Electron Cooling

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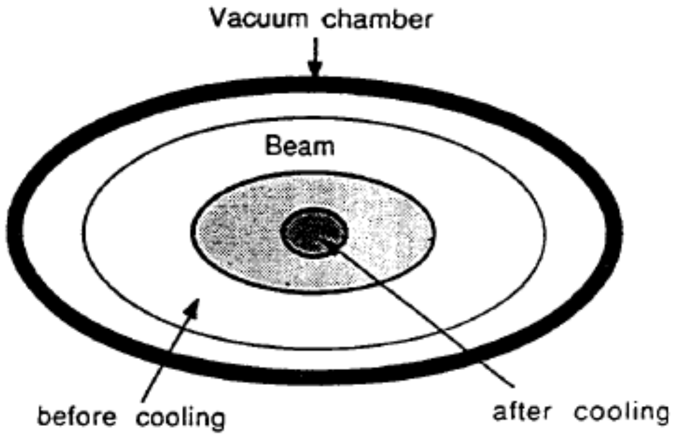
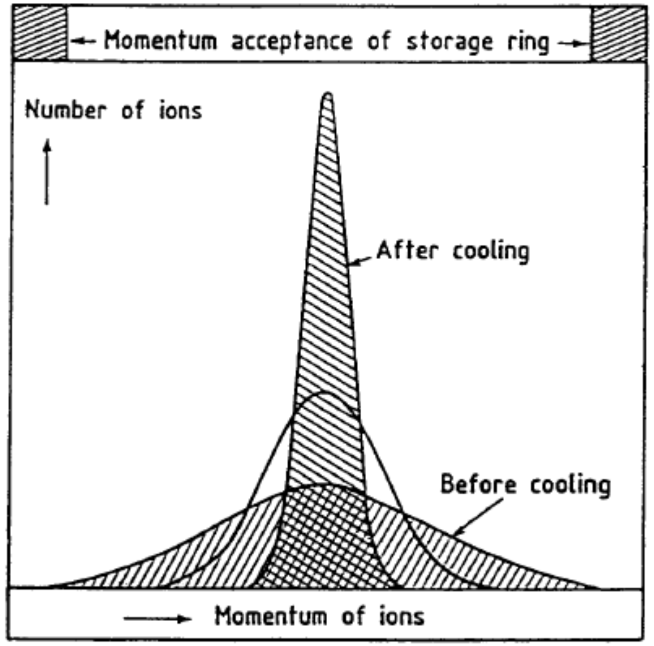
Motivation

- Why Beam Cooling?

❑Reduce beam emittance. reduce momentum spread

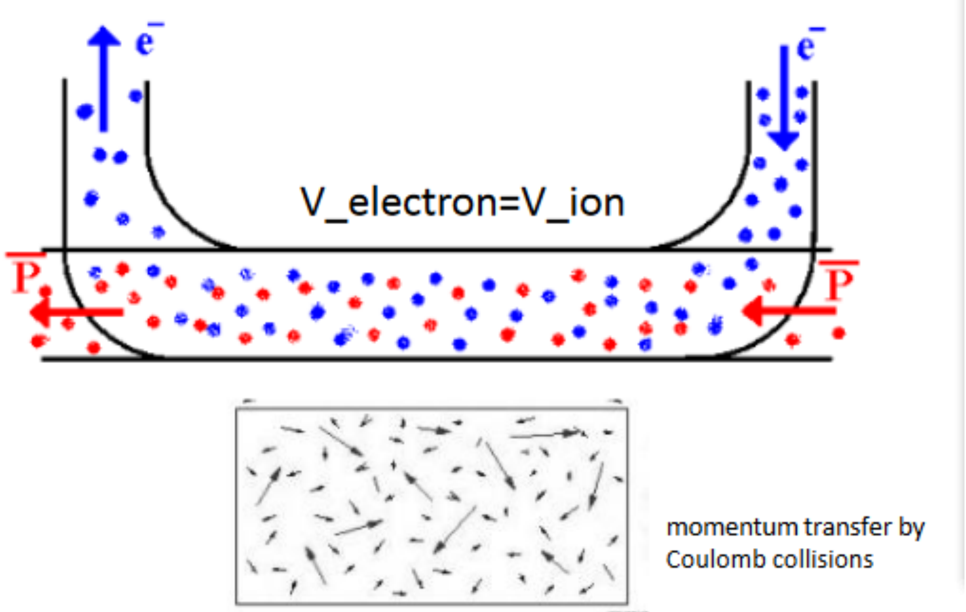




❑Improve beam quality (higher luminosity, higher brightness)

