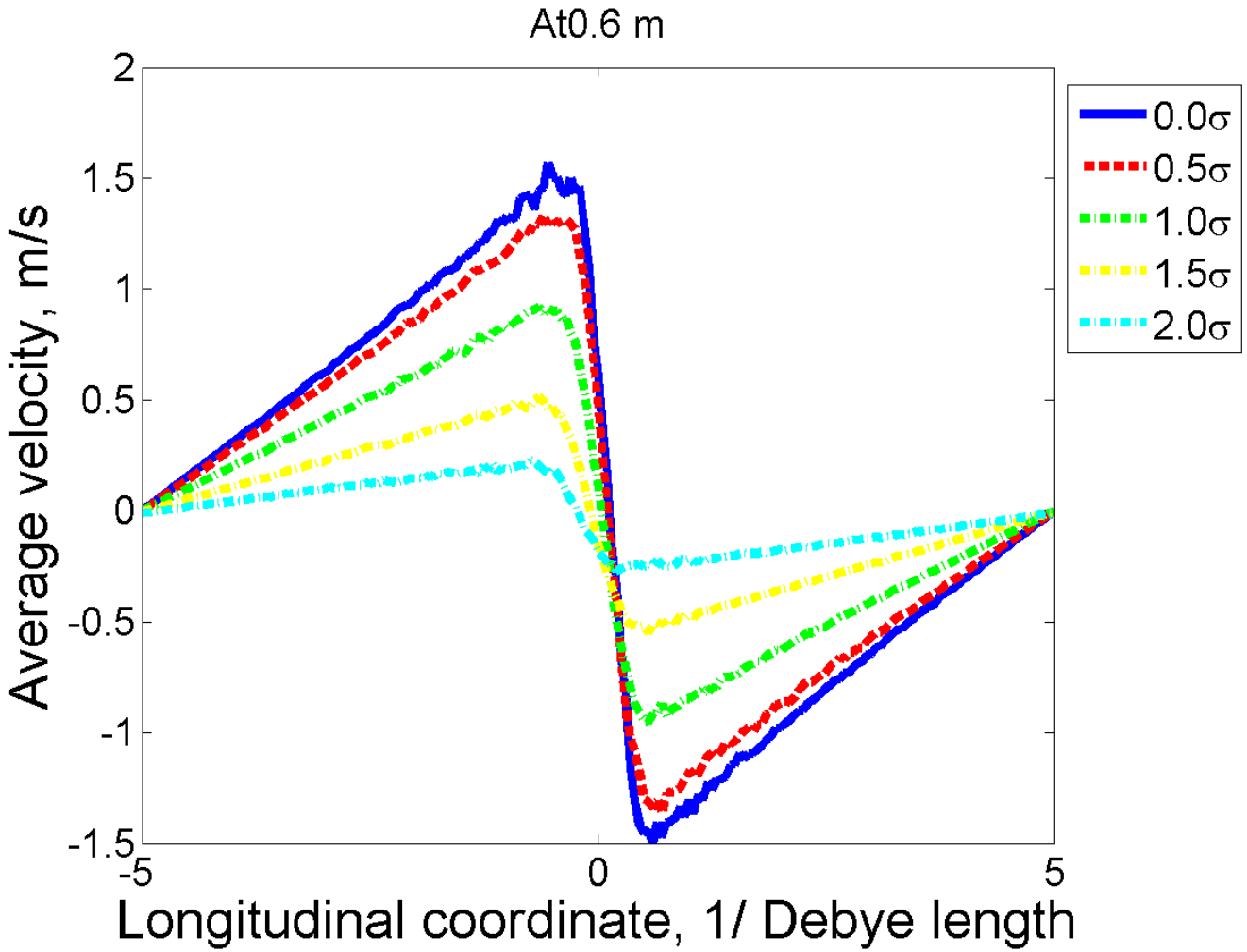

At2.4 m
Average velocity, m/s
Transverse coordinate y, 1/ Debye length
15
10
5
0
-5
-10
-15
-20
-10
-5
0
5
10
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

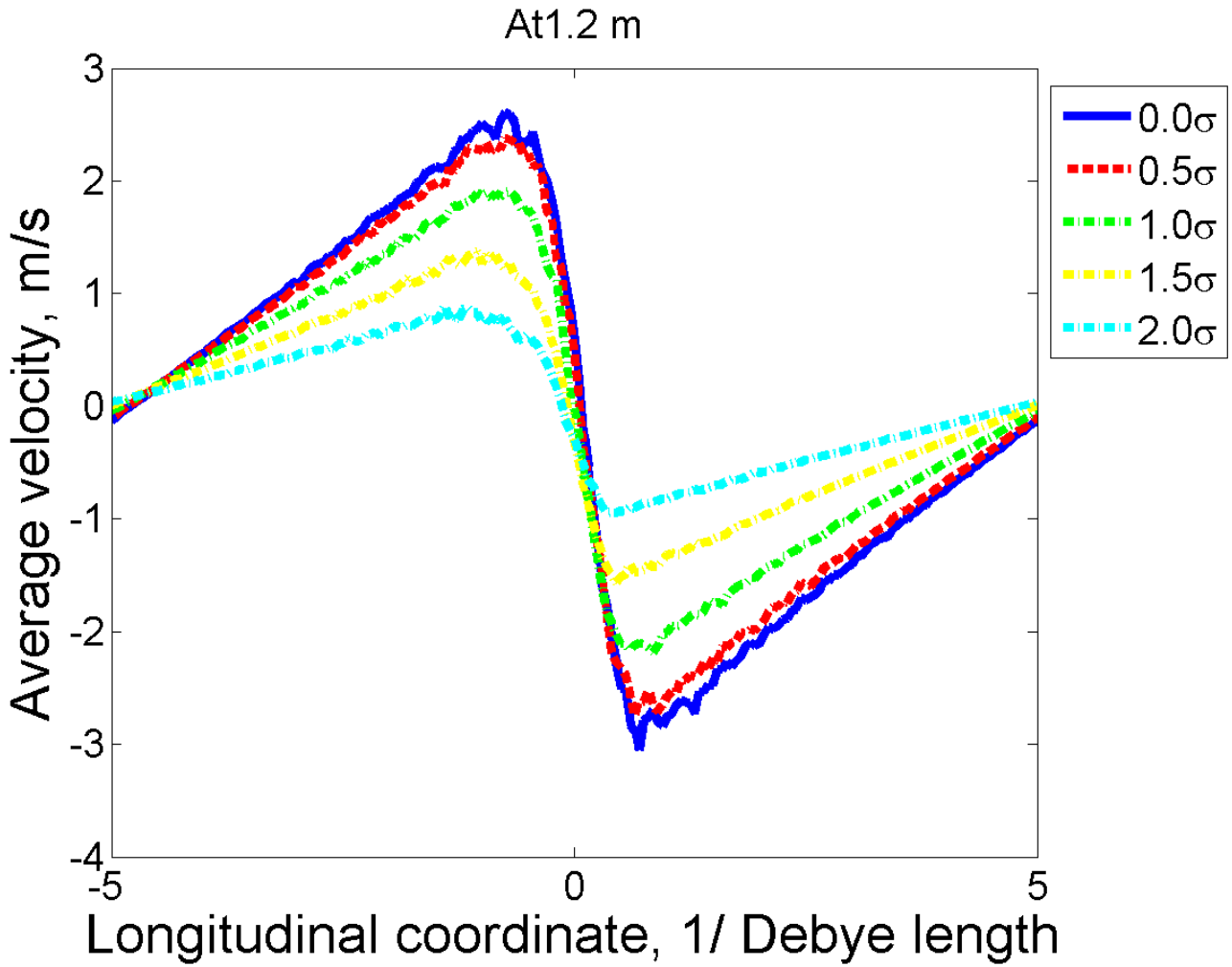

At0.6 m
Number of electrons per Debye length
Longitudinal coordinate, 1/ Debye length
6
5
4
3
2
1
0
-1
-5
0
5
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

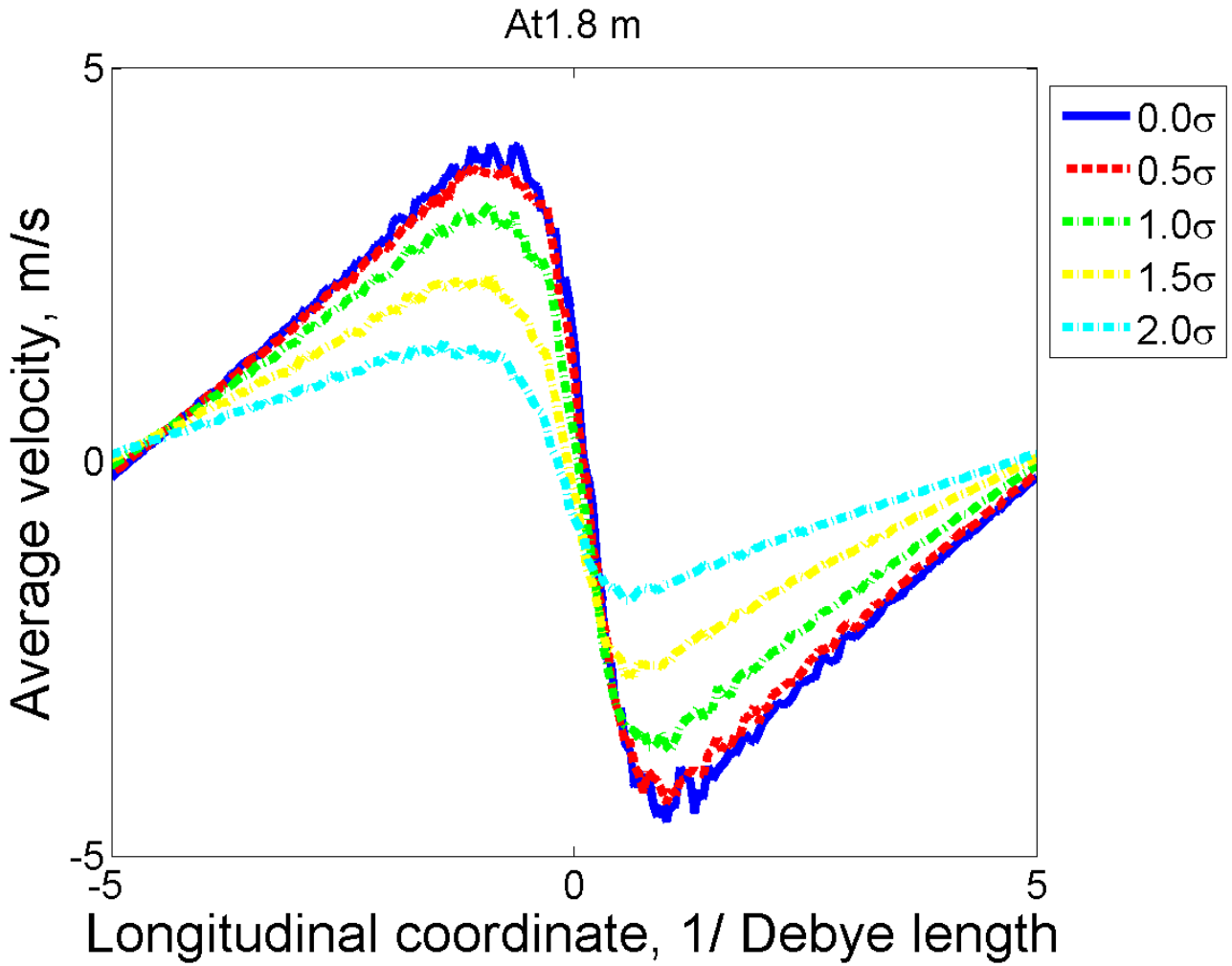

At1.2 m
Number of electrons per Debye length
Longitudinal coordinate, 1/ Debye length
20
15
10
5
0
-5
-5
0
5
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

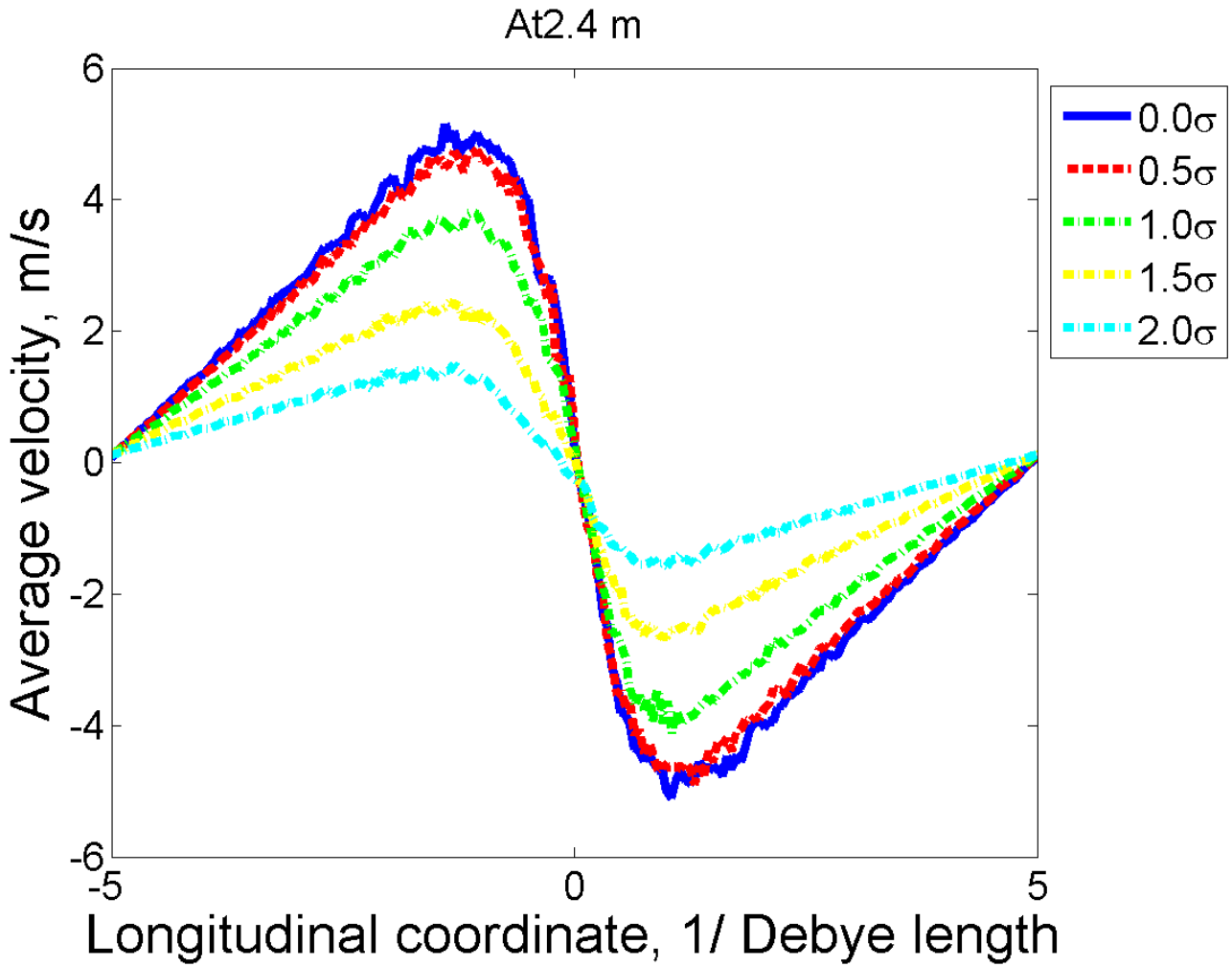

At1.8 m
Number of electrons per Debye length
Longitudinal coordinate, 1/ Debye length
35
30
25
20
15
10
5
0
-5
-5
0
5
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

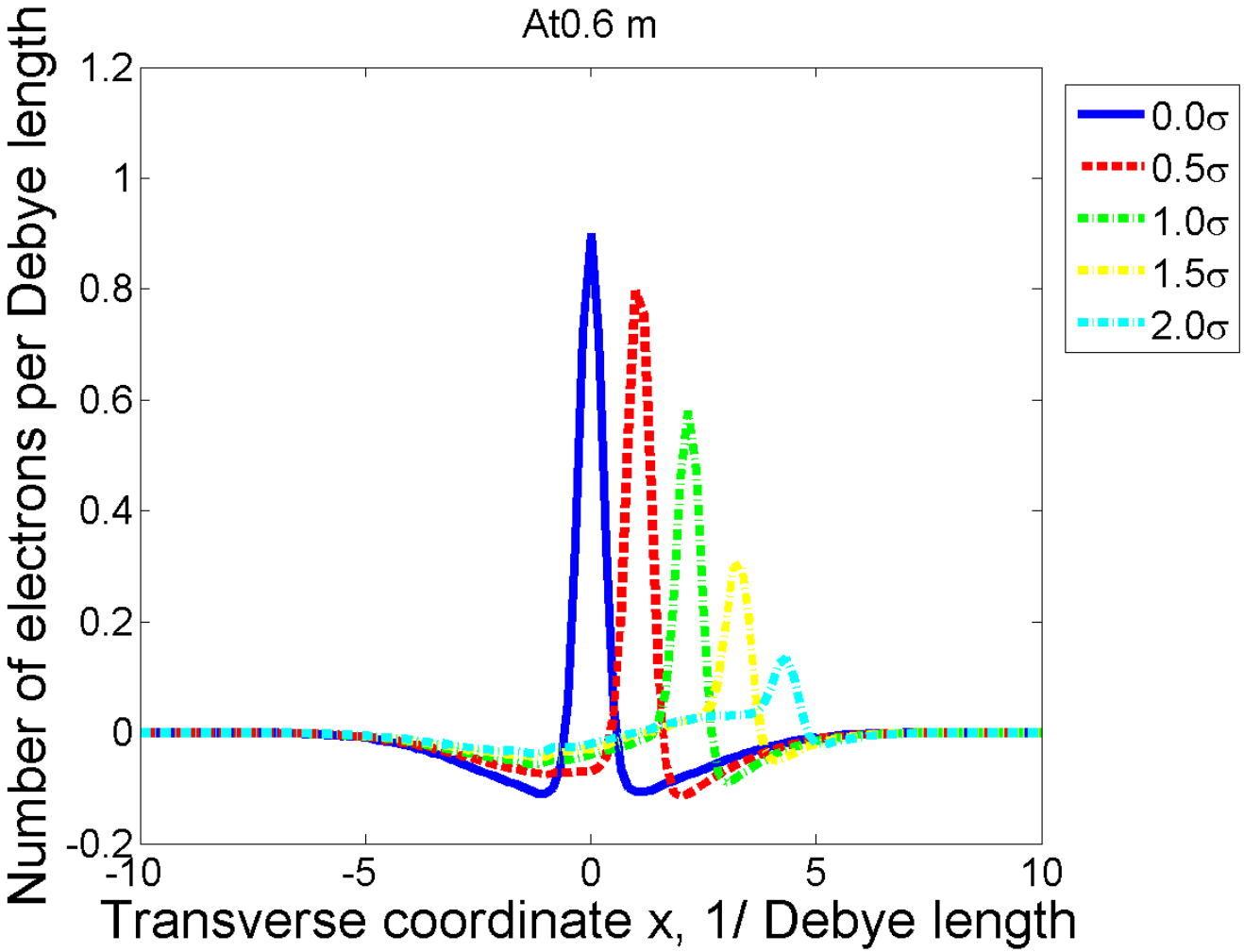

At2.4 m
Number of electrons per Debye length
Longitudinal coordinate, 1/ Debye length
60
50
40
30
20
10
0
-10
-5
0
5
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

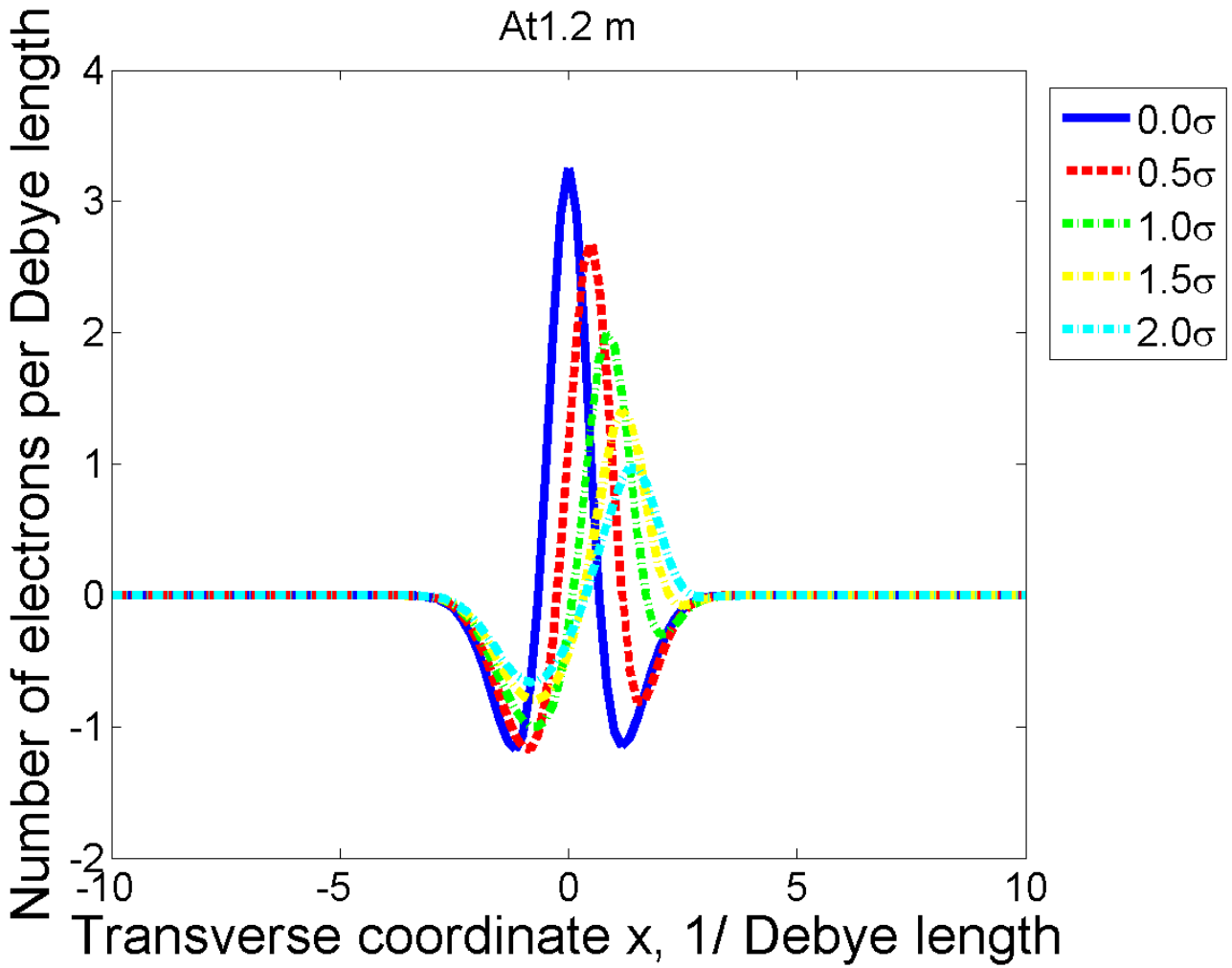

At0.6 m
Average velocity, m/s
Longitudinal coordinate, 1/ Debye length
2
1.5
1
0.5
0
-0.5
-1
-1.5
-5
0
5
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

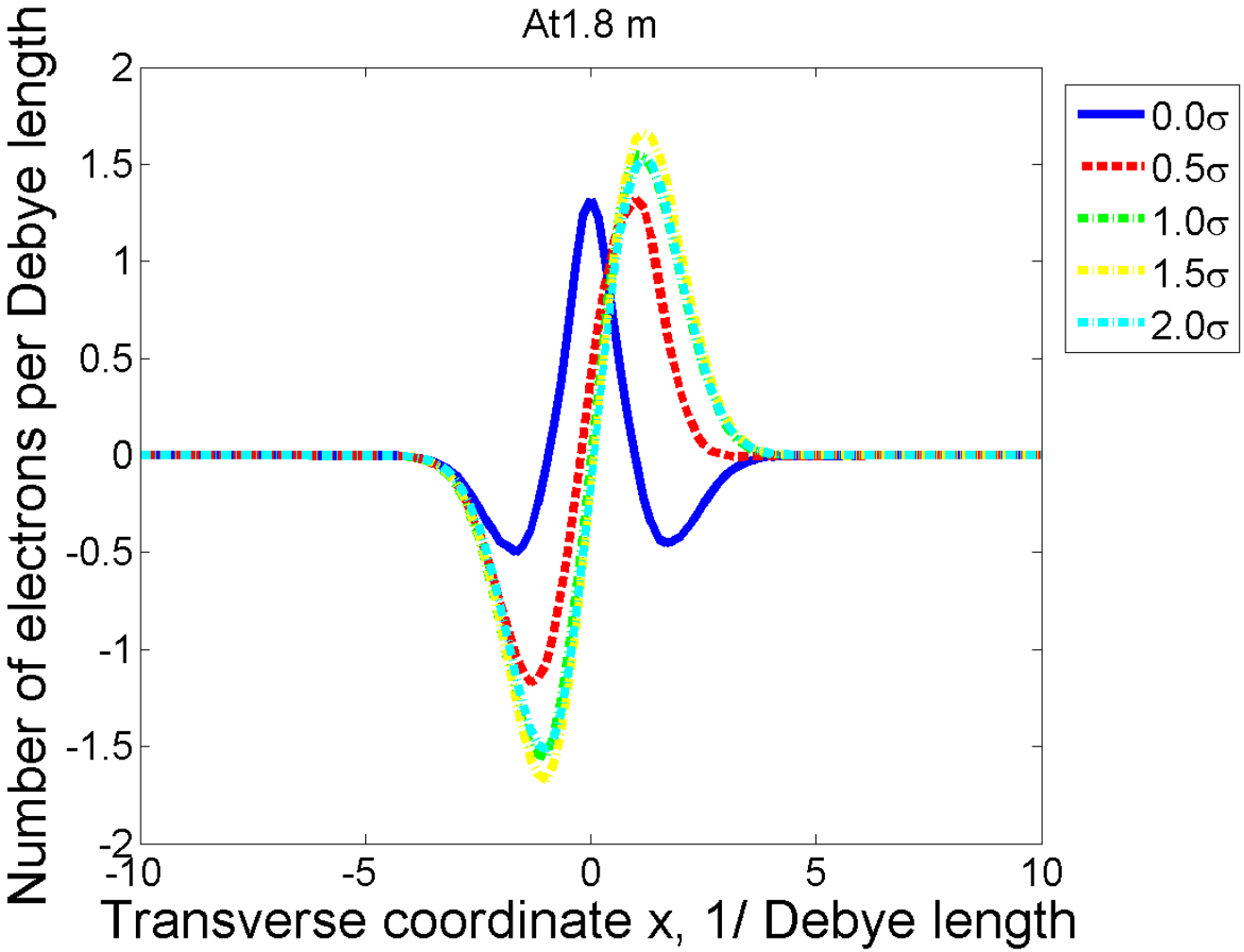

At1.2 m
Average velocity, m/s
Longitudinal coordinate, 1/ Debye length
3
2
1
0
-1
-2
-3
-4
-5
0
5
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

