Homework PHY 554 #5.

HW 1 (3 points): A multi-cell accelerating RF linac operating at 500 MHz in a standing wave π-mode (e.g., each cell has opposite sign of the accelerating voltage from the neighboring cell) is used to accelerate non-relativistic heavy ion (Z=2, A=79) moving with velocity v=c/3 (β=1/3).

(a) find the length of the cell required for resonant acceleration in such a linac – 1 point.
(b) at what velocity (ies) (and energy(ies) of the ion), the energy gain in 5-cell cavity would vanish (became zero) – 2 points

HW 2 (2 points): A N-cell standing wave cavity operates in π-mode with field on the axis describes as

$$E_z = E_o(z)\cdot\sin(\kappa z)\cdot\cos(\omega t+\varphi);\ \kappa=\omega/2c;$$

$$E_o(z)=\begin{pmatrix} E_o;\ 0\le z\le \dfrac{n\pi}{\kappa} \\ 0;\ \ z<0 \\ 0;\ \ z>\dfrac{n\pi}{\kappa}\end{pmatrix}$$

Find the energy gain and transit time factor in such a linac for particle moving with the speed of light.

Extra points: what will be modification if v= βc; β ≠1.

HW 3 (5 points): A *l*=0.3 m long 500 MHz pillbox cavity operates in fundamental accelerating TM_{010} mode with peak accelerating electric field of 20 MV/m.

(a) Find the energy stored in electric and magnetic fields as function of time.
(b) What is the total energy of EM field in the cavity? Does it change with time?
(c) What will be losses of the energy for Q-factor of 30,000?

HW 4 (5 points): RF cavity beam loading/unloading.

A short ultra-relativistic (1-v/c <<1) bunch with charge of 5 nC is passing through a 0.3 meter long 500 MHz pillbox accelerating cavity operating at the fundamental TM_{010} with peak accelerating field of 5 MV/m.

(1) Find the change of the cavity voltage ΔV/V (accelerating field) after the beam passes through it as function of the phase of the beam passing the cavity. What are the maximum and minimum ΔV/V?

(2) How the beam loading $\Delta V/V$ depends on the accelerating field? At what level of accelerating it reaches $\Delta V/V$ 1%?

(a) Assume that beam does not change velocity in the cavity;
(b) Hint – use energy conservation law
(c) Assume that relative change of the voltage $\Delta V/V$ is small, e.g. the beam loading can be treated as a perturbation.