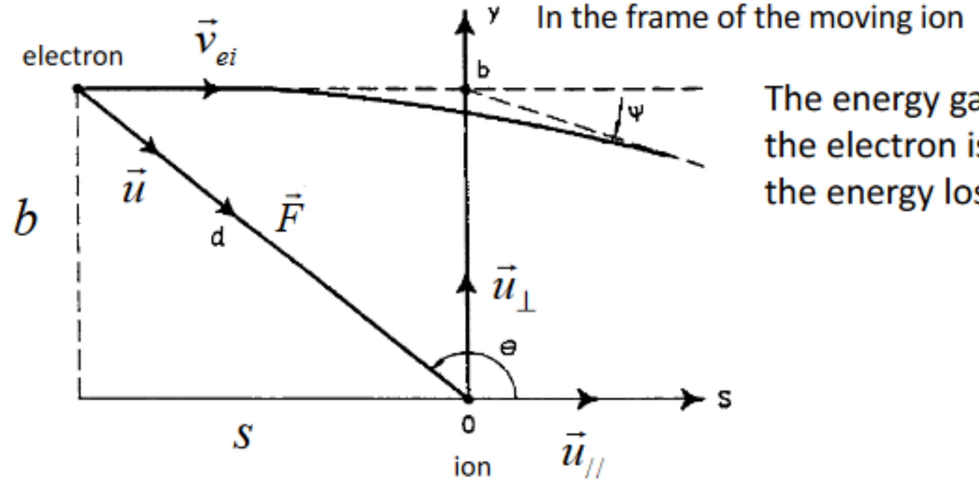
❑Accumulate the beam density through the weak particle source

Introduction: Electron Cooling (Non-magnetized)



- Electron cooling as a method increase luminosity for pp and pp-bar collision was invented by Gersh Budker in 1966.
- In beam moving frame, cold electrons interact with hot ions through Coulomb Scattering to exchange the heat away.

Non-magnetized Electron Cooling: Energy loss



The energy gained by the electron is equal to the energy loss by ion.

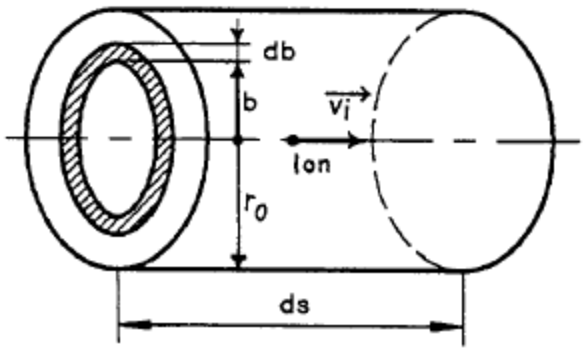
$$\vec{v}_{ei} = \vec{v}_e - \vec{v}_i$$

- In moving frame, the energy gained by electron is equal to the energy lost by ion.

$$\Delta E(b) \approx \frac{\Delta p_{\perp}^2}{2m_e} = \frac{2Z^2 e^4}{m_e v_{ei}^{\,2} (4\pi\varepsilon_0)^2 b^2} \qquad \Delta E_{loss}(b) = \frac{2Z^2 e^4}{m_e v_{ei}^{\,2} (4\pi\varepsilon_0)^2 b^2}$$

Non-magnetized Electron Cooling: friction force

- Coulomb Scattering: one electron sit at impact parameter b interact with a moving ion.



$$b_{\min} = \frac{Ze^2}{4\pi\varepsilon_0 m_e v_i^2}$$

$b_{\max} \sim$ Debye length

$$L_c \equiv \ln\left(\frac{b_{\max}}{b_{\min}}\right) \approx 10$$

Ion energy loss rate: $$\frac{dE}{ds} = 2\pi n_e \int_{b_{\min}}^{b_{\max}} b\Delta E_{loss}(b)\,db = \frac{4\pi n_e Z^2 e^4}{m_e v_{ei}^{\ 2}(4\pi\varepsilon_0)^2}\ln\left(\frac{b_{\max}}{b_{\min}}\right)$$

- Friction force: $$\vec{F} = \frac{dE}{ds}\frac{\vec{v}_e - \vec{v}_i}{|\vec{v}_e - \vec{v}_i|} = \frac{2Z^2 e^4 L_c}{m_e(4\pi\varepsilon_0)^2}\frac{\vec{v}_e - \vec{v}_i}{|\vec{v}_e - \vec{v}_i|^3}$$