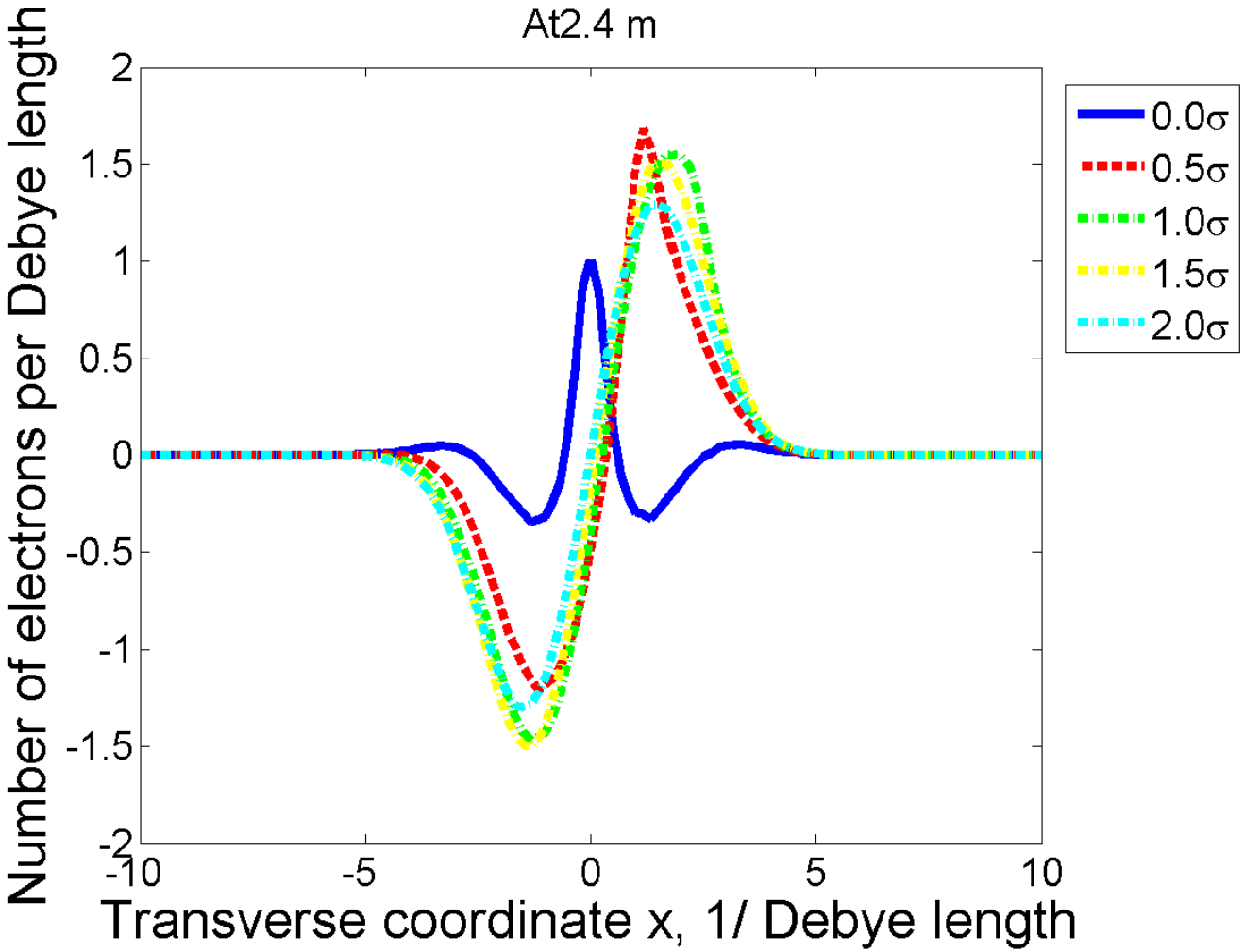

At1.8 m
Average velocity, m/s
Longitudinal coordinate, 1/ Debye length
5
0
-5
-5
0
5
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

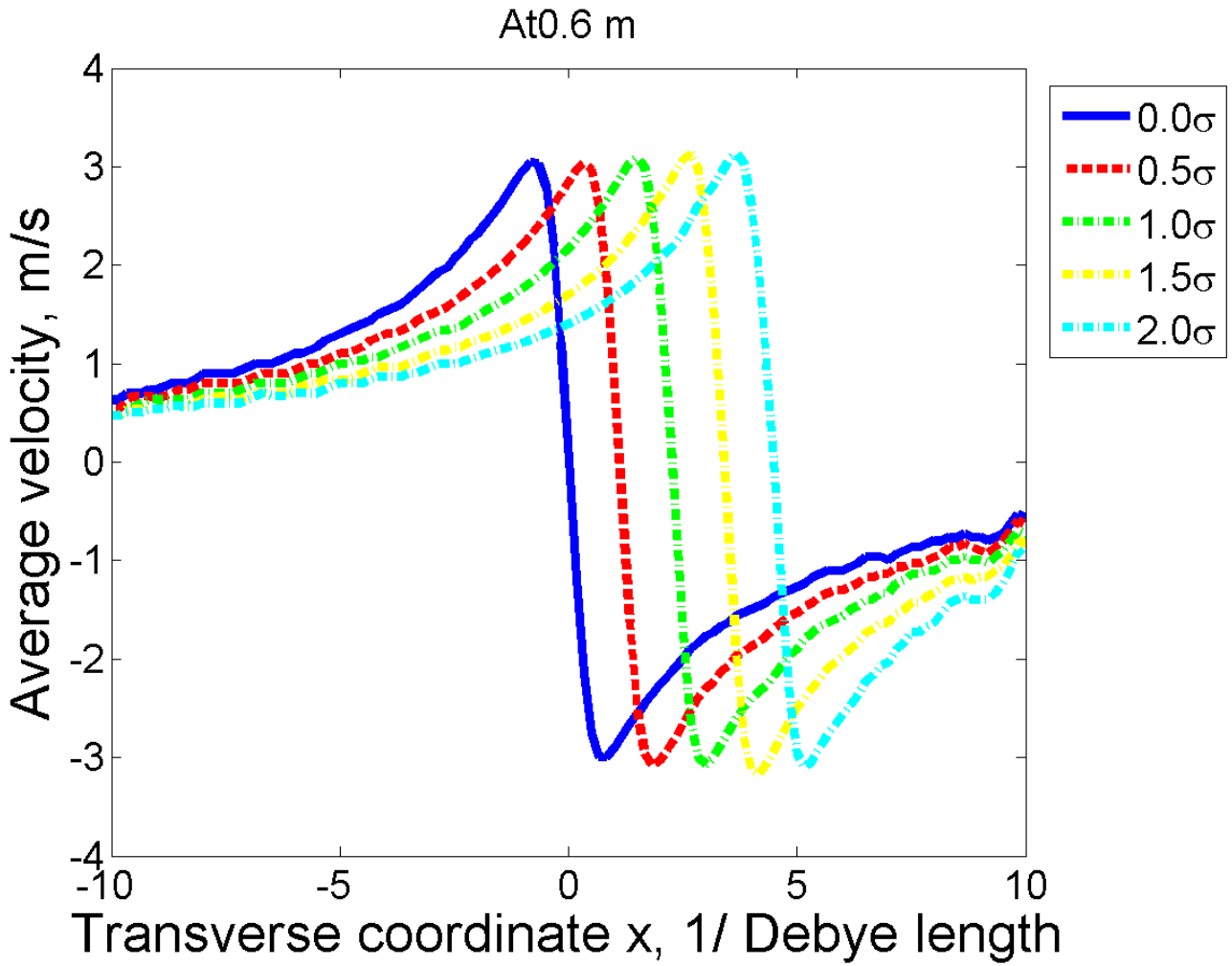

At2.4 m
Average velocity, m/s
Longitudinal coordinate, 1/ Debye length
6
4
2
0
-2
-4
-6
-5
0
5
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

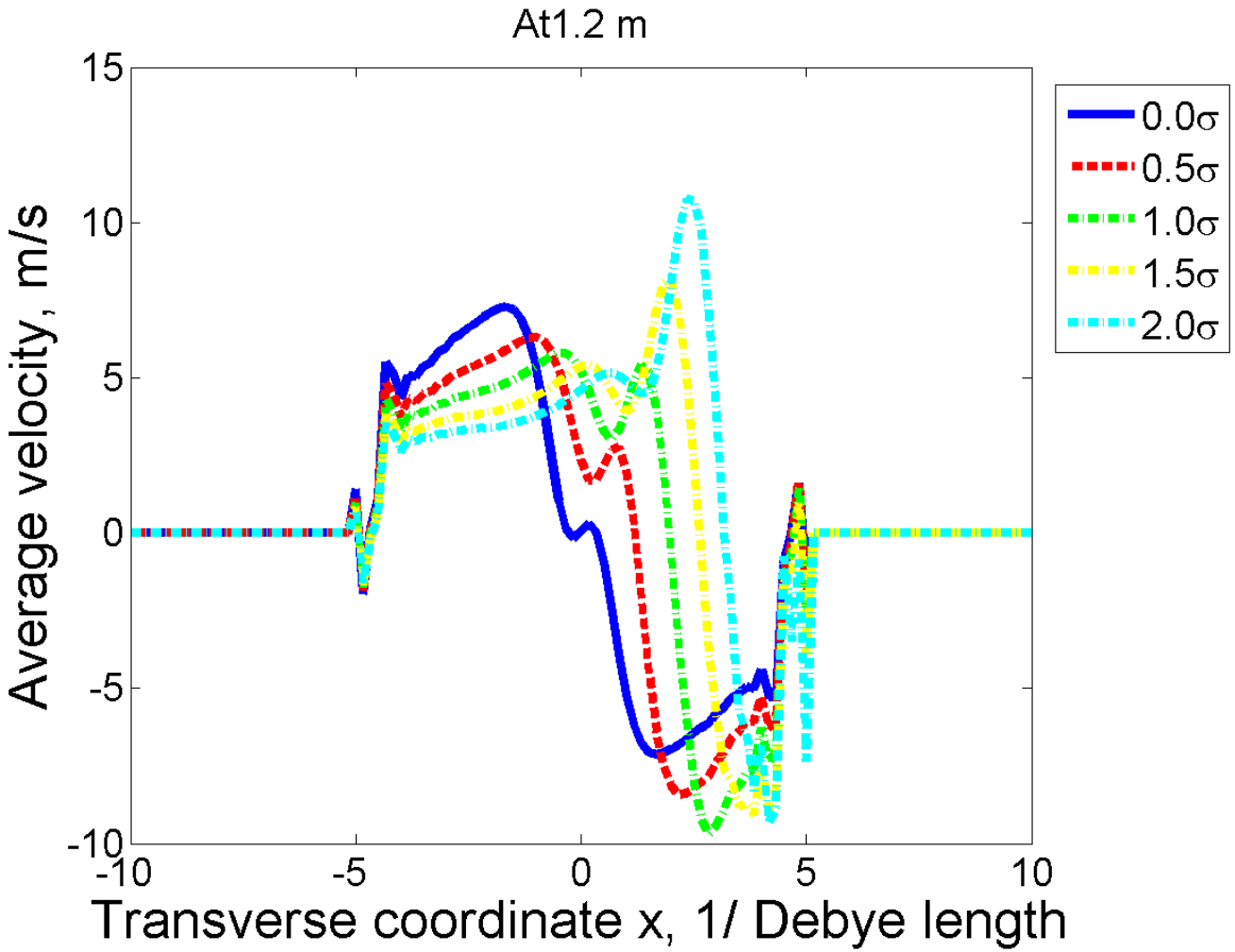

At0.6 m
Number of electrons per Debye length
Transverse coordinate x, 1/ Debye length
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-10
-5
0
5
10
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

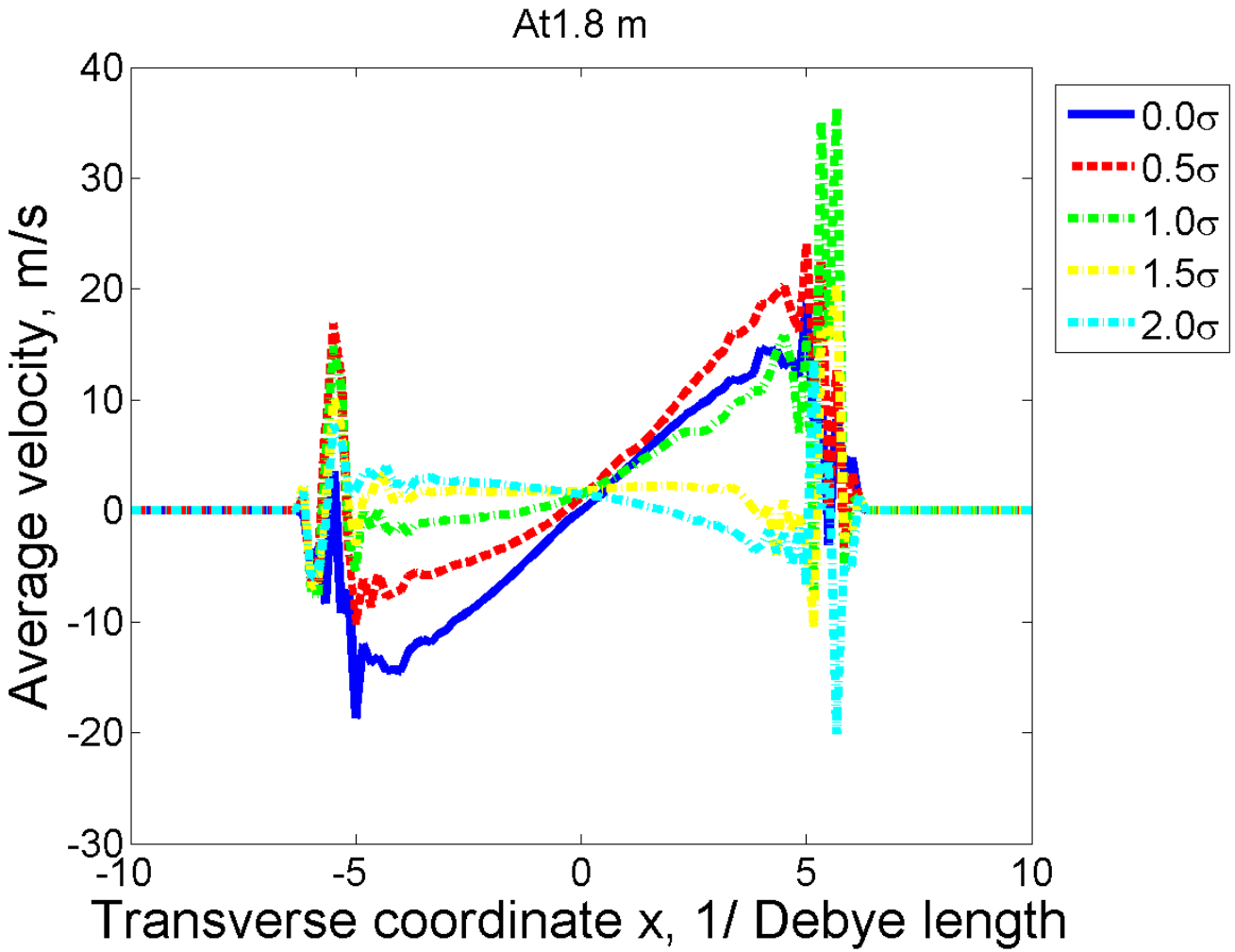

At1.2 m
Number of electrons per Debye length
Transverse coordinate x, 1/ Debye length
4
3
2
1
0
-1
-2
-10
-5
0
5
10
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

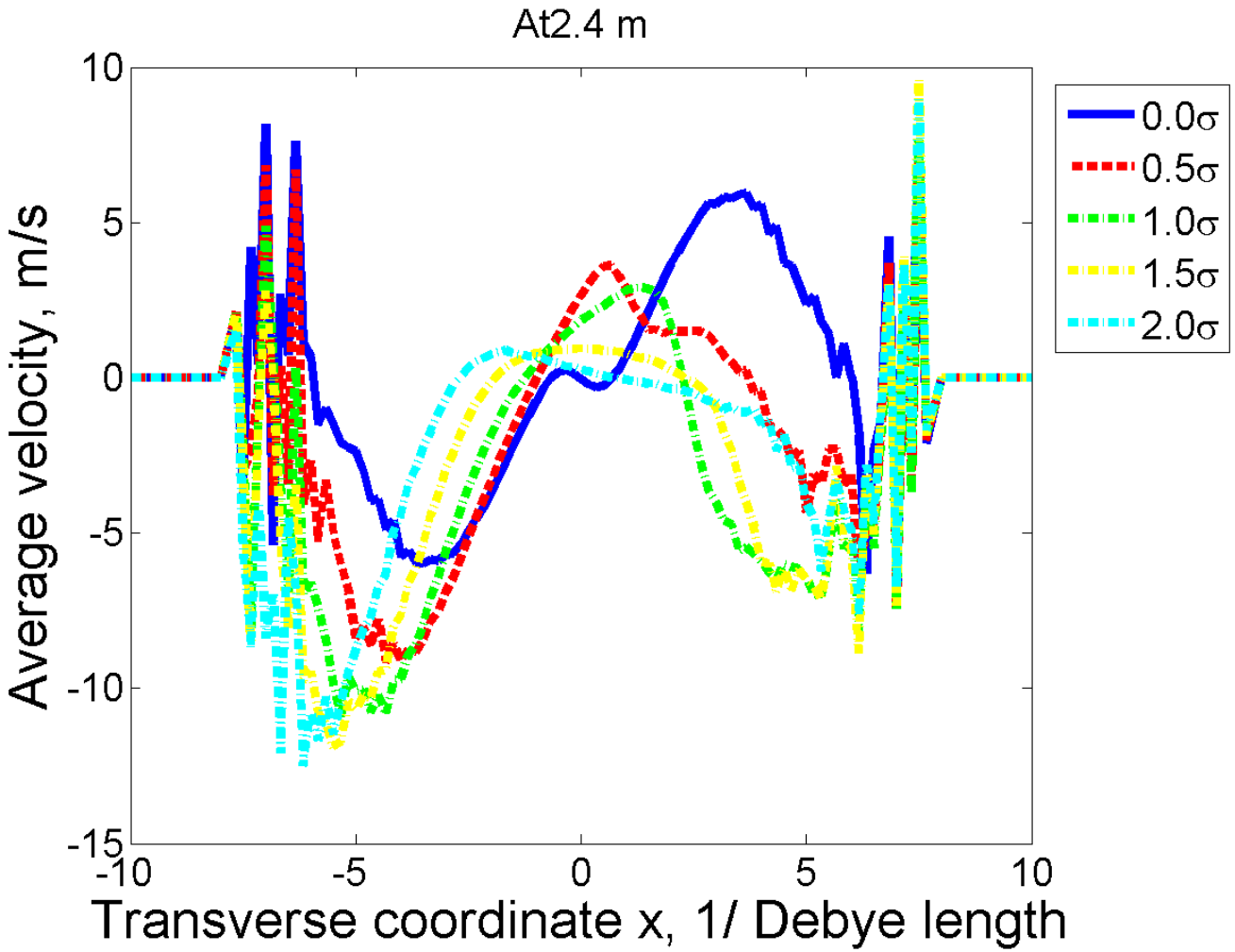

At1.8 m
Number of electrons per Debye length
Transverse coordinate x, 1/ Debye length
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-10
-5
0
5
10

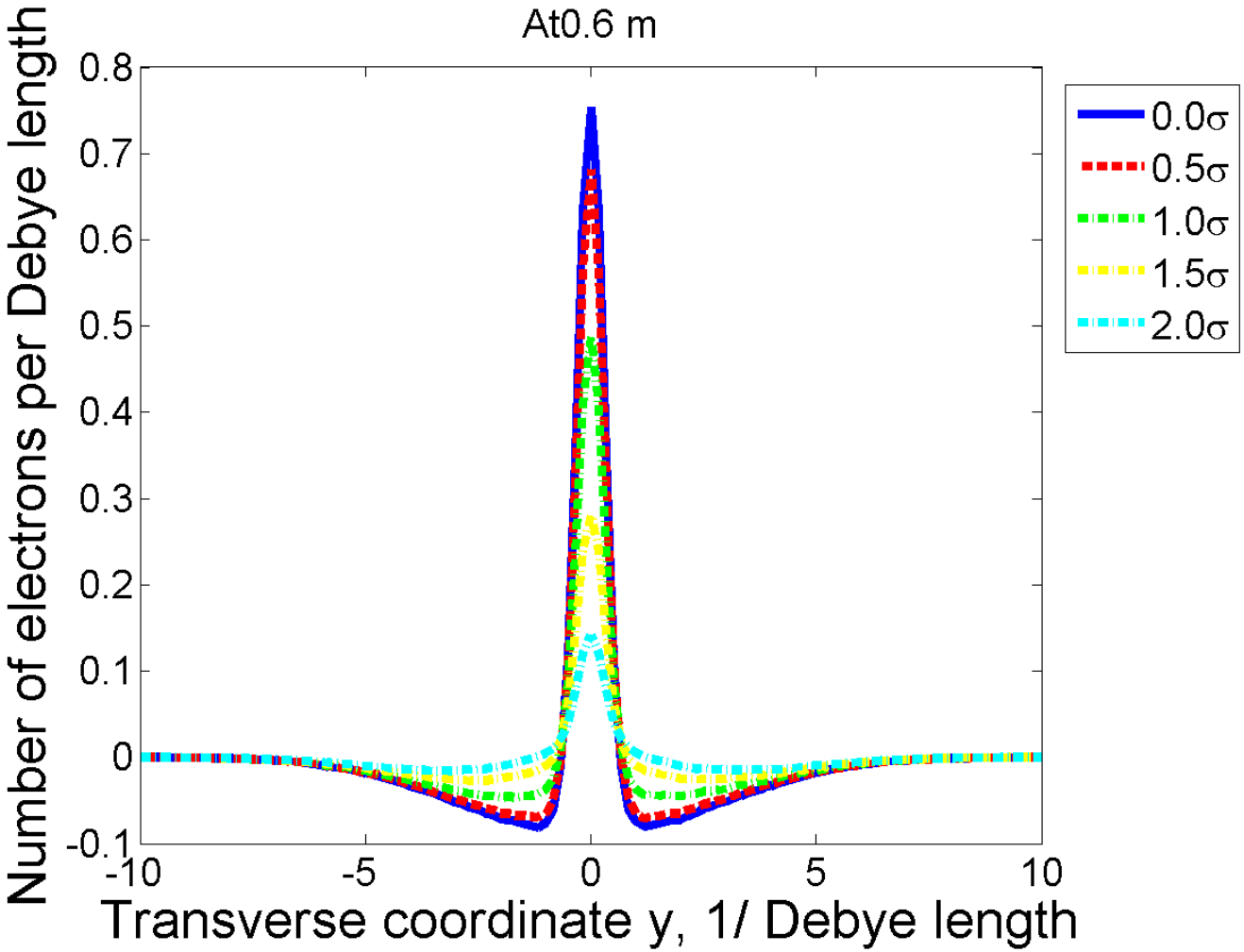

At2.4 m
Number of electrons per Debye length
Transverse coordinate x, 1/ Debye length
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-10
-5
0
5
10

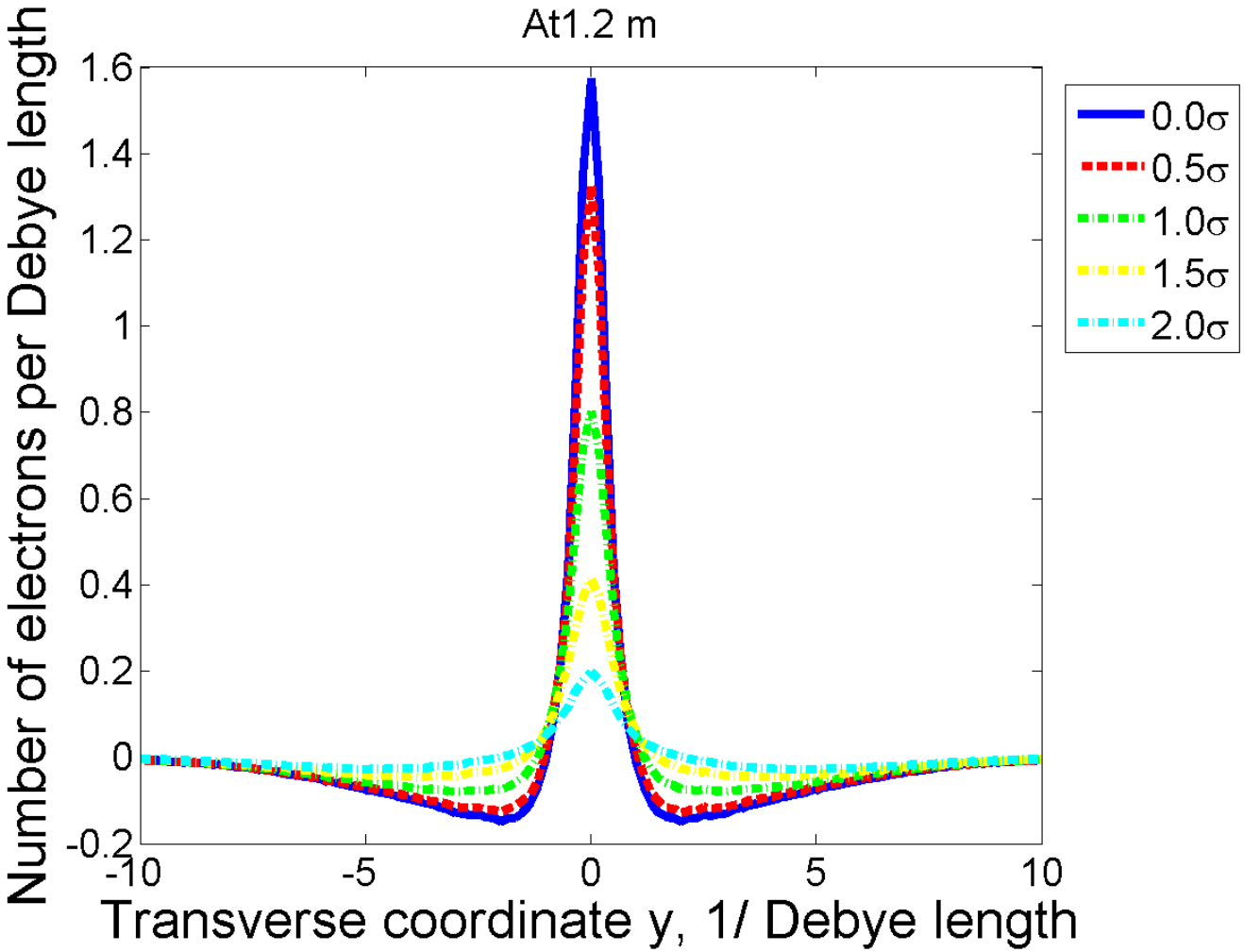

At0.6 m
Average velocity, m/s
Transverse coordinate x, 1/ Debye length
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ
4
3
2
1
0
-1
-2
-3
-4
-10
-5
0
5
10

