Today schedule:

1. IFEL project
2. Recall from two last classes
3. Simulation (HW2 and HW3)

PHY542. Emittance Measurements

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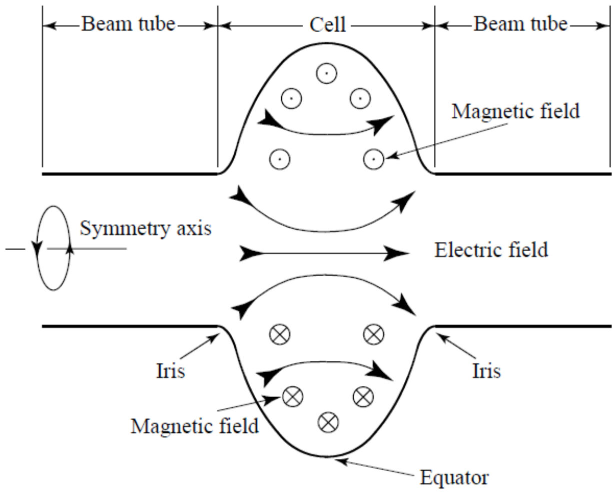
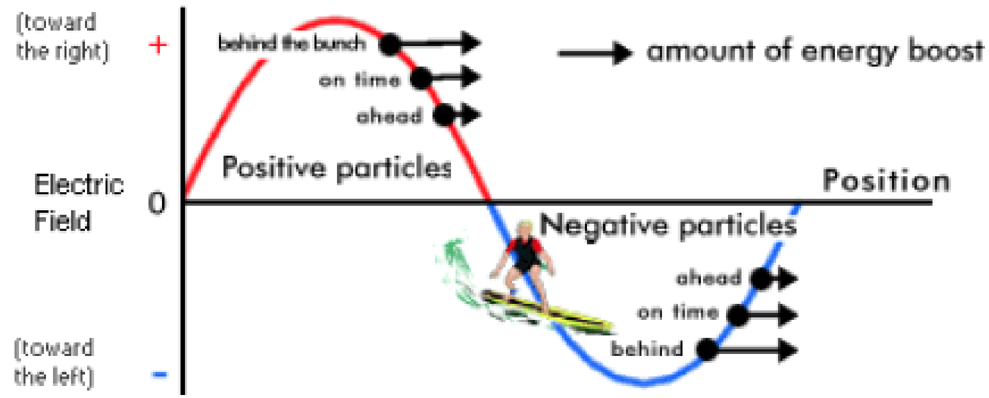
RF Field acceleration:





The RF field must be synchronous (correct phase relation) with the beam for a sustained energy transfer.

$$E_z(z,t) = E(z)\cos\left(\omega t - \int_0^z k(z)\,dz + \phi\right),$$

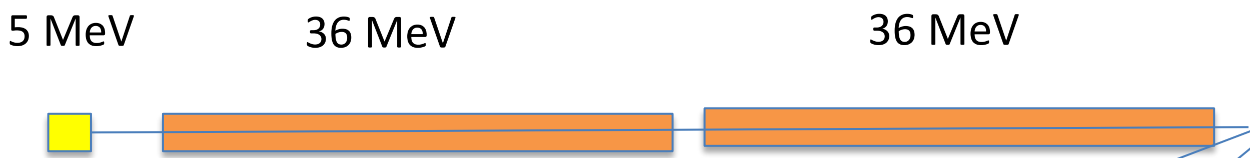
For efficient particle acceleration, the phase velocity of the wave must closely match the beam velocity. If we consider a particle of charge *q* moving along *+z* direction with a velocity at each instant of time equal the phase velocity of the traveling wave, then the electric force on the particle is given by

