HW II.1 (4 points): Pillbox cavity
(a) Calculate the RF surface resistance and skin depth of room-temperature copper at 500 MHz. Use DC resistivity $\rho = 1.7 \cdot 10^{-8}$ Ohm$\cdot$m.
(b) Calculate the RF surface resistance of superconducting niobium at 500 MHz at $T = 4.2$ K and $T = 2$ K. Assume a residual resistance $R_{\text{res}} = 10 \cdot 10^{-9}$ Ohm. What is the ratio of superconducting niobium to that of room-temperature copper?
(c) Design a cylindrical (pillbox) cavity that operates in the TM$_{010}$ mode at 500 MHz with an axial electric field of $E_0 = 1$ MV/m, and a length $l = \lambda/2$, where $\lambda$ is the RF wavelength in free space. Calculate the length and diameter of the cavity. Calculate the maximum $H$ and $E$ fields on the cavity wall. Where do they occur? Calculate the electromagnetic stored energy in the cavity.
(d) Calculate the power loss $P_c$, the quality factor $Q_0$, and the decay time $\tau$ for a room-temperature copper surface and for niobium surface at 4.2 K.

HW II.2 (1 point): Equivalent RLC circuit
A superconducting cavity with residual resistivity of 10 nOhm operates at a frequency of 1300 MHz. The geometry factor is 267 Ohm, and $R/Q = 900$ Ohm (accelerator definition). Calculate parameters of the equivalent parallel $RLC$ lumped-circuit model of this cavity.

HW II.3 (2 points): Anomalous skin effect
(a) Determine the improvement factor that can be expected for the $Q_0$ of a 500 MHz copper cavity if it is cooled down from room temperature to liquid helium temperature (4.2 K). What is the quality factor of the pillbox cavity from problem HW II.1 with copper walls cooled to 4.2 K? The $\rho l$ product of copper is $6.8 \cdot 10^{-16}$ Ohm$\cdot$m$^2$. The resistivity of copper at room temperature is $1.7 \cdot 10^{-8}$ Ohm$\cdot$m.
(b) Calculate the surface resistivity of niobium of RRR = 30 (reactor grade niobium) and RRR = 250 (high RRR niobium) at 500 MHz in the normal conducting state at 10 K (assume that RRR is given for this temperature.) What is the $Q_0$ of the pillbox cavity from problem HW II.1 with niobium walls at room temperature? What is the improvement factor for a niobium cavity on cooling from room temperature to 10 K? The resistivity of niobium at room temperature is $15 \cdot 10^{-8}$ Ohm$\cdot$m, and the $\rho l$ product of niobium is $6 \cdot 10^{-16}$ Ohm$\cdot$m$^2$. 