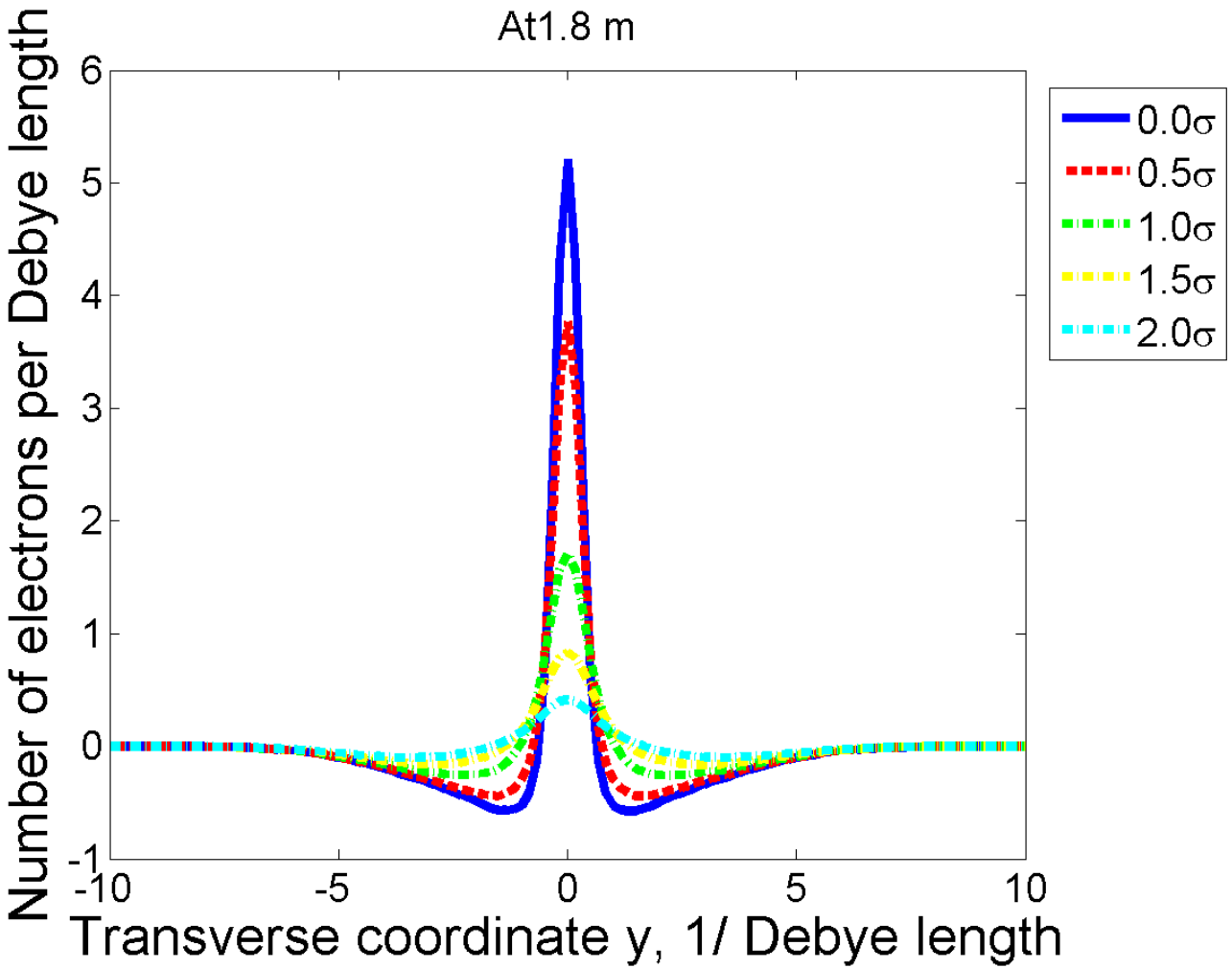

At1.2 m
Average velocity, m/s
Transverse coordinate x, 1/ Debye length
15
10
5
0
-5
-10
-10
-5
0
5
10
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

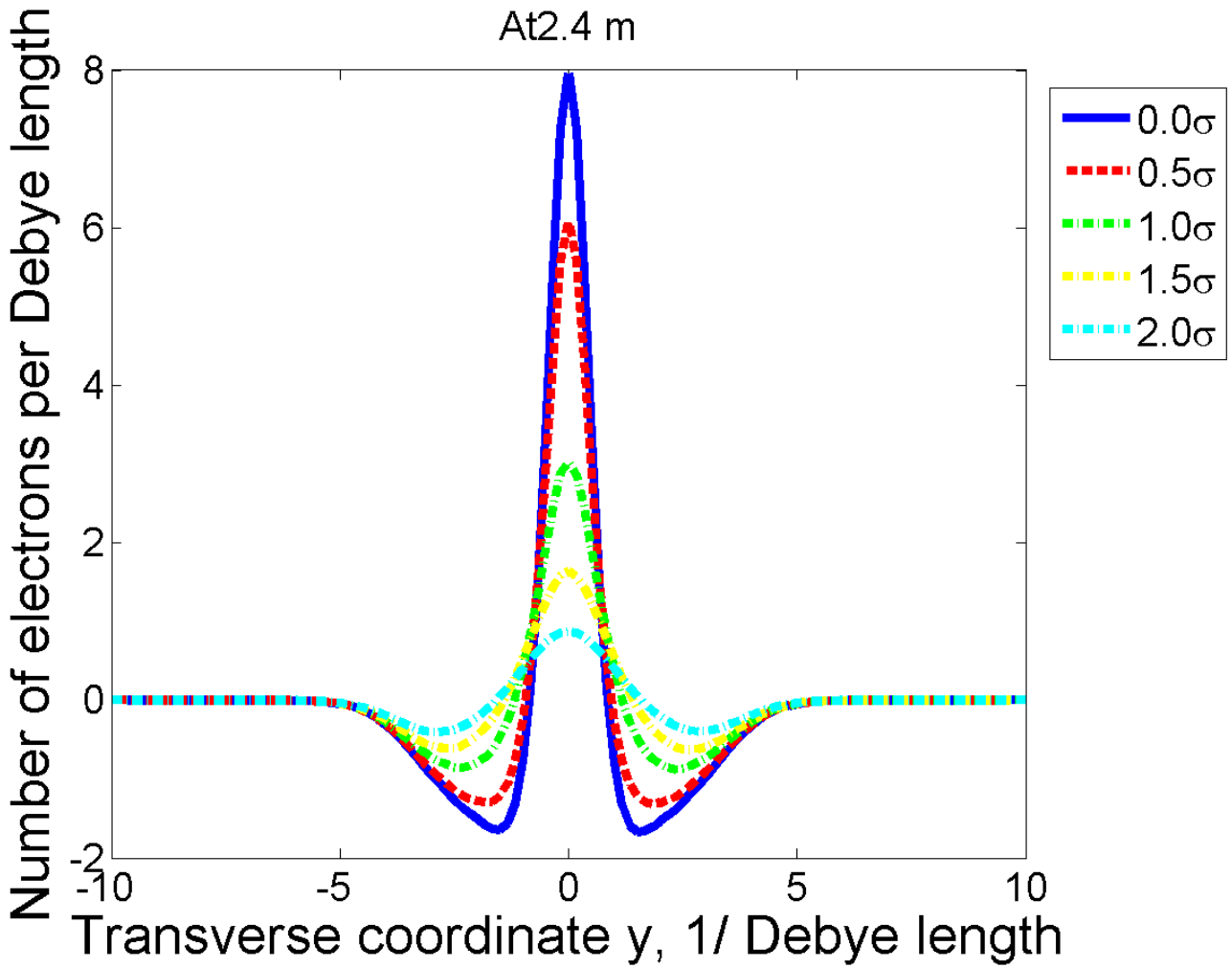

At1.8 m
Average velocity, m/s
Transverse coordinate x, 1/ Debye length
40
30
20
10
0
-10
-20
-30
-10
-5
0
5
10
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

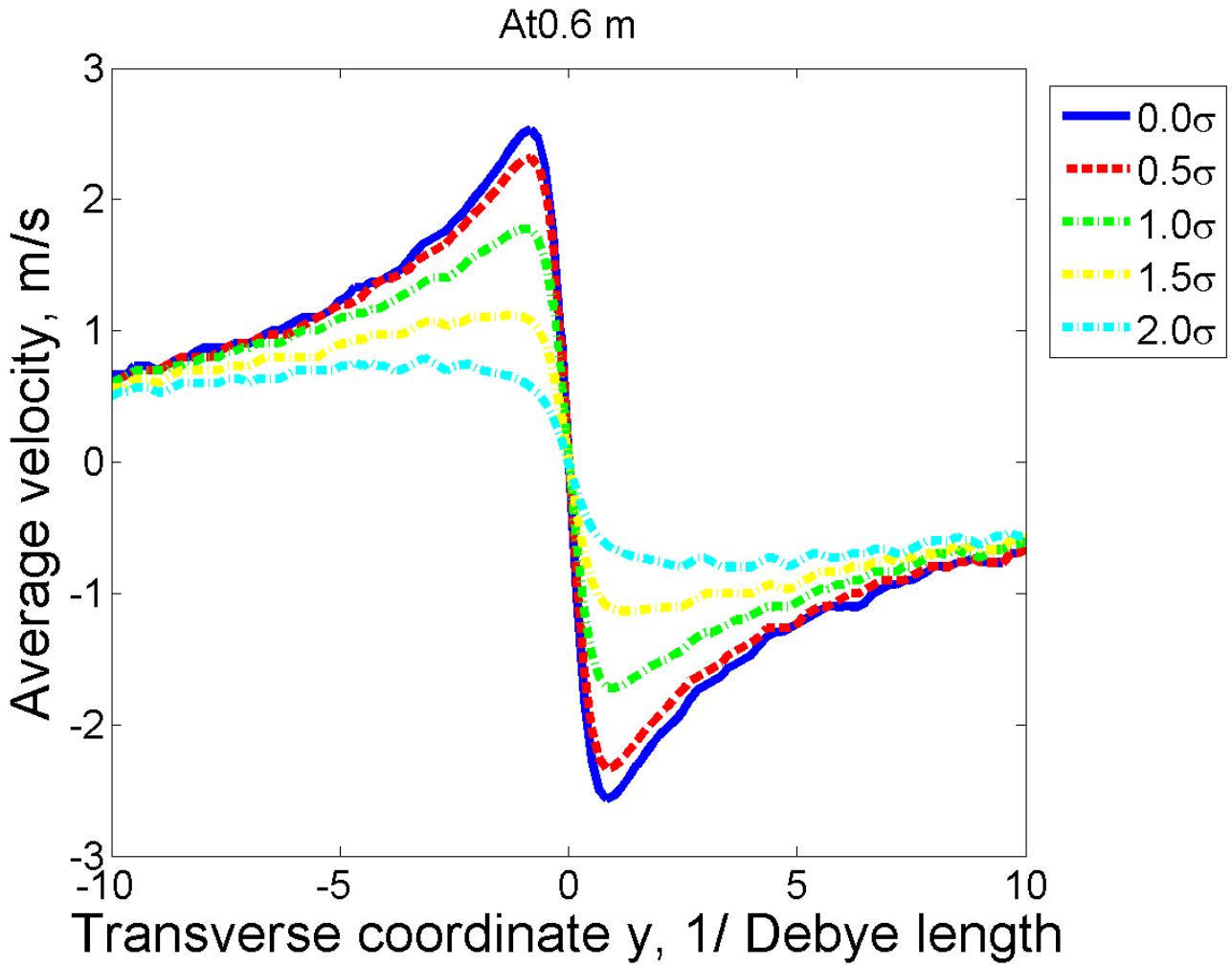

At2.4 m
Average velocity, m/s
Transverse coordinate x, 1/ Debye length
10
5
0
-5
-10
-15
-10
-5
0
5
10
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

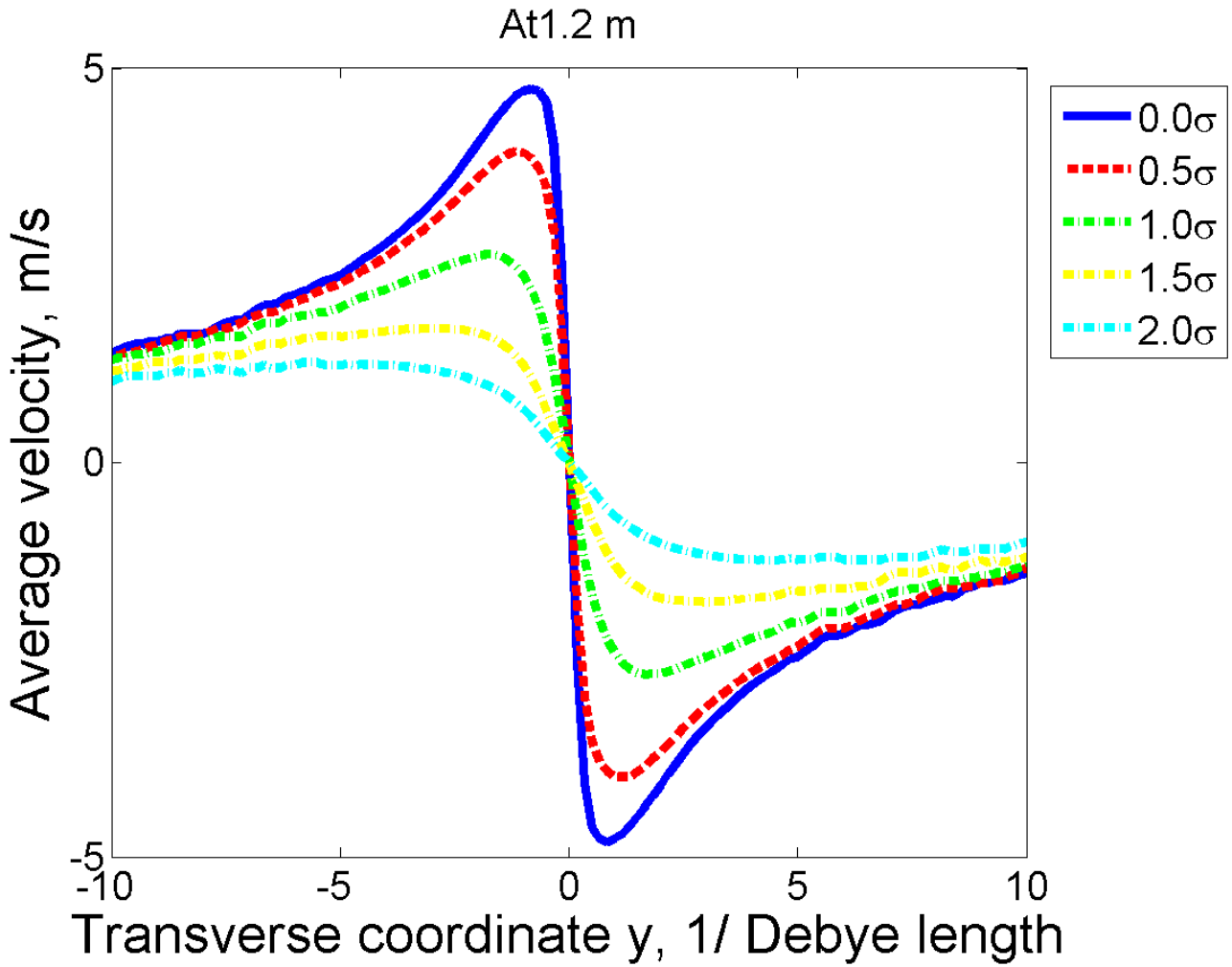

At0.6 m
Number of electrons per Debye length
Transverse coordinate y, 1/ Debye length
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-10
-5
0
5
10
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

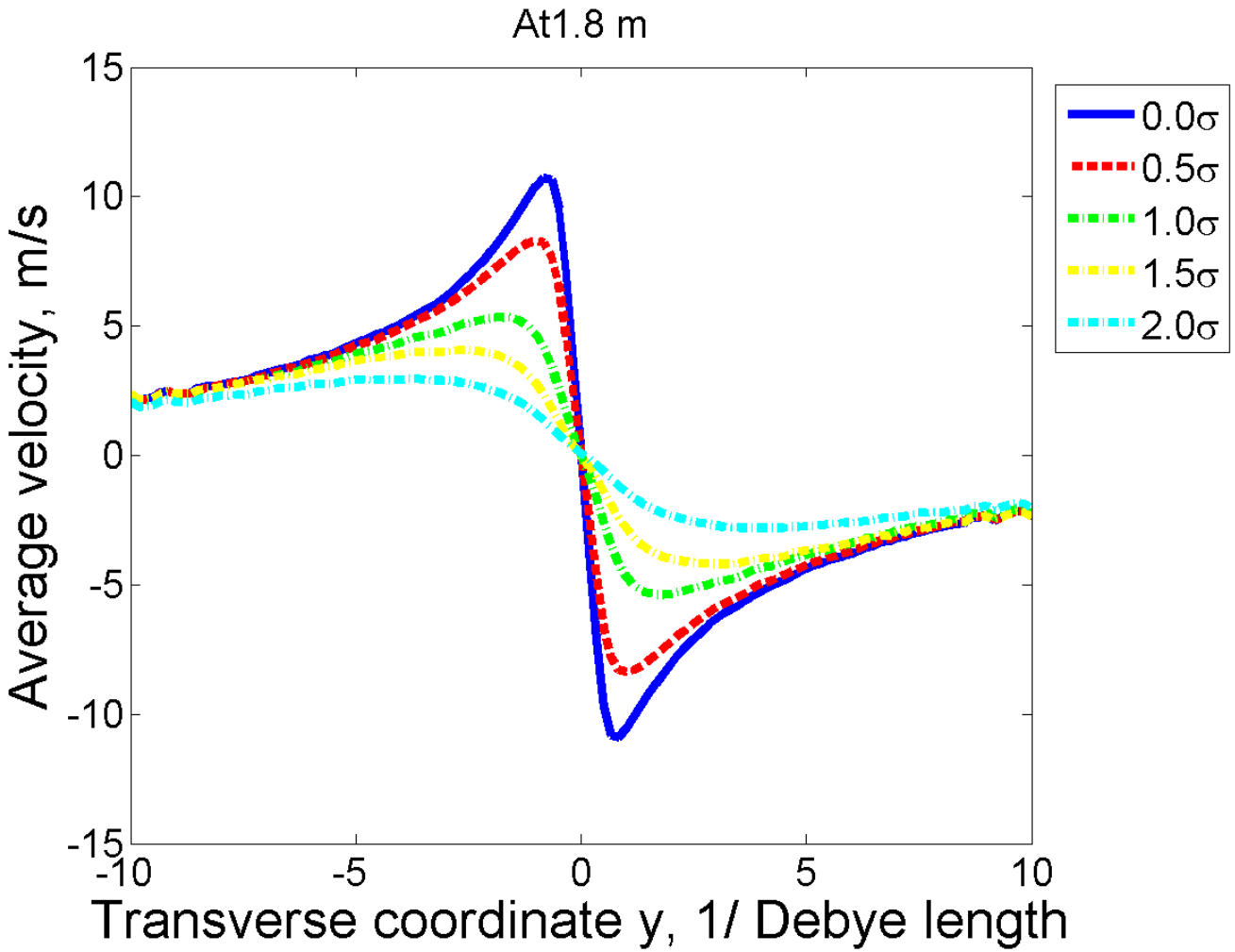

At1.2 m
Number of electrons per Debye length
Transverse coordinate y, 1/ Debye length
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-10
-5
0
5
10
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

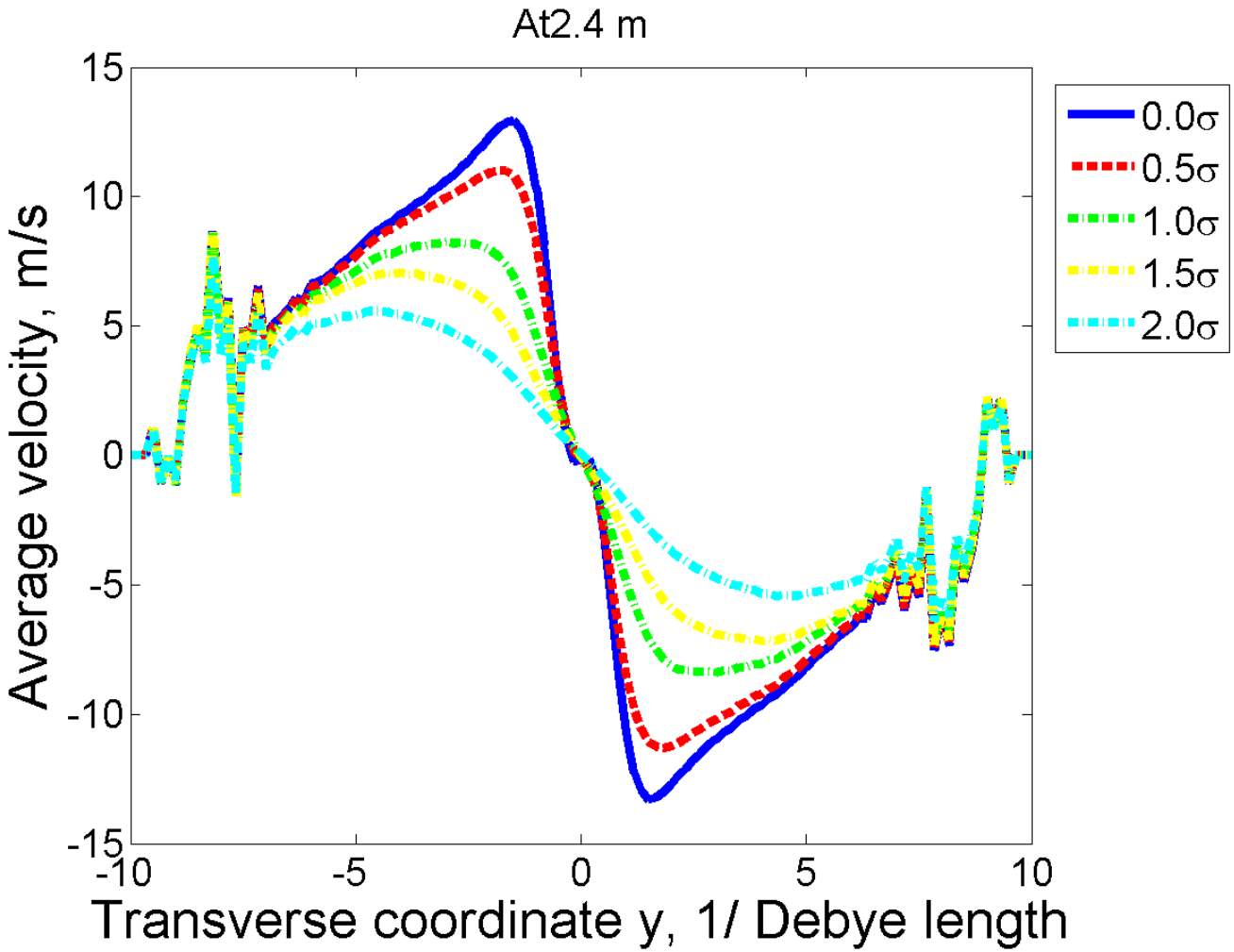

At1.8 m
Number of electrons per Debye length
Transverse coordinate y, 1/ Debye length
6
5
4
3
2
1
0
-1
-10
-5
0
5
10
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

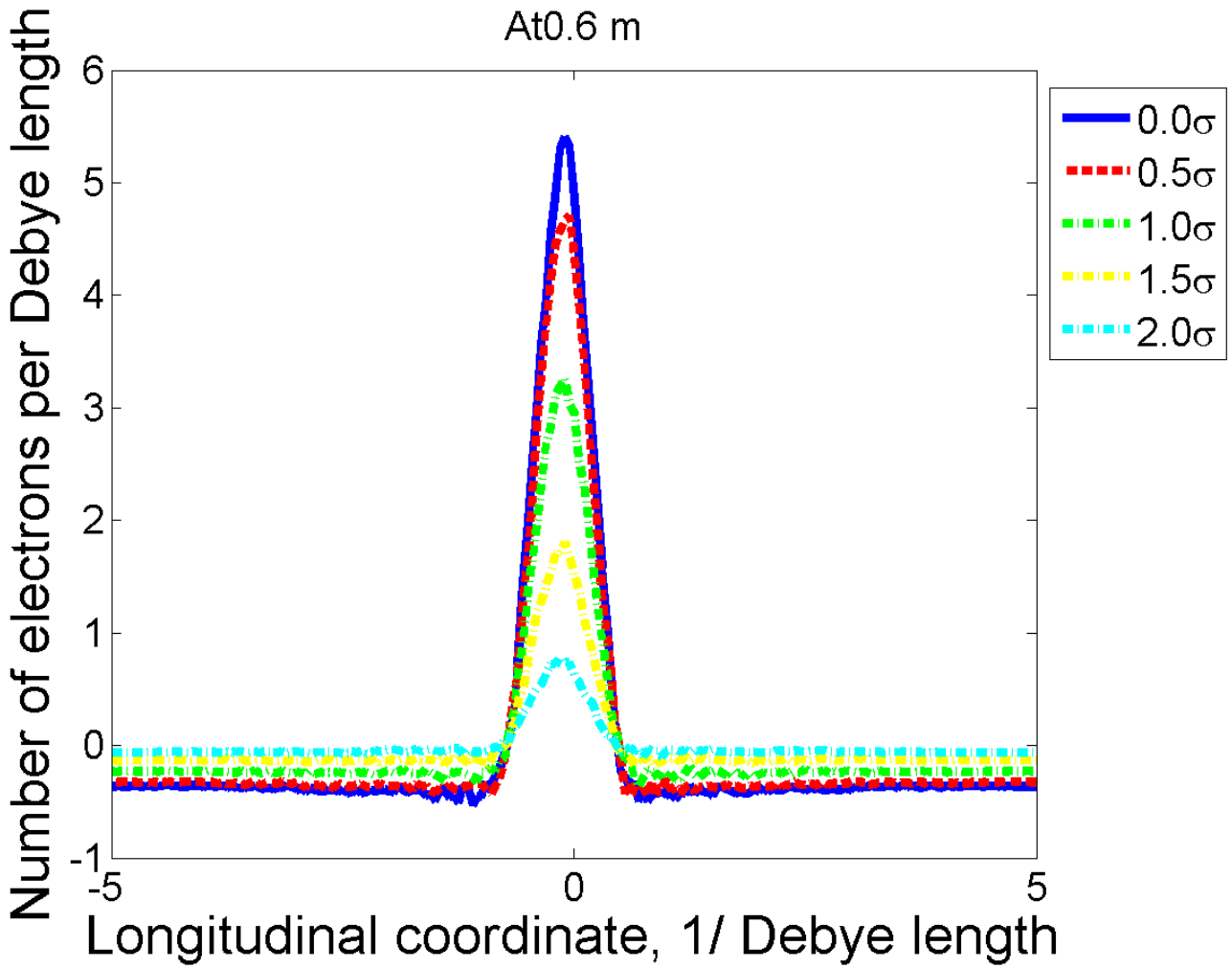

At2.4 m
Number of electrons per Debye length
Transverse coordinate y, 1/ Debye length
8
6
4
2
0
-2
-10
-5
0
5
10
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