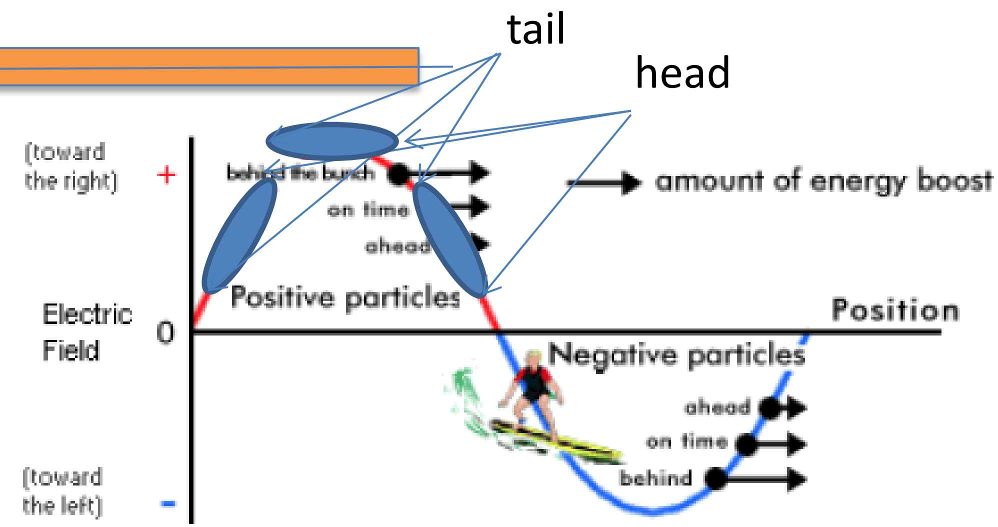
$$F_z = q\,E(z)\cos\phi$$

Energy gain

$$\Delta\mathcal{E} = q\int_{-L/2}^{L/2} E(0,z)\cos\left[\omega t(z) + \phi\right]\,dz$$

Multi linacs acceleration





$E=E_{inj}+Elinac1+Elinac2$

$Elinac1=eU1\cos(phi1)$

$Elinac2=eU2\cos(phi2)$

If there is enough voltage provided by one linac.
The final energy can be reached by combination different phases.

For ATF:
U1=U2=36MV, Einj=5MeV
Efinal=35MeV

phi1	phi2
65.4	65.4
65.4	-65.4
0.0	99.6
0.0	-99.6
90.0	33.6
-90.0	-33.6
33.6	90.0
-33.6	-90.0

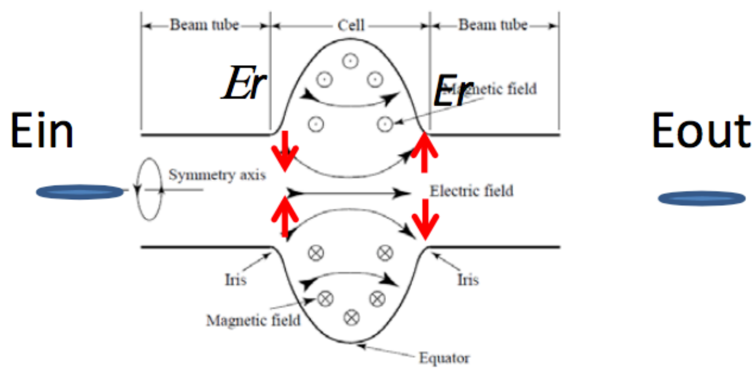
Why one operation could be better then others?

Few things to remember

- Space charge force depends on energy
  - Higher energy => less space charge effects

$$\sigma_x''(\zeta,s) + \kappa_\beta^2 \cdot \sigma_x(\zeta,s) = \frac{r_e \lambda(\zeta)}{2\gamma^3 \sigma_x(\zeta,s)} + \frac{\varepsilon_{n,x}^2}{2\gamma \sigma_x^3(\zeta,s)}$$

- Focusing due to entrance and exit of RF field
  - More energy gain => stronger focusing



$$\Delta p_r = \frac{e}{c} \int E_r dz$$

$$r'_{in} = \frac{\Delta p_{in\_r}}{p_{in\_z}} \qquad r'_{out} = \frac{\Delta p_{out\_r}}{p_{out\_z}}$$