Non-magnetized Electron Cooling: cooling rate

- For an exponential decrease of the ion velocity, we have

$$v_i(t) = v_i(t=0) \cdot \exp(-t / \tau_{v_i}) .$$

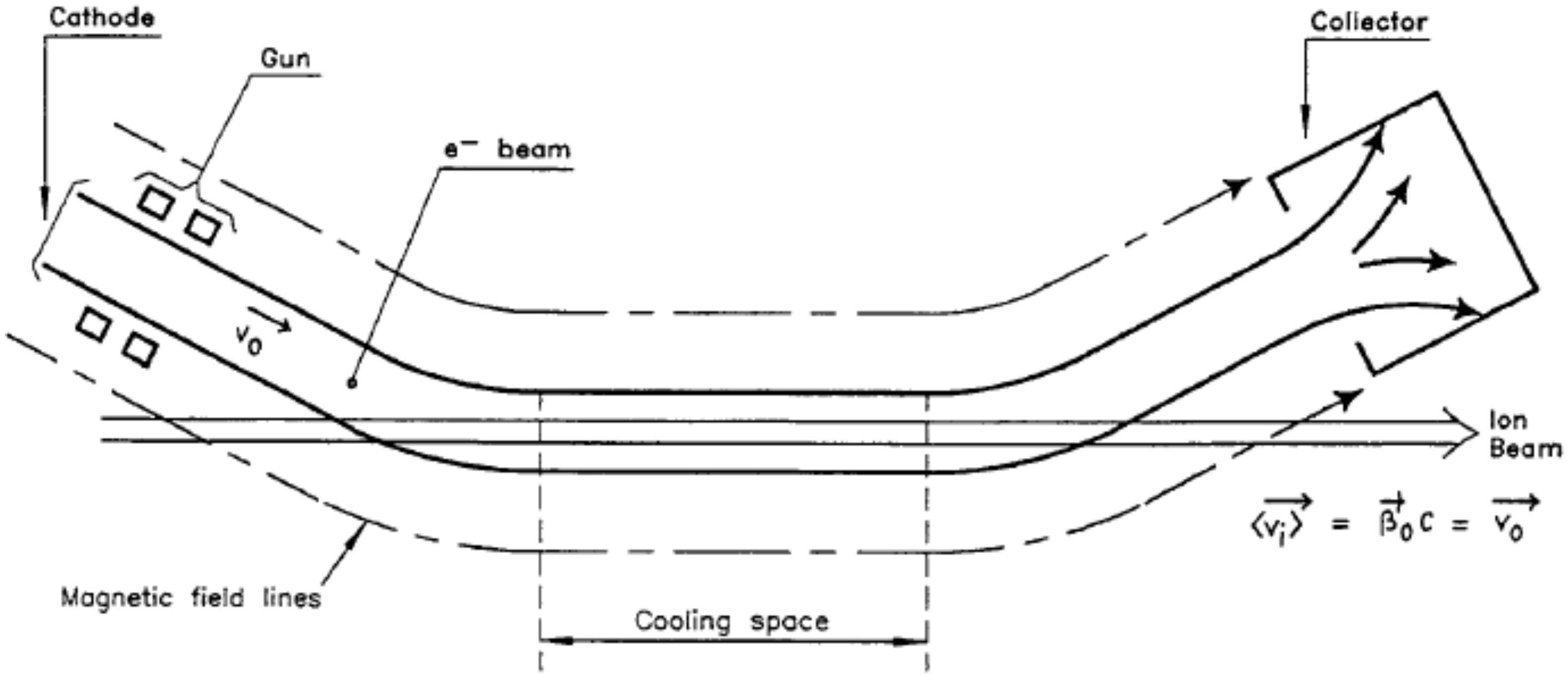
- The cooling time variable associates cooling rate (in moving frame) as

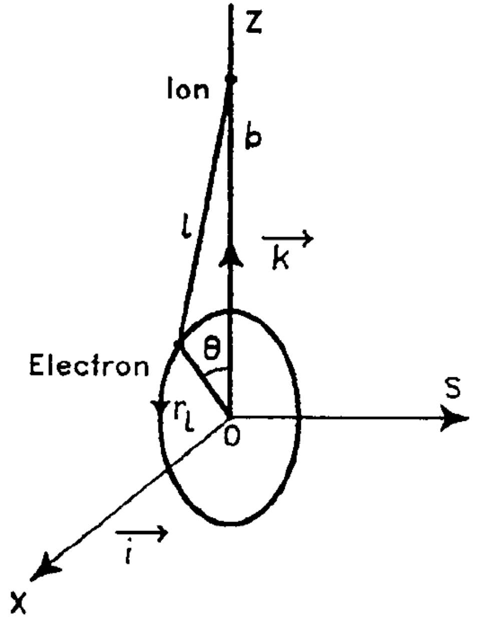
$$\frac{1}{\tau_{v_i}} = -\frac{1}{v_i}\frac{dv_i}{dt} = -\frac{1}{m_i \cdot v_i}\left(m_i \frac{dv_i}{dt}\right) = -\frac{F(v_i)}{p_i}$$

- In lab frame, the cooling time variable as

$$\tau^*_{v_i} = \gamma_0 \frac{1}{\eta_c} \tau_{v_i}$$

Introduction: Electron Cooling (Magnetized)





- Electrons experiences a longitudinal magnet field, it reduces cooling time at relative low ion velocity.

