Homework 5. Due September 23

Problem 1. 5 points. Following up HW4: you proved that simple combination of field multipoles can describe the edge of a magnet. You also learned that we can use Laplacian equation on effective field potential:

$$\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \phi = 0$$

Let expand the potential in transverse direction while keeping arbitrary dependence along the beam propagating axis (s=z)

$$\phi = \sum_{n+m=k}^{\infty} a_{nm}(z)x^n y^m$$

Derive the condition (connections) between functions $a_{nm}(z)$.

Problem 2. 8 points. Prove that

$$\det[I + \mathbf{\varepsilon A}] = 1 + \mathbf{\varepsilon} \cdot \text{Trace}[\mathbf{A}] + O(\mathbf{\varepsilon}^2)$$

where $I$ is unit $n \times n$ matrix, $\mathbf{A}$ is an arbitrary $n \times n$ matrix and $\mathbf{\varepsilon}$ is infinitesimally small real number. Term $O(\mathbf{\varepsilon}^2)$ means that it contains second and higher orders of $\mathbf{\varepsilon}$.

Hint: first, look on the diagonal elements $\prod_{m=1}^{n} (1 + \mathbf{\varepsilon} a_{mm})$ first, then see what contribution to determinant comes from non-diagonal terms $a_{im}, \ k \neq m$. 