SIMULATIONS FOR COHERENT ELECTRON COOLING

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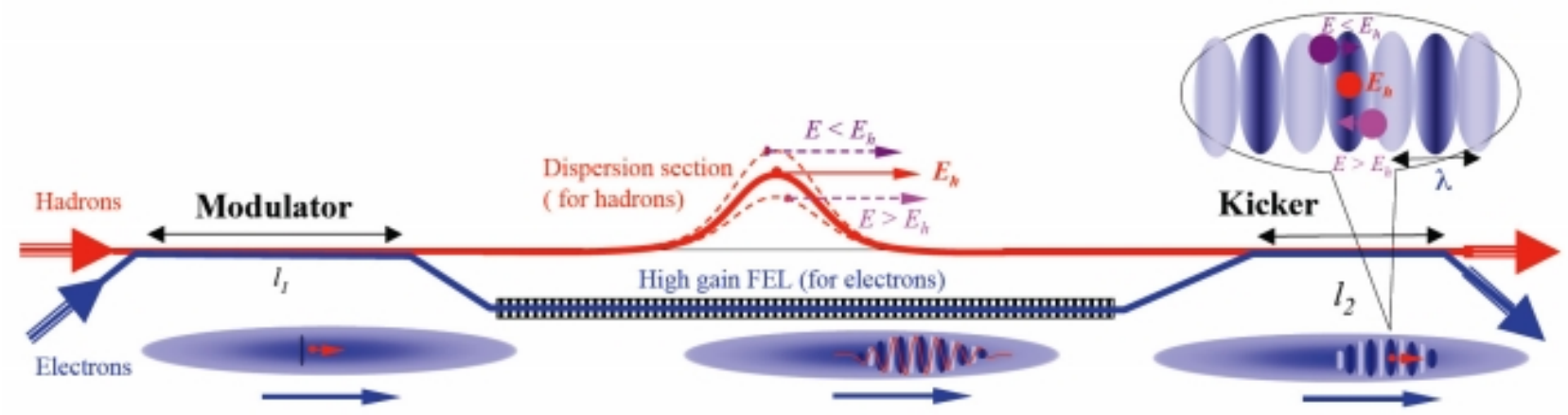
December 7, 2016

Schematic of CEC



Three sections of CEC

- Modulator : ion imprints a density wake on the electron distribution.
- Amplifier : density wake is amplified (high gain FEL).
- Kicker : amplified wake interacts with the ion.

Electron beam

- Energy : $\gamma = 42.9$
- Energy spread : 1e-3
- Peak current : 100[A]
- Full bunch length : 10[ps]
- Charge : 10[nC]
- RMS emittance : 5π[mm mrad]
- β at modulator and kicker : 4[m]

Ion beam

- Energy : $\gamma = 42.9$
- Species : Au^{+79}
- Bunch intensity : 1e+9
- RMS bunch length : 2[ns]
- RMS emittance : 2π[mm mrad]

Simulation tools

SPACE

- A parallel, relativistic, 3D electromagnetic Particle-in-Cell (PIC) code.
- Various boundary conditions : Dirichlet, periodic, open, mixed.
- Used for modulator and kicker simulations

GENESIS

- Used for FEL simulations (amplifier)

- Mechanism for modulation process is Coulomb interactions between ions and surrounding electrons.
- Relative density modulation is orders of magnitudes smaller than unity for relativistic beam energy.
- We treat each ion individually and use superposition principal to obtain net responses of electrons to all ions in the beam.
- One ion and a slice of electron beam (3e+7 electrons) are used in modulator simulations.

Thermal velocity

Kappa-2 velocity distribution is used to model the electron temperature.

$$f_0(\vec{v}) = \frac{n_0}{\pi^2 \beta_x \beta_y \beta_z} \left(1 + \frac{(v_x + v_{0x})^2}{\beta_x^2} + \frac{(v_y + v_{0y})^2}{\beta_y^2} + \frac{(v_z + v_{0z})^2}{\beta_z^2}\right)^{-2}$$

β_x, β_y and β_z describes the electron beam' s three-dimensional temperatures.

$\vec{v_0} = (v_{0x}, v_{0y}, v_{0z})$ is the ion velocity.

Shot noise

$$\frac{\text{shot noise}}{\text{modulation signal}} \approx 1e+5$$

- Perform two simulations with identical initial electron distribution.
- One simulation operates only with electron beam, the other simulation contains the electron beam and an ion.
- Take difference of the final electron distributions between the two simulations to obtain the influences of the ion.

Notations

- *nz* : Longitudinal density distribution
- *vz* : Longitudinal velocity distribution
- *nx* : Horizontal density distribution
- *vx* : Horizontal velocity distribution
- *ny* : Vertical density distribution
- *vy* : Vertical velocity distribution

Numerical convergence

- Identical initial particle distribution
- Different mesh refinements :

  5 grids per Debye Length

  20 grids per Debye Length
- Different number of macro electrons :

  3e+5 with representing number 100

  3e+7 with representing number 1

Numerical convergence, nz

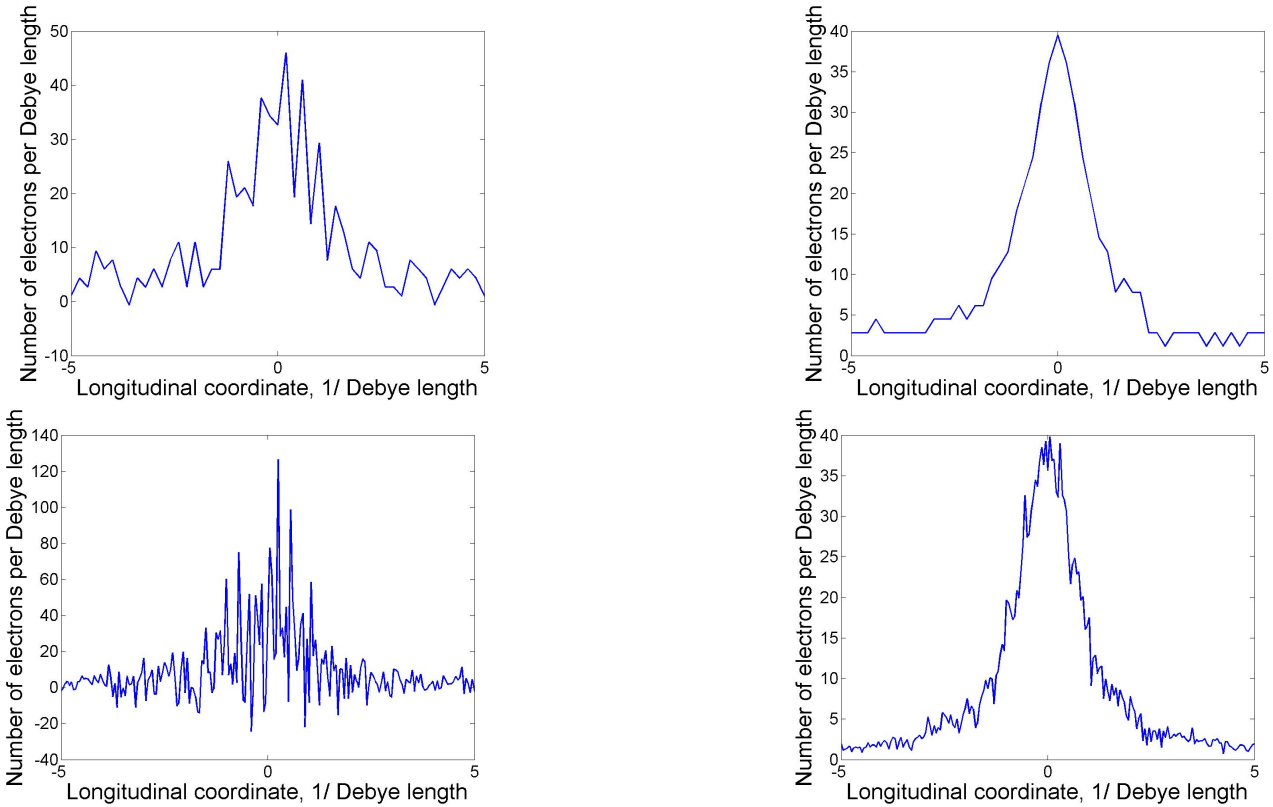


Figure: Coarse mesh (top), fine mesh (bottom), less macro particles (left), more macro particles (right).

Numerical convergence, vz

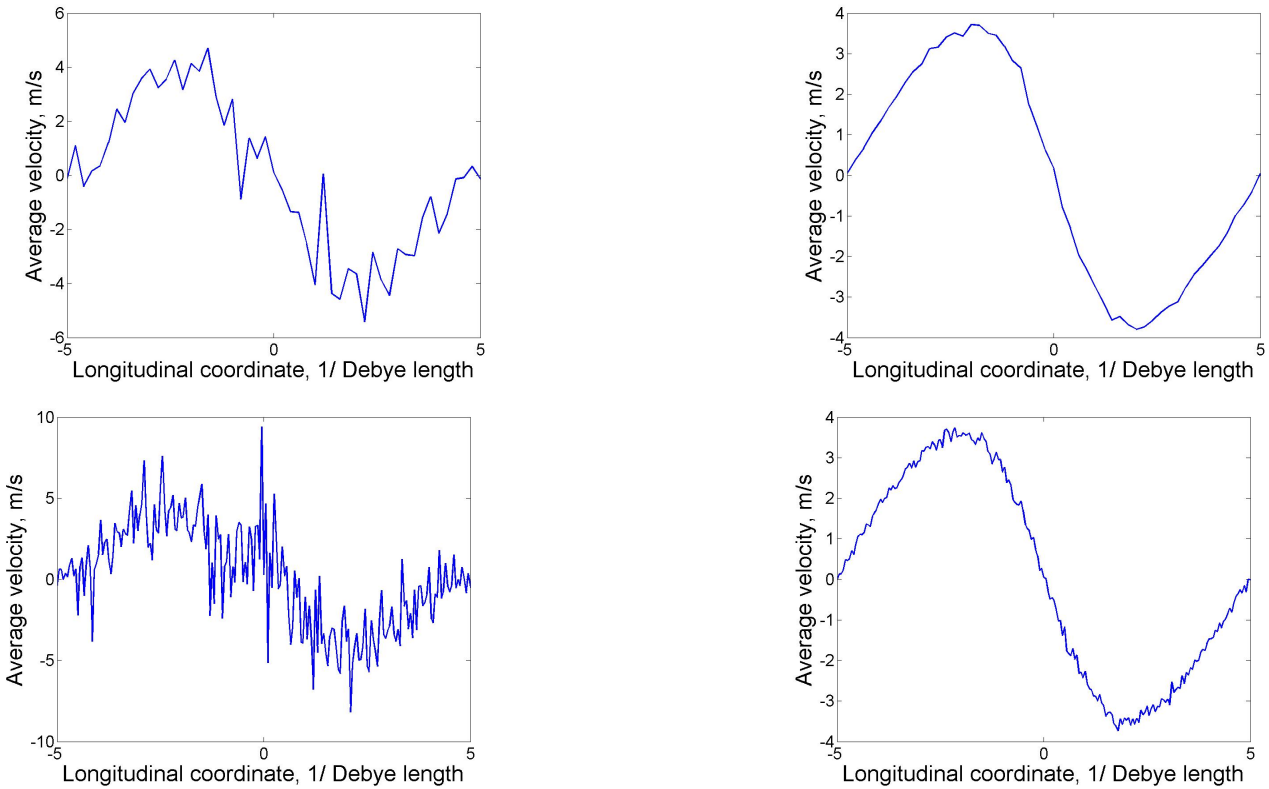


Figure: Coarse mesh (top), fine mesh (bottom), less macro particles (left), more macro particles (right).