Introduction: Electron Cooling (Magnetized)

- The electrons rotate around their axis at cyclotron frequency f_l (or angular frequency ω_l) with a radius equal to r_l. We define a threshold impact parameter b_t as $b_t = v_i/\omega_l$, which distinguish between two impact parameter region:

❑ $b_{min} < b < b_t$, Inner impact, fast collision region, the case becomes non-magnetized electron cooling.

❑ $b_t < b < b_{max}$, outer impact, adiabatic region, Coulomb Logarithm applies as $L_c^{ad} = ln(\lambda_d/r_e)$, λ_d is Debye length, r_e is Larmor radius as $r_e = \frac{m_e v_\perp}{|q|B}$.

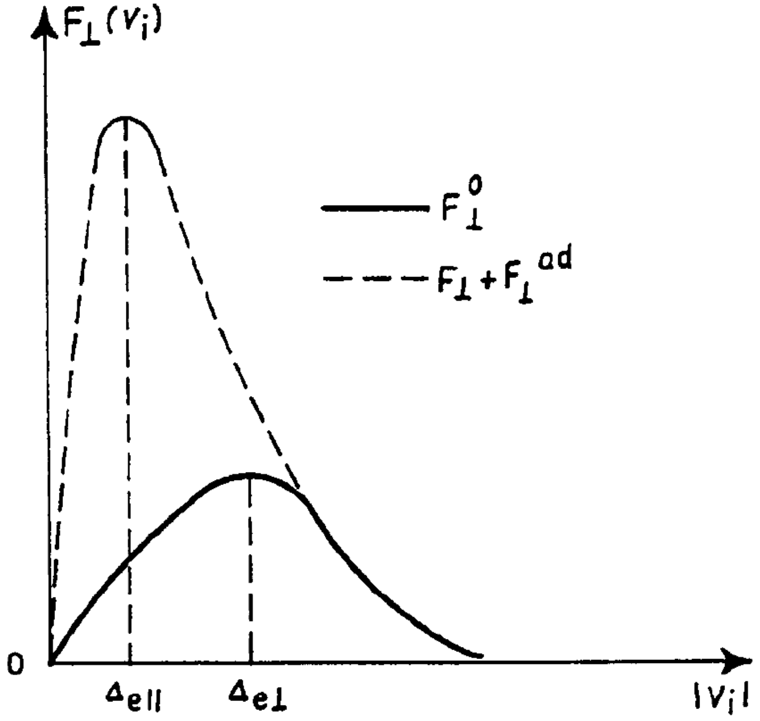
Introduction: Electron Cooling (Magnetized)

- In adiabatic region, for low ion velocity, there is no energy transfer with electrons' transverse motion, the exchange in energy mainly happens with electrons' longitudinal direction, since the electron longitudinal velocity spread is very small, the magnetization will significantly enhance the cooling force in case of low velocity ions.

$$F_{\ell,t} = -\int_{b_{\min}}^{b_{\max}} \Delta p_{\ell,t}\, n_e\, v_i\, 2\pi b\, db$$

$$F_\ell \cong -\frac{4\pi Z^2 e^4}{m_e} n_e\, L_c\, \frac{\sin^2\theta}{v_i^2}\,;\; F_t \cong -\frac{4\pi Z^2 e^4}{m_e} n_e\, L_c\, \frac{\sin\theta\cdot\cos\theta}{v_i^2}\,.$$

Introduction: Electron Cooling (Magnetized)



- Magnetic effect dominates mainly when relative ion velocity is small.

Experimental data (Magnetized electron cooling)

- The measurement of Longitudinal friction force vs. Co-moving velocity for protons at injection energy 48 MeV in CELSIUS.

TABLE III. Parameters of electron cooler for standard settings.

Magnetic field B	0.1 T
Cooler length	2.5 m
Electron beam radius	0.01 m
Transverse rms velocity spread of electrons	1.4×10^5 m/s
Major contributors to longitudinal rms velocity spread:	
longitudinal-longitudinal relaxation	$\sim3-5\times10^3$ m/s
high voltage power supply ripple	5.2×10^3 m/s
magnetic field errors	$\sim10^4$ m/s

Experimental data (Magnetized electron cooling)

- Current dependence: the friction force was measured for various current of electron beams.