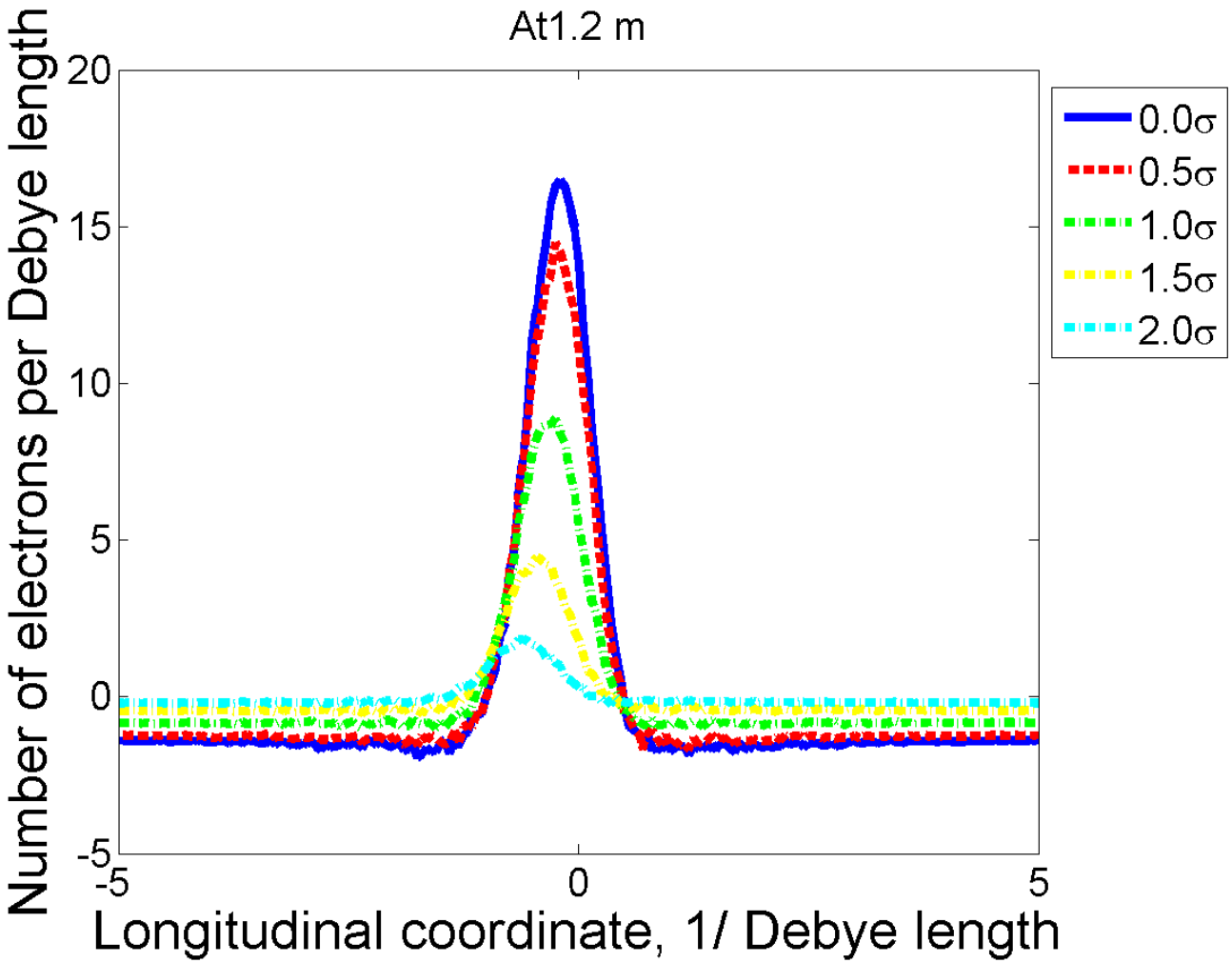

At0.6 m
Average velocity, m/s
Transverse coordinate y, 1/ Debye length
3
2
1
0
-1
-2
-3
-10
-5
0
5
10
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

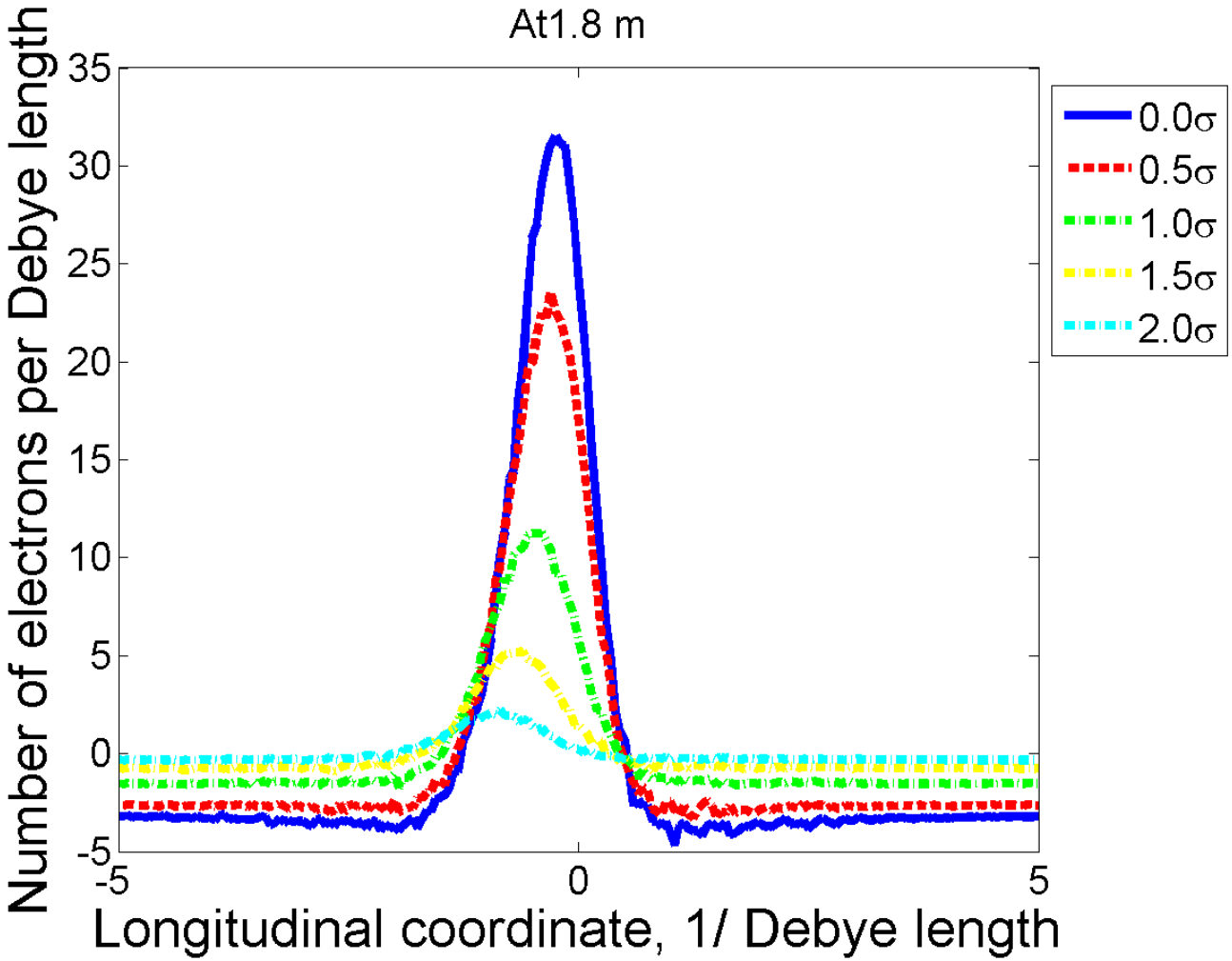

At1.2 m
Average velocity, m/s
Transverse coordinate y, 1/ Debye length
5
0
-5
-10
-5
0
5
10
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

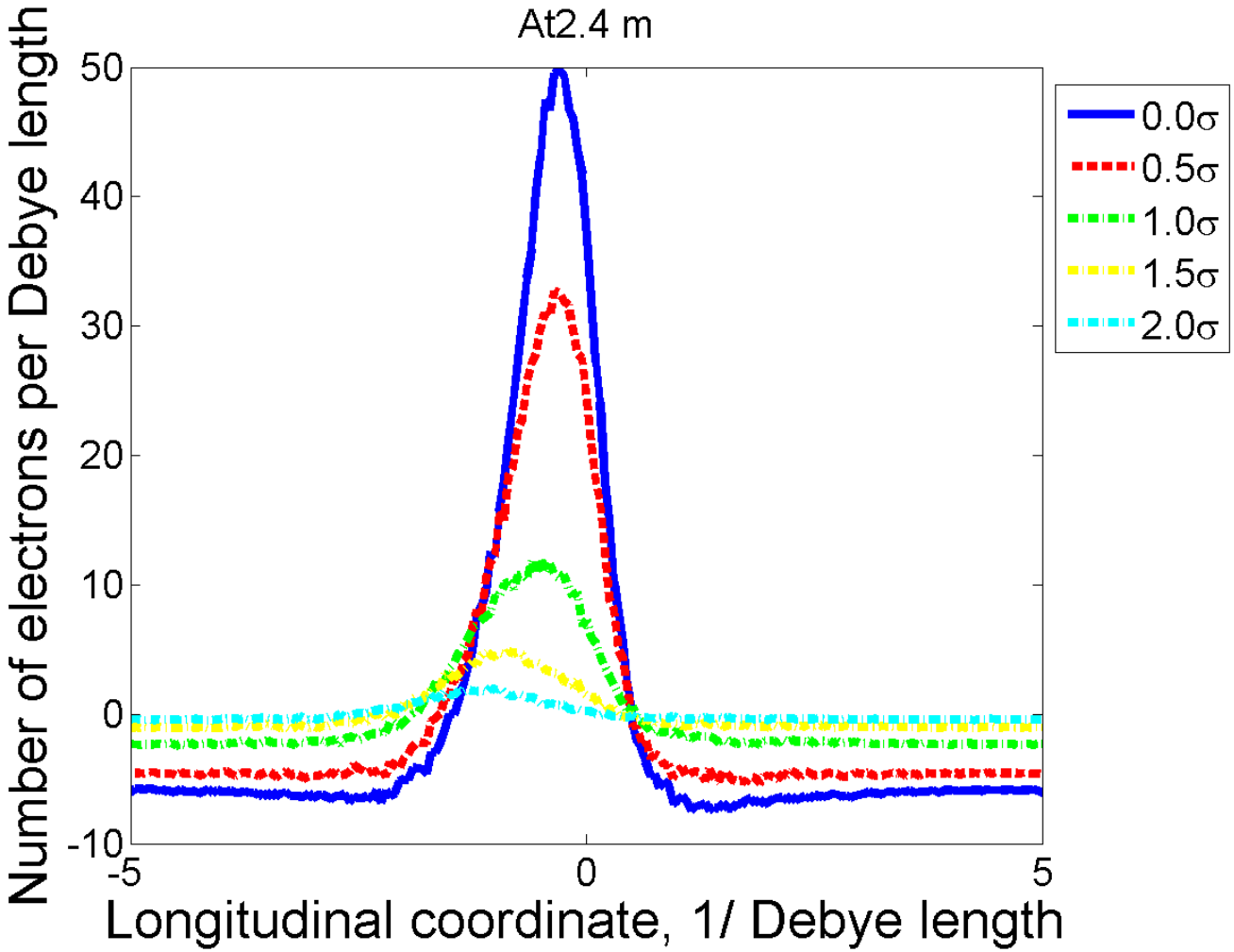

At1.8 m
Average velocity, m/s
Transverse coordinate y, 1/ Debye length
15
10
5
0
-5
-10
-15
-10
-5
0
5
10
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

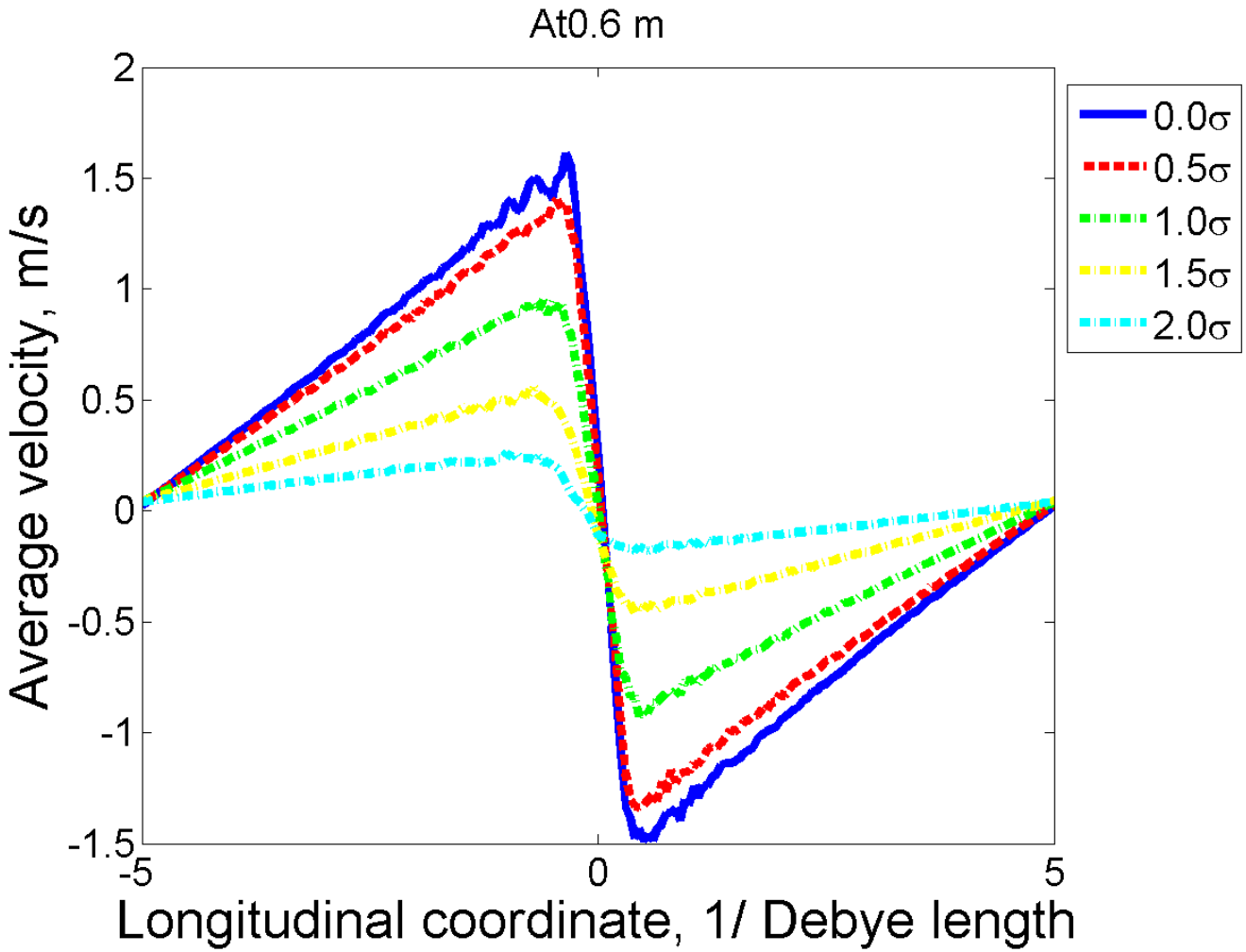

At2.4 m
Average velocity, m/s
Transverse coordinate y, 1/ Debye length
15
10
5
0
-5
-10
-15
-10
-5
0
5
10
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

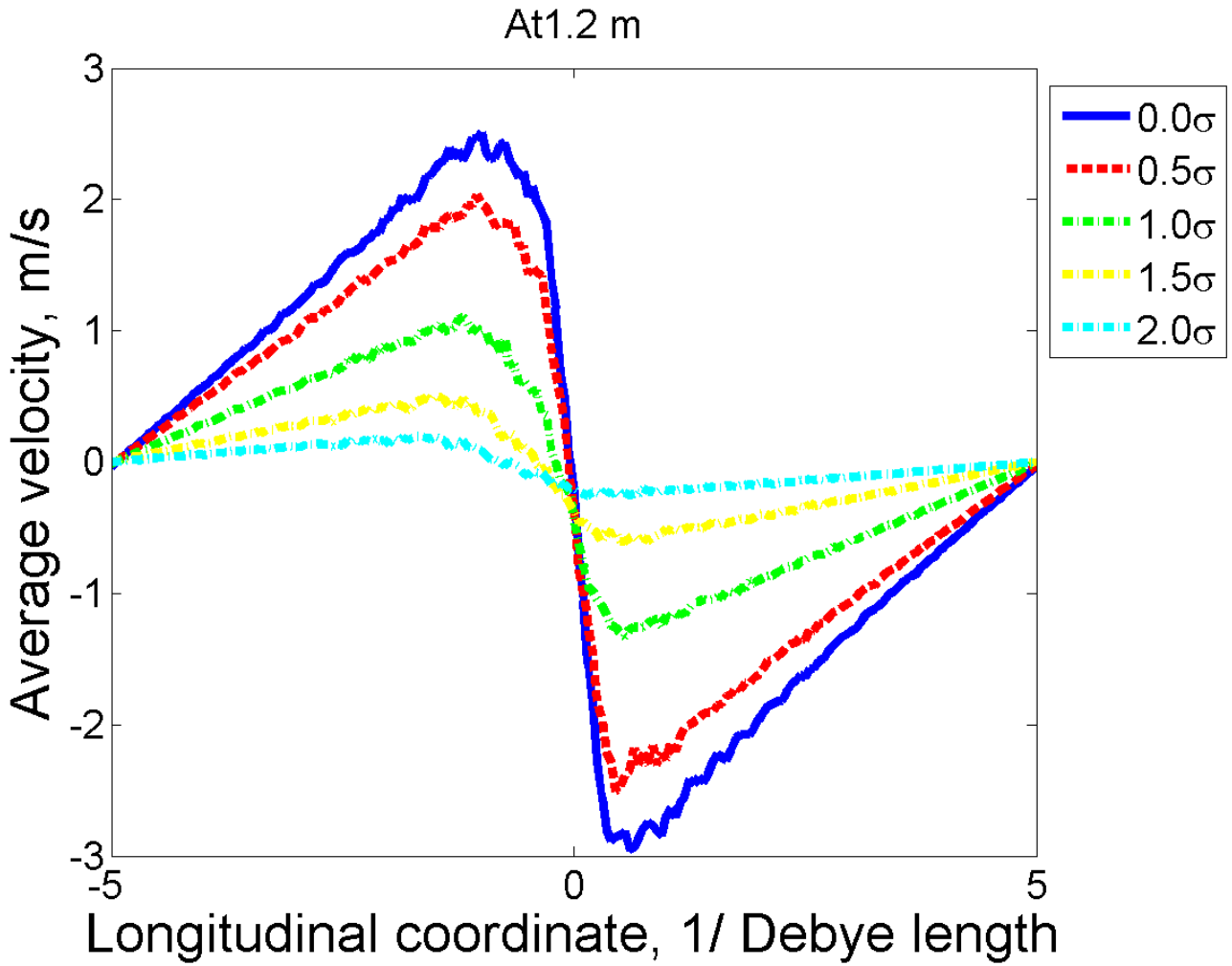

At0.6 m
Number of electrons per Debye length
Longitudinal coordinate, 1/ Debye length
6
5
4
3
2
1
0
-1
-5
0
5
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

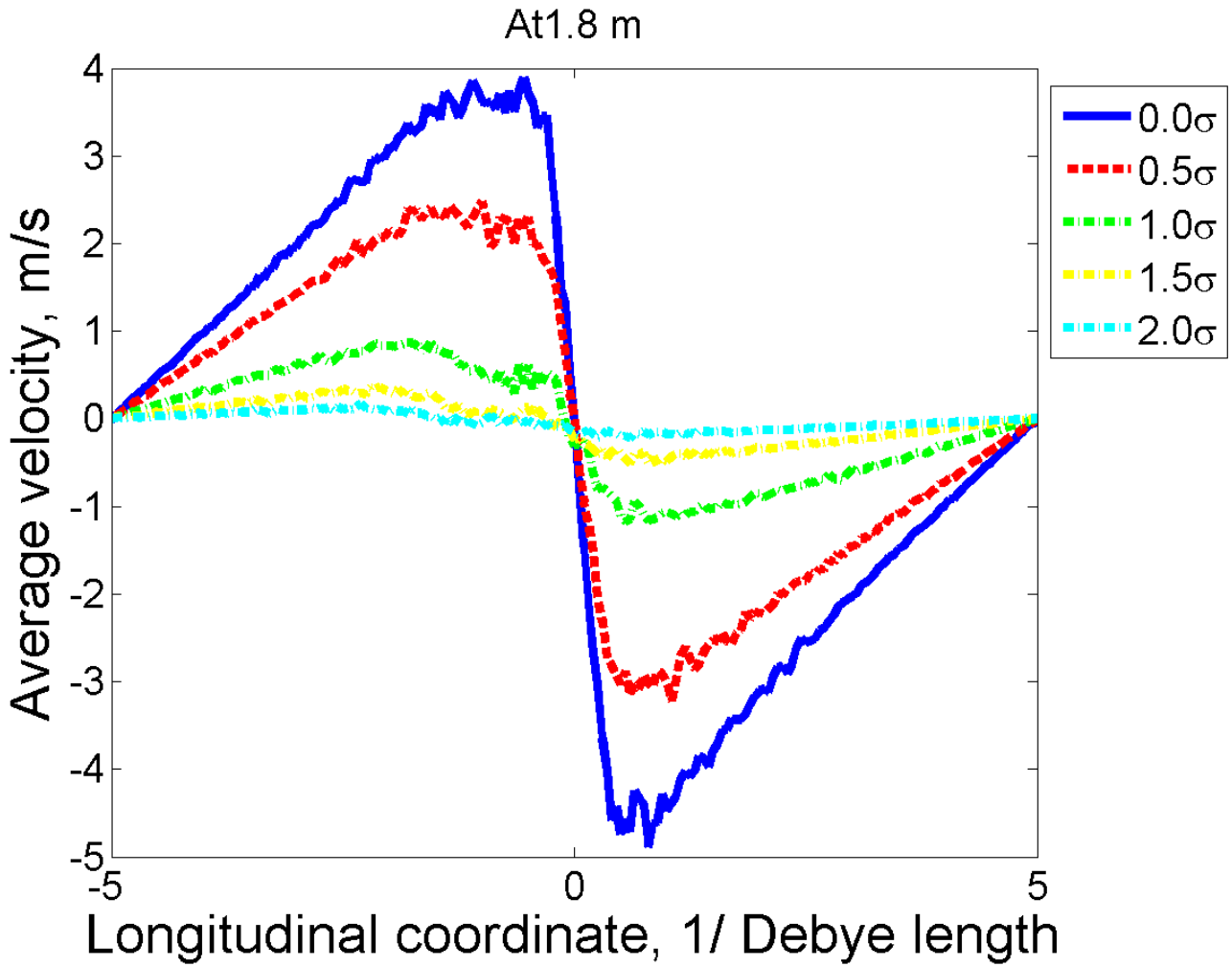

At1.2 m
Number of electrons per Debye length
Longitudinal coordinate, 1/ Debye length
20
15
10
5
0
-5
-5
0
5
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

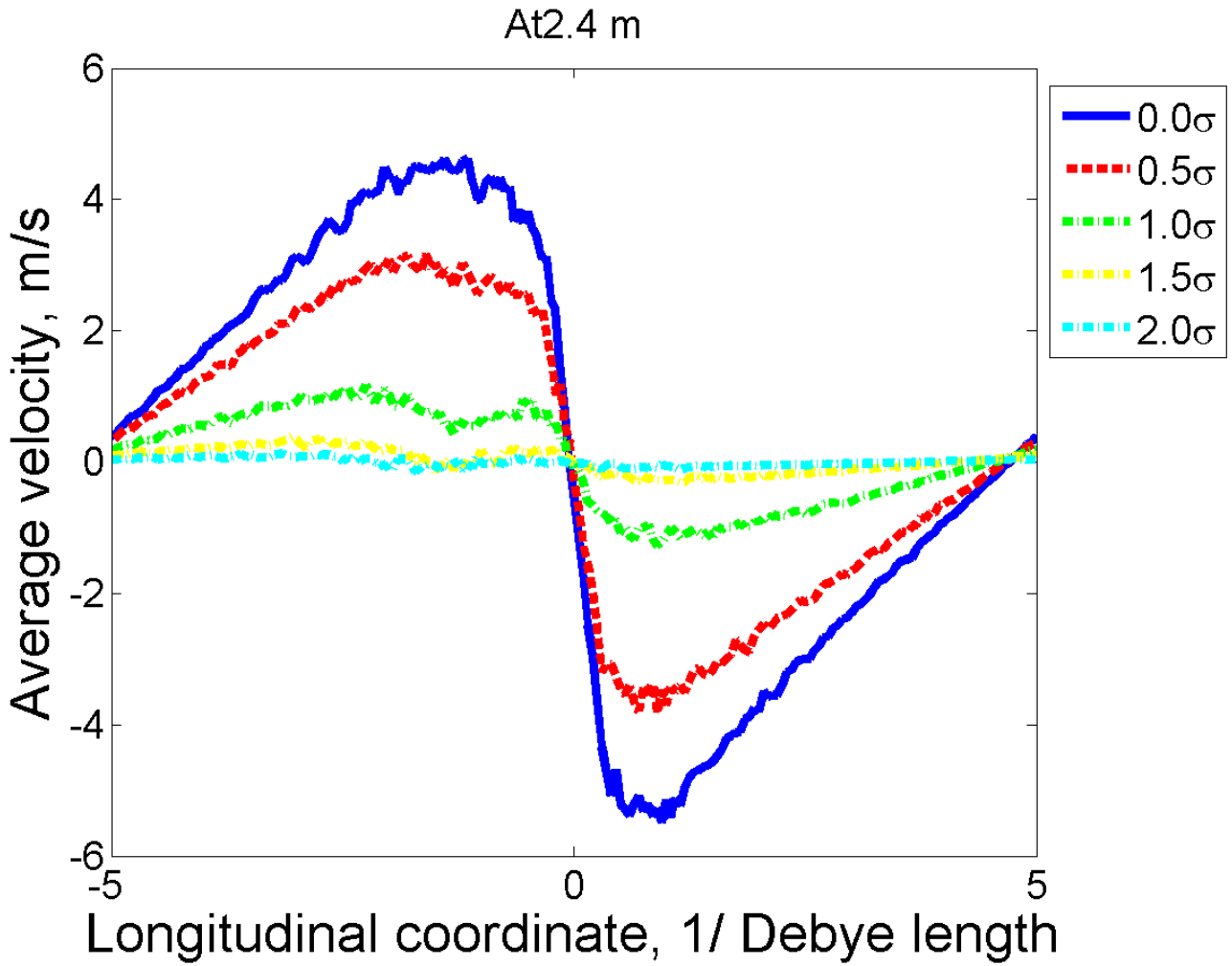

At1.8 m
Number of electrons per Debye length
Longitudinal coordinate, 1/ Debye length
35
30
25
20
15
10
5
0
-5
-5
0
5
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

