Homework 16 solution:

1. The longitudinal wakefield is given by the following equation

$$w_{//}(s) = \frac{1}{2\pi}\int_{-\infty}^{\infty} Z_{//}(\omega) e^{-i\omega s/c} d\omega . \tag{1}$$

Inserting the resistive wall impedance given in the problem into eq. (1) yields

$$\begin{aligned} w_{//}(s) &= \frac{c}{2\pi}\frac{(1-i)}{2\pi b}\sqrt{\frac{Z_0}{2\sigma}}\int_{-\infty}^{\infty} k^{1/2} e^{-iks} dk \\ &= \frac{Z_0 c}{2\pi b}\frac{1}{2\pi}\int_{-\infty}^{\infty}(1-i)\sqrt{\frac{k}{2\sigma Z_0}} e^{-iks} dk \\ &= \frac{Z_0 c}{2\pi b}\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{k}{\lambda} e^{-iks} dk \\ &= -\frac{c}{4\pi^{3/2} b |z|^{3/2}}\sqrt{\frac{Z_0}{\sigma}} \end{aligned} , \tag{2}$$

where the following relations

$$\lambda = i(1 - i\,\mathrm{sgn}(k))\sqrt{\frac{\sigma |k| Z_0}{2}} = i(1-i)\sqrt{\frac{\sigma k Z_0}{2}} , \tag{3}$$

and

$$\frac{k}{\lambda} = \frac{\sqrt{2k}}{i(1-i)\sqrt{\sigma Z_0}} = (1-i)\sqrt{\frac{k}{2\sigma Z_0}} \tag{4}$$

are used.

2. Following the definition of the loss factor and that of the impedance, we obtain

$$
\begin{aligned}
k_{//} &= \int_{-\infty}^{\infty} V_{//}(z)\lambda(z)\,dz \\
&= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \lambda(z_1)\,w_{//}(z_1-z)\lambda(z)\,dz_1 dz \\
&= \frac{1}{(2\pi)^3}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} Z_{//}(\omega_2)e^{i(\omega_2-\omega)z/c}\tilde{\lambda}(\omega)\tilde{\lambda}(\omega_1)e^{-i(\omega_1+\omega_2)z_1/c}\,d\omega_2 d\omega_1 d\omega dz_1 dz \\
&= \frac{c^2}{(2\pi)}\int_{-\infty}^{\infty} Z_{//}(\omega)\tilde{\lambda}(\omega)\tilde{\lambda}(-\omega)\,d\omega \\
&= \frac{c^2}{2\pi}\int_{-\infty}^{\infty} Z_{//}(\omega)\tilde{\lambda}(\omega)\tilde{\lambda}^*(\omega)\,d\omega \\
&= \frac{c^2}{2\pi}\int_{-\infty}^{\infty} Z_{//}(\omega)\left|\tilde{\lambda}(\omega)\right|^2 d\omega
\end{aligned}
\quad , (5)
$$

where I used $\tilde{\lambda}^*(\omega)=\tilde{\lambda}(-\omega)$ since $\lambda(z)$ is real.