Three sets of modulator simulations

- Distribution : Uniform

  Boundary : Periodic

  External field : No

  Analytical solution exists

- Distribution : Uniform in z, Gaussian in x and y

  Boundary : Periodic in z, open in x and y

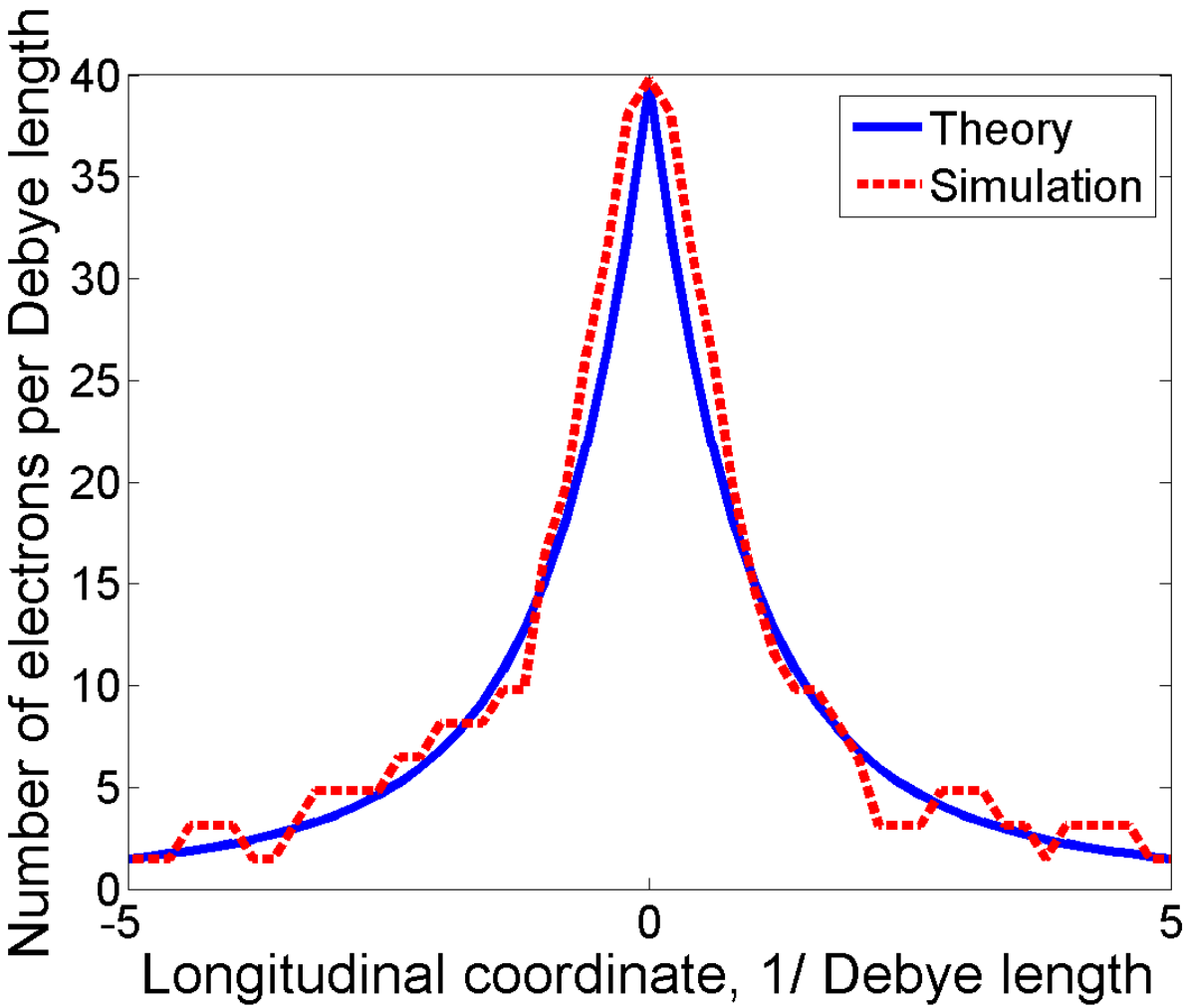
  External field : Continuous focusing

- Distribution : Uniform in z, Gaussian in x and y

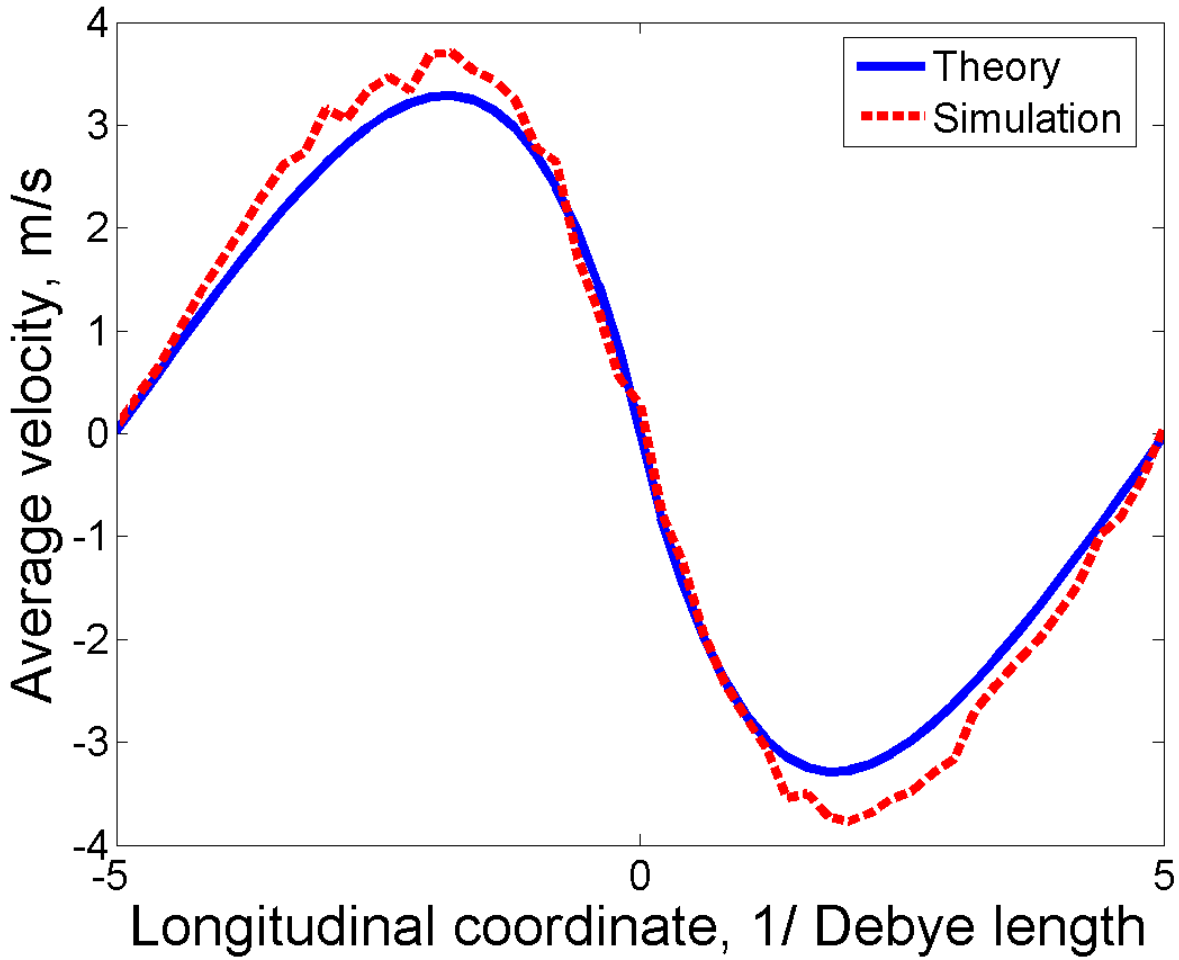
  Boundary : Periodic in z, open in x and y

  External field : Quadrupoles focusing

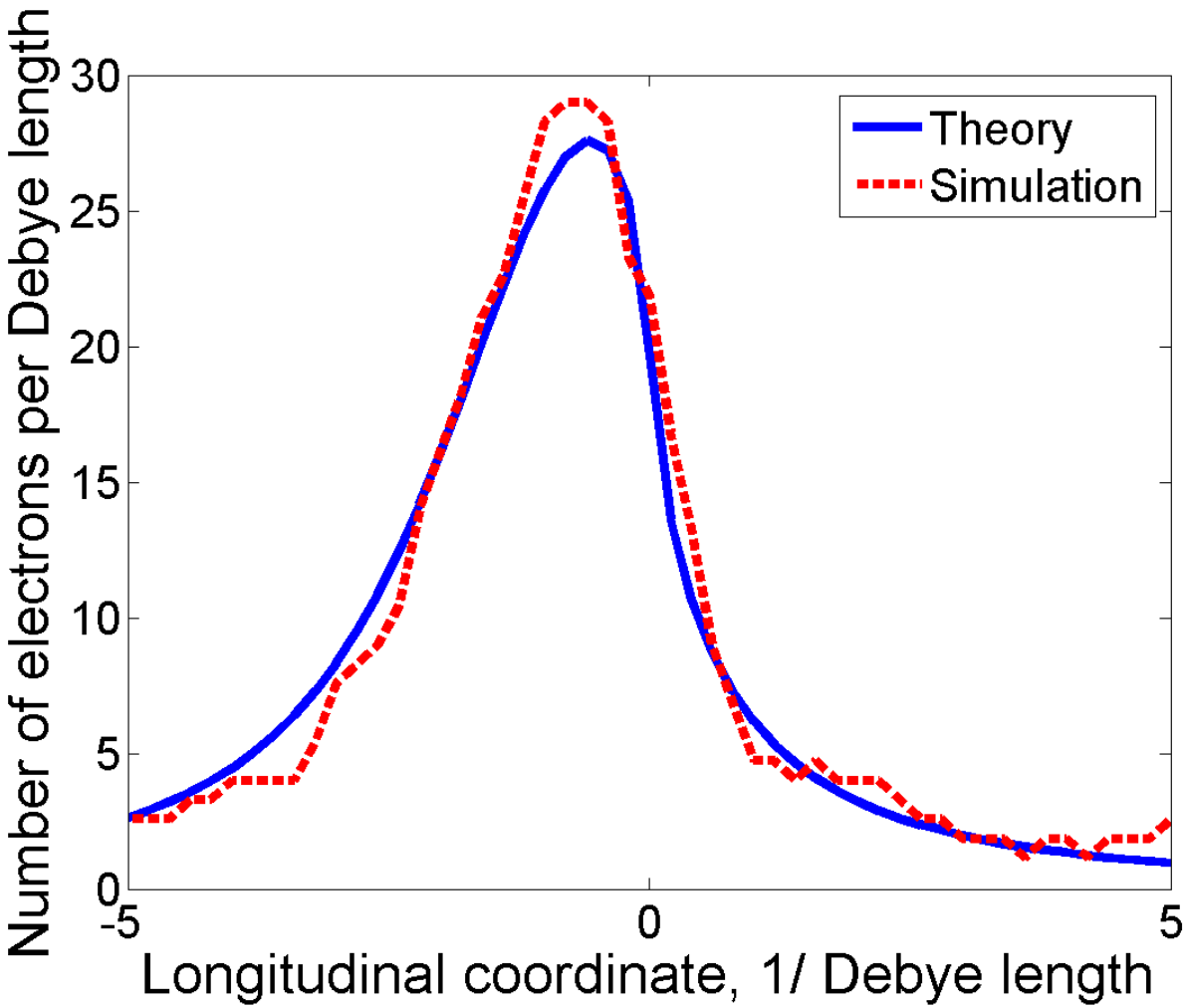
Comparison of nz, stationary ion



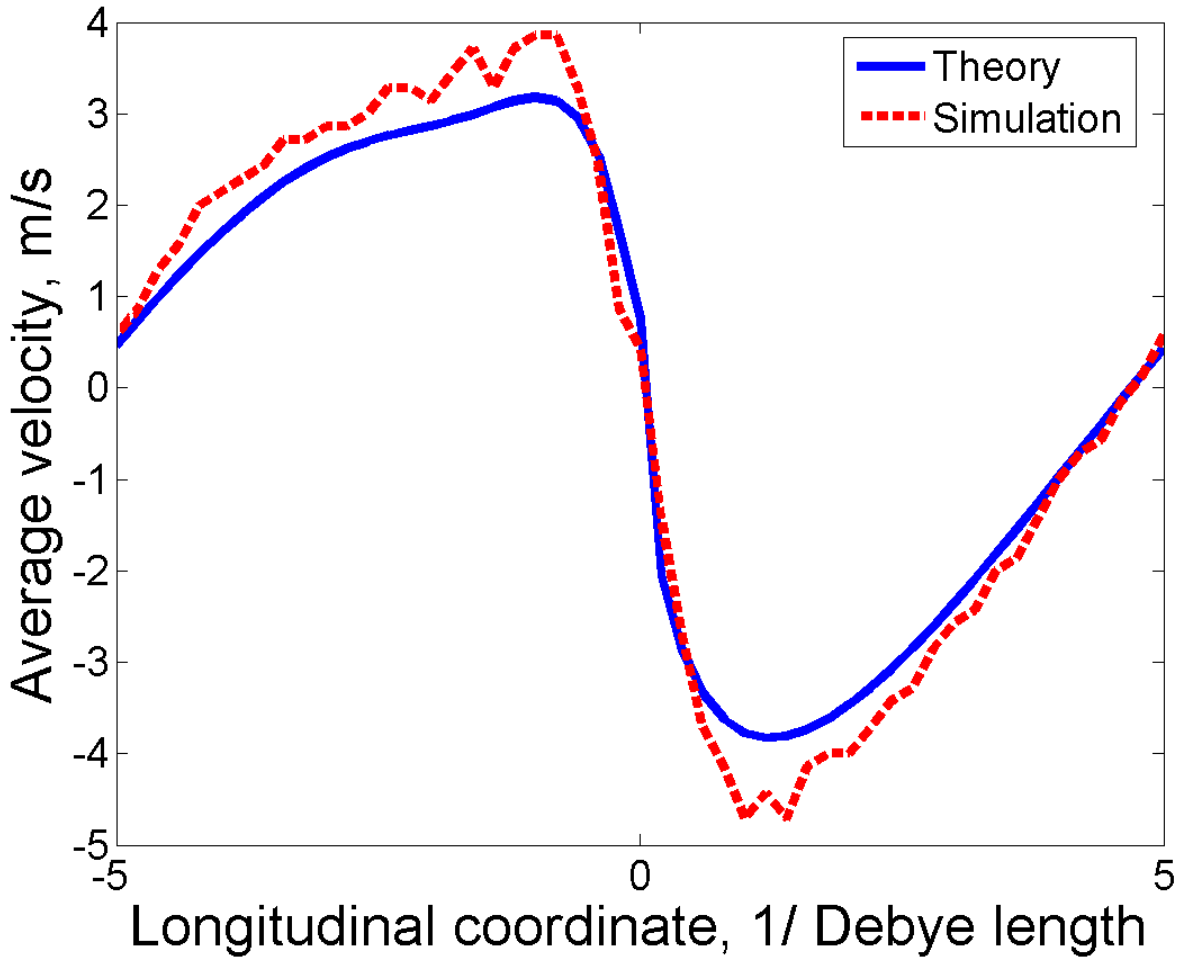
Comparison of vz, stationary ion



Comparison of nz, moving ion



Comparison of vz, moving ion



Electron beam expands in transverse because of

- Thermal velocity
- Space charge effect

Focusing field

- Focusing field compensating thermal expansion

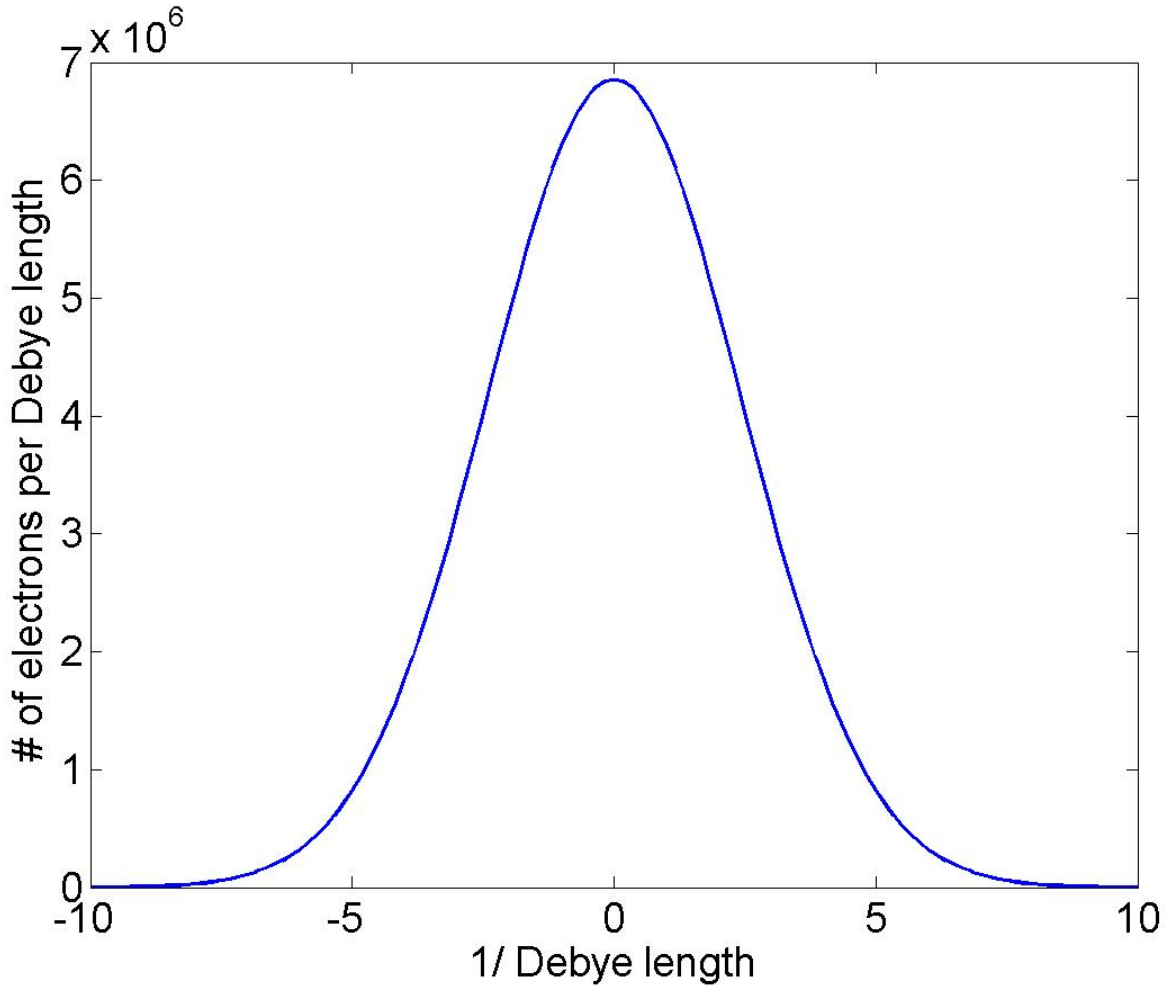
$$E_0(x) = \frac{m_e}{e}\frac{\beta^2}{r_0^2}(x - x_0)$$

- Focusing field compensating space charge expansion

$$E_1(x) = \frac{q}{2\pi\varepsilon_0(x - x_0)}\left(1 - e^{-(x-x_0)^2/2r_0^2}\right)$$

- x is radial coordinate, x_0 and r_0 are center and RMS of x, β is RMS velocity of electron beam, q is line charge density of electron beam