$$\Delta p_{in\_r} \sim -\Delta p_{out\_r}$$

Eout= Ein+ΔE

Entrance kick is larger then exit kick

PHY 542
COMPUTATIONAL EXERCISE – RF Linac

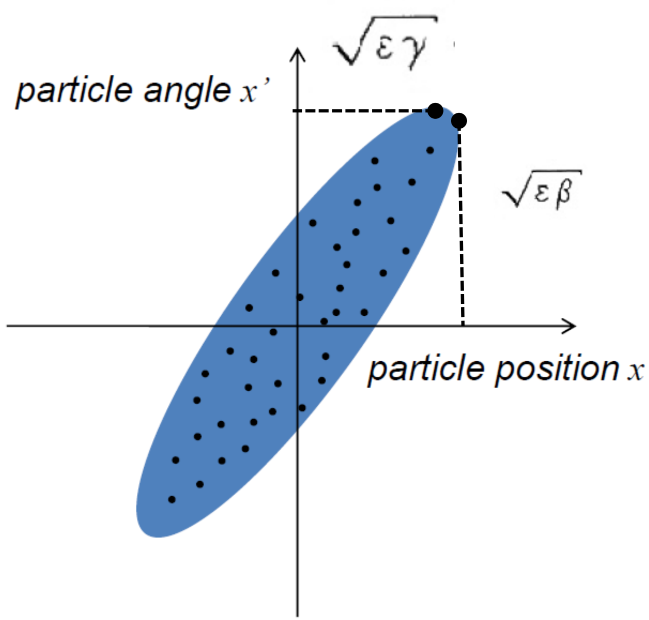
Exercise: RF linac accelaration

1. Open file *ATF_LINAC.in.* Find acceleration linac line description. There are two linacs. Make sure that the both cavities gradient is sufficient to accelerate e-beam on 36 MeV by each cavity. Change adjust maximum gradient (maxE parameter).
   (*hint: set acceleration phase to 0 in both linacs and run ASTRA for this project*)
2. Search for optimum linac set points for fix energy gain 30 MeV. Set up linac acceleration gradient 16 MV/m. Set the same phase for both linac to accelerate 15 MeV each. (phi=65 deg). Find final energy spread and emittance.
3. Repeat step 2 for different linac phases:
   a. Linac Phase1=65 LinacPhase2=-65
   b. Linac_Phase1=34 Linac Phase2=90
   c. Linac Phase1=90 LinacPhase2=34 (have you got the same energy?)
   d. Linac Phase1=0 LinacPhase2=100
4. What linacs phase settings provide minimum **emittance**?
5. What linacs phase settings provide minimum **energy spread**?

Same exercise without space charge:

6. Try turn off space charge and repeat steps 2-5.
7. Why final emittance is different without space charge?

Emittance, what is it ?



