PHY 554. Homework 6.

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HW 1, 3 points. Using representation of transport matrices using β-functions from Lecture, and a weak quadrupole error:

$$M_o\left(s_1|s_2\right)=\begin{bmatrix}\sqrt{\frac{\beta_2}{\beta_1}}\left(\cos\Delta\psi_{12}+\alpha_1\sin\Delta\psi_{12}\right) & \sqrt{\beta_1\beta_2}\sin\Delta\psi_{12}\\ -\frac{1+\alpha_1\alpha_2}{\sqrt{\beta_1\beta_2}}\sin\Delta\psi_{12}-\frac{\alpha_1-\alpha_2}{\sqrt{\beta_1\beta_2}}\cos\Delta\psi_{12} & \sqrt{\frac{\beta_1}{\beta_2}}\left(\cos\Delta\psi_{12}-\alpha_2\sin\Delta\psi_{12}\right)\end{bmatrix};$$

$$\Delta\psi_{12}=\psi_2-\psi_1; M_\delta\left(s_1\right)=\begin{bmatrix}1 & 0\\ -k\left(s_1\right)ds & 1\end{bmatrix};$$

$$M\left(s_2|s_2+C\right)=M_o\left(s_1|s_2+C\right)M_\delta\left(s_1\right)M_o\left(s_2|s_1\right);\beta_i\equiv\beta_o\left(s_i\right);\psi_i\equiv\psi_o\left(s_i\right)=\nu\phi_o\left(s_i\right);$$

$$\delta M_{12}\left(s_2|s_2+C\right)=M\left(s_2|s_2+C\right)-M_o\left(s_2|s_2+C\right);$$

prove the modification of the transport matrix element M_{12} is indeed what we used in Lecture 8:

$$\delta M_{12}\left(s_2|s_2+C\right)=-\beta_1\beta_2k\left(s_1\right)ds\cdot\sin\left(\psi_1-\psi_2\right)\cdot\sin\left(\mu_o-\psi_1+\psi_2\right)$$

$$=\frac{1}{2}\beta_1\beta_2k\left(s_1\right)ds\cdot\left[\cos\mu_o-\cos\left(\mu_o-2\left(\psi_1-\psi_2\right)\right)\right]$$

HW2: 3 points. Prove that relative value of β-beat has the forced socillator equation with doubel betatron frequency:

$$f\left(s\right)=\frac{\delta\beta\left(s\right)}{\beta_o\left(s\right)}=-\frac{1}{2\sin\mu_o}\int_{\psi(s)}^{\psi(s)+\mu}\beta_o^2\left(z\right)k\left(z\right)\cdot\cos\left(\mu_o+2\left(\psi-\varphi\right)\right)d\varphi;d\varphi=\frac{ds}{\beta_o};$$

$$\frac{d^2}{d\psi^2}f\left(s\right)+4f\left(s\right)=-2\beta_o^2\left(s\right)k\left(s\right).$$

HW3, 4 points: Prove that it is impossible to compensate both horizontal and verticla chromaticity in a storage ring with uniform weak focusing.

Hints:

(a) Use the fact that β-functions are constants;
(b) Prove that both natural chromaticities are negative;
(c) Show hat dispersion fnction is constant and positive;
(d) Use this fact to show that sextupoles have equal opposite effect on tow chromaticities independently of locaion