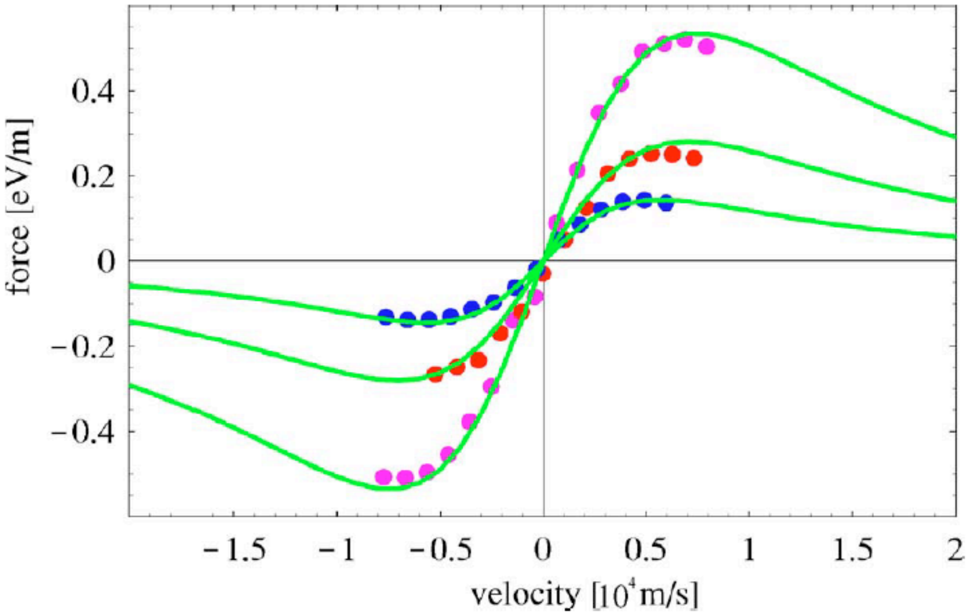


FIG. 1. (Color online) Longitudinal friction force in [eV/m] vs velocity [$\times 10^4$ m/s] for three currents of electron beam. Measured data: 250 (pink dots, upper curve), 100 (red dots, middle curve), and 50 (blue color) mA (B=0.1 T).

Experimental data (Magnetized electron cooling)

- Alignment between beams: the dependence of the effective velocity on the alignment angle between ion and electron beam.

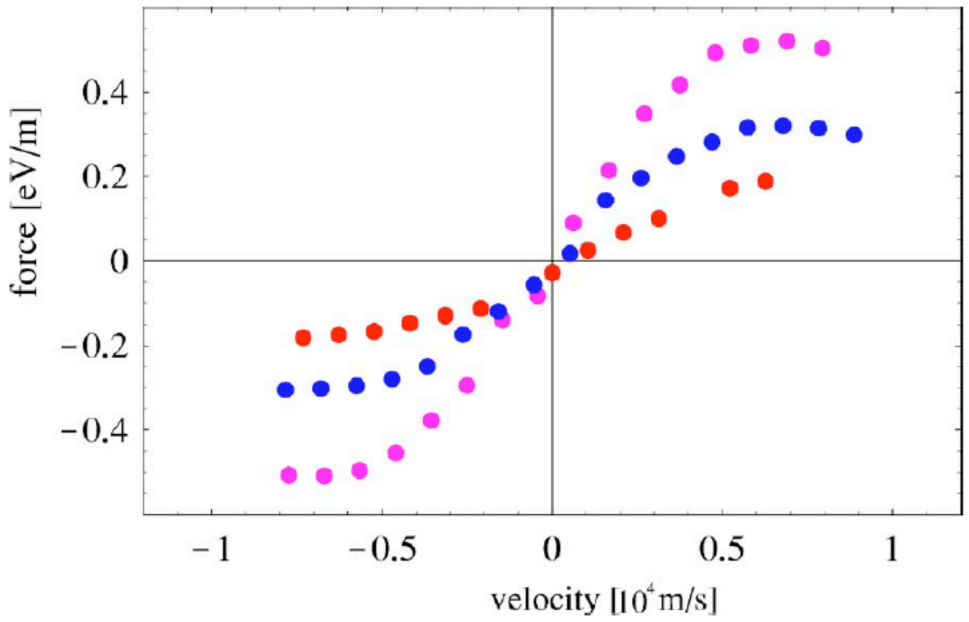


FIG. 2. (Color online) Longitudinal friction force in [eV/m] vs velocity [$\times 10^4$ m/s] for a misalignment angle between the beams in the horizontal direction: 0 (pink, upper curve), 0.4 (blue, middle curve), 0.8 (red) mrad (B=0.1 T, I_e=250 mA).

