Homework 2.

1 (8 point): Cooling time in LEReC

Use the beam parameter table for LEReC in lecture and estimate the cooling time/rates for Au ions. Compare with experimental measurements in year 2020 and 2021. Comment on the reasons for possible discrepancies.

Solution: in the lecture, the cooling time (assuming continuous cold e-beam, i.e., no velocity spread in the beam) can be written as

$$\tau_{lab} = \gamma_0^5 \frac{A}{Z^2} \frac{Se}{4\pi r_e r_p L_c} \theta_i^3 \frac{1}{I_b}$$

where I_b is the average beam current, and θ_i is the angular spread of ion beam (which can use the electron spread for better matching).

Where $r_{e,p} = \frac{1}{4\pi\varepsilon_0} \frac{e^2}{m_{e,p}c^2}$ are the classical electron/proton radii.

Assuming $S = \pi\sigma_x\sigma_y$ being the cross section area and Lc=10 as the coulomb logarithm, one can plug in the number and calculate the cooling time (for Au + at 3.85 GeV/nucleon)

$$\tau_{lab,3.85} = 72 \text{ ms},$$

note that this is for a continuous beam with delta function velocity spread (cold beam).

In the real case, the cooling section occupies only 20 m over the RHIC circumference of 3.8 km, and there is another reduction of cooling rate from synchrotron motion (take 2 as an example for linear force, nonlinear would be higher). And finally the estimation of transverse size with 1 sigma is too small, if use 6 sigma to include 95% of the beam for a Gaussian distribution, then the final cooling time reads

$$\tau_{lab,3.85} = 16.35 \text{ mins}$$

This when comparing with the measured cooling time is still shorter because we did not consider the IBS growth rate which is obvious when there is no cooling (bunch size grows over time).

Performing the same calculation for Au + at 4.6 GeV/nucleon case, one can get

$$\tau_{lab,4.6} = 144 \text{ ms}$$

with all corrections, the cooling time for 4.6 GeV/nucleon case is

$$\tau_{lab,4.6} = 32.7 \text{ mins}$$

2 (8 point): Cooling rate reduction for non-linear cooling force

Derive the cooling rate reduction by integrating the reduction of action (similar to what is done for the linear forces) over one synchrotron oscillation and show that for non-linear cooling force the reduction due to synchrotron motion is higher than a linear force.

Hint: one can assume the force is in Gaussian model

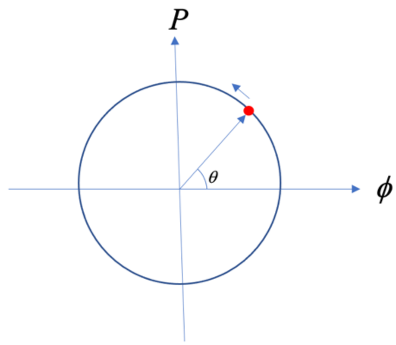
$$\Delta\delta\gamma_c = -\zeta_0 T_{rev}\delta\gamma \exp\left(-\frac{\delta\gamma^2}{2\sigma_l^2}\right)$$

and show that the reduction factor (over one oscillation period) is

$$\bar{\zeta}(I) = \frac{1}{2}\exp\left(-\frac{I}{2\sigma_P^2}\right)\left[I_0\left(\frac{I}{2\sigma_P^2}\right) - I_1\left(\frac{I}{2\sigma_P^2}\right)\right]$$

Solution:

We start with the definition of action and angle, same as for the linear force



$$P \equiv -h\frac{|\eta|}{\nu_s}\frac{\Delta p}{p} = \sqrt{2I}\sin\theta \qquad \phi \equiv \omega_{rf}\tau = \sqrt{2I}\cos\theta$$

Following the same derivation of the change of momentum P due to cooling, we get for a nonlinear force assuming a Gaussian distribution

$$\Delta\delta\gamma_c = -\zeta_0 T_{rev}\delta\gamma \exp\left(-\frac{\delta\gamma^2}{2\sigma_l^2}\right)$$