ε = Area in *x, x'* plane occupied by beam particles divided by π

Beam ellipse and its orientation is described by 4 parameters

$$\varepsilon = \gamma x^2 + 2\,\alpha\, x\, x' + \beta\, x'^2$$

$\sqrt{\beta\varepsilon}$ Is the beam half width

$\sqrt{\gamma\varepsilon}$ Is the beam half divergence

α Describes how strong x and x' are correlated

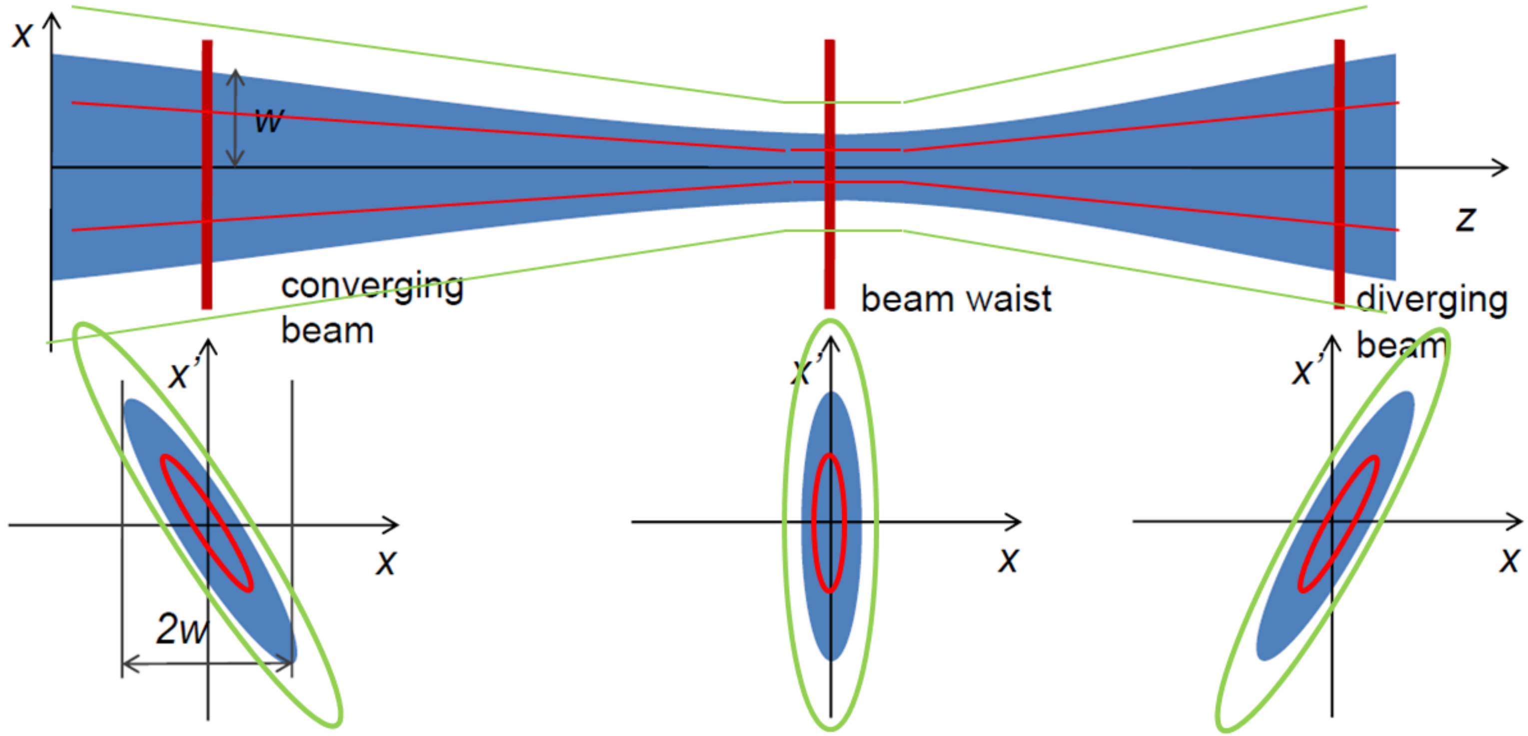
a<0 beam diverging

a>0 beam converging

a=0 beam size is maximum or minimum(waist)

The three orientation parameters are connected by the relation $\gamma = \dfrac{1+\alpha^2}{\beta}$

Beam envelope along a beamline.

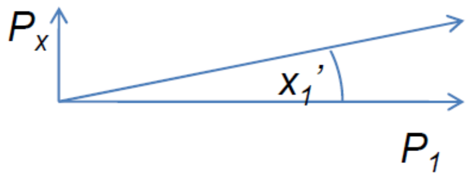


Along a beamline the orientation and aspect ratio of the beam ellipse in x, x' changes, but area (emittance) remains constant.
Alike initial beam distributions have similar phase space dynamics

Beam width along Z is described $$w(z) = \sqrt{\beta(z)\,\varepsilon}$$

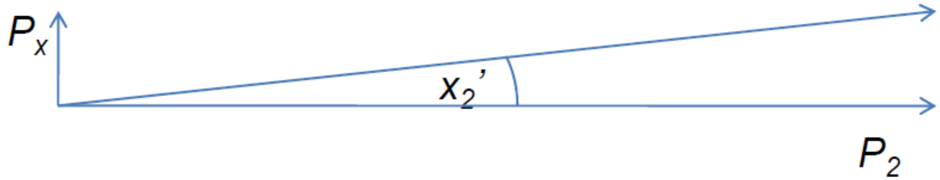
$\beta(z)$ describes the beam line, ε – describes beam quality

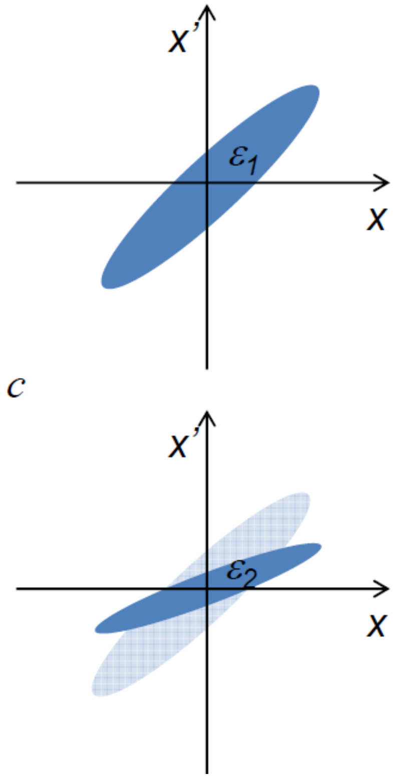
Geometrical emittance is only constant in beamlines without acceleration



acceleration

$$\varepsilon_2 = \varepsilon_1 \frac{P_1}{P_2} \quad \text{with} \quad P = m\beta_{rel}\gamma_{rel}\, c$$