Initial nz


x 10^6
7
6
5
4
3
2
1
0
# of electrons per Debye length
-10
-5
0
5
10
1/ Debye length

vz in continuous focusing

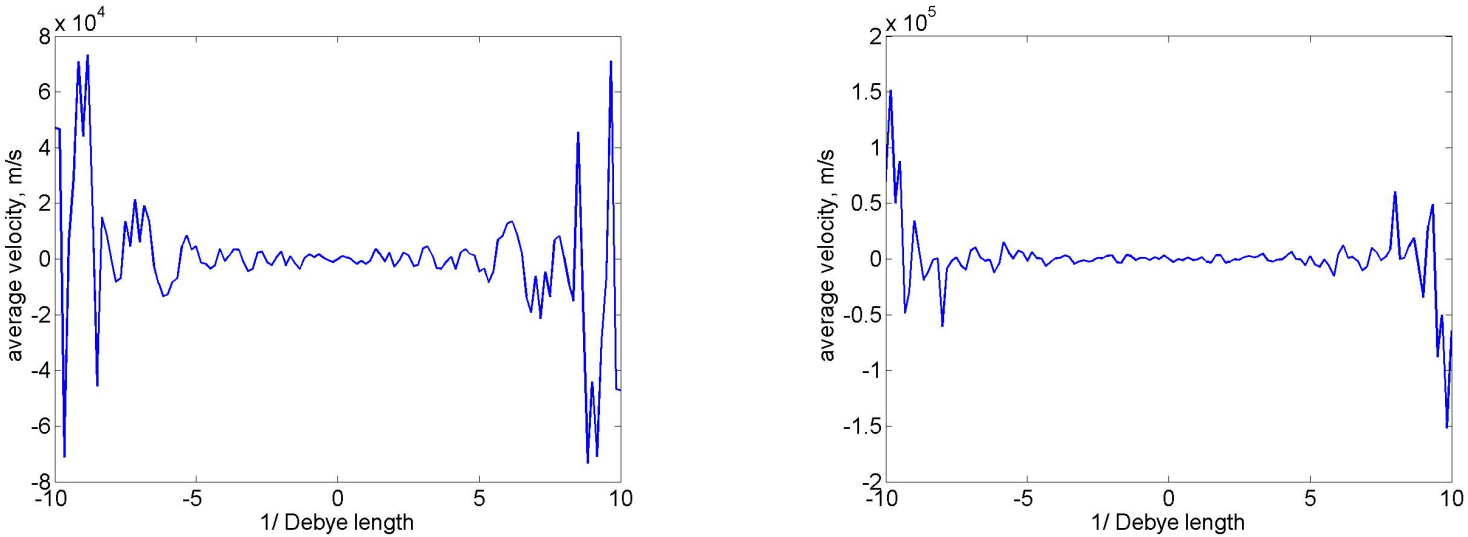
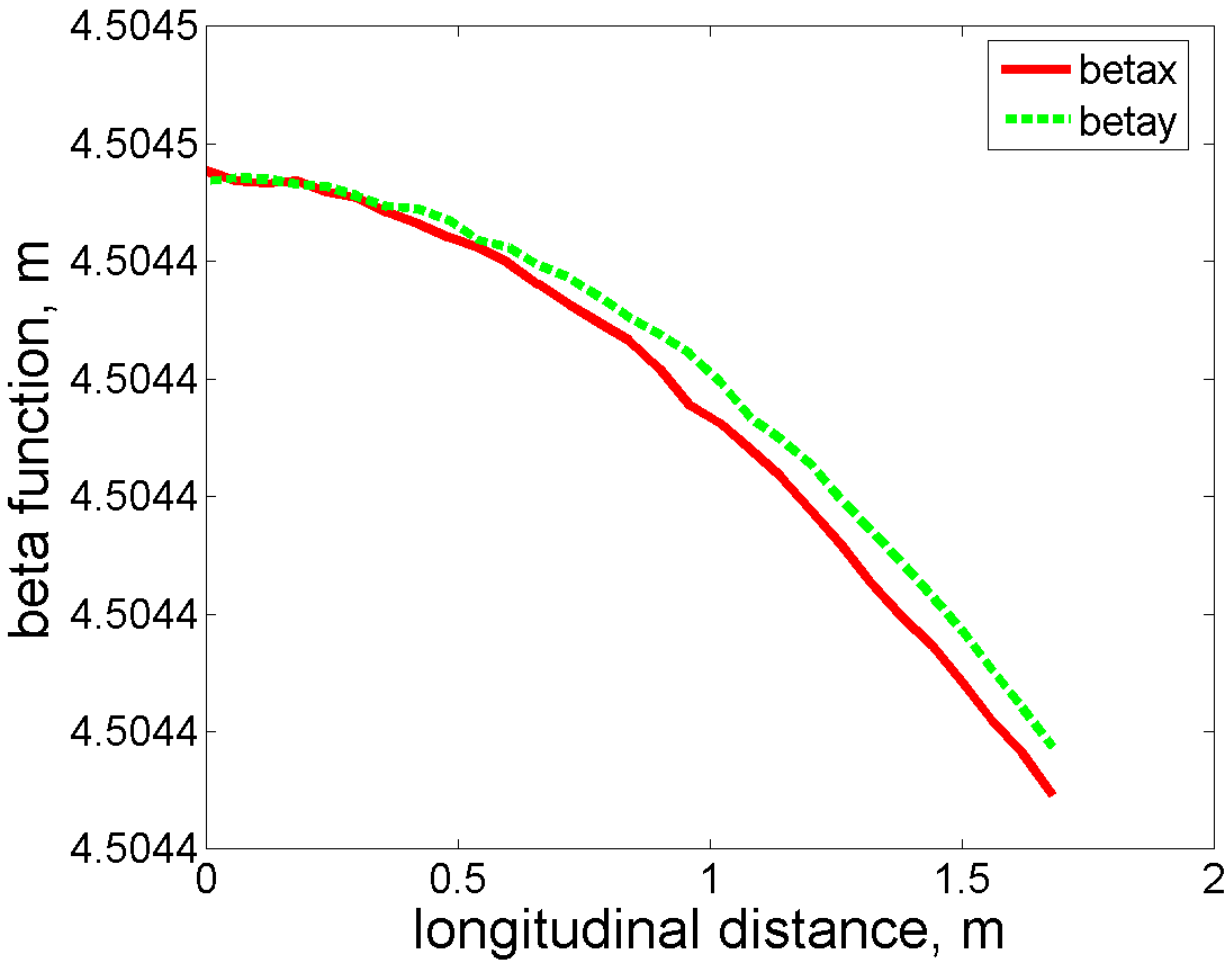
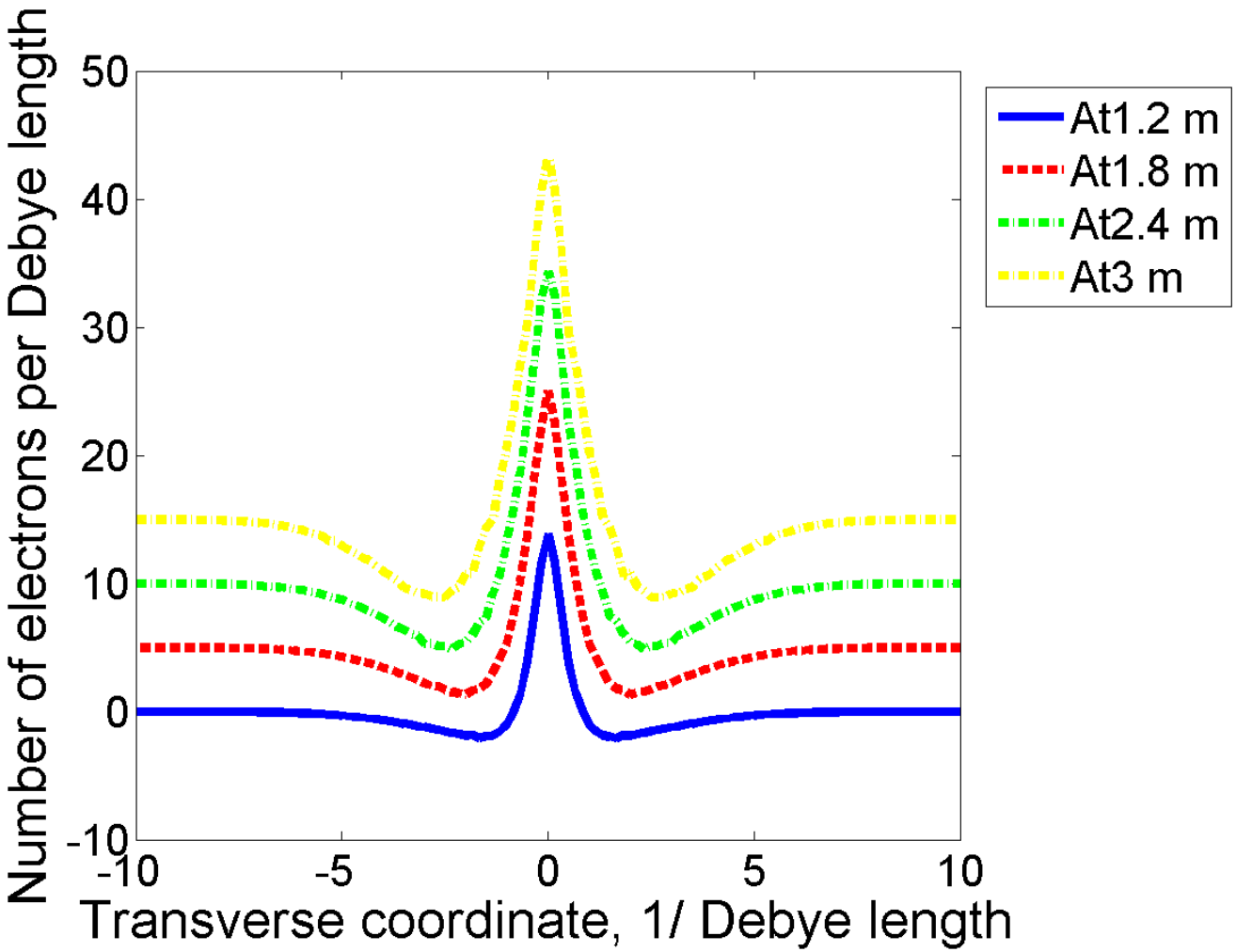


