

Homework 10. Due October 18

Problem 1. 3x5 points. Beam envelope in straight section.

For a one-dimensional motion consider beam propagating in a straight section starting at s_0 and having length L . Let's eigen vector (beam envelope) at s_0 is given by:

$$Y(s_0) = \begin{bmatrix} w_0(s) \\ w_0'(s) + \frac{i}{w_0(s)} \end{bmatrix};$$
$$\beta_0(s_0) \equiv w_0^2(s_0); \quad \alpha_0(s_0) = -\frac{\beta_0'(s_0)}{2} \equiv -w_0(s_0)w_0'(s_0); \quad (1)$$

(a) Propagate the eigen vector along the straight section. Show that β -function can be expressed as

$$\beta(s) = \beta^* + \frac{(s - s^*)^2}{\beta^*};$$

where β^*, s^* can be found from initial conditions (1). Hint, use derivative of β -function to find s^* . β^* is frequently used in colliders to describe the beam envelope in detectors.

(b) Calculate the (betatron) phase advance acquired in the straight section. Express it using β^*, s^* . Write expression for $x(s)$ and $x'(s)$. Show that $x' = \text{const}$.

(c) What is the maximum possible phase advance in a straight section (e.g. when s_0, L are unlimited)?