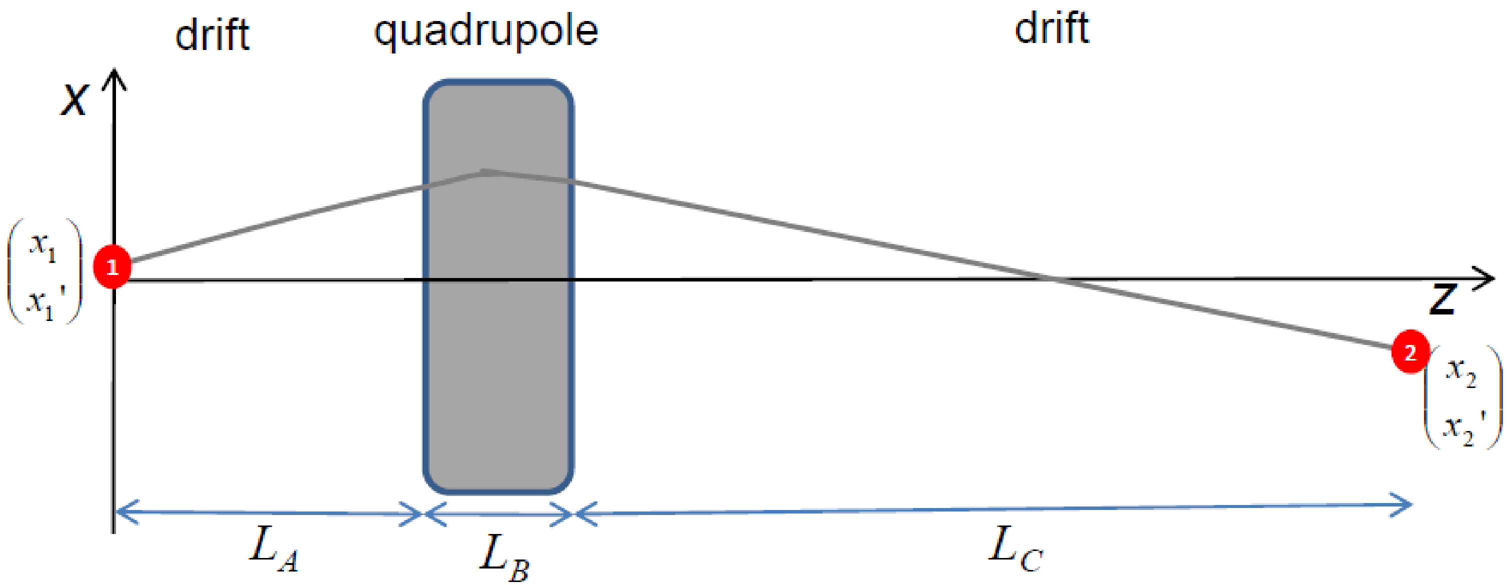




Normalized emittance preserved with acceleration

$$\varepsilon_N = \beta_{rel}\gamma_{rel}\ \varepsilon$$

Transport of single particle described with matrix



$$M_{Drift} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \qquad M_{Quadrupole} = \begin{pmatrix} \cos\left(\sqrt{k}L\right) & 1/\sqrt{k}\sin\left(\sqrt{k}L\right) \\ -\sqrt{k}\sin\left(\sqrt{k}L\right) & \cos\left(\sqrt{k}L\right) \end{pmatrix}$$

$$\begin{pmatrix} x_2 \\ x_2' \end{pmatrix} = M \cdot \begin{pmatrix} x_1 \\ x_1' \end{pmatrix}$$ There $$M = \begin{pmatrix} 1 & L_C \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos\left(\sqrt{k}L_B\right) & 1/\sqrt{k}\sin\left(\sqrt{k}L_B\right) \\ -\sqrt{k}\sin\left(\sqrt{k}L_B\right) & \cos\left(\sqrt{k}L_B\right) \end{pmatrix} \cdot \begin{pmatrix} 1 & L_A \\ 0 & 1 \end{pmatrix}$$