Figure: Initial (left) and final (right) vz

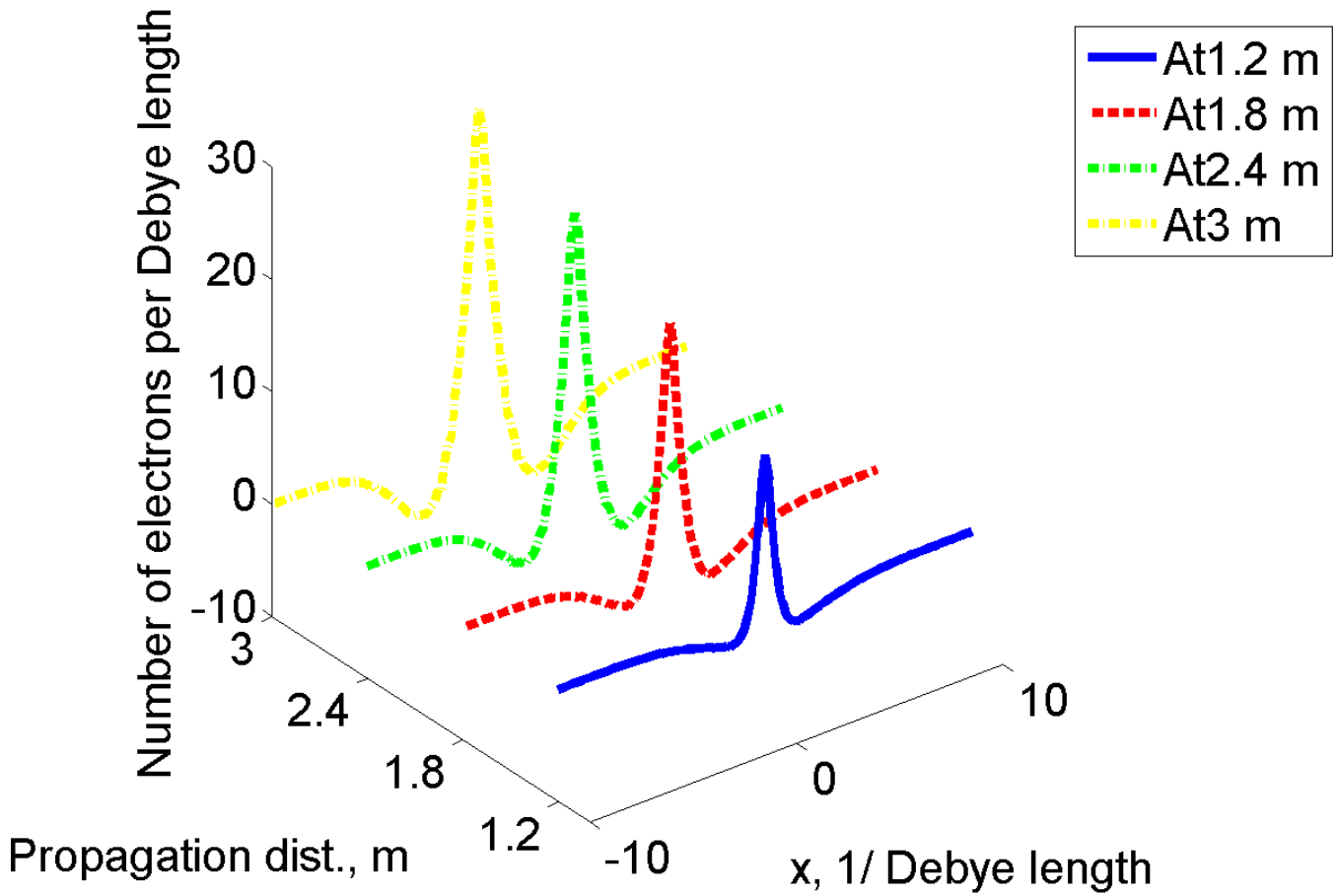
Beam size in continuous focusing



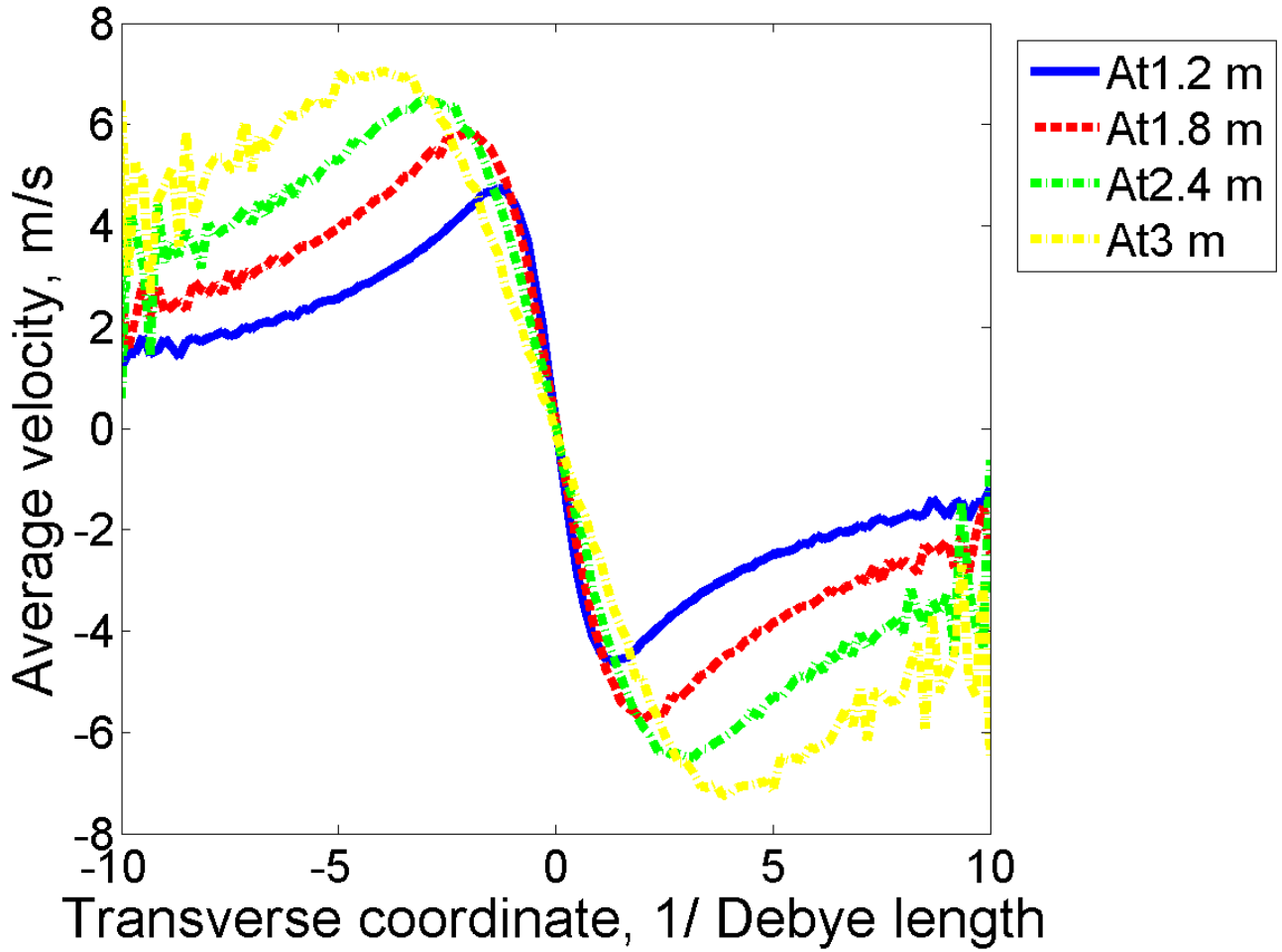
nx, ion at center



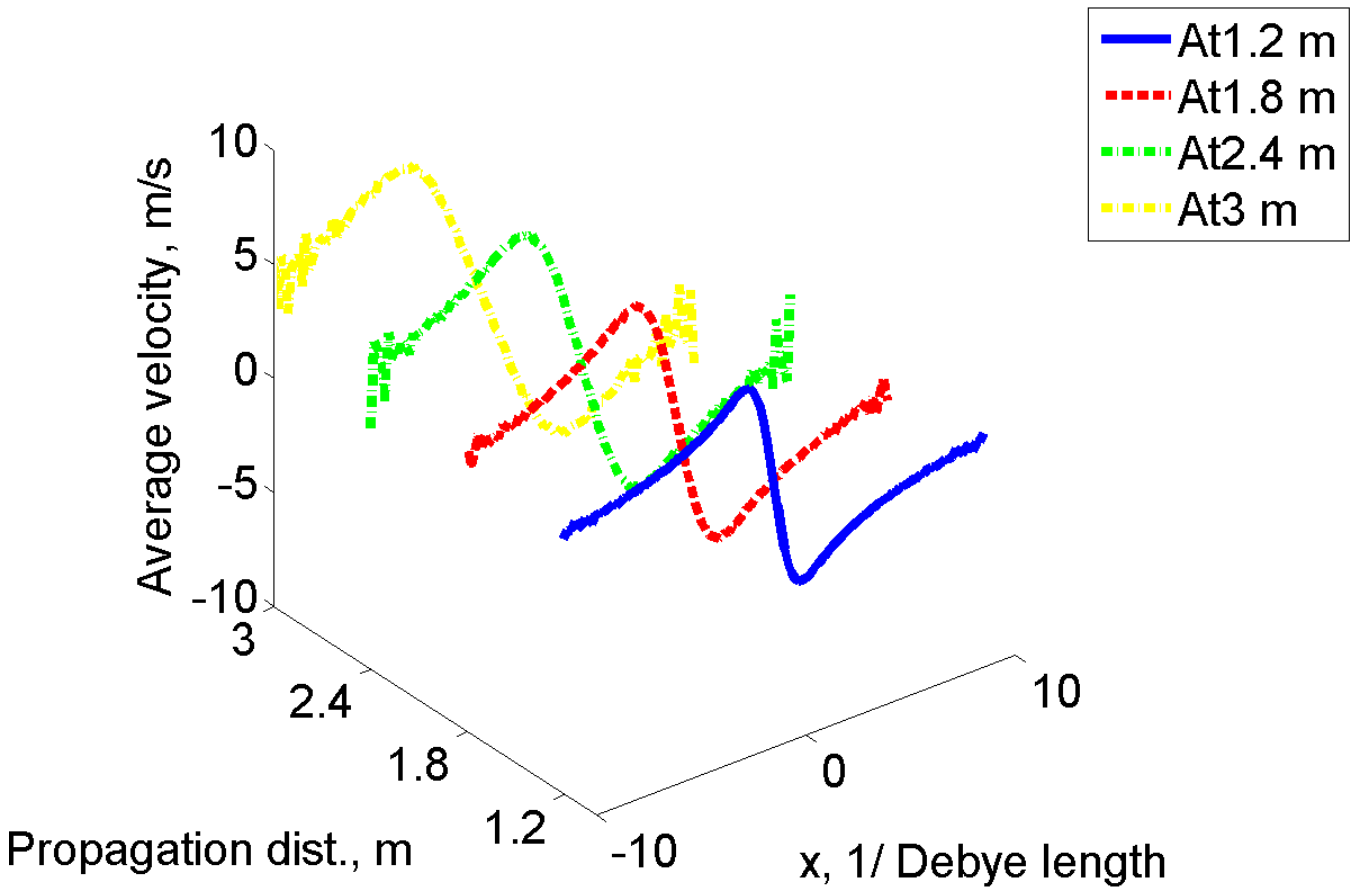
nx, ion at center



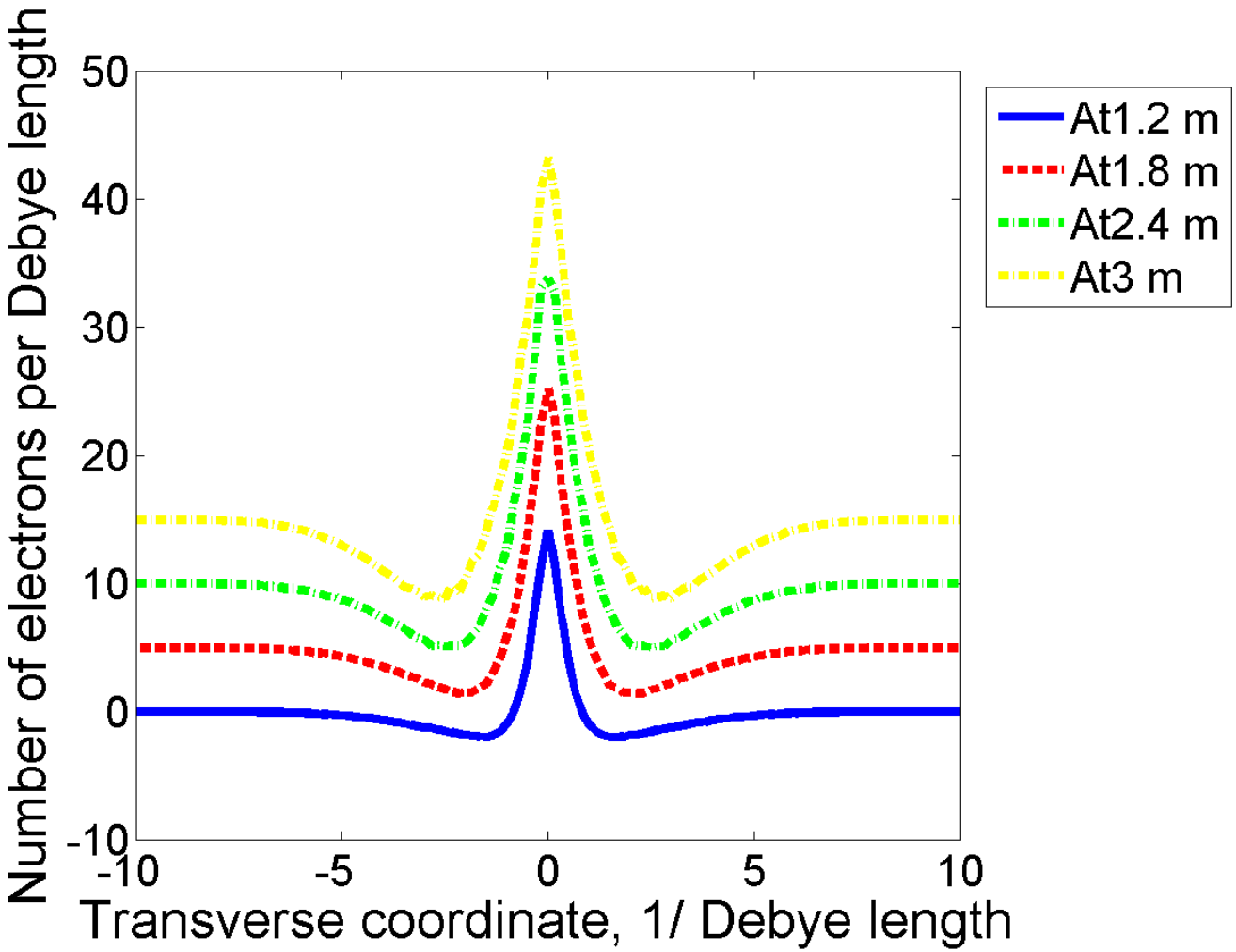
vx, ion at center



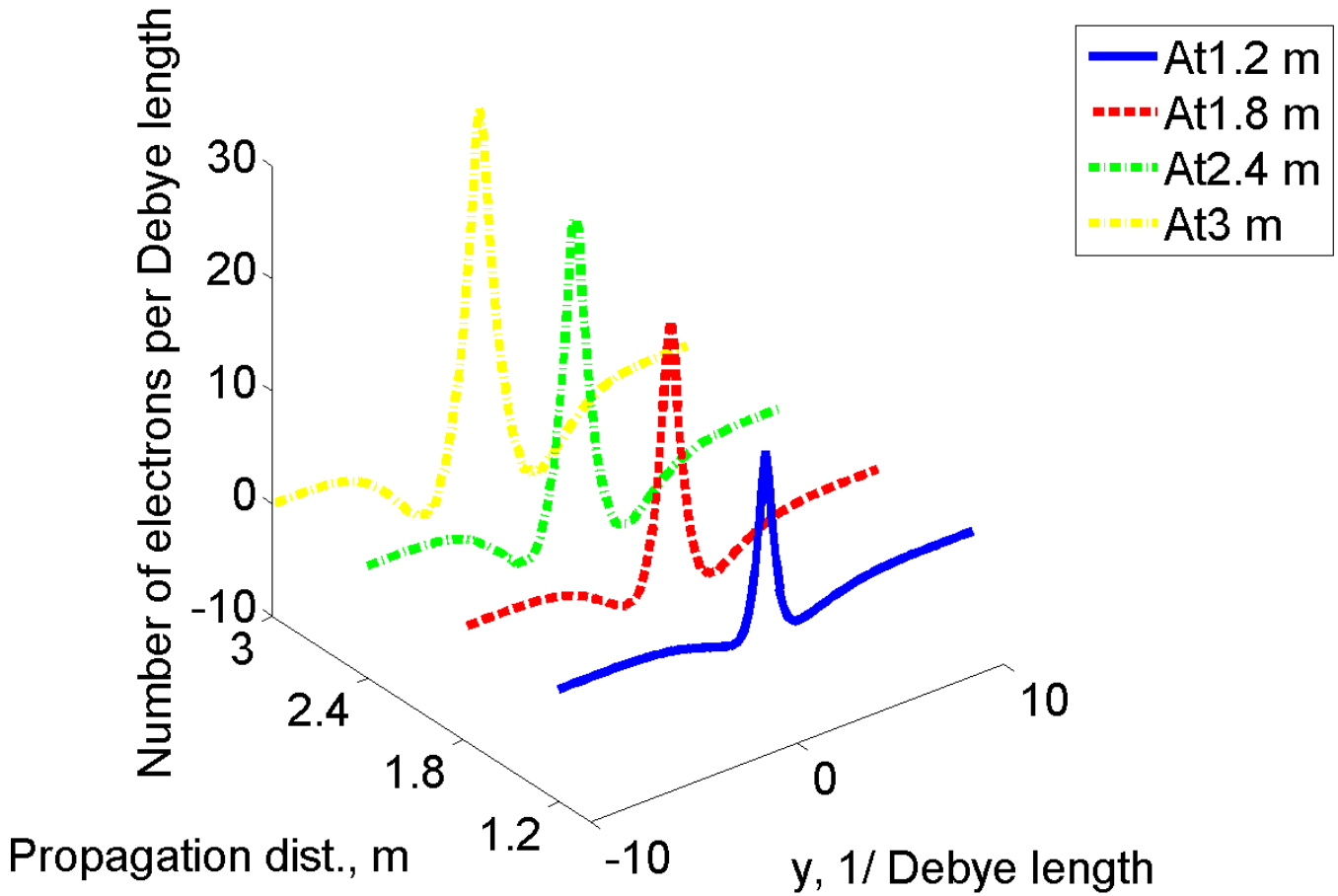
vx, ion at center