Experimental data (Magnetized electron cooling)

- Different regimes of magnetization: The dependence of Magnetization on longitudinal friction force.

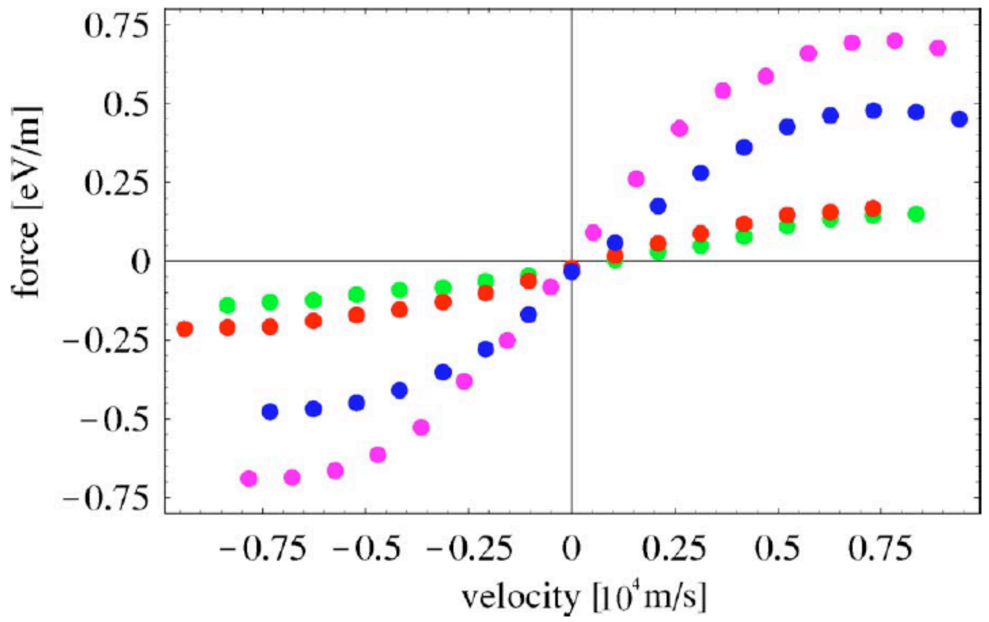


FIG. 4. (Color online) Longitudinal friction force in [eV/m] vs velocity [$\times 10^4$ m/s] for different magnetic fields with parameter α=1.3 (pink dots, upper curve), 1.0 (blue), 0.9 (red), 0.8 (green, lower curve) from top to bottom, respectively.

Experimental data (Magnetized electron cooling)

- If the maximum impact parameter in the magnetized collisions is not large compared to the Larmor circle one may encounter a strong nonlogarithmic dependence on both the magnetic field and the transverse velocity of the electrons.

$$F = C\frac{4\pi n_e Z^2 e^4}{m_e}\frac{V}{(V^2+\Delta_{eff}^2)^{3/2}}L_M, \qquad (3)$$

where V is the relative ion velocity, Δ_{eff} is the longitudinal effective velocity spread of the electrons. The magnetized Coulomb logarithm is defined as $L_M=\ln(\rho_{max}/\rho_L+1)$ with ρ_{max} being the maximum impact parameter in binary ion-electron collisions, and ρ_L being the Larmor radius, n_e and m_e are the density and mass of electrons, and C is some numeric coefficient. The value of this numeric coefficient C is discussed in Sec. IV.

For the purpose of the experiment discussed in this paragraph we are concerned only with the parametric dependence on the magnetic field in Eq. (3), which enters through the Larmor radius $\rho_L=m_e\Delta_{e\perp}/eB$, where $\Delta_{e\perp}$ is the transverse rms velocity spread of the electron distribution. If the maximum impact parameter in magnetized collisions is not large compared to the Larmor circle there is a possibility of a nonlogarithmic dependence on both the magnetic field and the transverse velocity of electrons. In such a case with $\alpha=\rho_{max}/r_L<1$ one can replace $\ln(\alpha+1)$ by α. As a result, the force has a linear dependence on the magnetic field and the transverse velocity of electrons which leads to a strong reduction in its absolute value.

Magnetized electron cooling: Comparison with theory

- Fitting with a single-particle expression: For a low electron current, it goes through the measured data points. For a high electron current, the experimental data are significantly lower than the curve utilizing the effective velocity determined from the low current data.