$$\Delta P_c = -\zeta_0 T_{rev} P \exp\left(-\frac{P^2}{2\sigma_P^2}\right)$$

where $\sigma_P \equiv h\frac{|\eta|}{\nu_s}\frac{\sigma_l}{\gamma}$

so the change in action Ic reads

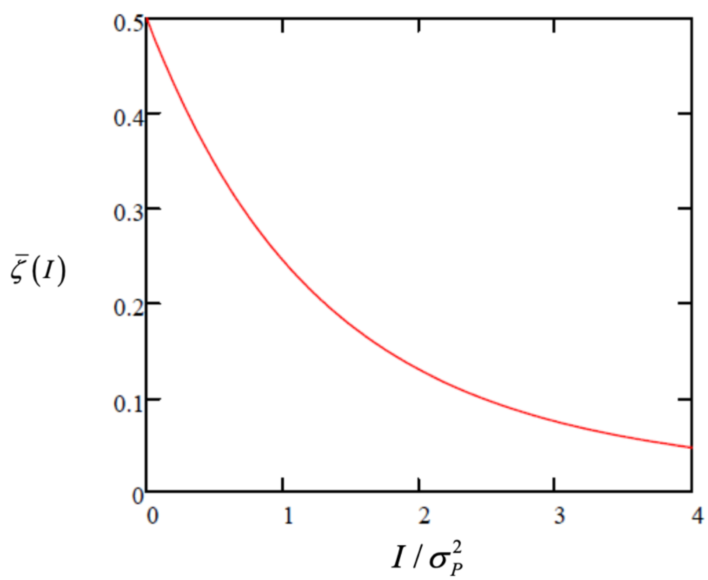
$$\Delta I_c = -\zeta_0 T_{rev} P^2 \exp\left(-\frac{P^2}{2\sigma_P^2}\right) = -2I\zeta_0 T_{rev} sin^2\theta \exp\left(-\frac{I}{\sigma_P^2} sin^2\theta\right)$$

With $\zeta(I) = -\frac{1}{I}\left\langle\frac{\Delta I_c}{T_{rev}}\right\rangle_{T_s} = \zeta_0\bar{\zeta}(I)$

One can get

$$\bar{\zeta}(I) = \frac{1}{2\pi}\int_0^{2\pi} sin^2\theta \exp\left(-\frac{I}{\sigma_P^2} sin^2\theta\right) d\theta$$
$$= \frac{1}{2}\exp\left(-\frac{I}{2\sigma_P^2}\right)\left[I_0\left(\frac{I}{2\sigma_P^2}\right) - I_1\left(\frac{I}{2\sigma_P^2}\right)\right]$$

Thus the reduction with normalized parameters can be shown



Always lower than 0.5.

3 (9 point): Design an electron cooler ring for the future electron ion collider (EIC).

Fill the table at your choices with beam parameters (and give reasons) for an electron cooler ring for the future EIC.

Keep in mind (while choosing beam parameters) that the bunch charge is limited by the incoherent space charge tune shift (AKA, Laslett formula) as

$$\Delta\nu_{e(x,y)} = \frac{I_e}{4\pi I_a \gamma^3} \int_0^C \frac{\beta(s)}{\sigma_e^2(s)} ds = \frac{I_e C}{4\pi I_a \gamma^3 \varepsilon_{x,y}},$$

which should be less than 0.2 to avoid particle loss), where Ia is Alfven current (17 kA) and C being the cooling section length. The goal is to have cooling time less than 5 hours (show calculation/justification).

Solution:

The suggested table can be found as follows:

parameter	**value**
p-bunch length σ_z [cm]	6
Proton beam energy (GeV)	275
protons: $\sigma_{\delta p}$	6.8e-4
protons: N_p	6.88e10
protons: $\varepsilon_{xp,yp}$ [nm]	11.3, 1.0
protons: $\beta_{xp,yp}$ [m]	168, 1900
Electron beam energy (MeV)	150
Electrons per bunch: N_e	1.4e11
Cooling section length (m)	170

e- geo. Emittance (rms): $\varepsilon_{xe,ye}$ [nm]	7,7
e- betatron function: $\beta_{xe,ye}$ [m]	270, 270
e- bunch length (rms): L_{ze} [cm]	36
e- energy spread (frac.): $\sigma_{\delta e}$	9e-4
e- angular spread:	5e-6
Cooling times $\boldsymbol{\tau}$[min]	270 (⊥), 180 (∥)

This is an open question as there is no unique solution to satisfies the requirements. This suggested table (used in real design) is one for a symmetric e- beam design.

Some considerations when choosing parameters

1. The number of electrons per bunch is limited by the space charge tune shift and thus for larger emittance, one can accommodate more charge.
2. The beam emittances have to be chosen together with the beta functions to match the electron beam and ion beam transverse sizes.
3. High emittance with low beta function will generate large angular spread thus increase the transverse temperature of the electron beam thus reducing the transverse cooling.
4. Another source of the angular spread is coming from the poor matching of lattice design thus it is wise to choose the beta function close to the length of the cooling section which is limited by the available space in machine, in this case the RHIC tunnel can

hold up to 200 m straight section for the cooler.

5. Transverse temperature is always higher than the longitudinal temperature thus the cooling time is longer in transverse planes. If higher cooling rate is desired, one can propose to use dispersion function to couple some of the longitudinal cooling into transverse directions.