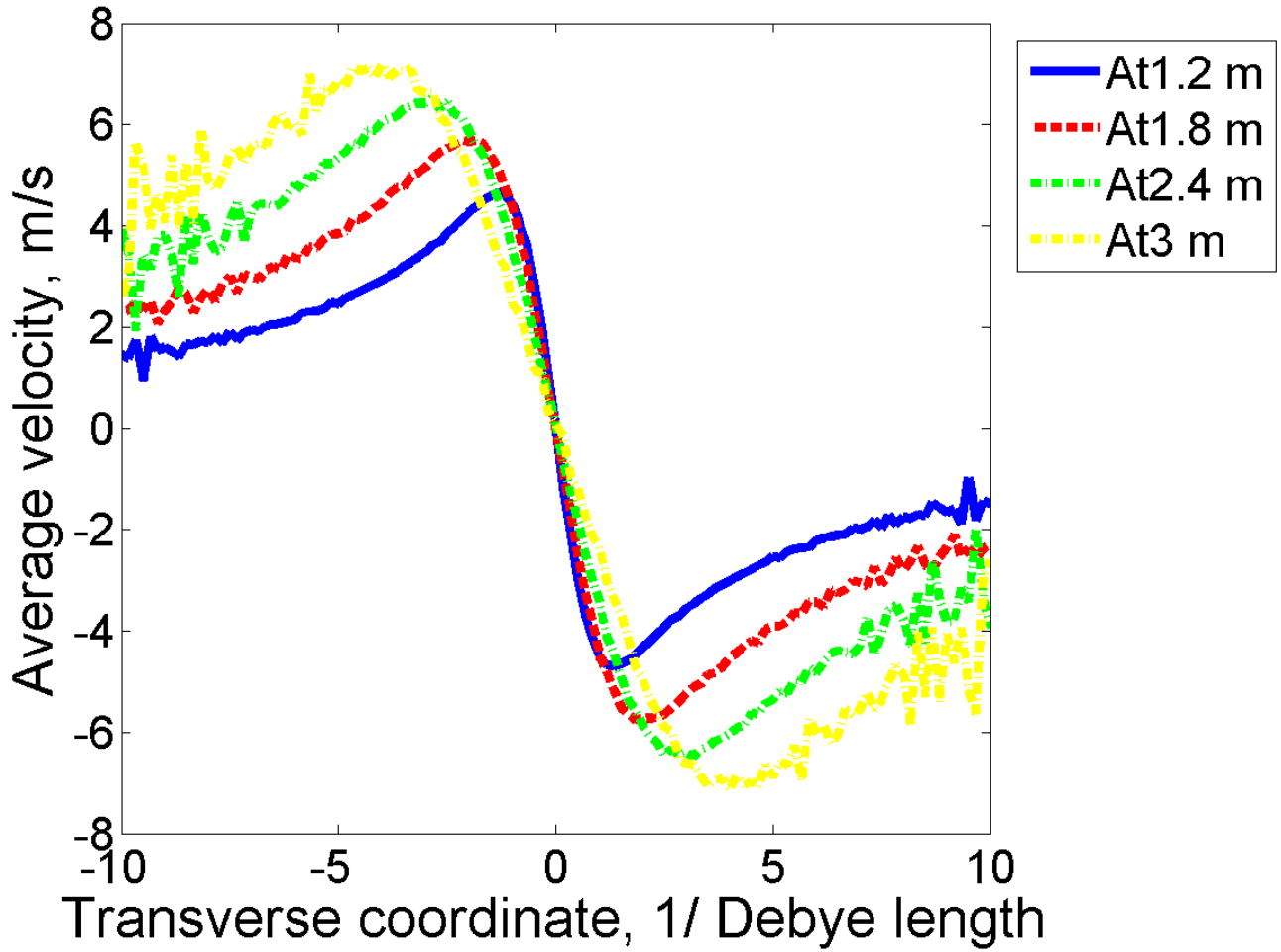
ny, ion at center



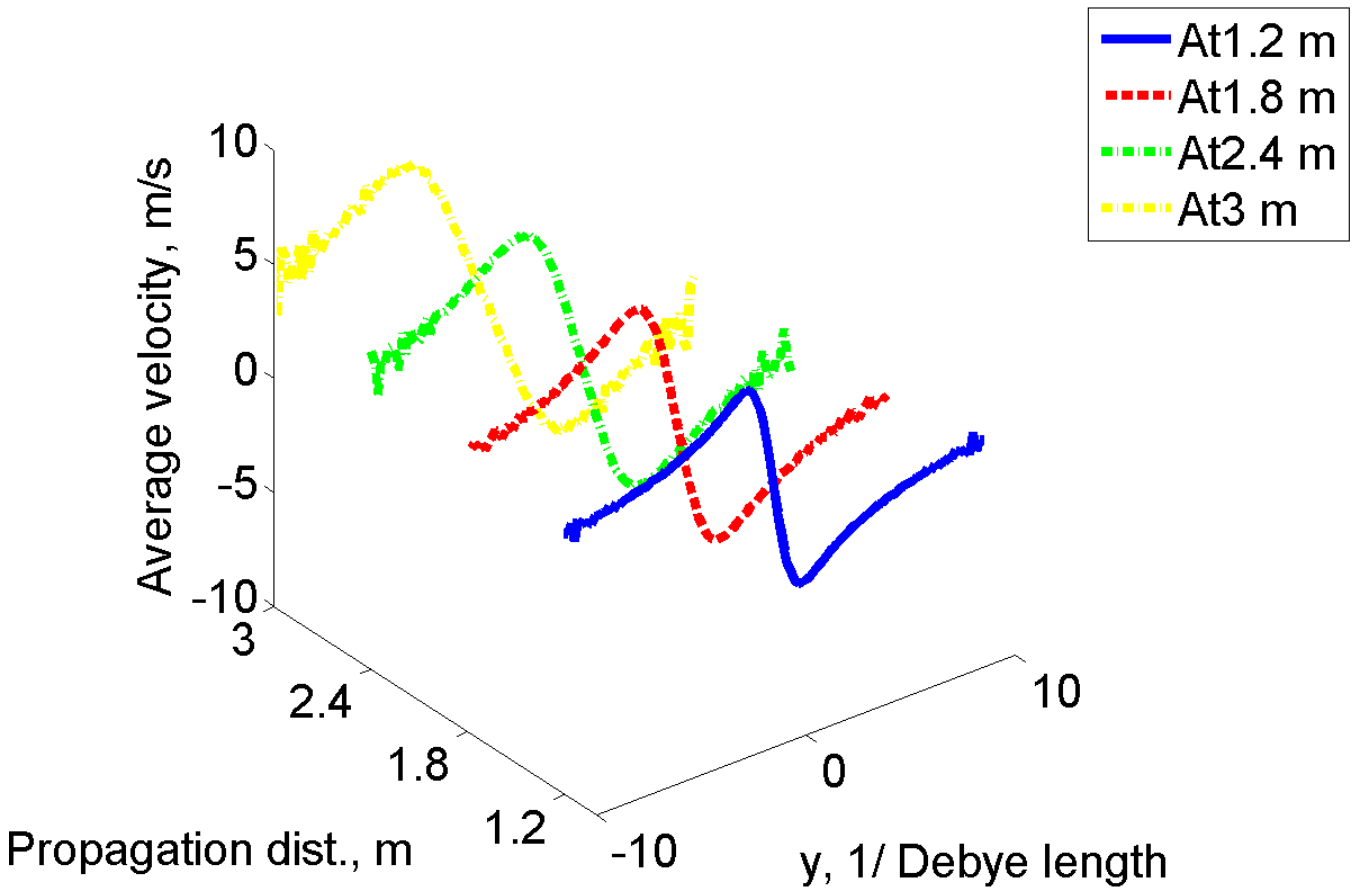
ny, ion at center



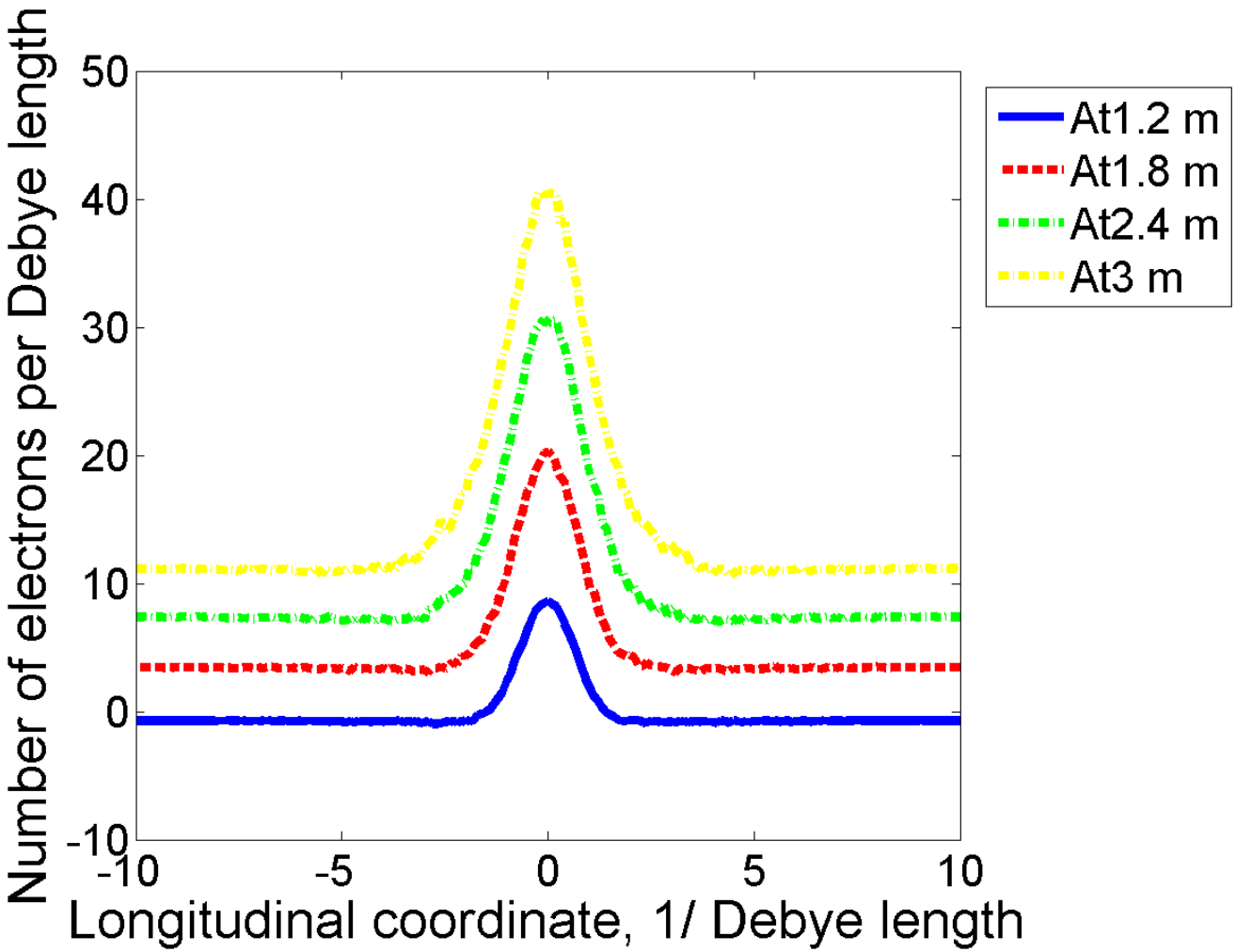
vy, ion at center



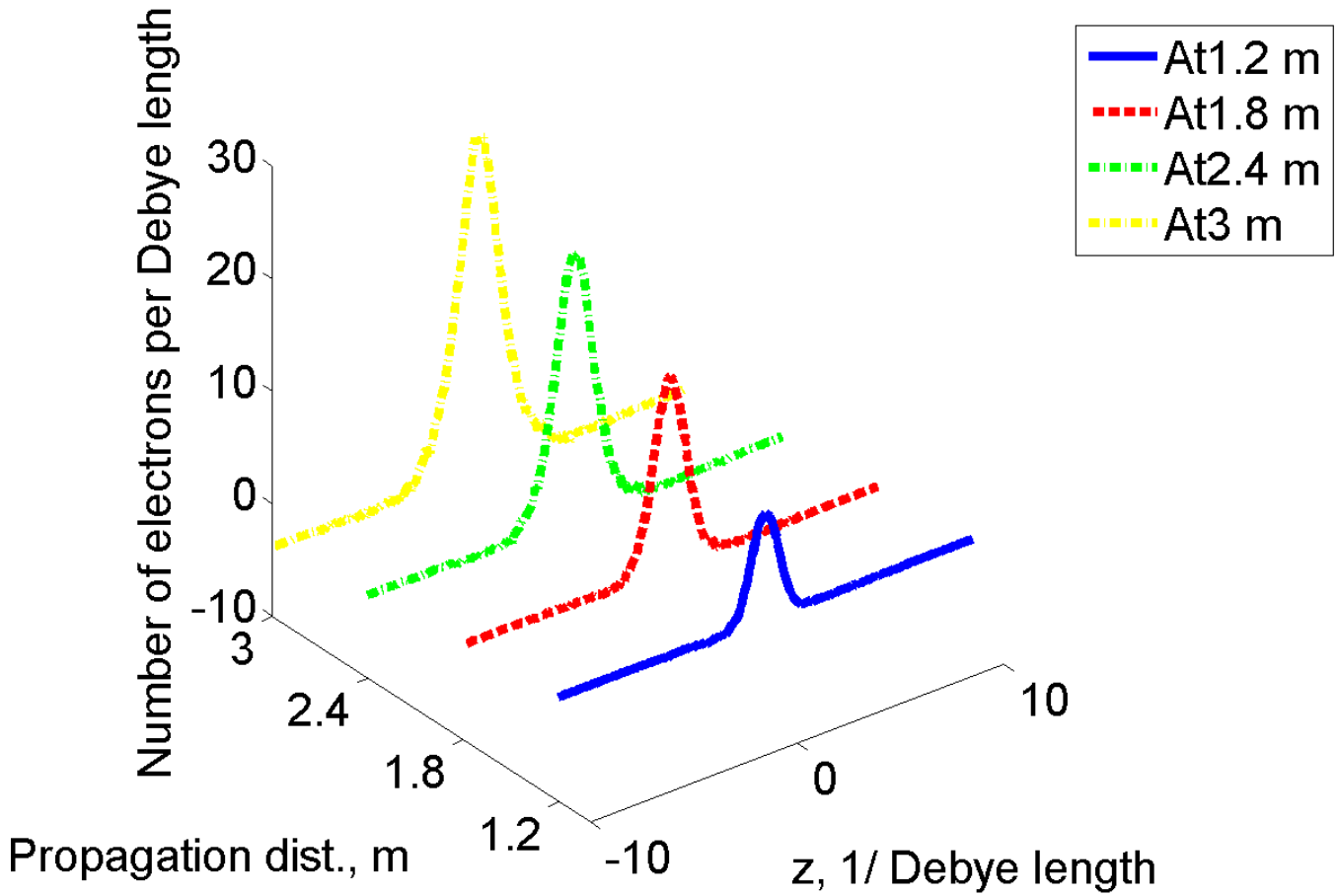
vy, ion at center



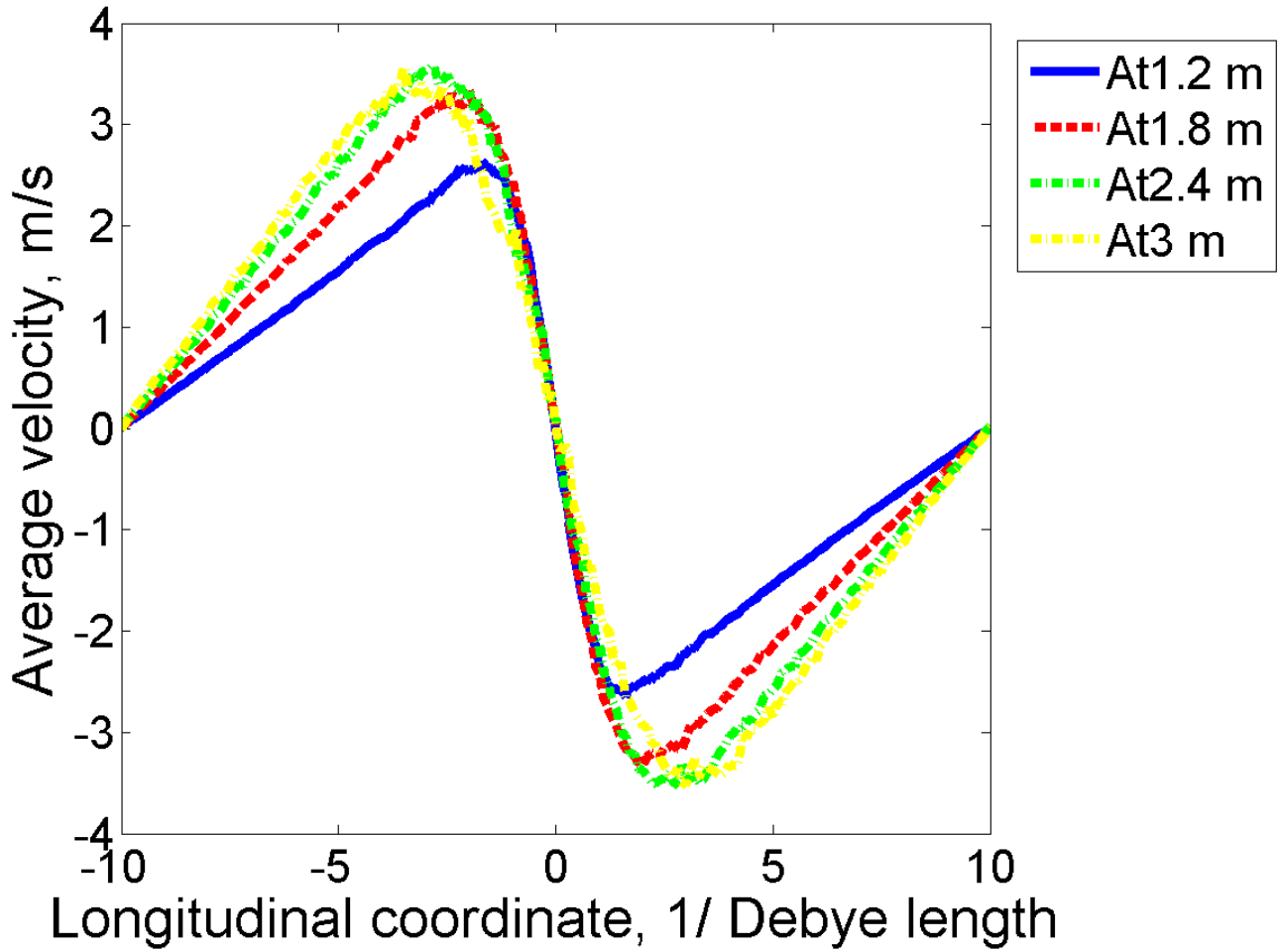
nz, ion at center



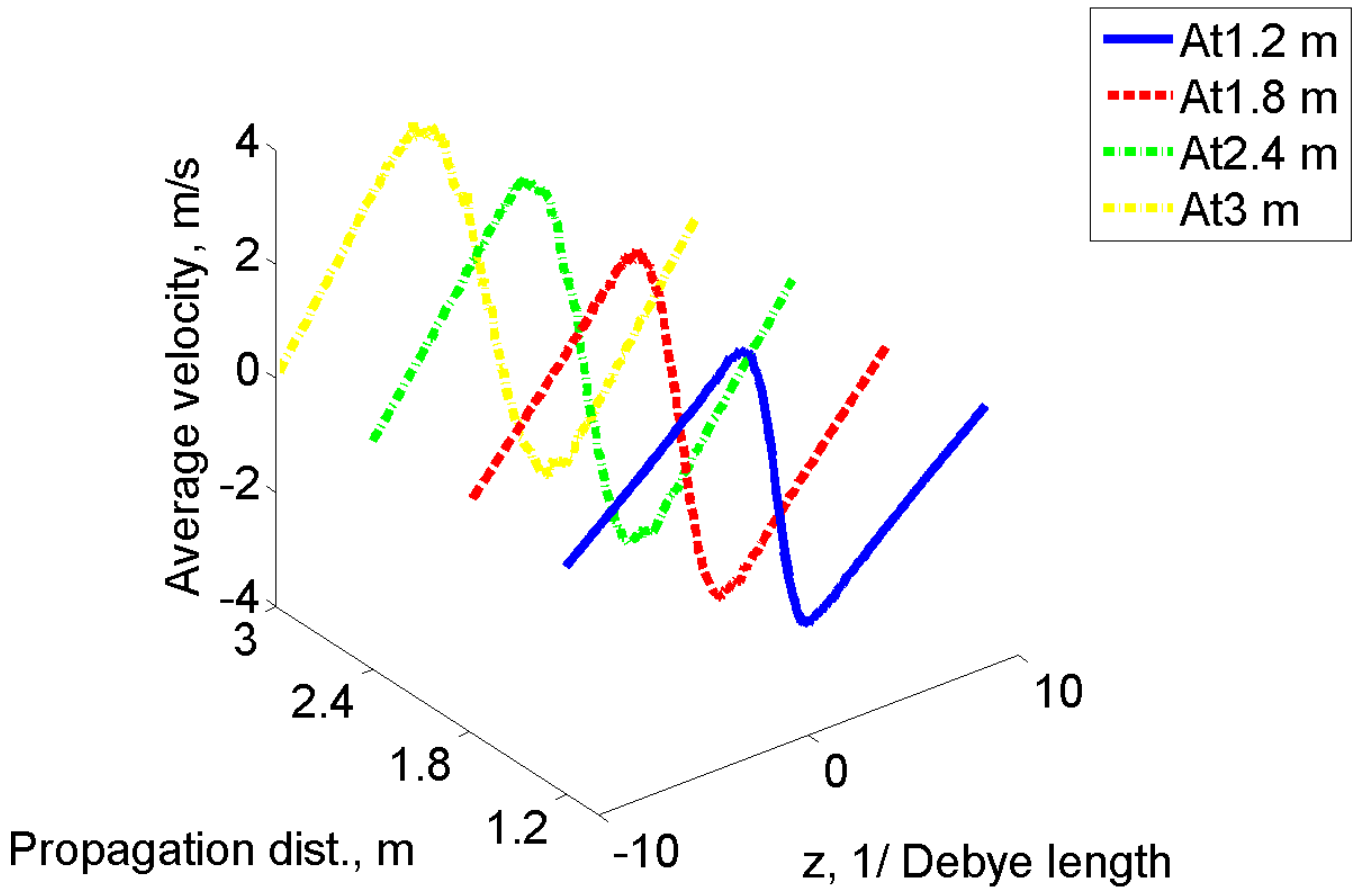
nz, ion at center