Twiss parameters and matrix transformations

- http://uspas.fnal.gov/materials/09VU/Lecture6.pdf
- http://uspas.fnal.gov/materials/09VU/Lecture7.pdf

$$\varepsilon = \gamma x^2 + 2\,\alpha\, x\, x' + \beta\, x'^2 \qquad \varepsilon = (x, x')_0 \begin{pmatrix} \gamma & \alpha \\ \alpha & \beta \end{pmatrix}_0 \begin{pmatrix} x \\ x' \end{pmatrix}_0 = (x, x')_1 \begin{pmatrix} \gamma & \alpha \\ \alpha & \beta \end{pmatrix}_1 \begin{pmatrix} x \\ x' \end{pmatrix}_1$$

$$\begin{pmatrix} x \\ x' \end{pmatrix}_1 = \begin{pmatrix} C & S \\ C' & S' \end{pmatrix}_{0to1} \begin{pmatrix} x \\ x' \end{pmatrix}_0 = M \begin{pmatrix} x \\ x' \end{pmatrix}_0$$

$$\varepsilon = (x, x')_0 \begin{pmatrix} \gamma & \alpha \\ \alpha & \beta \end{pmatrix}_0 \begin{pmatrix} x \\ x' \end{pmatrix}_0 = (x, x')_0\, M^T \begin{pmatrix} \gamma & \alpha \\ \alpha & \beta \end{pmatrix}_1 M \begin{pmatrix} x \\ x' \end{pmatrix}_0$$

This is true for any (x, x') $\Rightarrow$

$$\begin{pmatrix} \gamma & \alpha \\ \alpha & \beta \end{pmatrix}_1 = M^{-1T} \begin{pmatrix} \gamma & \alpha \\ \alpha & \beta \end{pmatrix}_0 M^{-1}$$

Emittance, how to measure it ?

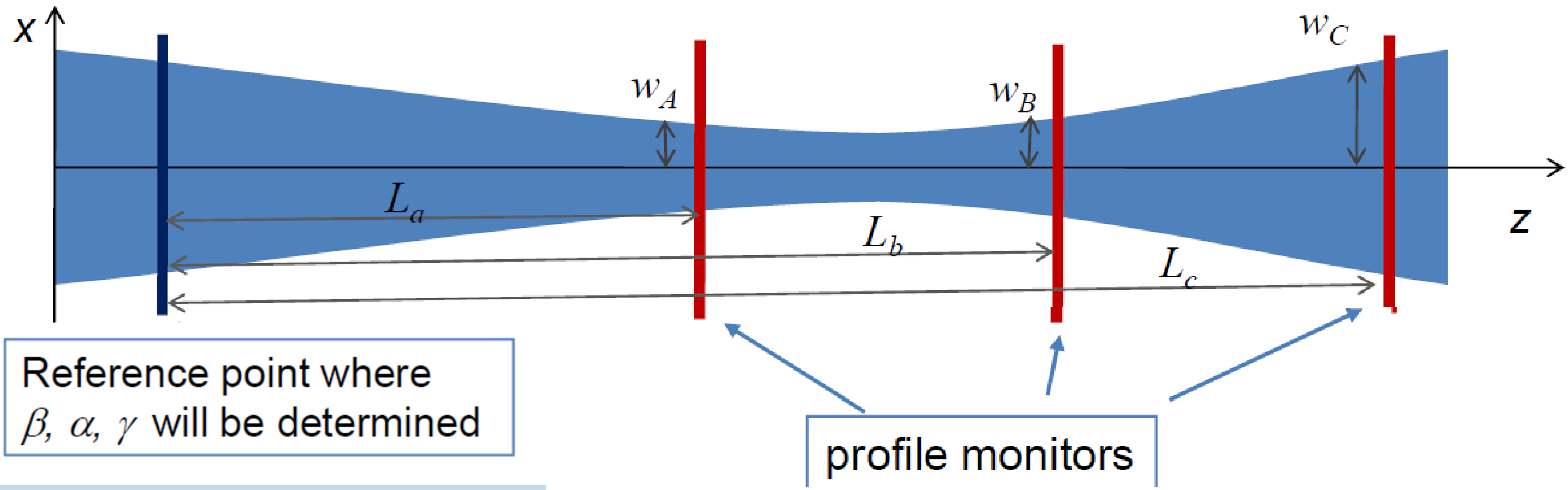
- Methods based on transverse beam profile measurements
  - Different location beam profile measurements
  - Quadrupole scan

  Slit and pepper pot(multi slit) methods

Emittance measurement in transfer line or linac

Twiss parameters α,β,γ are a priori not known, they have to be determined together with emittance ε