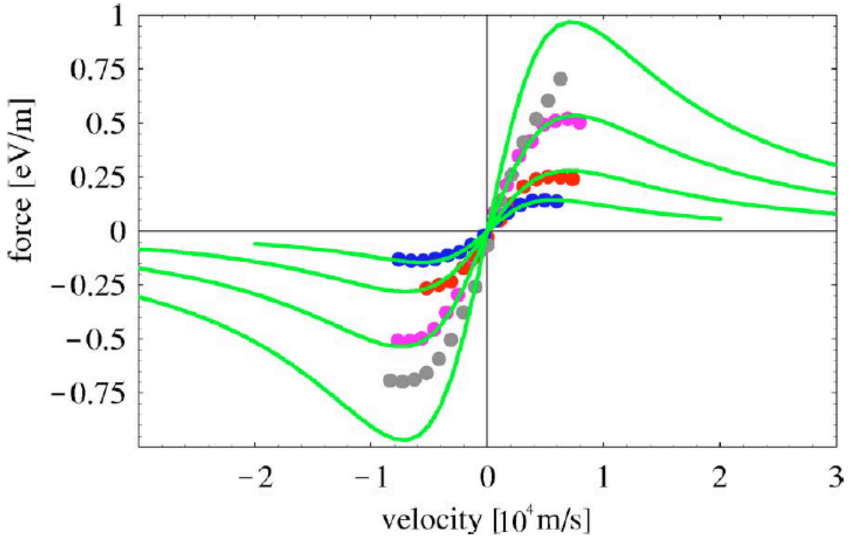


FIG. 6. (Color online) Longitudinal friction force in [eV/m] vs velocity [$\times 10^4$ m/s] for electron currents of 50 (blue, lowest set of data), 100 (red), 250 (pink), 500 mA (black, highest set of data) and a magnetic field in the cooling solenoid $B = 0.1$ T.

Conclusion

- Present low-energy electron coolers can be used for a detailed study of the friction force.

- The parameters of a low-energy cooler can be chosen in a manner to reproduce regimes expected in future high-energy operation.

- Electron cooling is suitable for cooling the intense, relatively cool, low energy ion beams.

References

- J. Bosser, Electron Cooling, CERN 95-06 Vol.I 22 November 1995

- Prof. G. Wang, Beam cooling (Hadron), PHY 554 Fall 2016 Lecture 23 Slides, Nov 2016

- A. V. Fedotov, B. Gålnander, V. N. Litvinenko, T. Lofnes, A. Sidorin, A. Smirnov, and V. Ziemann, Experimental studies of the magnetized friction force, PHYSICAL REVIEW E 73, 066503 (2006)

Thank You!

Backup Slides 1 :Electron Cooling (Magnetized)I

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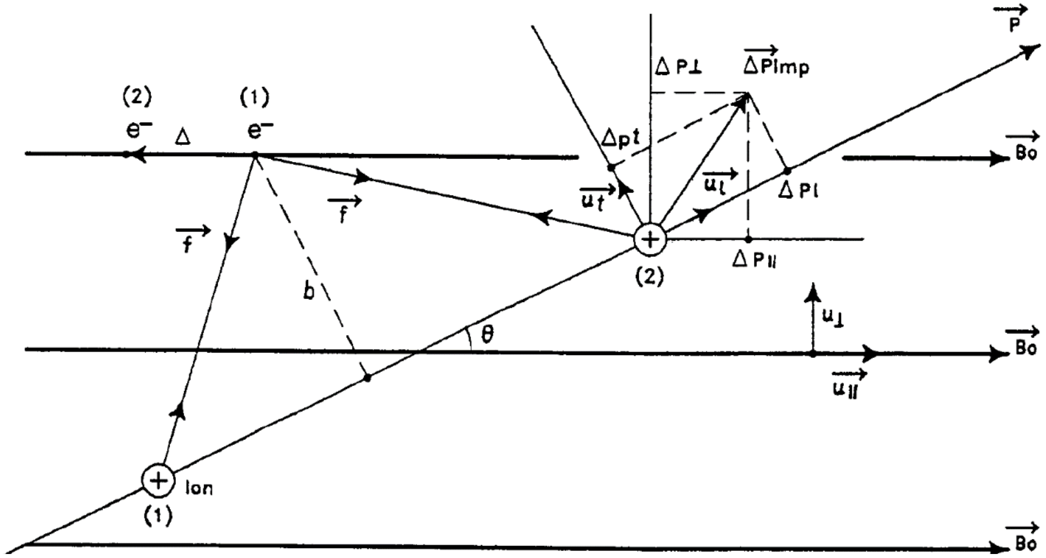

Starting from position (1), the ion and electron will interact. After a time t, when in position (2), the electron will have executed a small displacement $\Delta << b$ such that:

$$\Delta = \frac{\text{acceleration} \cdot t^2}{2} = \frac{f}{m_e} \sin\theta \cdot \frac{t^2}{2} \cong \frac{Ze^2}{2m_e b^2} \sin\theta \cdot \left(\frac{b}{v_i}\right)^2 = \frac{Ze^2}{2m_e v_i^2} \sin\theta \, .$$

This gives a small change in the impact parameter and it is easily shown that $\Delta b \cong \Delta$.