vz, ion at center

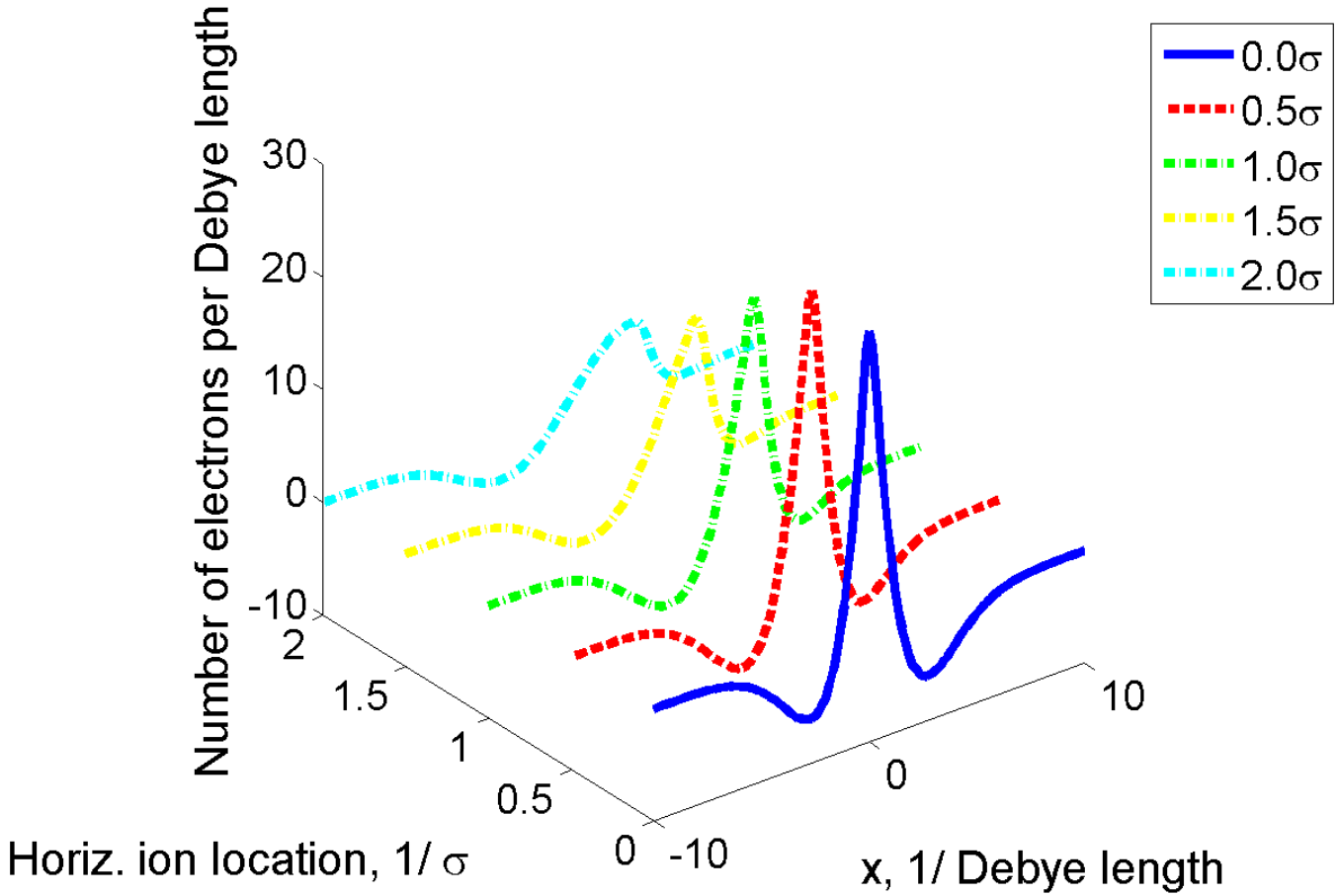


vz, ion at center

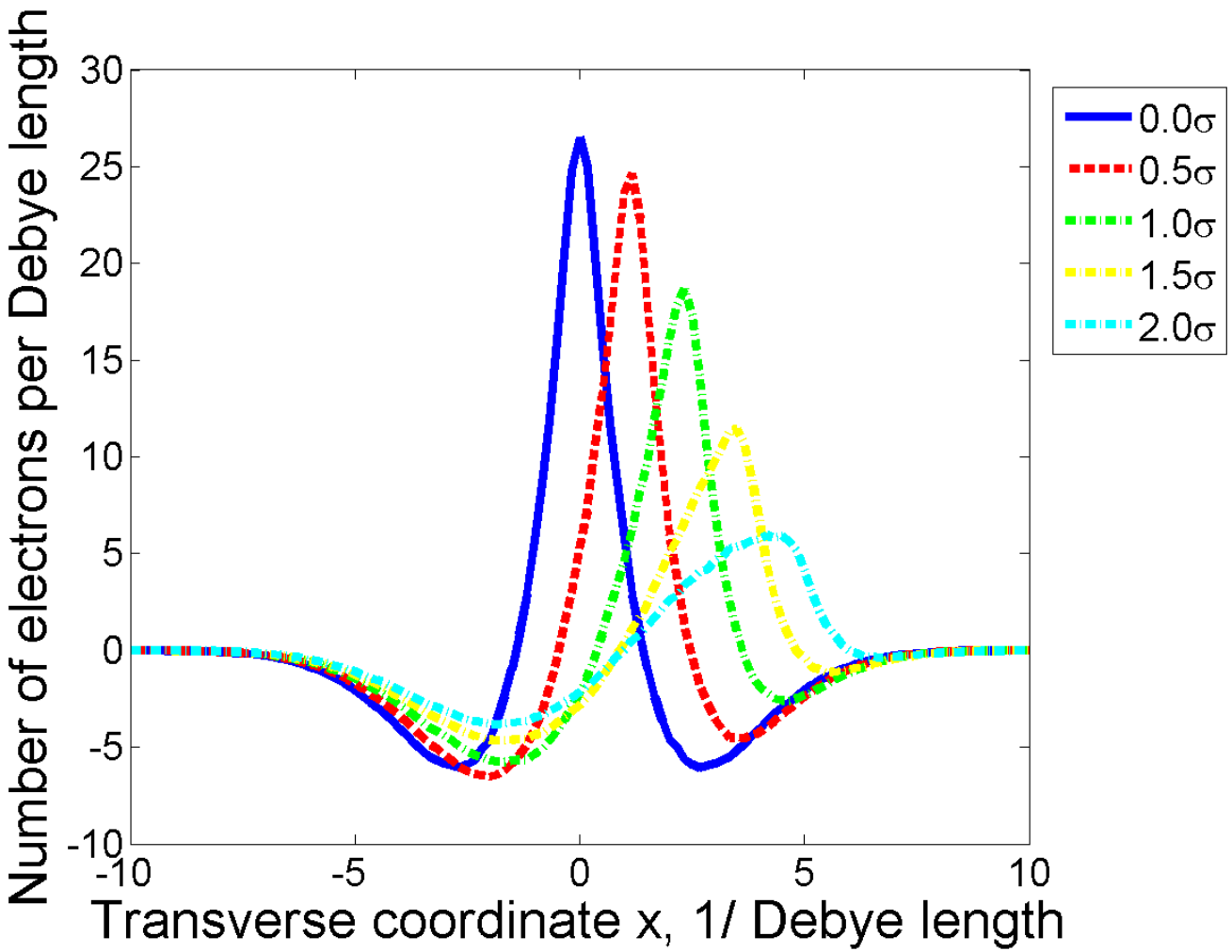


- Electron beam has Gaussian distribution in transverse
- Location of ion affects modulation process
- Typical ion locations are 0.5σ, 1.0σ, 1.5σ and 2.0σ off center of the Gaussian electron beam, where σ is RMS transverse size of electron beam