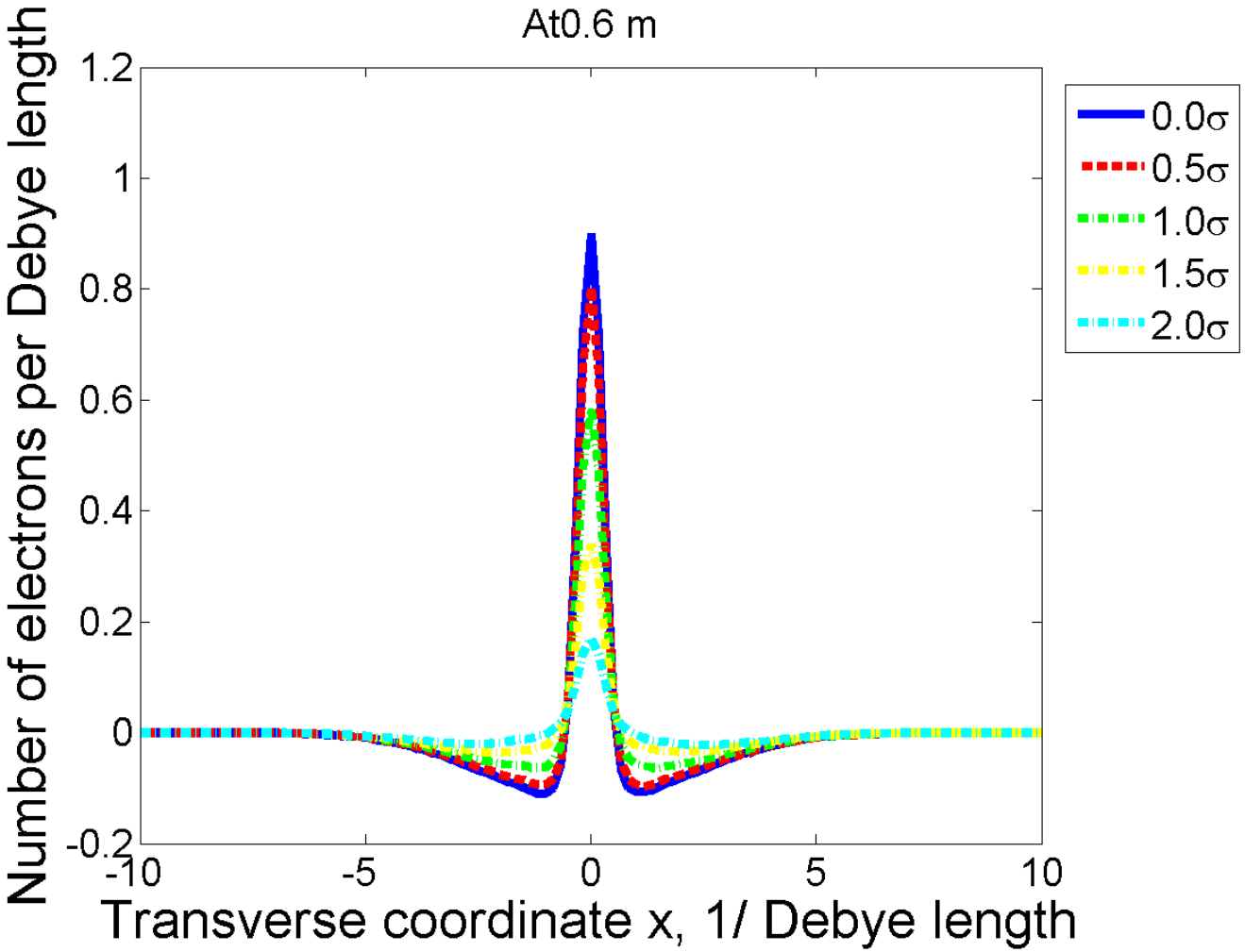

At2.4 m
Number of electrons per Debye length
Longitudinal coordinate, 1/ Debye length
50
40
30
20
10
0
-10
-5
0
5
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

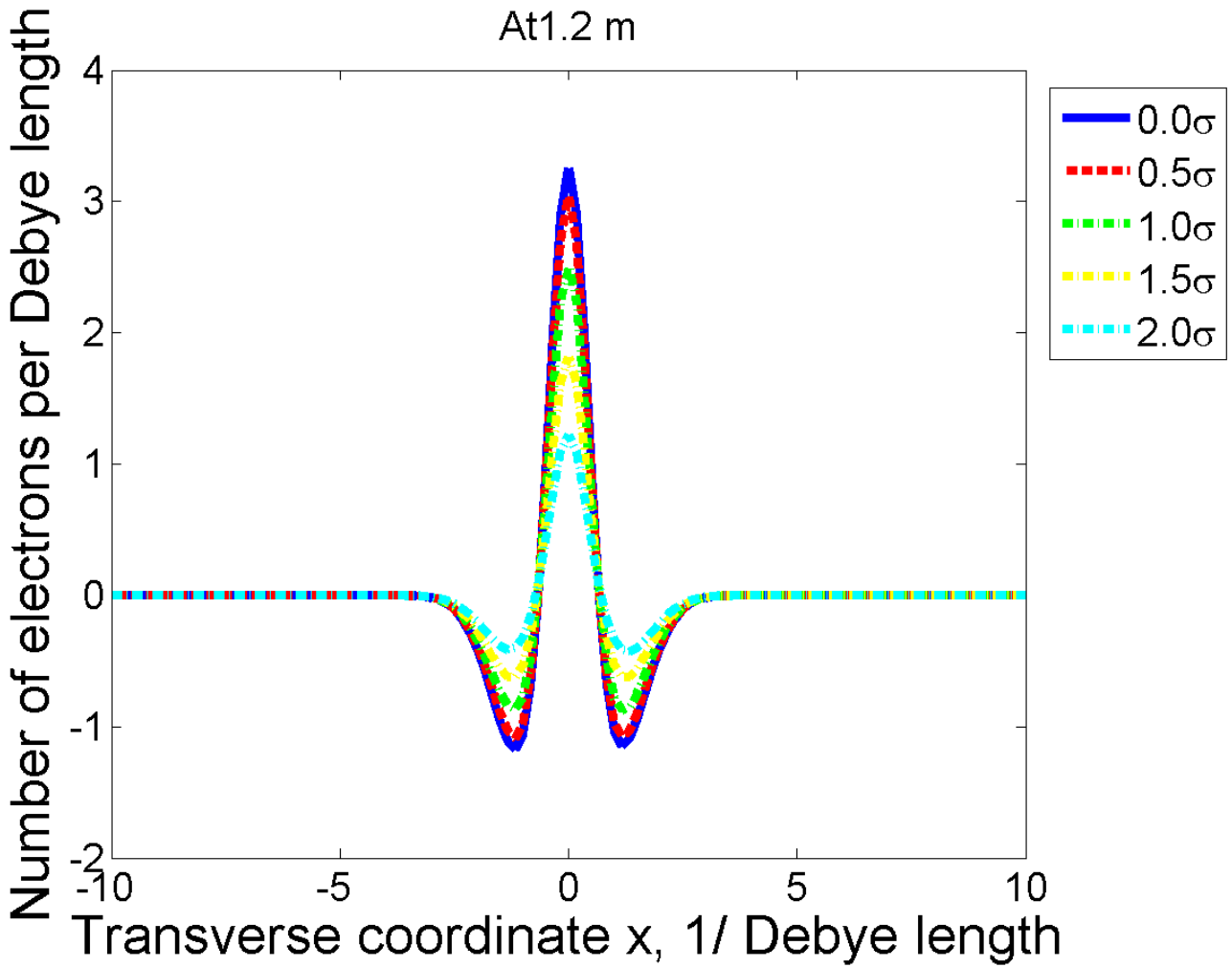

At0.6 m
Average velocity, m/s
Longitudinal coordinate, 1/ Debye length
2
1.5
1
0.5
0
-0.5
-1
-1.5
-5
0
5
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

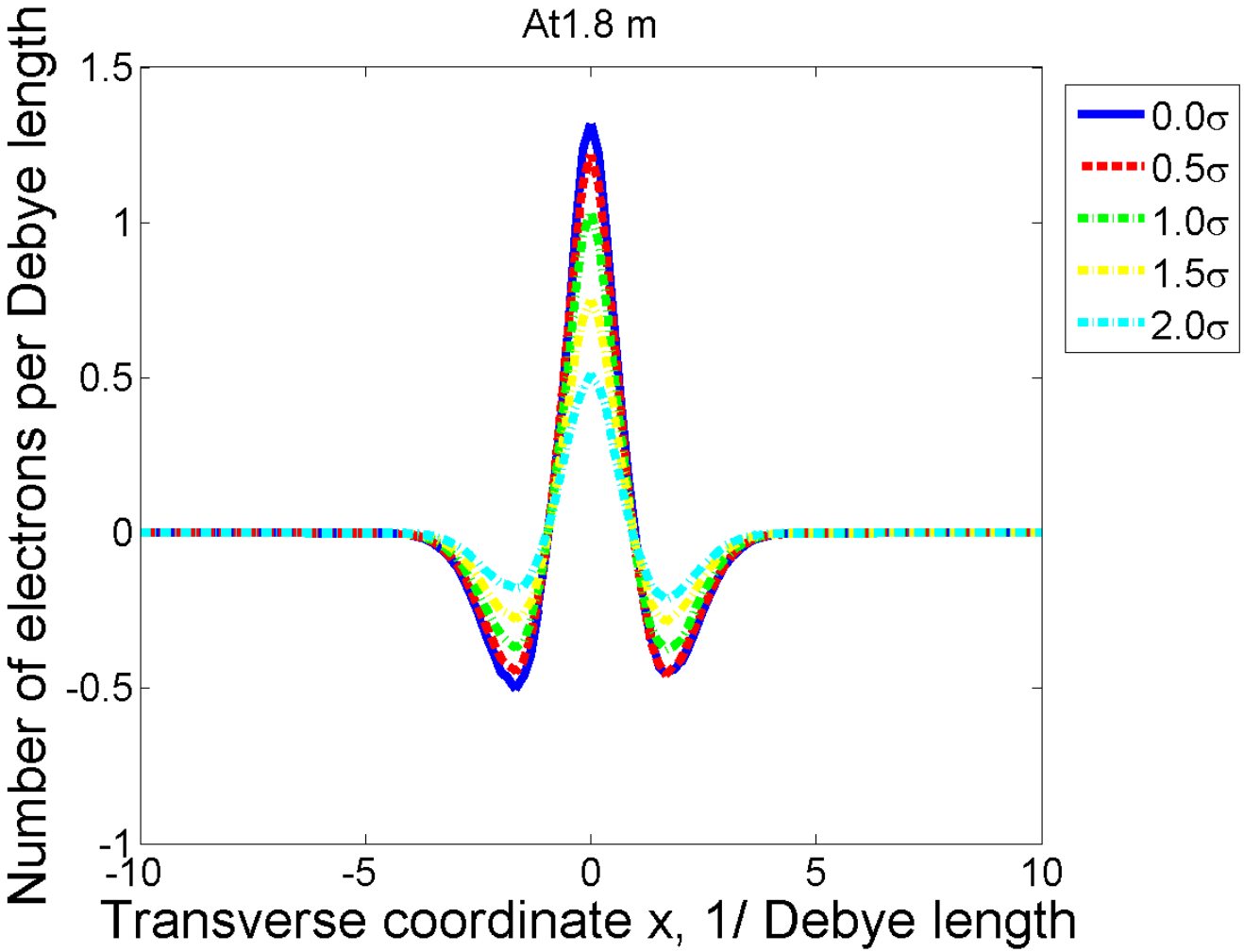

At1.2 m
Average velocity, m/s
Longitudinal coordinate, 1/ Debye length
3
2
1
0
-1
-2
-3
-5
0
5
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

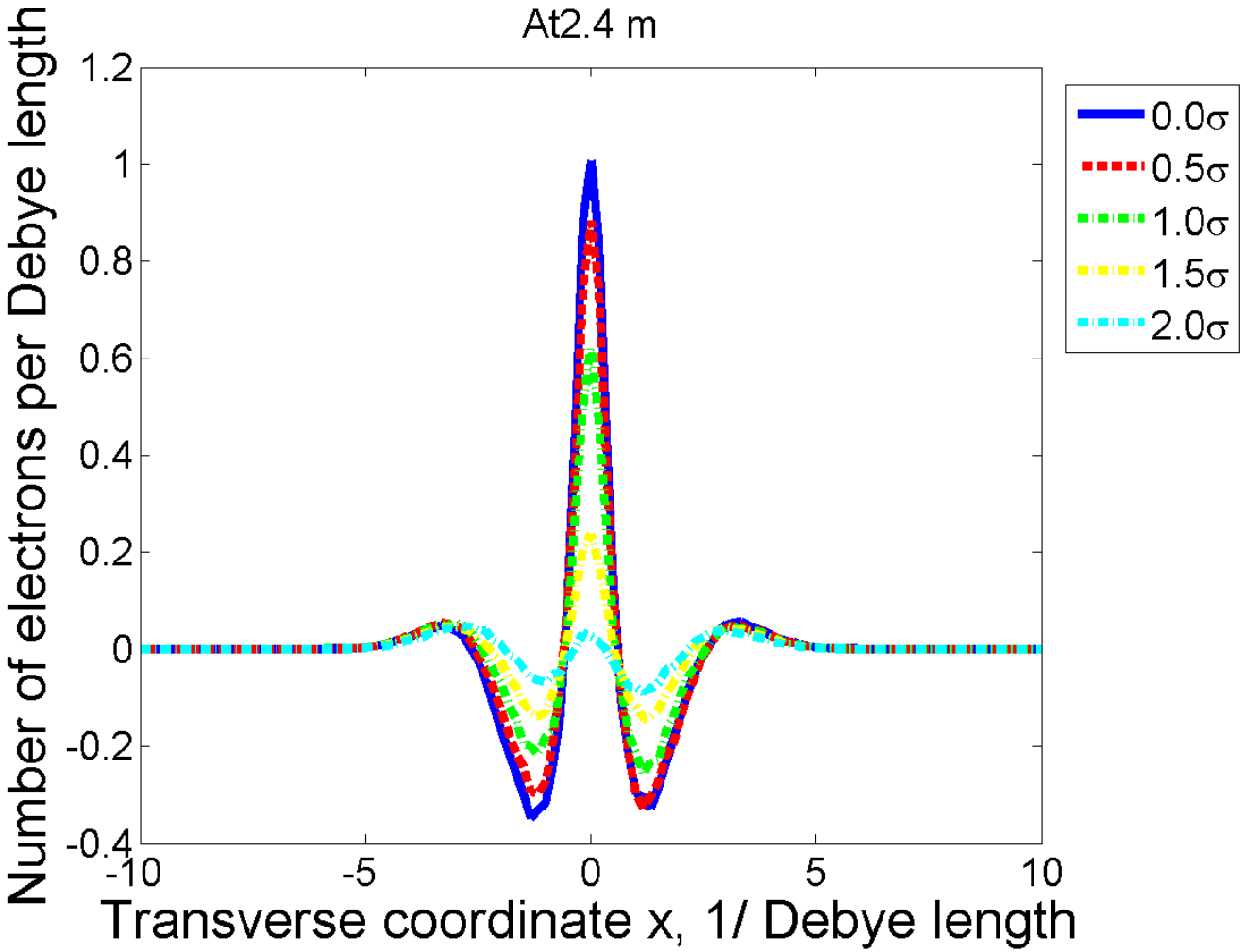

At1.8 m
Average velocity, m/s
Longitudinal coordinate, 1/ Debye length
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ
4
3
2
1
0
-1
-2
-3
-4
-5
-5
0
5

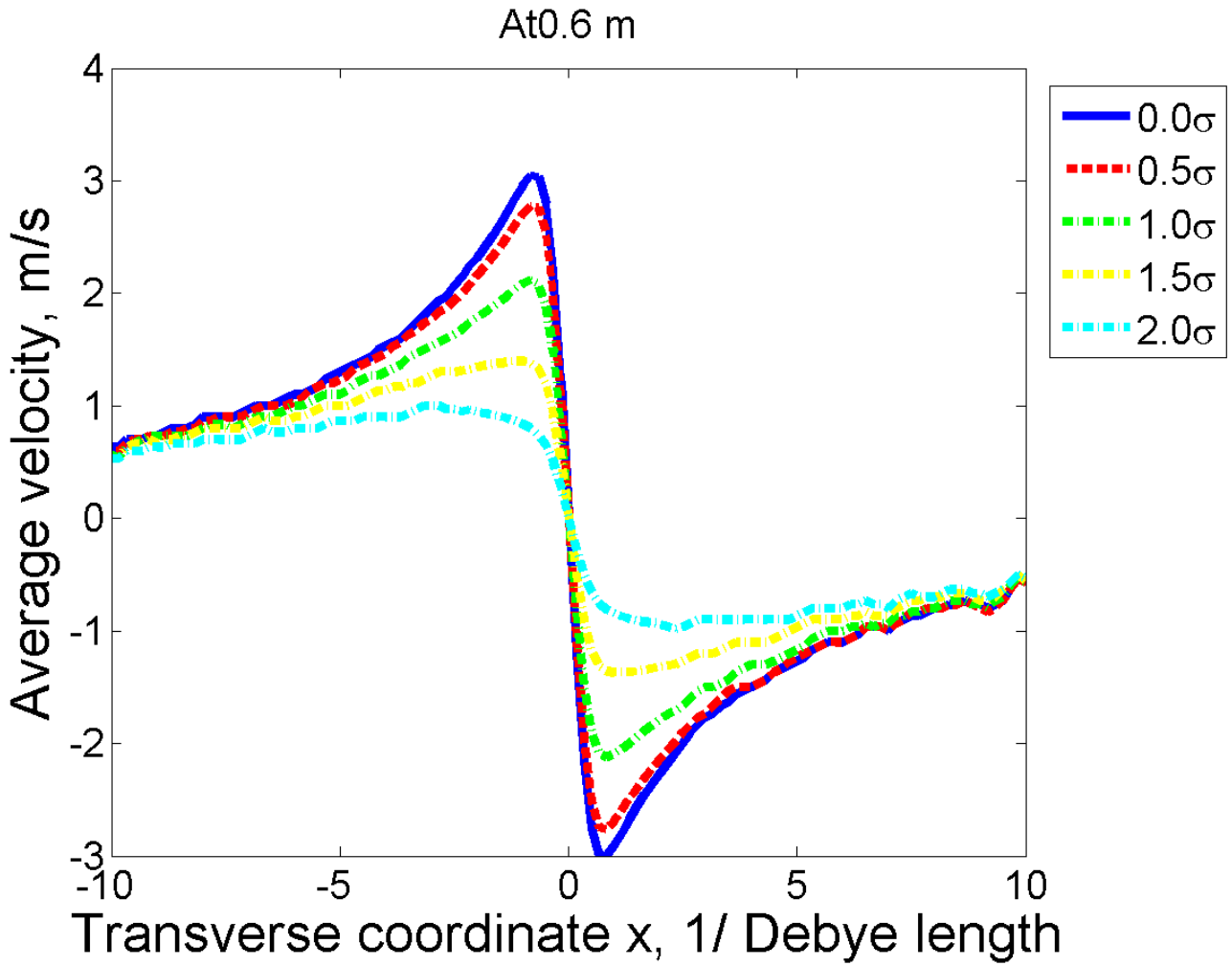

At2.4 m
Average velocity, m/s
Longitudinal coordinate, 1/ Debye length
6
4
2
0
-2
-4
-6
-5
0
5
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

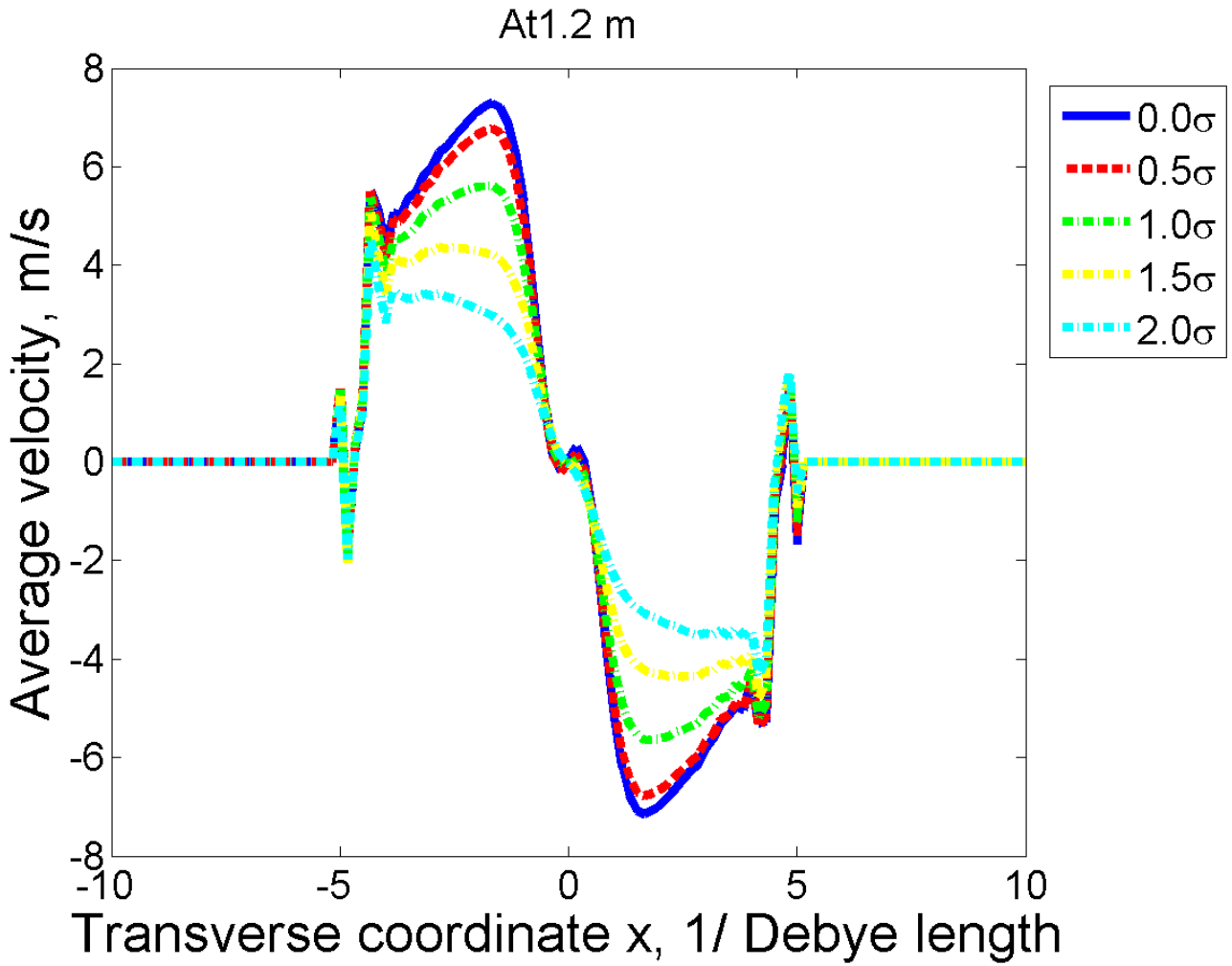

At0.6 m
Number of electrons per Debye length
Transverse coordinate x, 1/ Debye length
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-10
-5
0
5
10
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

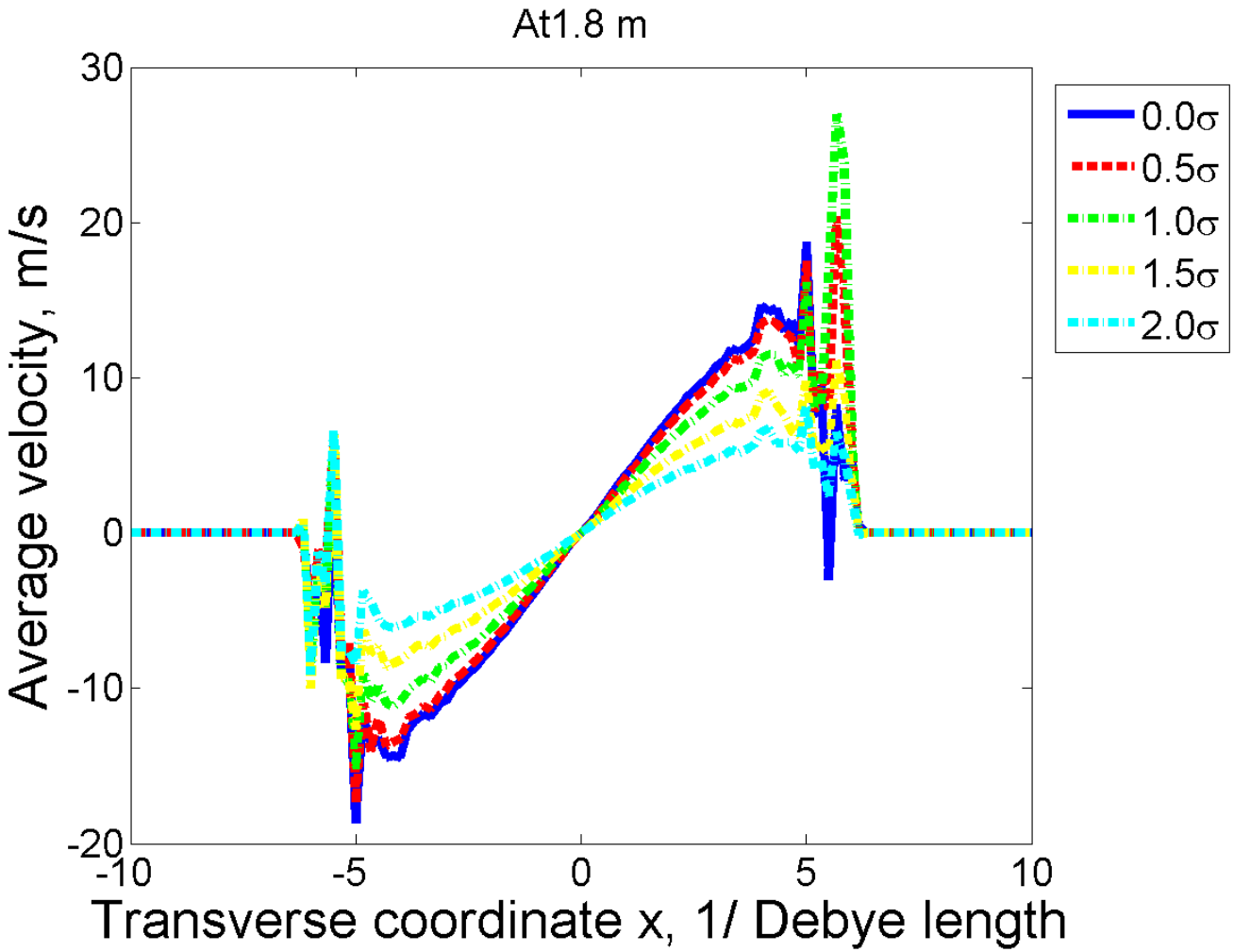

At1.2 m
Number of electrons per Debye length
Transverse coordinate x, 1/ Debye length
4
3
2
1
0
-1
-2
-10
-5
0
5
10
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