This small difference will result in a change in the drag force depending on whether the ion is on the first (1) or second (2) half of its trajectory. The momentum transfer induced by the collision is:

$$\Delta\vec{p}_{imp} = \Delta p_{\parallel} \vec{u}_{\parallel} + \Delta p_{\perp} \vec{u}_{\perp} = \Delta p_{\ell} \vec{u}_{\ell} + \Delta p_{t} \vec{u}_{t}$$

Backup Slides 2 :Electron Cooling (Magnetized)II

with $\Delta p_{\parallel} << \Delta p_{\perp}$ since the phenomenon is the same as for a collision of a gas molecule with a hard wall.

The result is:

$$\Delta p_{\ell} = \Delta p_{\perp} \sin\theta + \Delta p_{\parallel} \cos\theta \cong \Delta p_{\perp} \sin\theta$$

$$\Delta p_{t} = \Delta p_{\perp} \cos\theta - \Delta p_{\parallel} \sin\theta \cong \Delta p_{\perp} \cos\theta \,.$$

To get an estimate of $\Delta p_{\perp}$ we can write (see section 5.1):

$$\Delta p_{\perp} \cong \int_{-\infty}^{\infty} 2\left(\frac{Ze^2}{b^2} - \frac{Ze^2}{(b+\Delta b)^2}\right)\frac{ds}{v_i^2} \cong \frac{4Ze^2}{v_i}\frac{\Delta b}{b^2} \cong \frac{2Z^2e^4}{m_e v_i^3}\frac{\sin\theta}{b^2}$$

since the integration is effective over the distance b only.

We then come to the simplified form:

$$\Delta p_{\ell} = \frac{2Z^2e^4}{m_e v_i^3}\sin^2\theta \,;\, \Delta p_t \cong \frac{2Z^2e^4}{m_e v_i^3}\frac{\sin\theta}{b^2}\cos\theta \,.$$

The friction force resulting from the electron cloud is:

$$F_{\ell,t} = -\int_{b_{min}}^{b_{max}} \Delta p_{\ell,t}\, n_e\, v_i\, 2\pi b\, db$$

such that:

$$F_{\ell} \cong -\frac{4\pi Z^2 e^4}{m_e} n_e\, L_c\, \frac{\sin^2\theta}{v_i^2} \,;\, F_t \cong -\frac{4\pi Z^2 e^4}{m_e} n_e\, L_c\, \frac{\sin\theta\cdot\cos\theta}{v_i^2} \,.$$

Backup Slides 3: Friction Force measurement I

- Define rms velocity spread as effective velocity, for relative velocities higher than the effective velocity, the cooling force has a nonlinear dependence.

❑ In linear region, the phase shift between the rf system and the ion beam causes by competition of the weak rf voltage and the longitudinal friction force.

$$F_{\parallel} = \frac{ZeU_{rf}\sin\Delta\phi_s}{L_c},$$

$$v_{\parallel} = \frac{\beta c}{\eta_p}\frac{\Delta f}{f_0} = \frac{C_r}{\eta_p}\frac{\Delta f}{h},$$

❑ In raw measurements, the relative phase shift $\Delta\phi$ vs the frequency change Δf (10-Hz steps of 1129 kHz) for the protons at 48 MeV, which are converted to the plots of the drag force $F_{\parallel}$ vs $v_{\parallel}$, using equations above, and parameters in Tables I and II.

Backup Slides 4: Friction Force measurement II

TABLE I. Estimate of errors for $v_{\parallel}$.

Value	Error	Comment
C_r 81.76 m	±0.1%	circumference
η_p 0.783	±0.5%	from optics
Δf	±0.01%	frequency generator
$v_{\parallel}$	±0.5%	total error

TABLE II. Estimate of errors for $F_{\parallel}$.

Value	Error	Comment
U_{rf} 10.2 V	±7%	measurement
$\Delta\phi$	±1%	phase discriminator
L_c 2.50 m	±10%	cooler length
$F_{\parallel}$	±12%	total error

High precision measurements are done by:

- ❑Measure the phase difference accurate with the phase discriminator.
- ❑Changing rf frequency instead of the electron voltage step to vary the relative velocity.