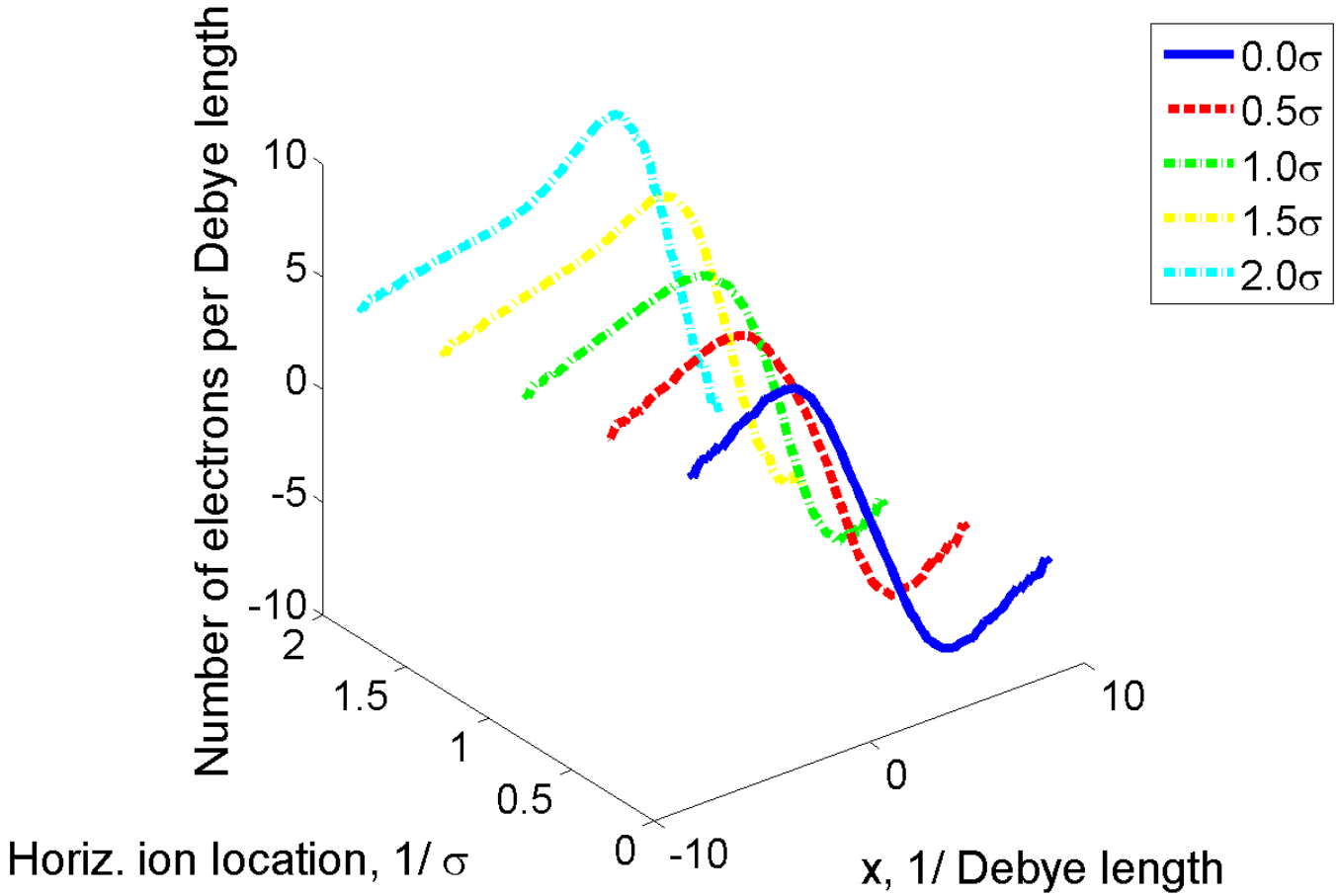
nx, various ion locations



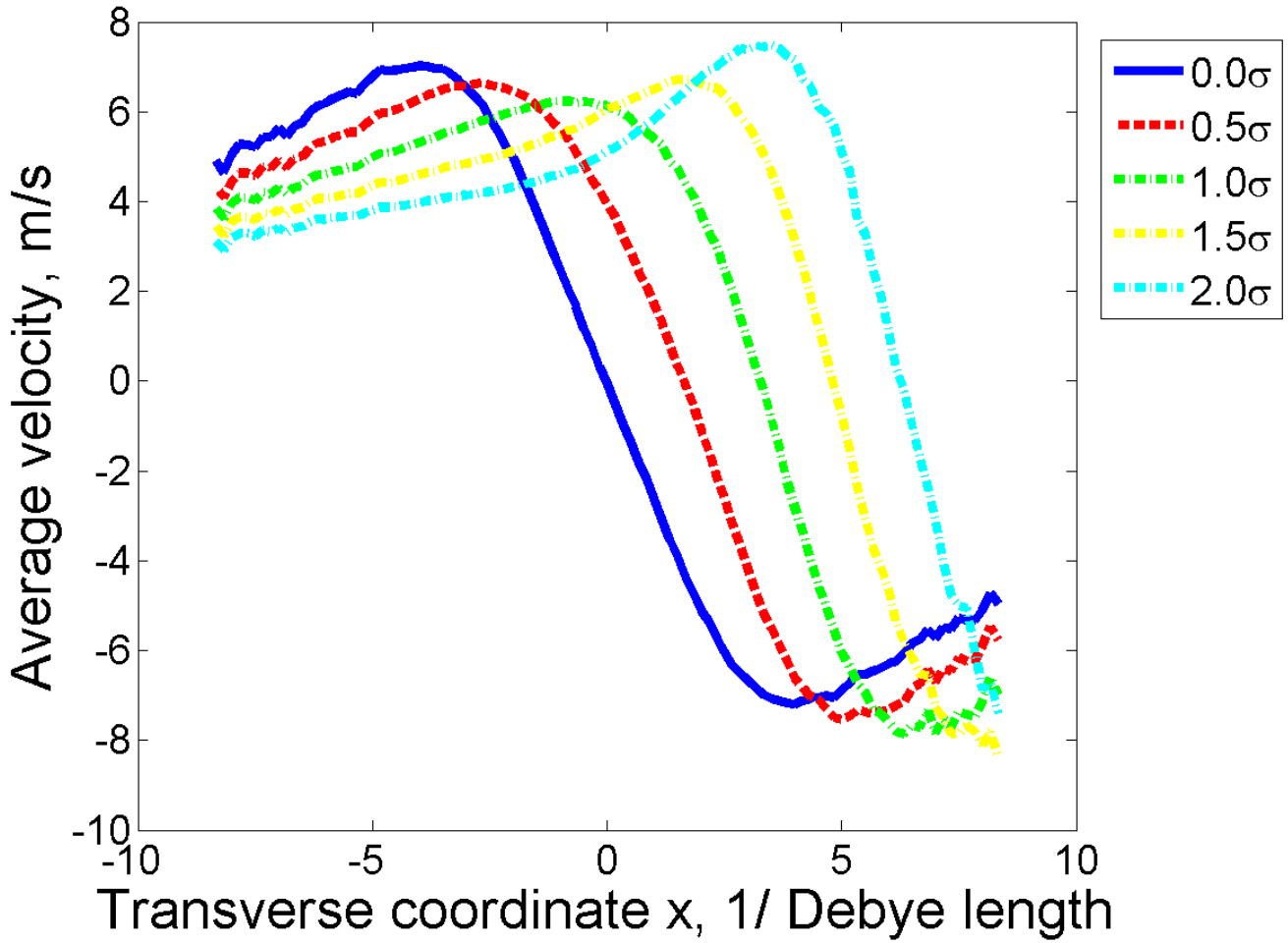
nx, various ion locations



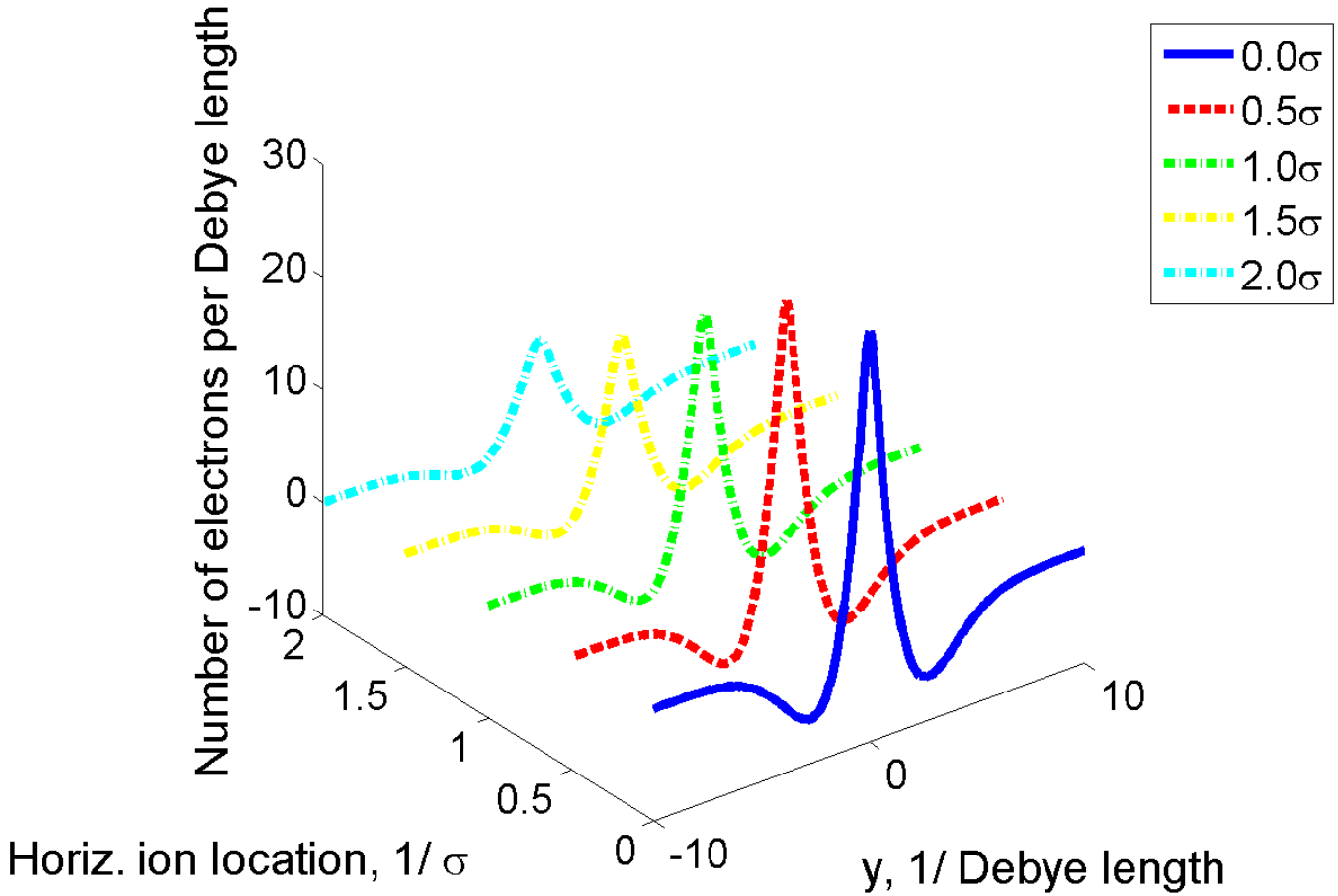
vx, various ion locations



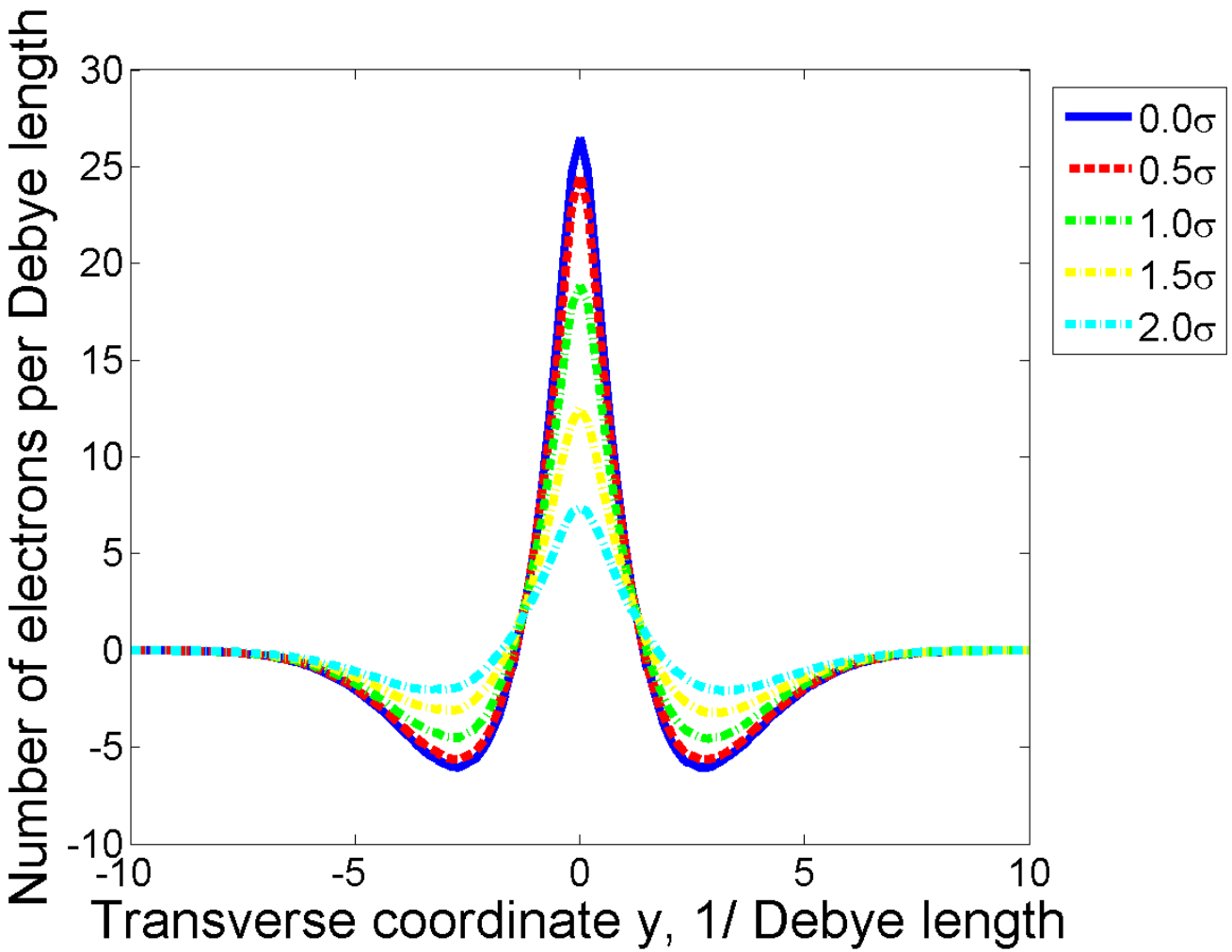
vx, various ion locations



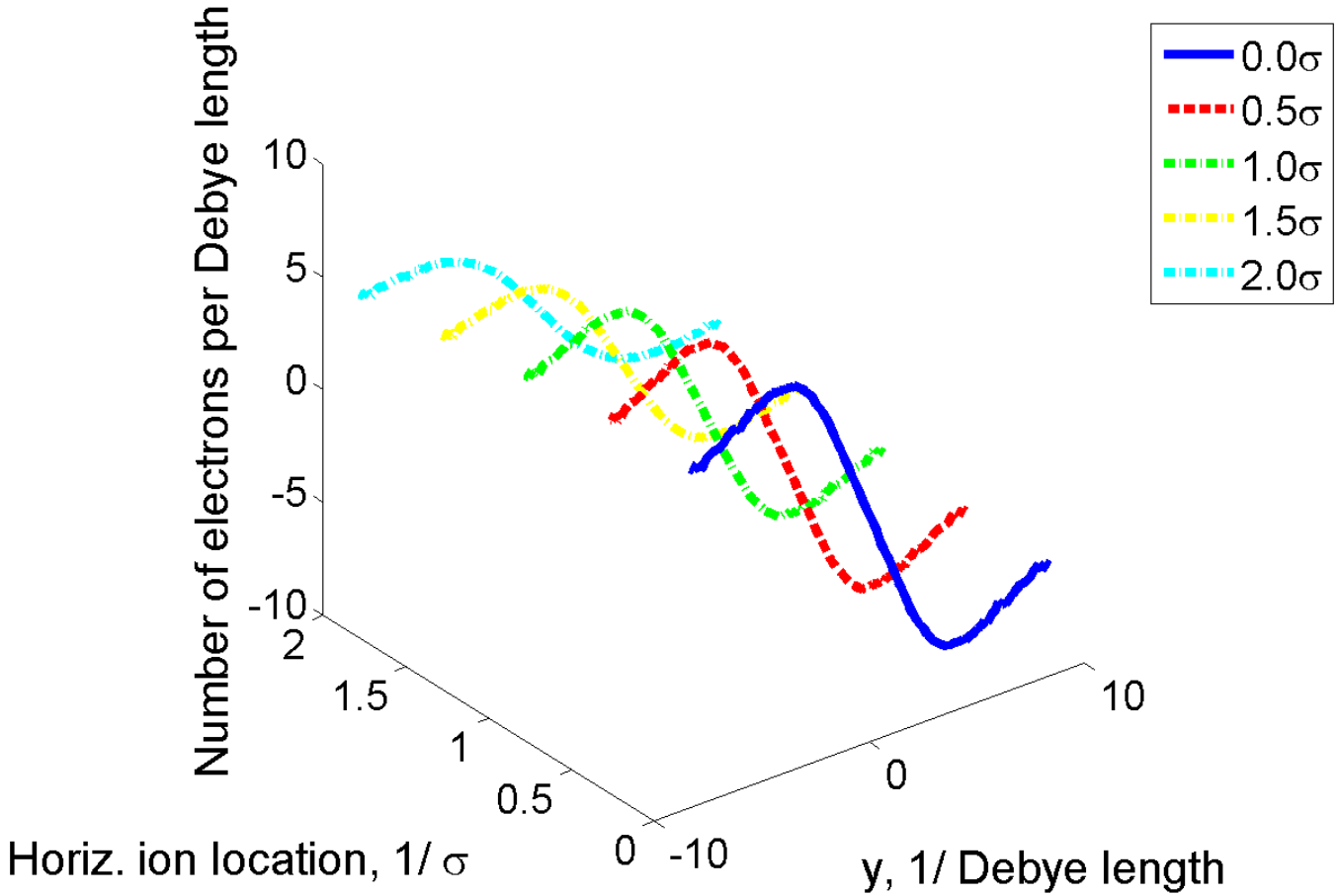