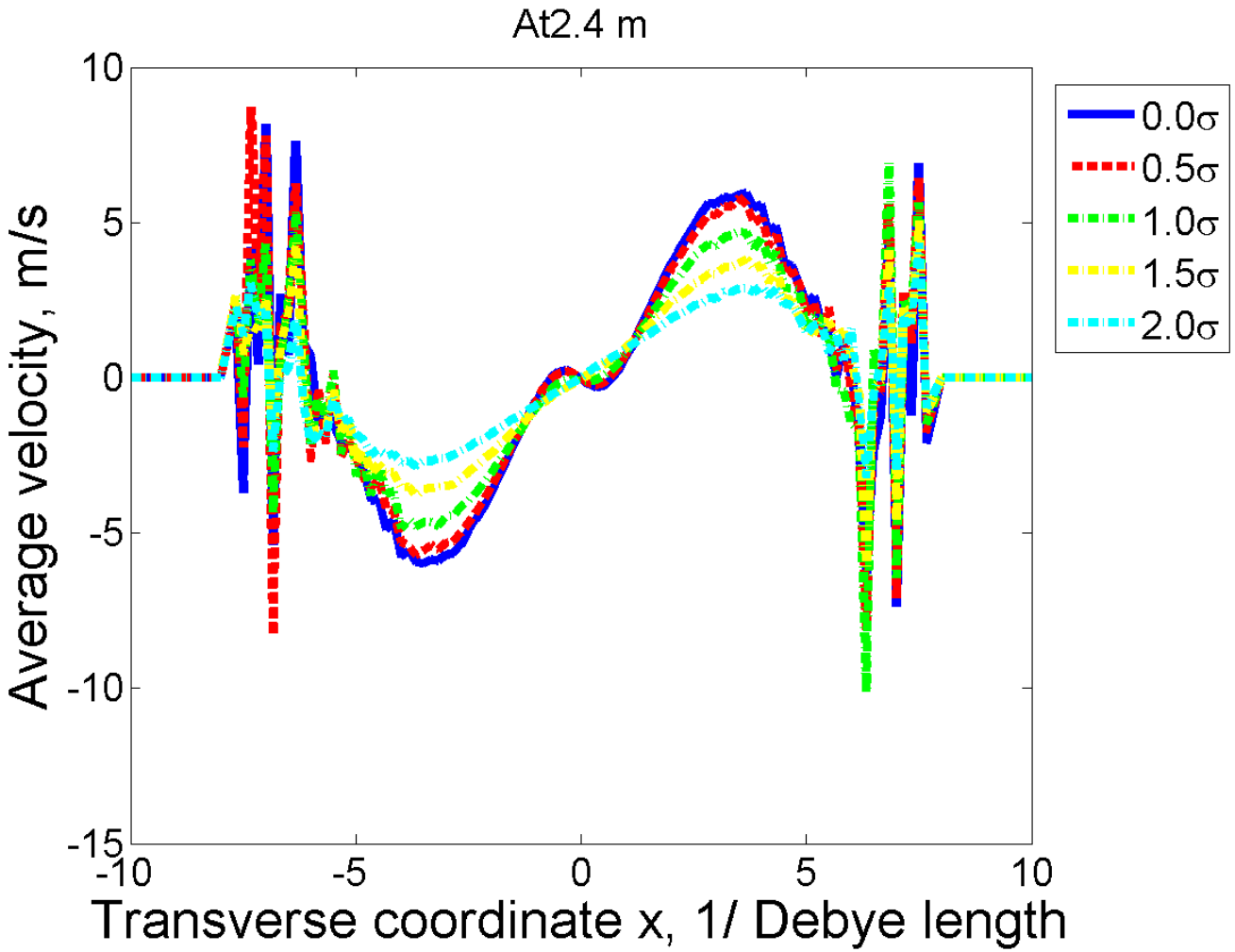

At1.8 m
Number of electrons per Debye length
Transverse coordinate x, 1/ Debye length
1.5
1
0.5
0
-0.5
-1
-10
-5
0
5
10
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

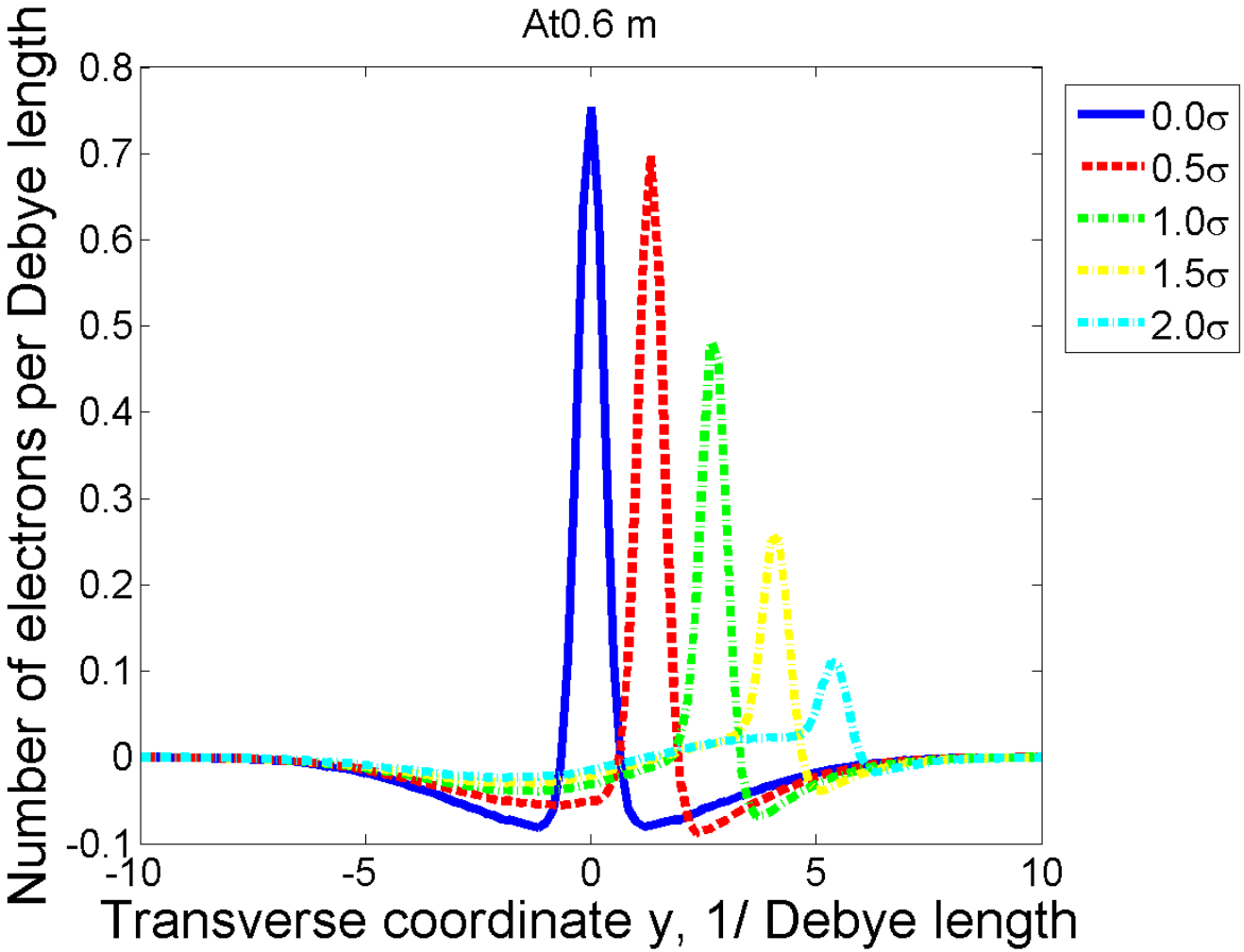

At2.4 m
Number of electrons per Debye length
Transverse coordinate x, 1/ Debye length
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-10
-5
0
5
10
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

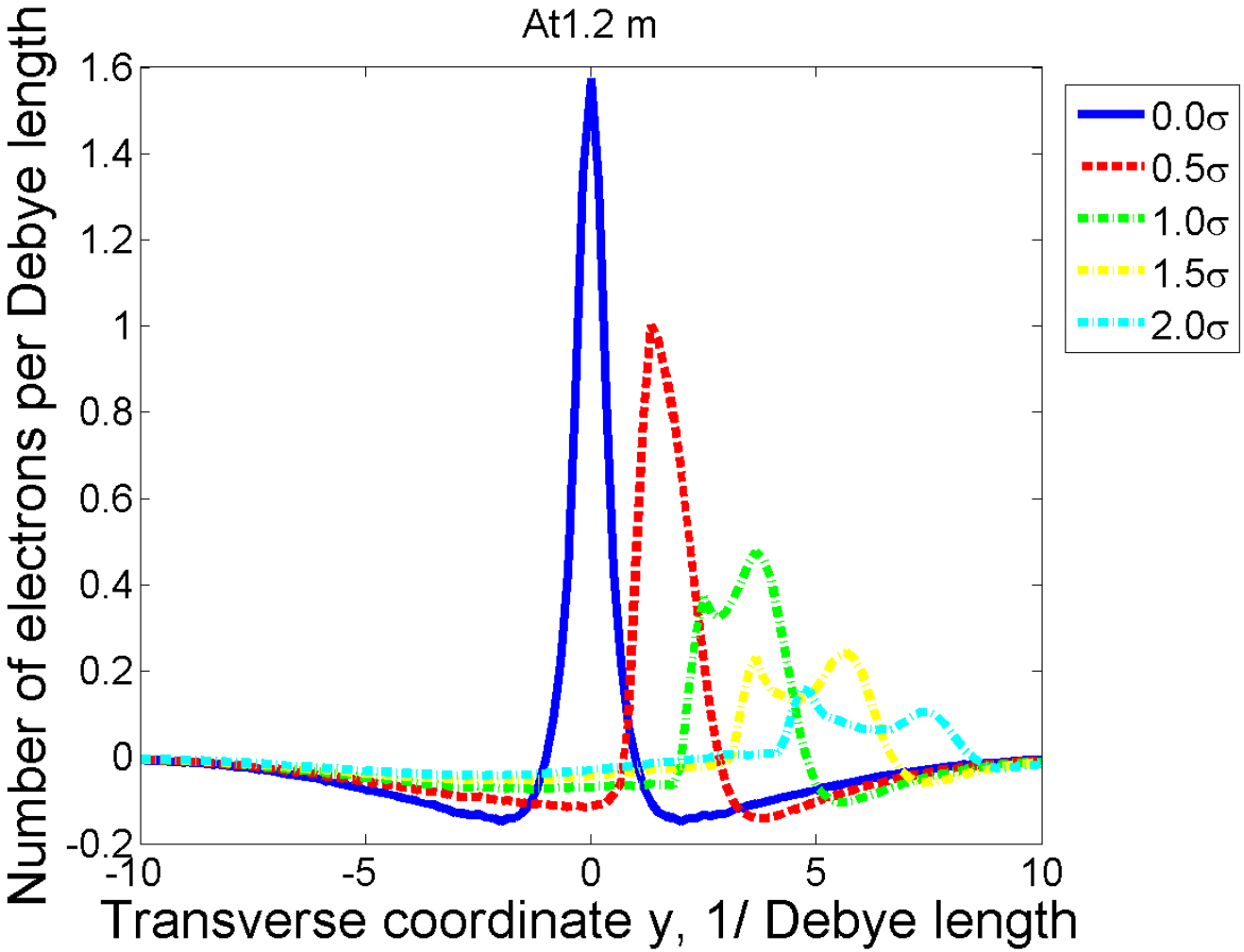

At0.6 m
Average velocity, m/s
Transverse coordinate x, 1/ Debye length
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ
4
3
2
1
0
-1
-2
-3
-10
-5
0
5
10

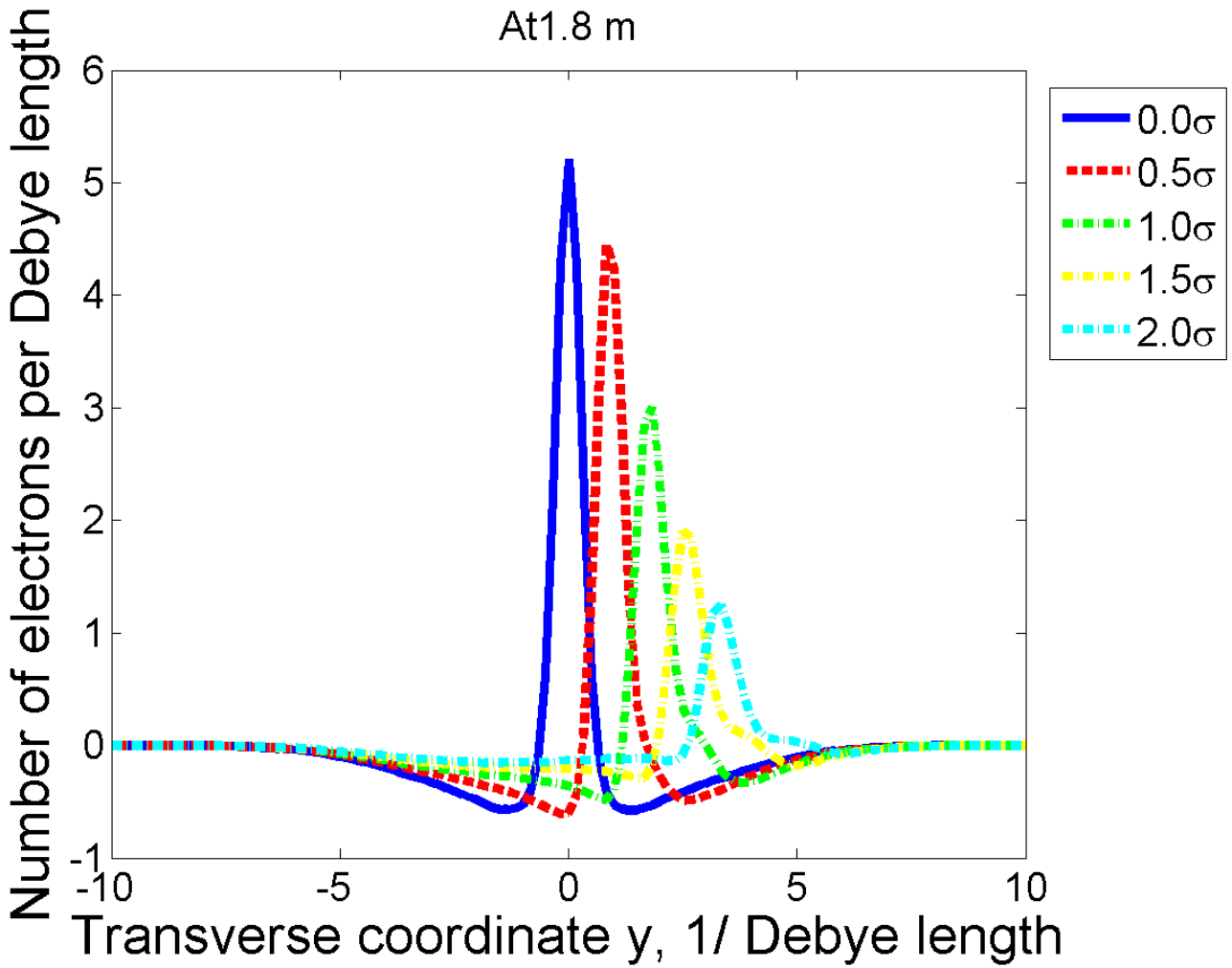

At1.2 m
Average velocity, m/s
Transverse coordinate x, 1/ Debye length
8
6
4
2
0
-2
-4
-6
-8
-10
-5
0
5
10
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

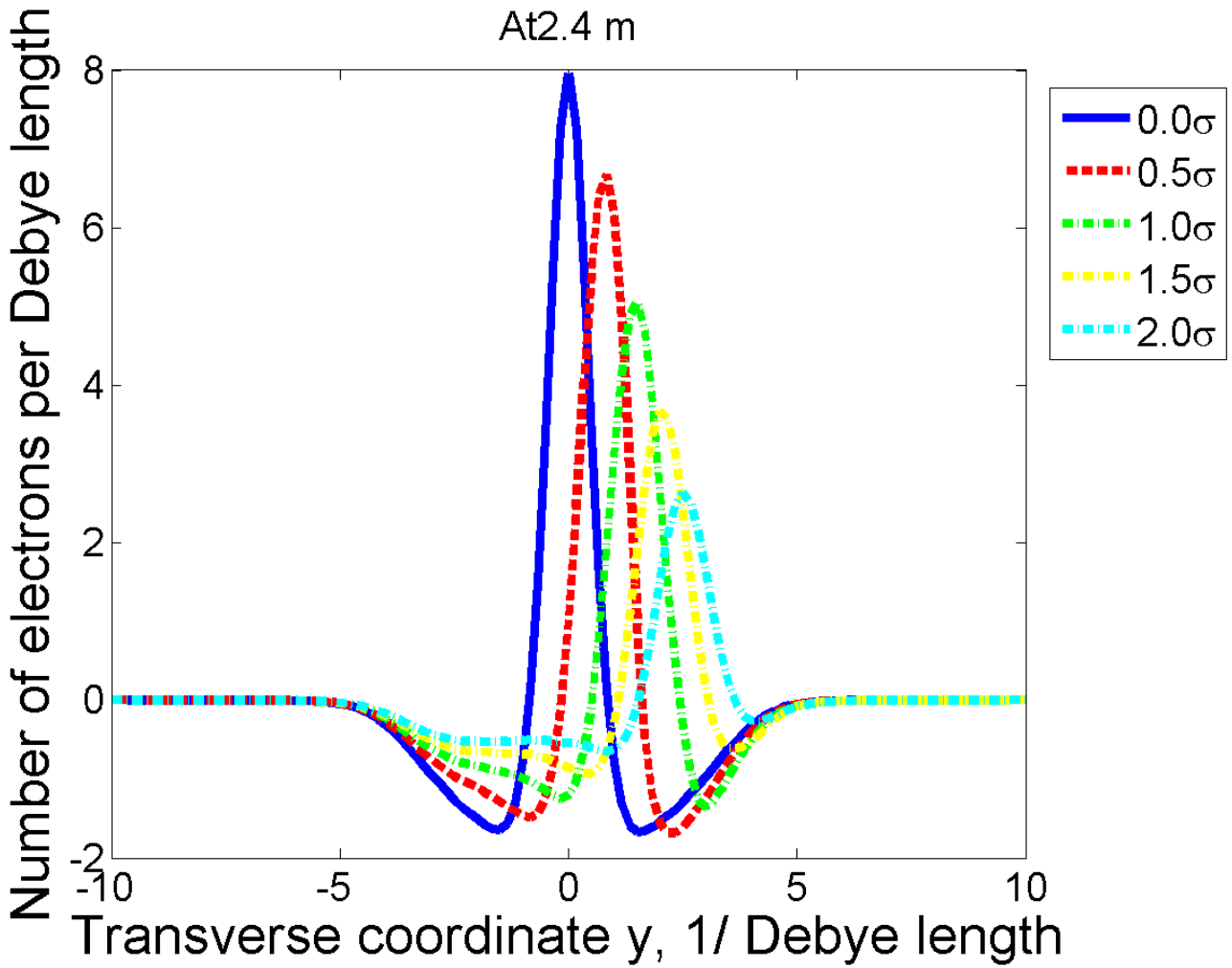

At1.8 m
Average velocity, m/s
Transverse coordinate x, 1/ Debye length
30
20
10
0
-10
-20
-10
-5
0
5
10
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

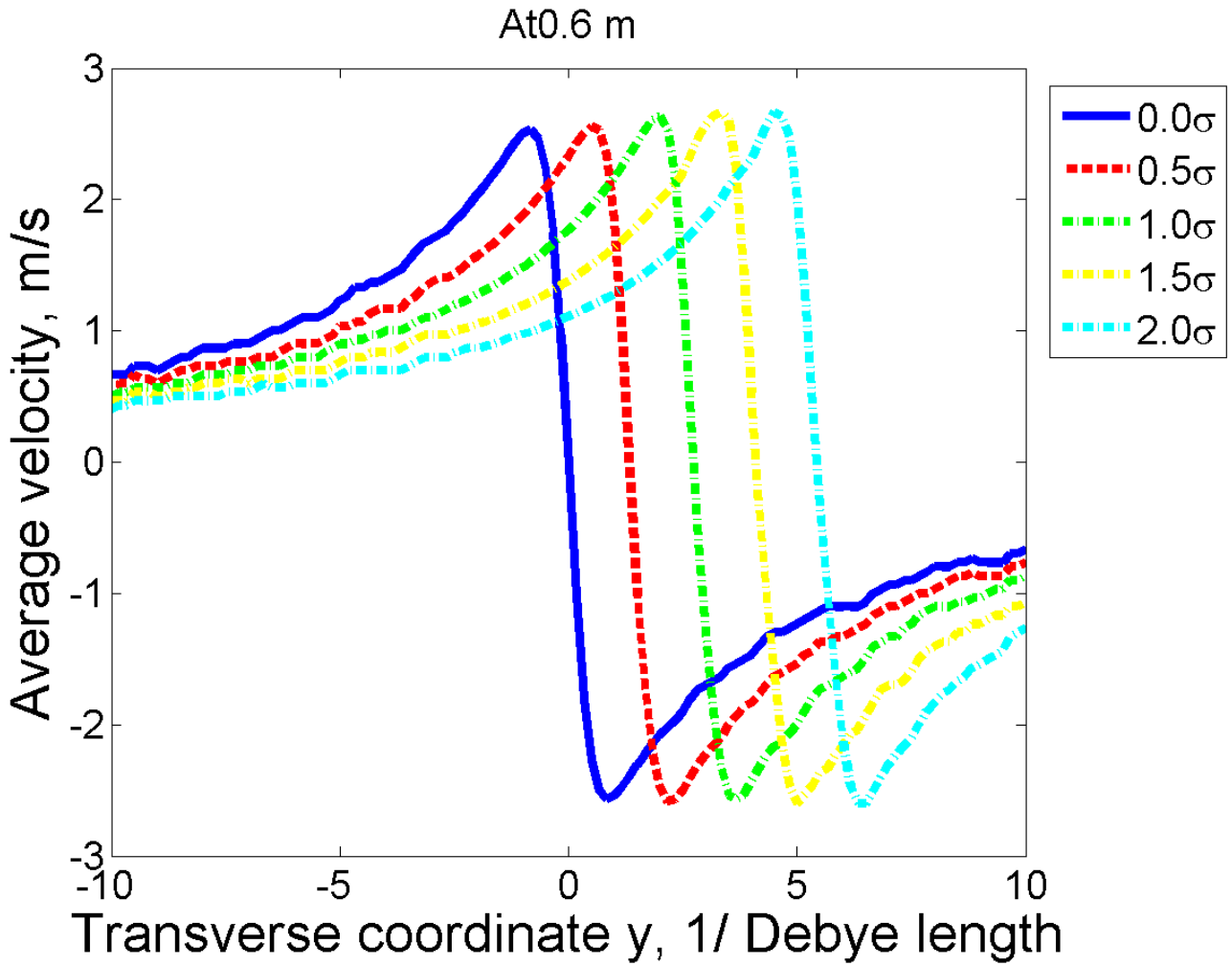

At2.4 m
Average velocity, m/s
Transverse coordinate x, 1/ Debye length
10
5
0
-5
-10
-15
-10
-5
0
5
10
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

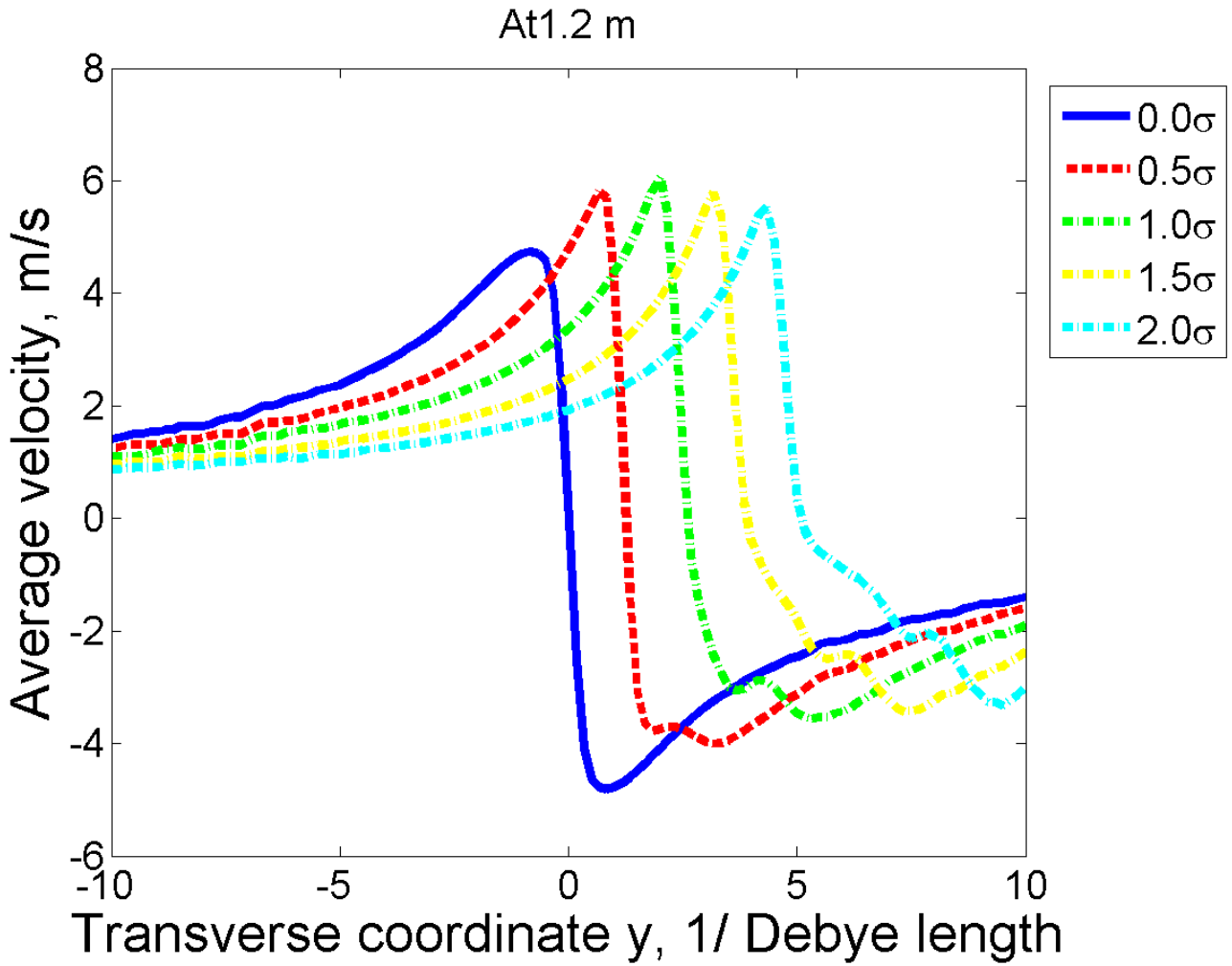

At0.6 m
Number of electrons per Debye length
Transverse coordinate y, 1/ Debye length
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-10
-5
0
5
10
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