Method A



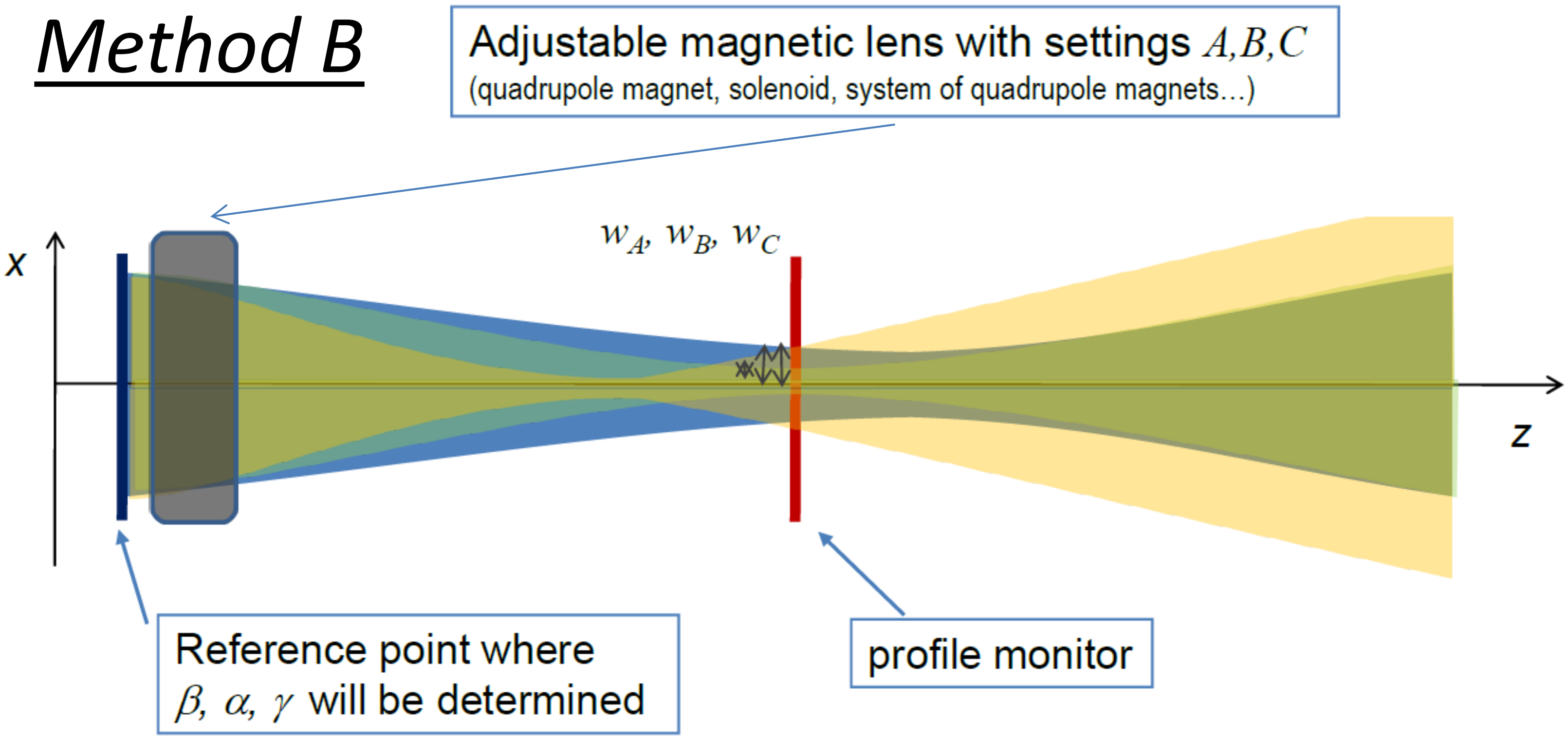
$$w_A^2 = \beta\,\varepsilon - 2\ L_A\ \alpha\ \varepsilon + L_A^2\ \gamma\,\varepsilon$$
$$w_B^2 = \beta\,\varepsilon - 2\ L_B\ \alpha\ \varepsilon + L_B^2\ \gamma\,\varepsilon$$
$$w_C^2 = \beta\,\varepsilon - 2\ L_C\ \alpha\ \varepsilon + L_C^2\ \gamma\,\varepsilon$$

