Problem 1. Chromaticity compensation: total 50 points

1a. 10 points. Consider a weak-focusing storage ring with bending radius ρ and equations of motion

$$x'' + K_x \cdot x = 0;\ \ K_x = \frac{1-n}{\rho^2};\ n = -\frac{\rho}{B_y}\frac{\partial B_y}{\partial x};$$

$$y'' + K_y \cdot y = 0;\ \ K_y = \frac{n}{\rho^2};\ \ 0 < n =< 1.$$

Derive expressions for horizontal (x) and vertical (y) β and D functions:

$$D'' + \frac{1-n}{\rho^2} \cdot D = \frac{1}{\rho},$$

and show that β and D functions are simply zeros. Calculate chromaticity of vertical and horizontal oscillation in such a ring

1b. 15 points. Show that adding sextuple component of the magnetic field

$$\frac{B_2}{B_0} = \frac{b}{2} \cdot \left(\hat{x} \cdot \left(x^2 - y^2 \right) - 2\hat{y} \cdot xy \right)$$

cannot compensate both chromaticity, i.e. that depending on sign of S it can reduce one chromaticity while increasing the other. In other words you should prove that chromaticity can be compensated only in lattice where β and D function are not constant, for example in strong focusing FODO lattice.

1c. 25 points. Consider a FODO lattice with thing quadrupole lenses which are combined with thin sextuples with integrated strength of (note sign change in definition – you would need "defocusing" sextuple to compensate vertical chromaticity)

$$S_F = \int_{QF} K_2 ds;\ \ S_D = -\int_{QD} K_2 ds;$$

As you may remember from the lectures, horizontal β-function as well as (horizontal) dispersion D reach their maxima in focusing quadrupole (QF) and minima in the defocusing quadrupoles (QD). Similarly, value of vertical β-function is minimal in QF and maximal in QD. Let's assume that – <u>you do not need to derive this</u>! – that

$$\beta_{xF}, \beta_{xD}, \beta_{yF}, \beta_{yD}, D_F, D_D$$

are the optics functions in F and D quadrupoles. Furthermore, let's assume – <u>you do not need to derive this</u>! – that C_x and C_y are horizontal and vertical chromaticities of the FODO cell, which you need to compensate. Find necessary strength of S_F and S_D and identify conditions – i.e. combination of $\beta_{xF}, \beta_{xD}, \beta_{yF}, \beta_{yD}$ - when such compensation is impossible. Also show that stable FODO lattice provides condition for chromaticity compensations.

Problem 2. 20 points. CERN is considering building 100 TeV proton collider. They plan to build circular storage ring with circumference of 100 km colliding 50 GeV proton beams. Assuming 70% filling factors by bending dipoles – total length of the bending magnets is 70 km! -, find the following:

i. Bending radius and necessary magnetic filed in the dipoles
ii. Energy loss of protons for synchrotron radiation and critical wavelength of radiation
iii. Assuming iso-magnetic lattice, calculate damping times for synchrotron (energy) and betatron oscillations

Problem 3 30 points. Design diffraction-limited FODO light source for hard-X-rays. The diffraction limited X-ray source requires transverse beam emittance to satisfy

$$\varepsilon_{x,y} \leqslant \frac{\lambda}{4\pi}$$

where $\varepsilon_{x,y}$ are geometric (not normalized) beam emittances. Assume that λ=1 angstrom (0.1 nm) and that coupling provides for equal splitting (κ=1) of natural emittance induced by quantum fluctuation of synchrotron radiation. Assume that dipoles (part of FODO cells) have the uniform magnetic field and occupy 2/3 of the circumference ring circumference *C*.

i. Derive equation for necessary number of the FODO cells and bending angle and length of each dipole magnet as function of the beam energy and ring circumference
ii. Calculate bending angle and length of each dipole magnet for two cases:
   a. APS storage ring: E=6 GeV, C=1,104 m
   b. NSLS II storage ring E=3 GeV, C=792 m