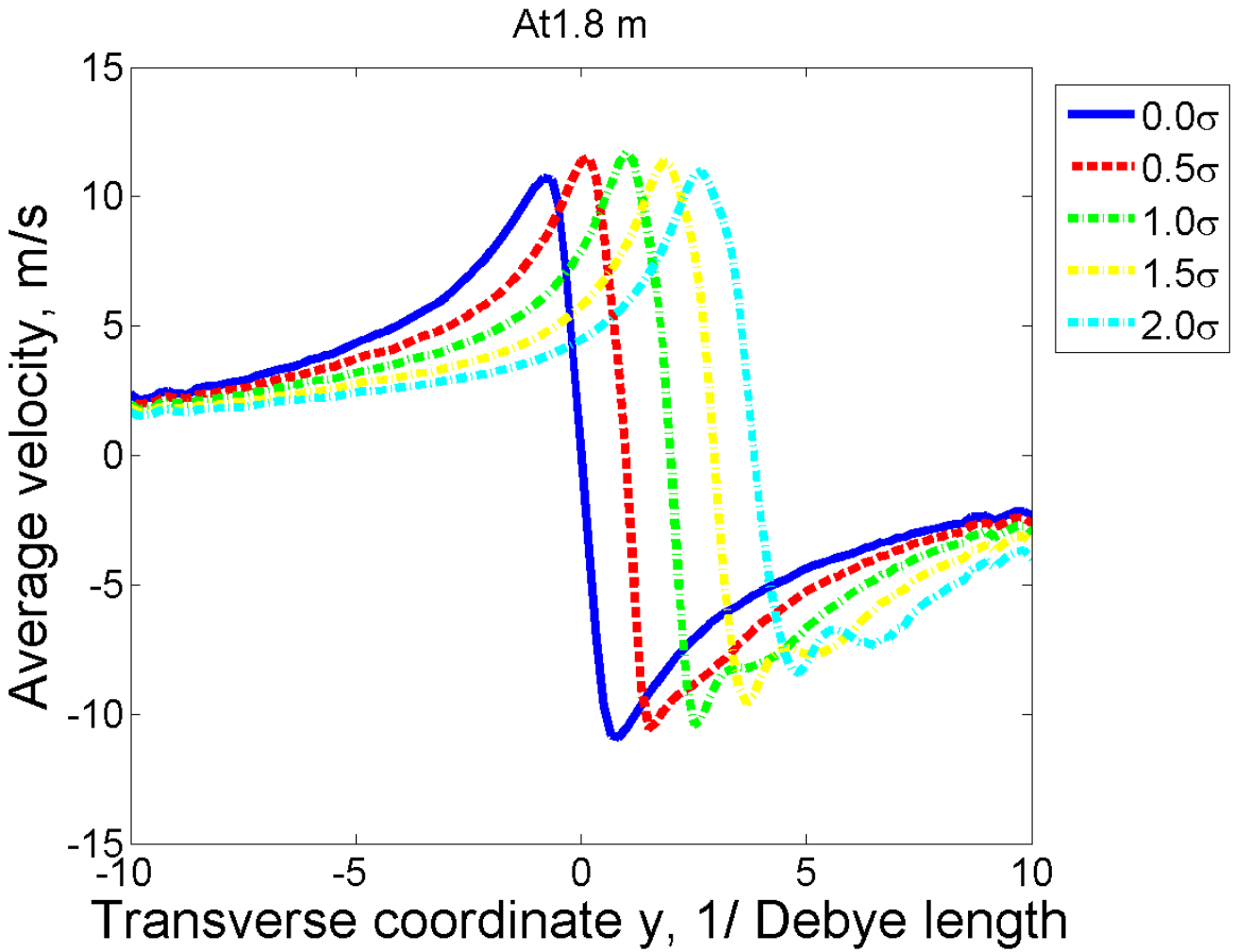

At1.2 m
Number of electrons per Debye length
Transverse coordinate y, 1/ Debye length
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-10
-5
0
5
10
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

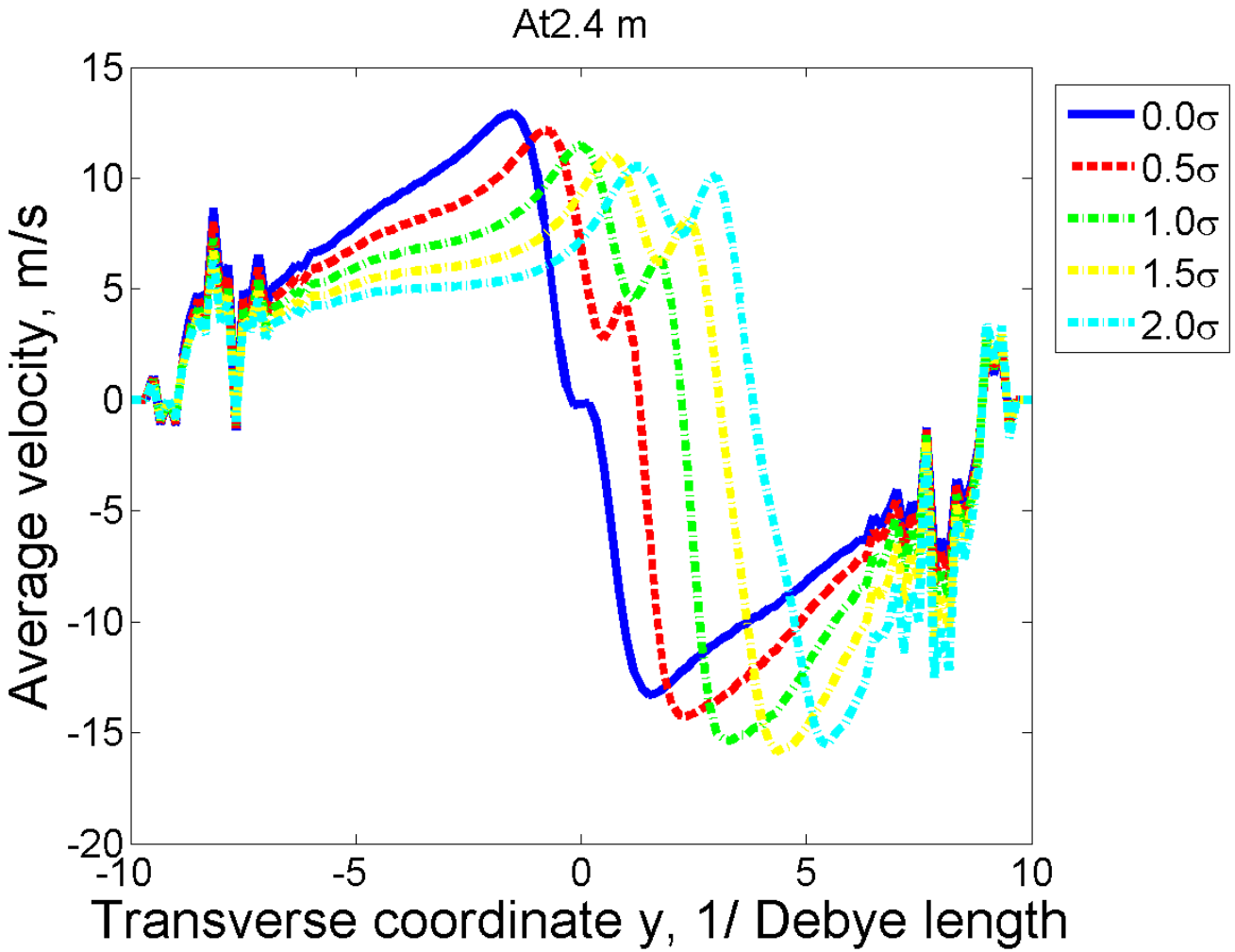

At1.8 m
Number of electrons per Debye length
Transverse coordinate y, 1/ Debye length
6
5
4
3
2
1
0
-1
-10
-5
0
5
10
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

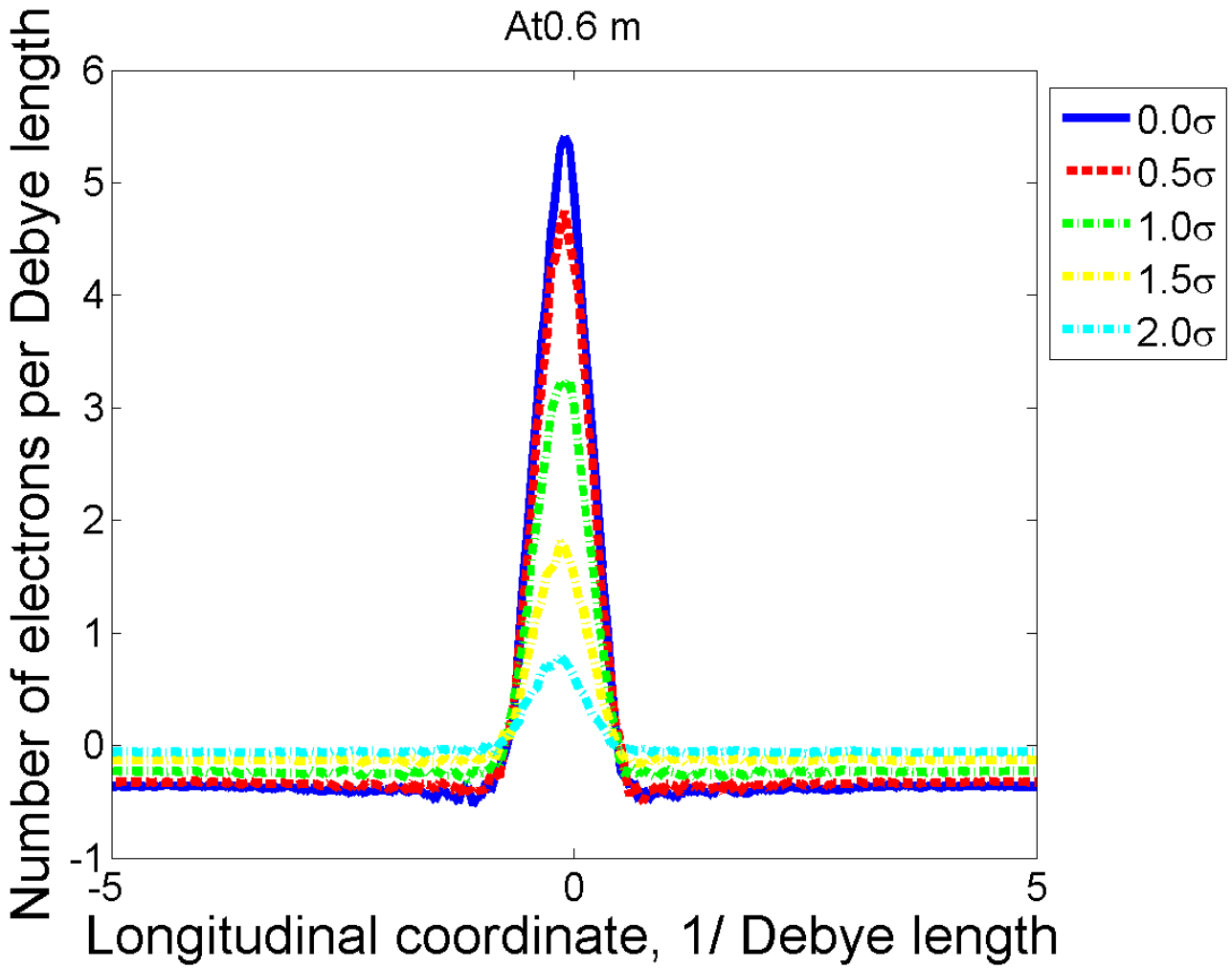

At2.4 m
Number of electrons per Debye length
Transverse coordinate y, 1/ Debye length
8
6
4
2
0
-2
-10
-5
0
5
10
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

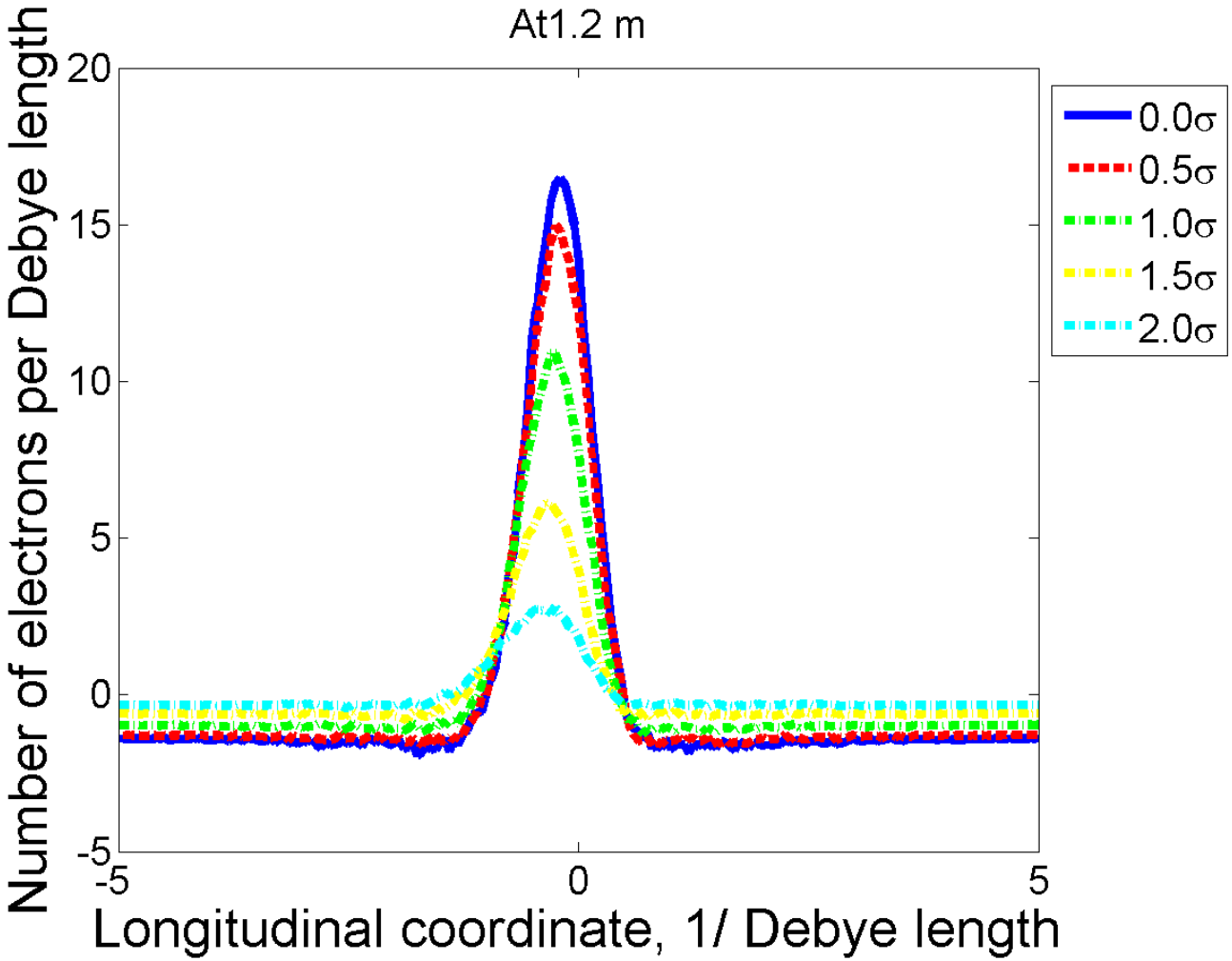

At0.6 m
Average velocity, m/s
Transverse coordinate y, 1/ Debye length
3
2
1
0
-1
-2
-3
-10
-5
0
5
10
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

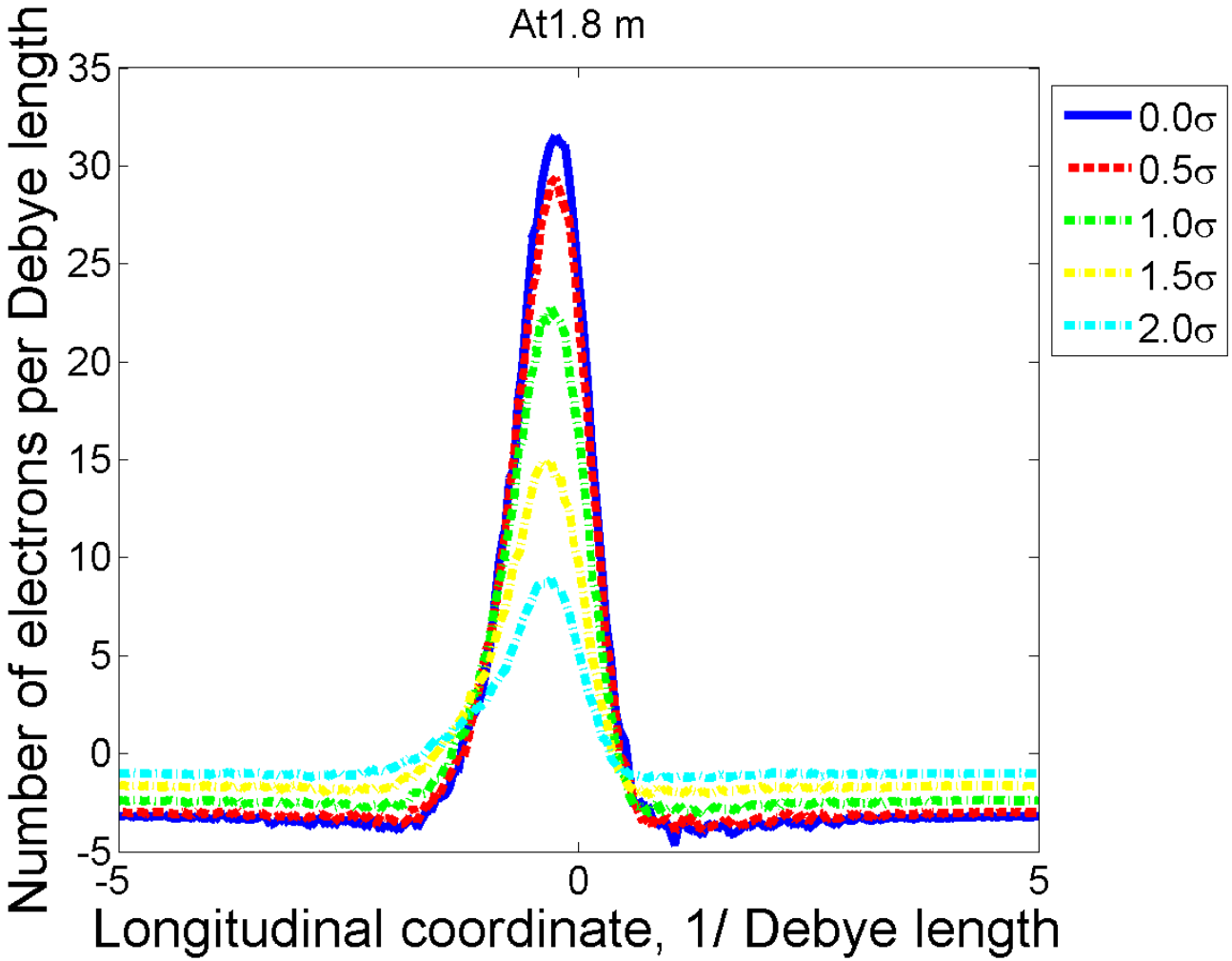

At1.2 m
Average velocity, m/s
Transverse coordinate y, 1/ Debye length
8
6
4
2
0
-2
-4
-6
-10
-5
0
5
10
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

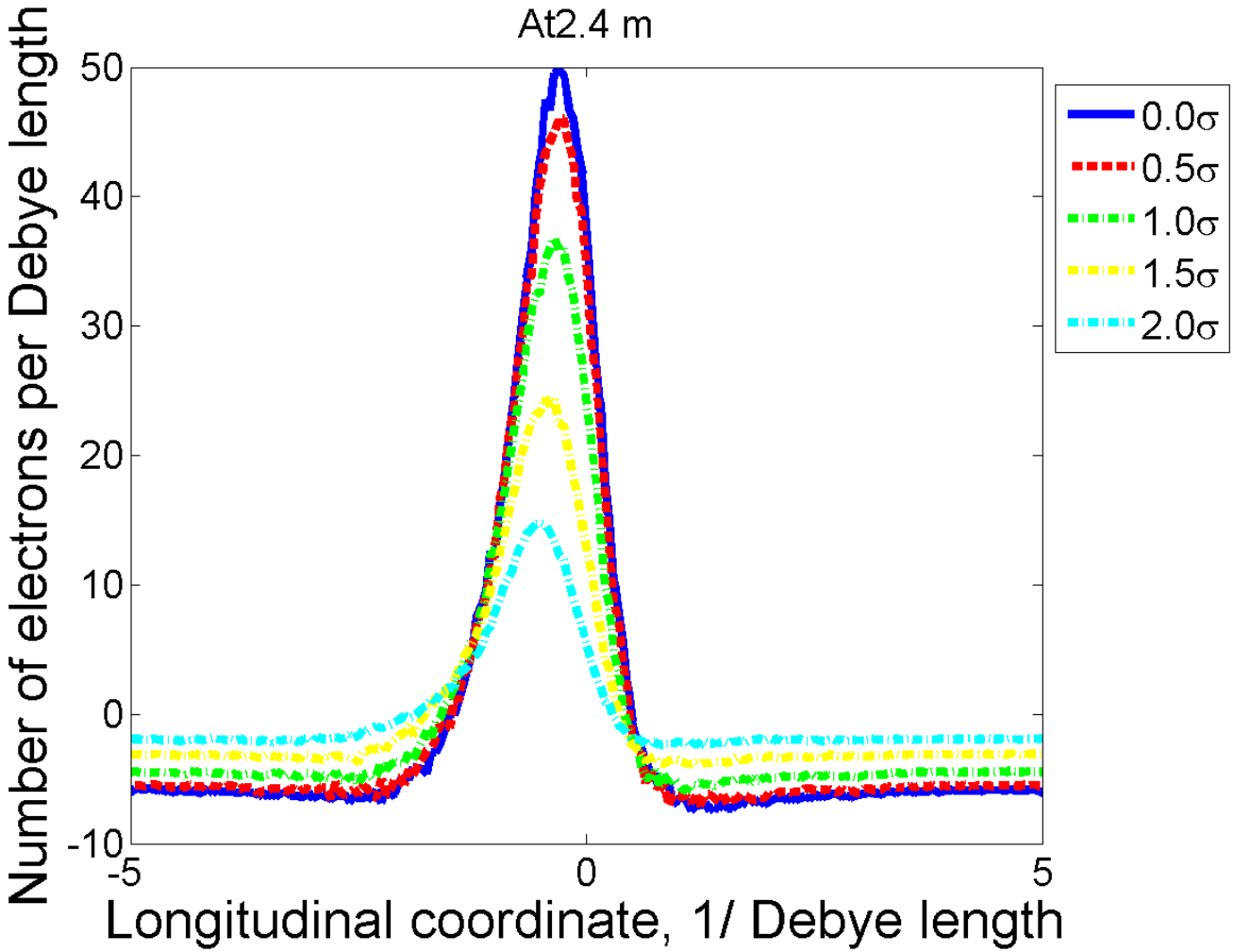

At1.8 m
Average velocity, m/s
Transverse coordinate y, 1/ Debye length
15
10
5
0
-5
-10
-15
-10
-5
0
5
10
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

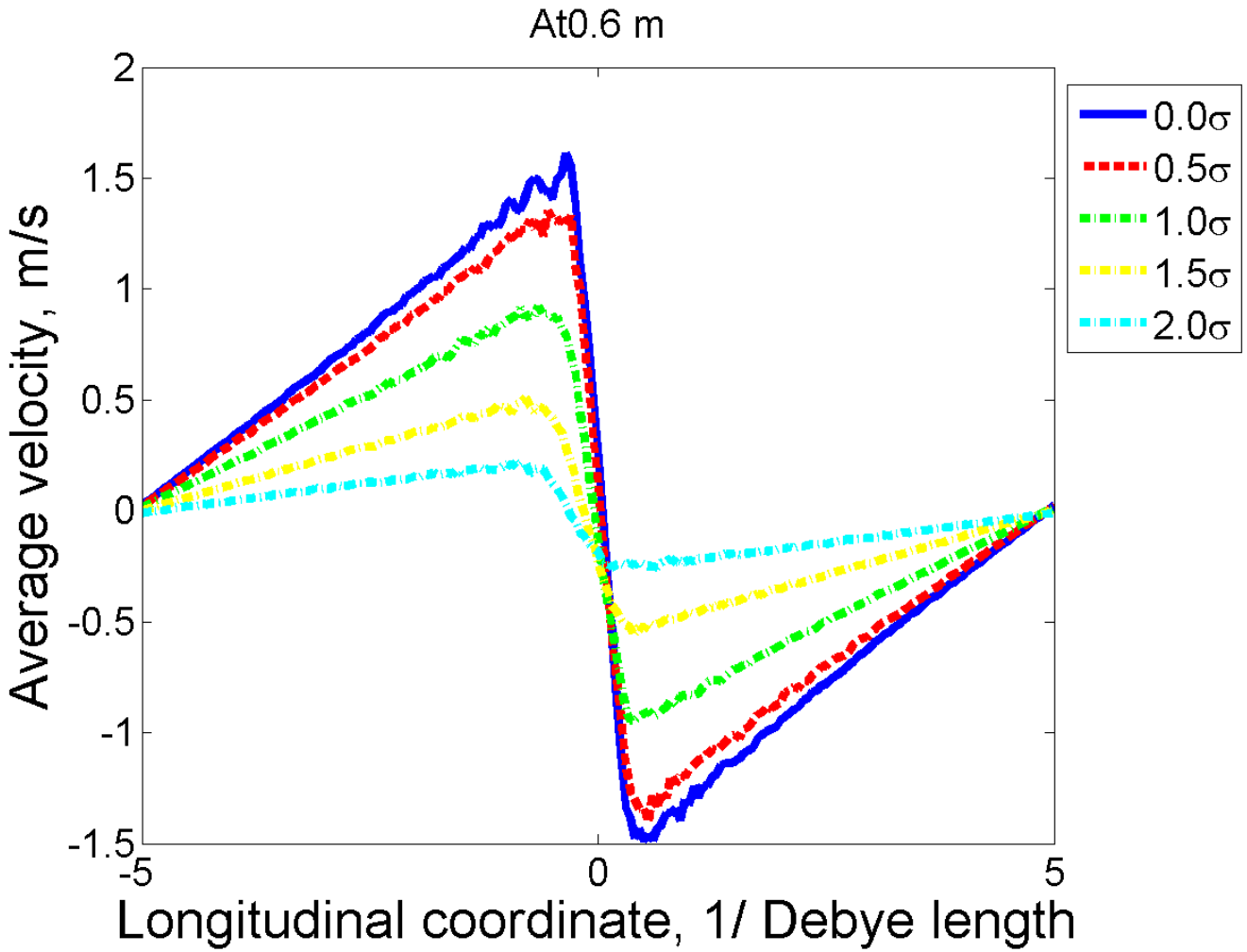

At2.4 m
Average velocity, m/s
Transverse coordinate y, 1/ Debye length
15
10
5
0
-5
-10
-15
-20
-10
-5
0
5
10
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