3 linear equations, 3 independent variable
Solved by inverting matrix.

$$\beta\varepsilon\cdot\gamma\varepsilon - (\alpha\varepsilon)^2 = \varepsilon^2\left(\beta\cdot\gamma - \alpha^2\right) = \varepsilon^2 \quad \Rightarrow \quad \sqrt{\beta\varepsilon\cdot\gamma\varepsilon - (\alpha\varepsilon)^2} = \varepsilon, \quad \beta = \frac{\beta\varepsilon}{\varepsilon}, \quad \alpha = \frac{\alpha\varepsilon}{\varepsilon}$$

Emittancemeasurement in transfer line or linac, (count.)

- ***Method B***



$$w^2 = c^2\ \beta\varepsilon - 2cs\ \alpha\ \varepsilon + s^2\ \gamma\varepsilon, \qquad \begin{pmatrix} c & s \\ c' & s' \end{pmatrix} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} m_{11}(I_{mag}) & m_{12}(I_{mag}) \\ m_{21}(I_{mag}) & m_{22}(I_{mag}) \end{pmatrix}$$

$$w_A^2 = c_A^2 \beta\varepsilon - 2c_A s_A\ \alpha\ \varepsilon + s_A^2\ \gamma\varepsilon$$
$$w_B^2 = c_B^2 \beta\varepsilon - 2c_B s_B\ \alpha\ \varepsilon + s_B^2\ \gamma\varepsilon$$
$$w_C^2 = c_C^2 \beta\varepsilon - 2c_C s_C\ \alpha\ \varepsilon + s_C^2\ \gamma\varepsilon$$

3 linear equations, 3 independent variable
Solved by inverting matrix.

$$\beta\varepsilon\cdot\gamma\varepsilon - (\alpha\varepsilon)^2 = \varepsilon^2\left(\beta\cdot\gamma - \alpha^2\right) = \varepsilon^2 \quad \Rightarrow \quad \sqrt{\beta\varepsilon\cdot\gamma\varepsilon - (\alpha\varepsilon)^2} = \varepsilon, \quad \beta = \frac{\beta\varepsilon}{\varepsilon}, \quad \alpha = \frac{\alpha\varepsilon}{\varepsilon}$$

Summary beam profile technics

- To determine ε, β, α at a reference point in a beamline one needs at least three *w* measurements with different transfer matrices between the reference point and the *w* measurements location.
- Different transfer matrices can be achieved with different profile monitor locations, different focusing magnet settings or combinations of both.
- Once β, α at one reference point is determined the values of β, α at every point in the beamline can be calculated.

Things to do in ATF control room (emittance measurements)

Today (demonstration)

1. Learn how to measure emittance using 4 screens.
2. Measure emittance for different RF gun solenoid current settings Collect several data for each solenoid settings (minimum 10 solenoid settings).

HW1: Plot these data. Find the optimum solenoid settings and minimum emittance. Conduct error analysis.

Next class (demonstration)

3. For some solenoid settings (1-3) perform quadrupole scan.

HW2: Plot profile vs quadrupole current. Calculate emittance (see next slide) for each solenoid settings (units?).

HW3: Based on quadrupole scan find quadrupole calibration coefficient K[1/(m*A)] to convert quadrupole current to quadrupole focus strength (1/f[m]=K*I[A]). For given energy

Quadrupole scan

The quadrupole scan technique is a standard technique used in accelerator facilities to measure the transverse emittance. It is based on the fact that the squared rms beam radius (x_{rms}^2) is proportional to the quadrupole "strength" or inverse focal-length f squared, so

$$x_{rms}^2 = < x^2 > = A\left(\frac{1}{f^2}\right) - 2AB\left(\frac{1}{f}\right) + (C + AB^2) \qquad (1)$$

where A, B, C are constants and f is the focal length defined as

$$\frac{1}{f} = \kappa l, \qquad (2)$$

$$k\left[\frac{1}{cm^2}\right] = \frac{G[\frac{\text{Gauss}}{\text{cm}}]}{\text{Brho}[\text{Gauss} \cdot \text{cm}]}$$

here κ is the magnet focusing strength in units of 1 over length squared and l is the effective length of the magnet.

The emittance can be estimated according to

$$\varepsilon = \frac{\sqrt{AC}}{d^2} \qquad (3)$$

where d is the distance from the magnet you scan to the point you calculate the beam rms radius.