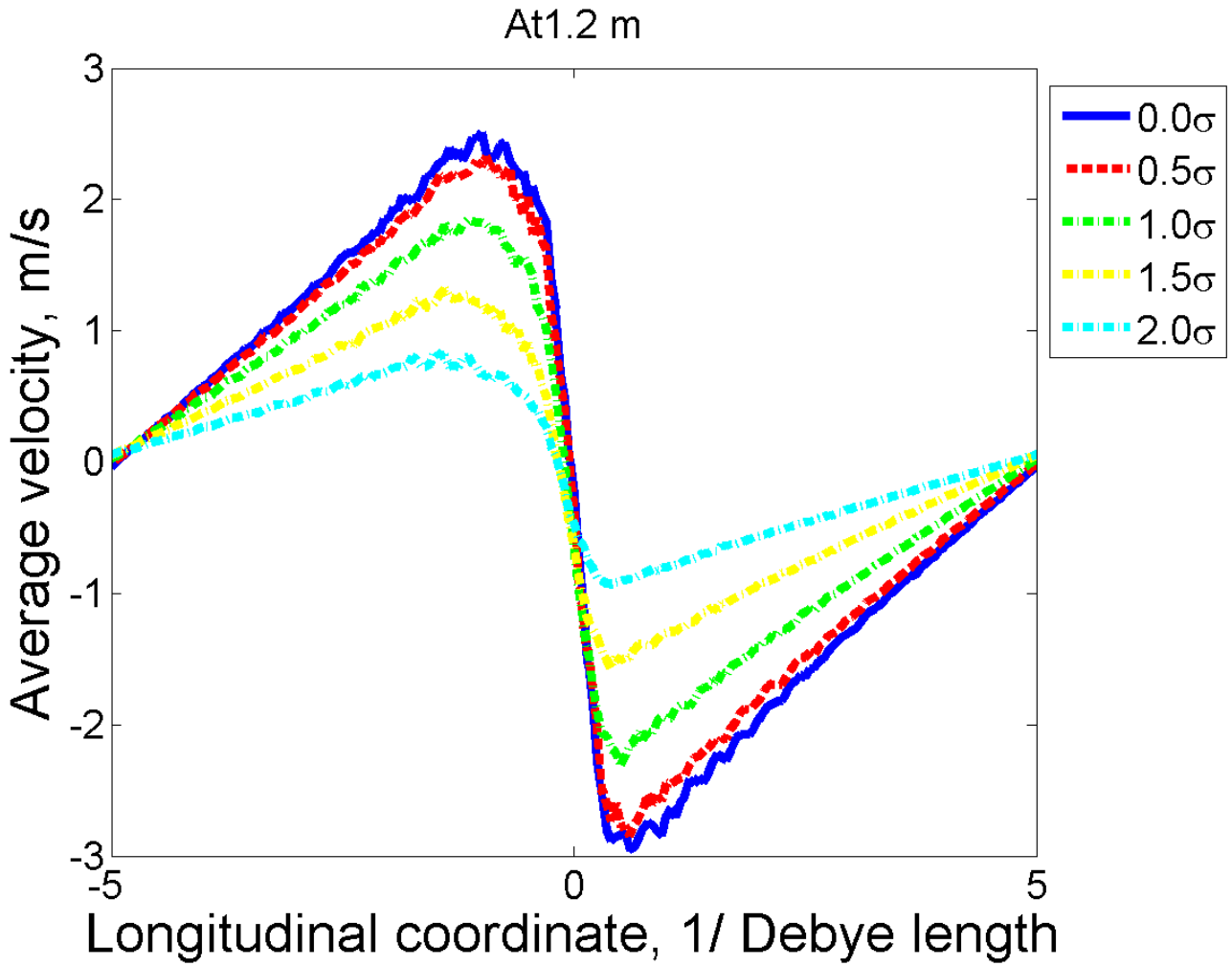

At0.6 m
Number of electrons per Debye length
Longitudinal coordinate, 1/ Debye length
6
5
4
3
2
1
0
-1
-5
0
5
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

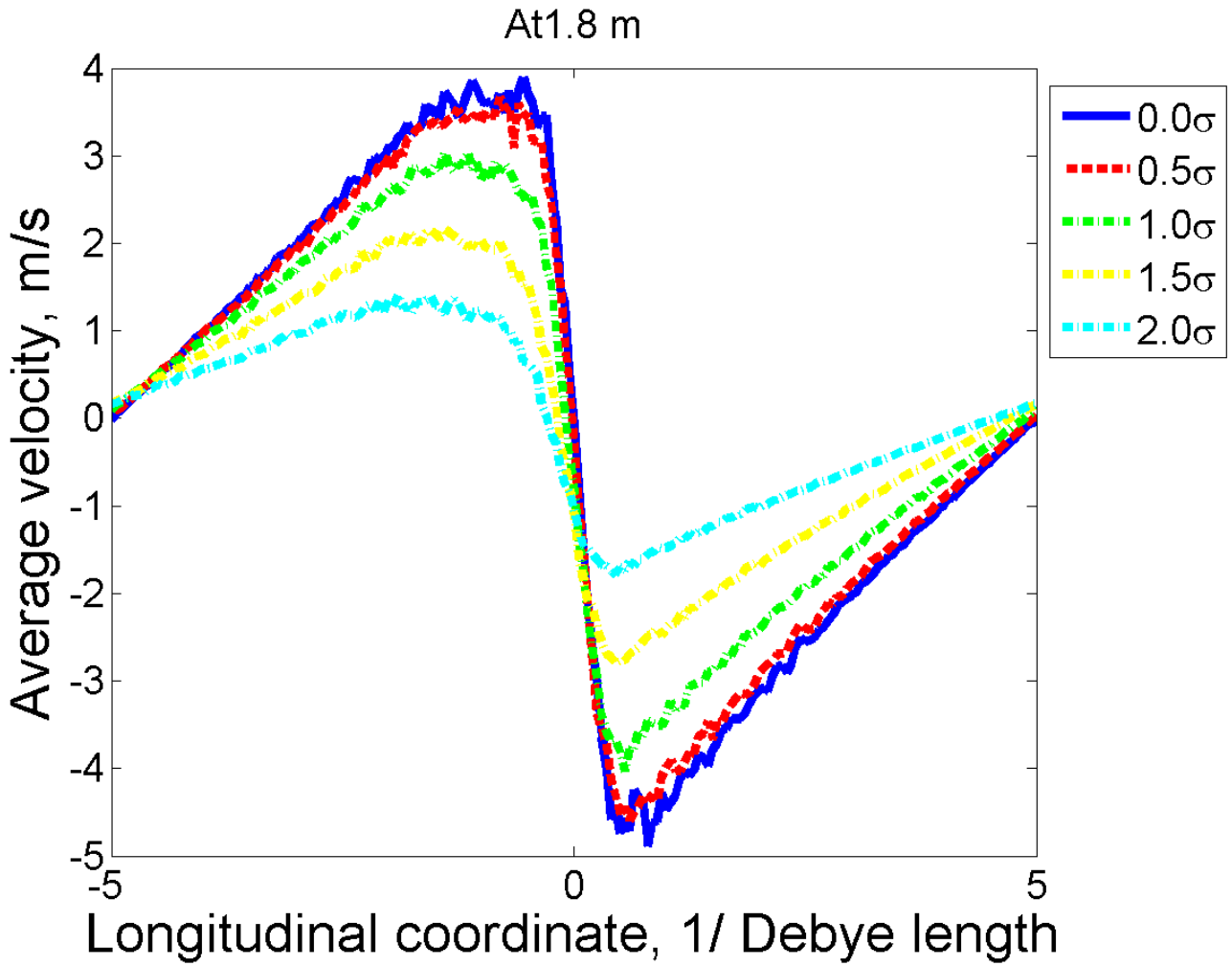

At1.2 m
Number of electrons per Debye length
Longitudinal coordinate, 1/ Debye length
20
15
10
5
0
-5
-5
0
5
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

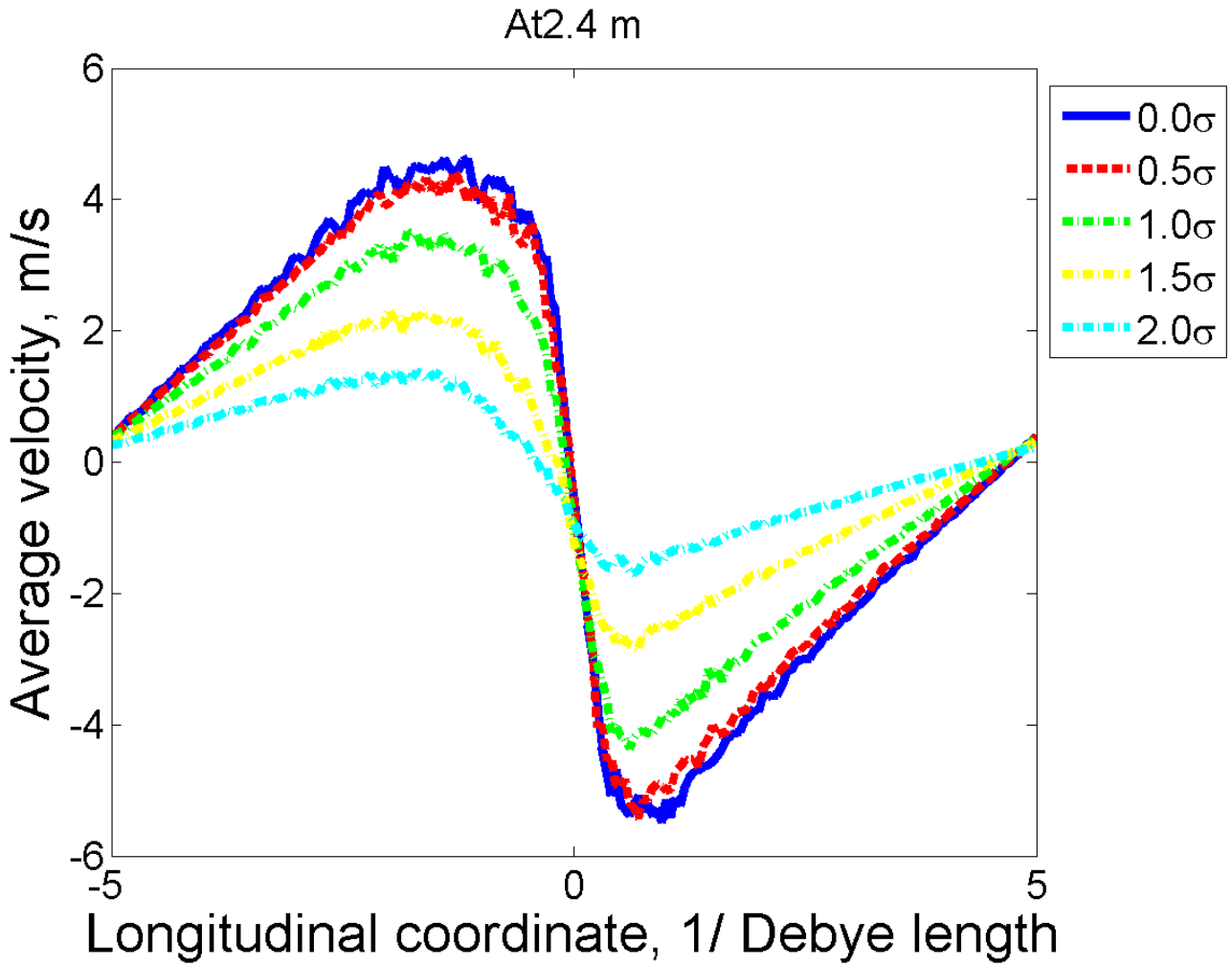

At1.8 m
Number of electrons per Debye length
Longitudinal coordinate, 1/ Debye length
35
30
25
20
15
10
5
0
-5
-5
0
5
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ


At2.4 m
Number of electrons per Debye length
Longitudinal coordinate, 1/ Debye length
50
40
30
20
10
0
-10
-5
0
5
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ


At0.6 m
Average velocity, m/s
Longitudinal coordinate, 1/ Debye length
2
1.5
1
0.5
0
-0.5
-1
-1.5
-5
0
5
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

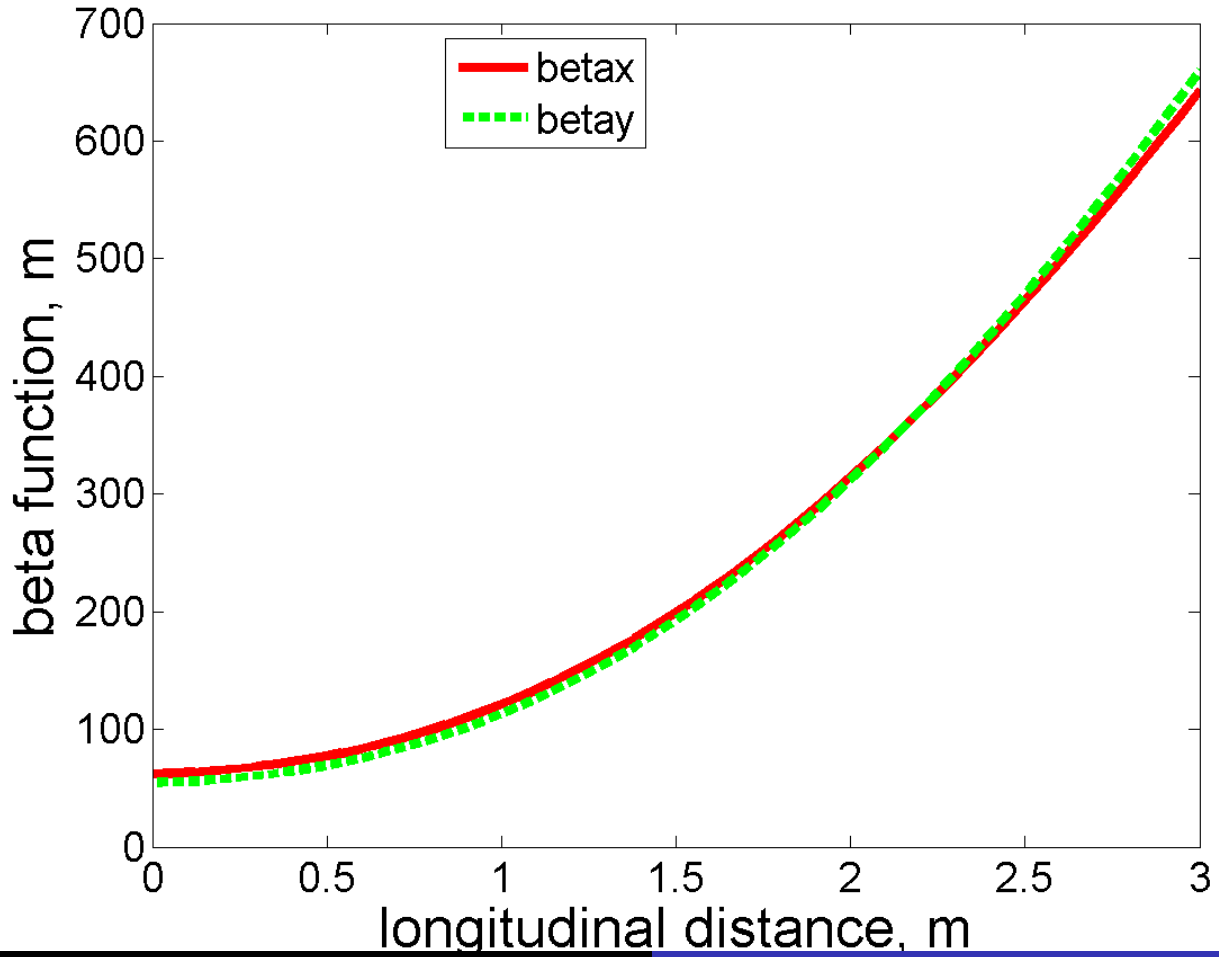

At1.2 m
Average velocity, m/s
Longitudinal coordinate, 1/ Debye length
3
2
1
0
-1
-2
-3
-5
0
5
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ


At1.8 m
Average velocity, m/s
Longitudinal coordinate, 1/ Debye length
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ
4
3
2
1
0
-1
-2
-3
-4
-5
-5
0
5

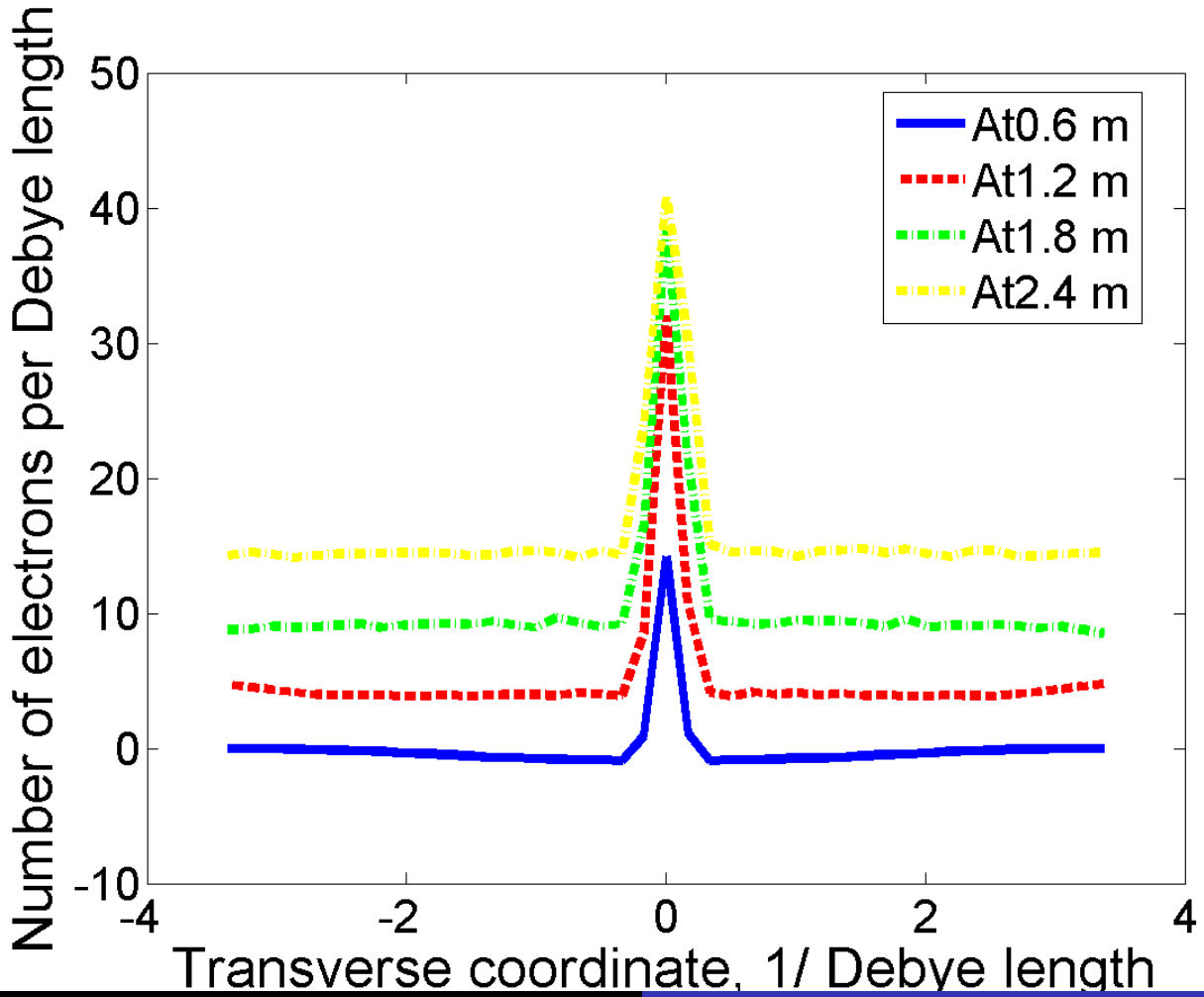

At2.4 m
Average velocity, m/s
Longitudinal coordinate, 1/ Debye length
6
4
2
0
-2
-4
-6
-5
0
5
0.0σ
0.5σ
1.0σ
1.5σ
2.0σ

- Calculate beam parameters from beam dynamics simulation
- Generate particles using the beam parameters

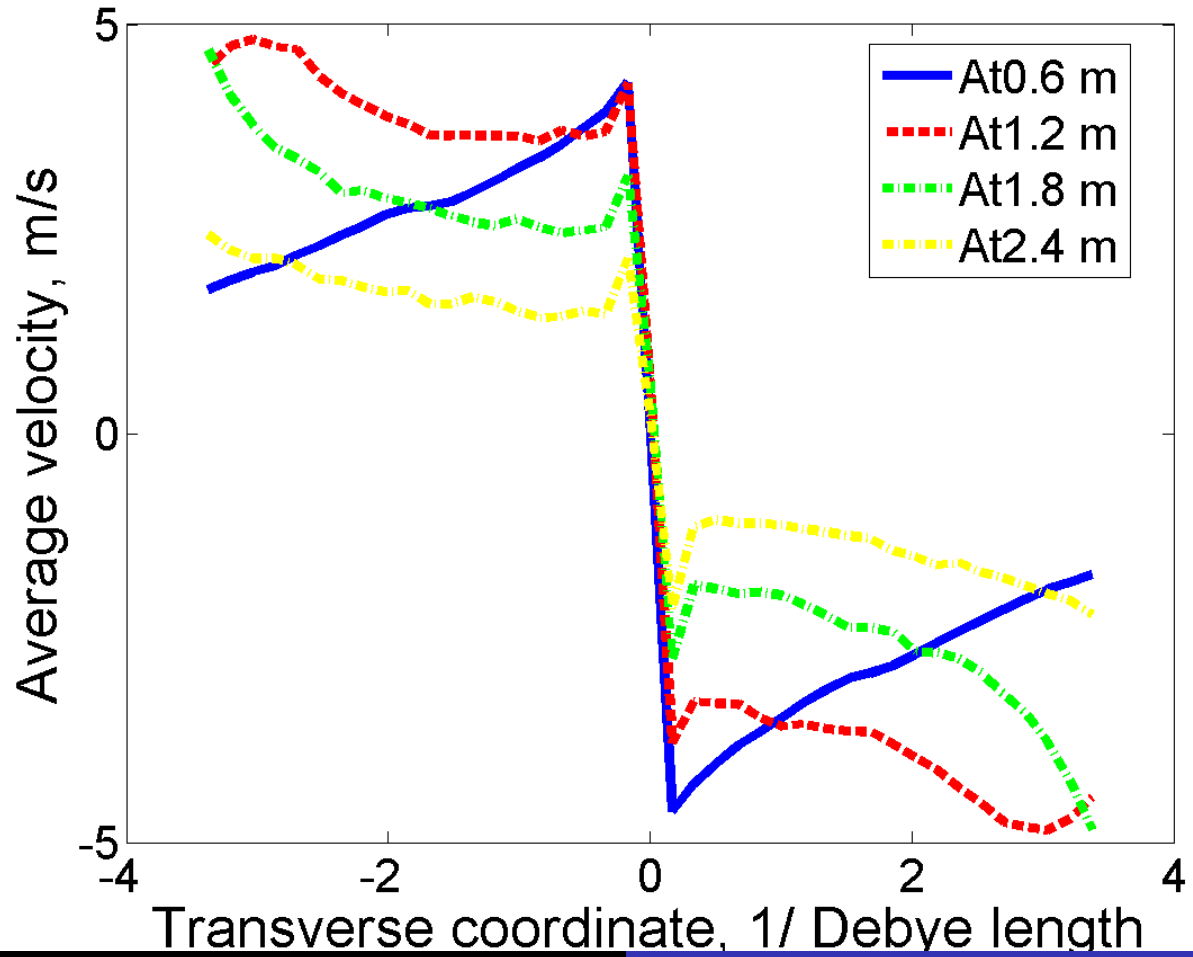
Beta function change



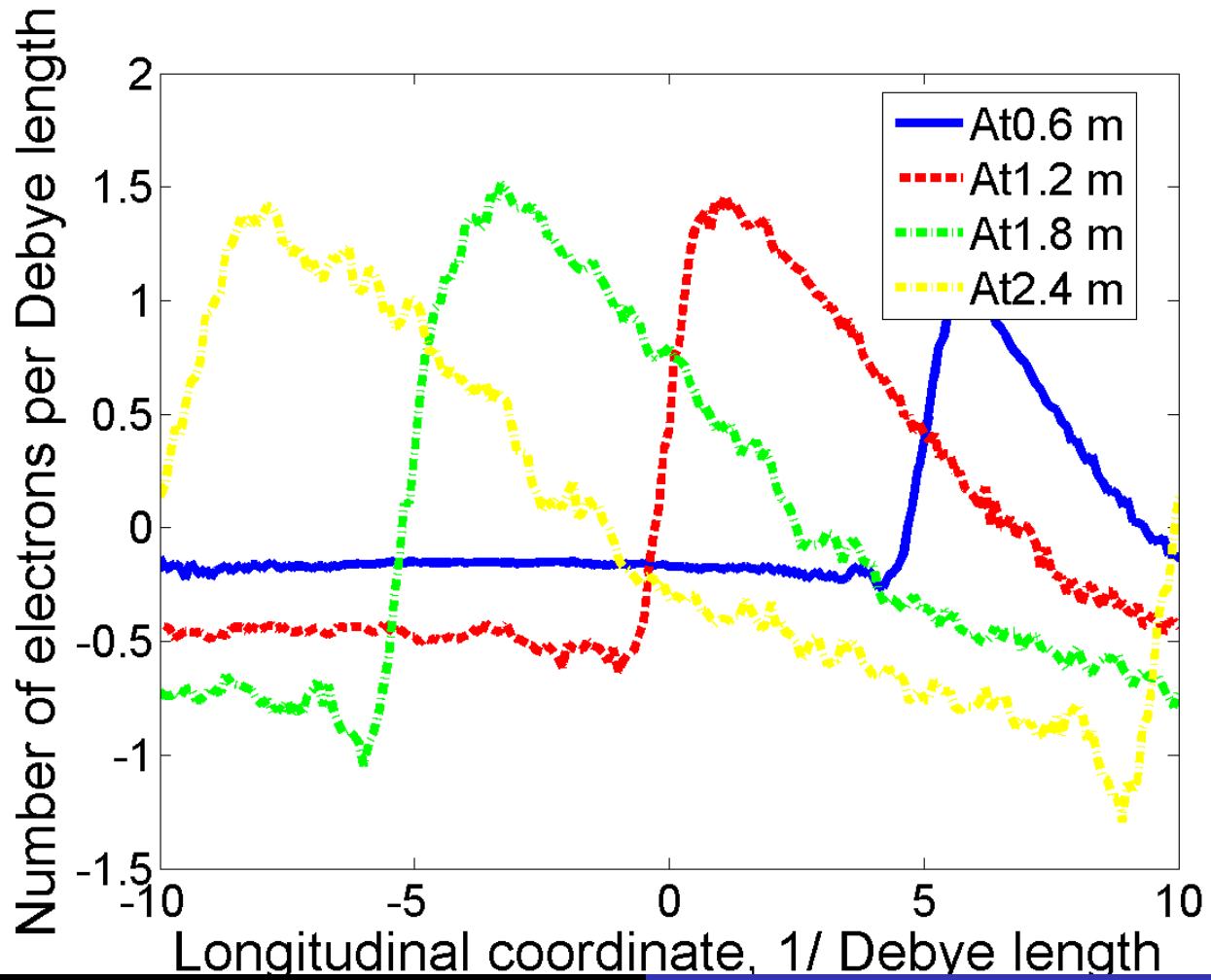
nx


Number of electrons per Debye length
Transverse coordinate, 1/ Debye length
At0.6 m
At1.2 m
At1.8 m
At2.4 m
50
40
30
20
10
0
-10
-4
-2
0
2
4

VX


At0.6 m
At1.2 m
At1.8 m
At2.4 m
Average velocity, m/s
Transverse coordinate, 1/ Debye length
5
0
-5
-4
-2
0
2
4

nz


Number of electrons per Debye length
Longitudinal coordinate, 1/ Debye length
At0.6 m
At1.2 m
At1.8 m
At2.4 m
2
1.5
1
0.5
0
-0.5
-1
-1.5
-10
-5
0
5
10