ny, various ion locations



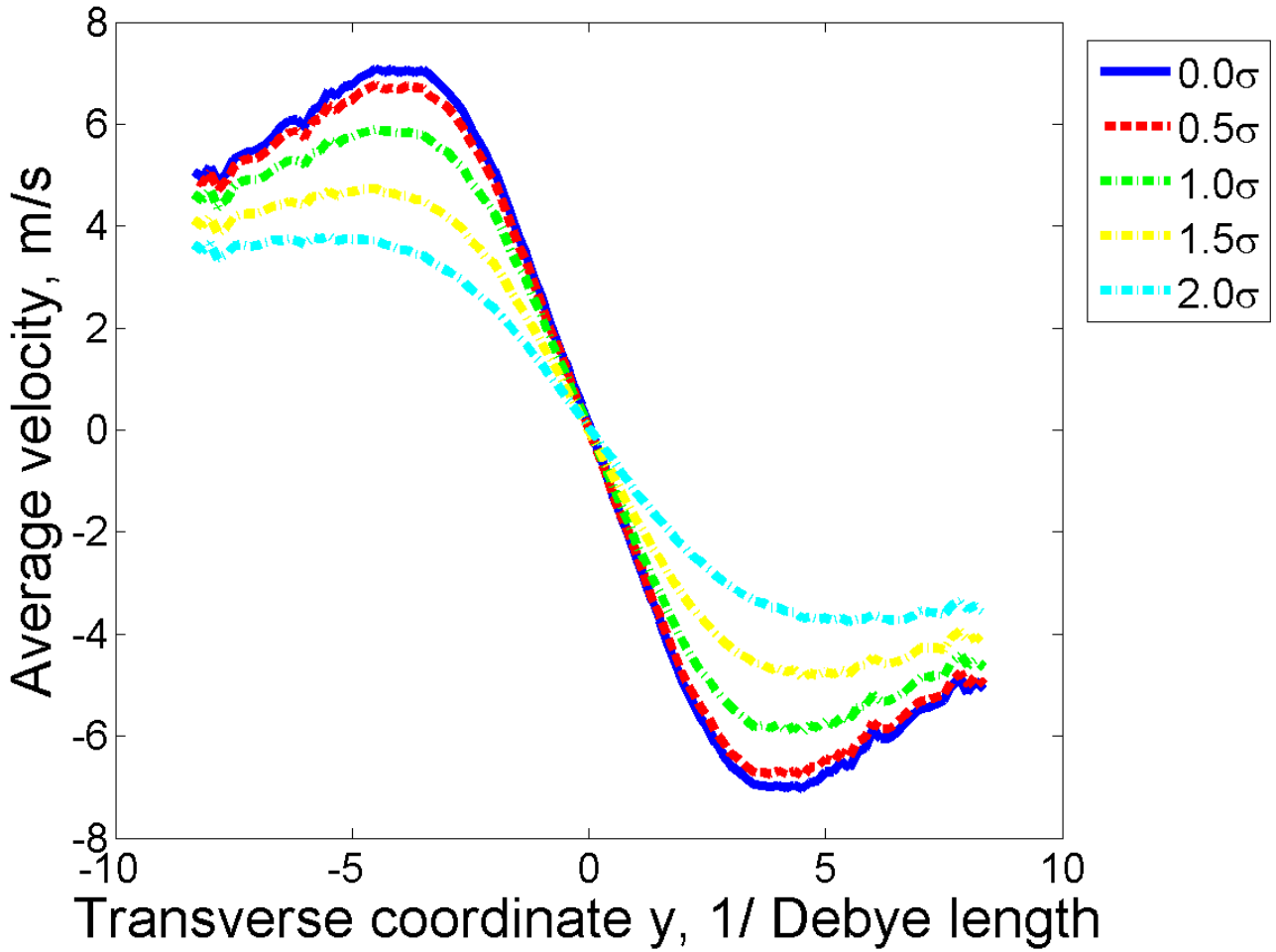
ny, various ion locations



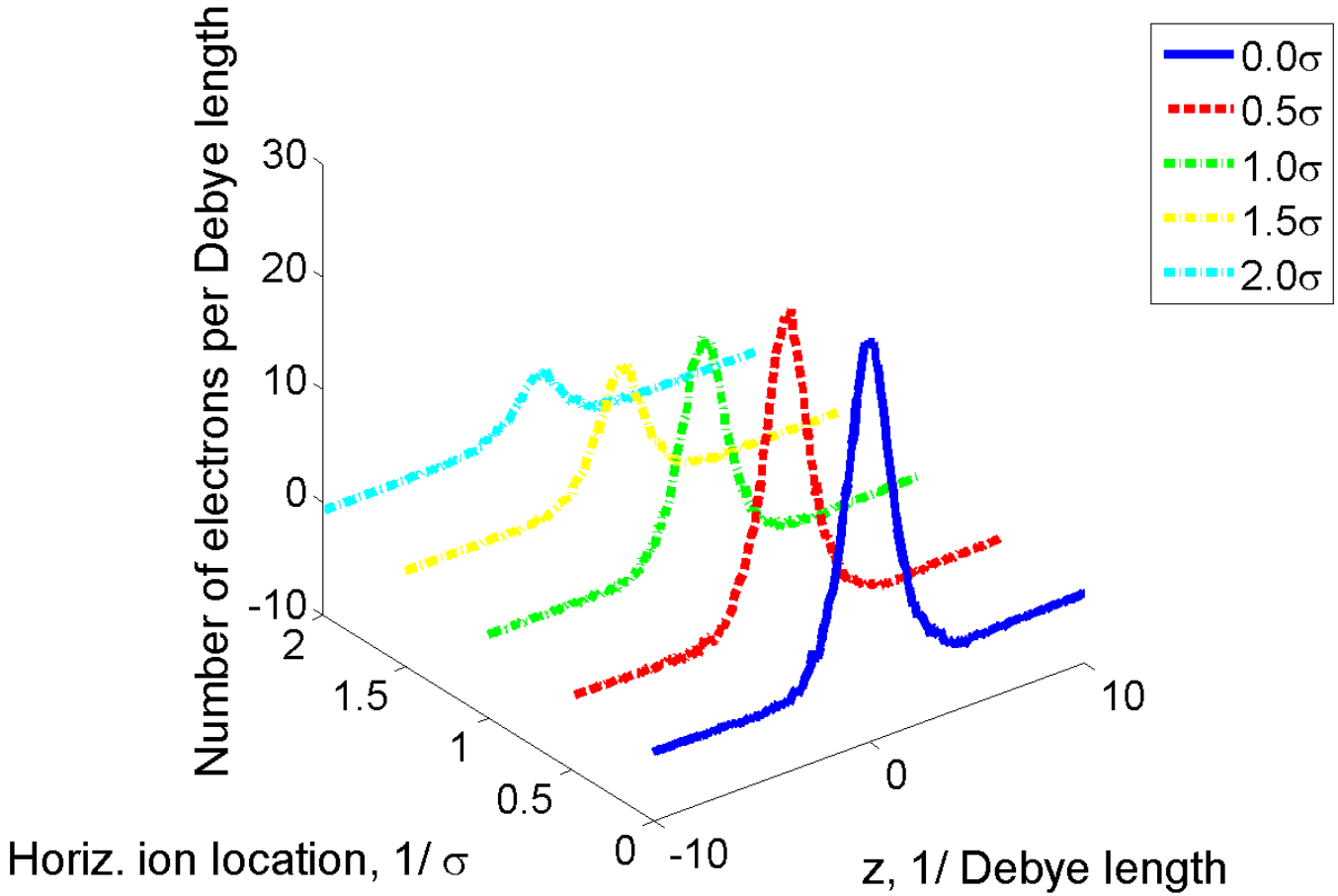
vy, various ion locations



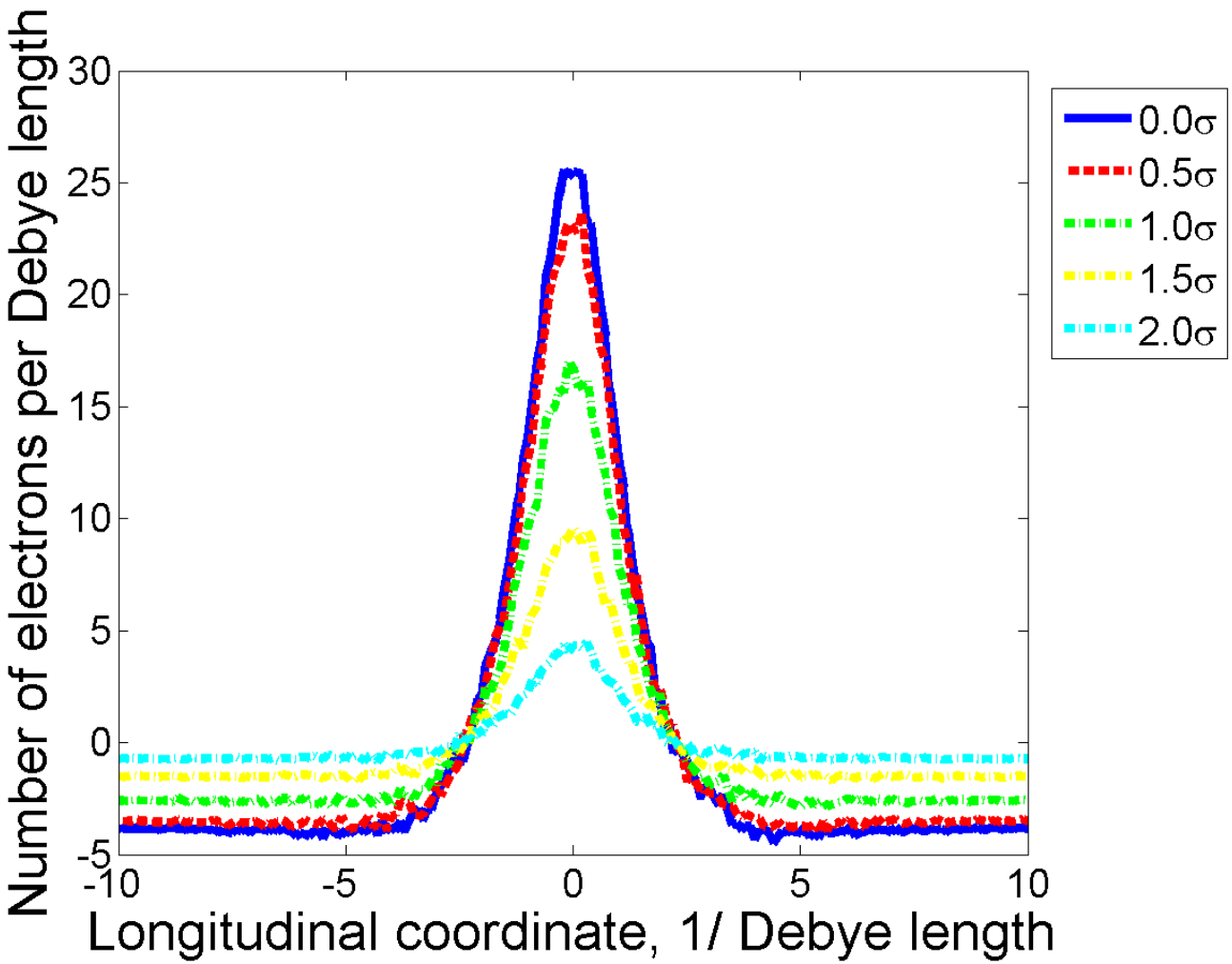
vy, various ion locations



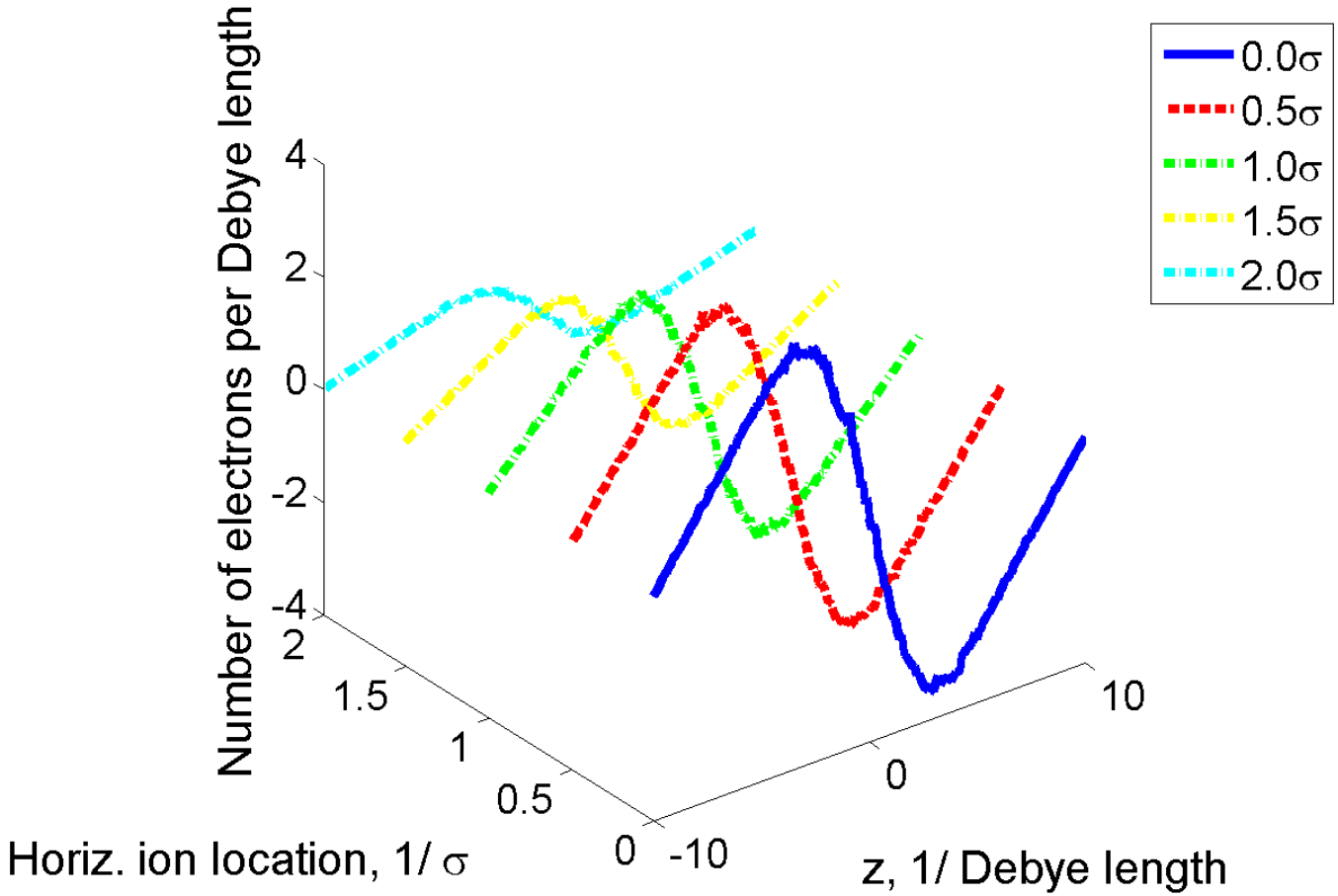
nz, various ion locations



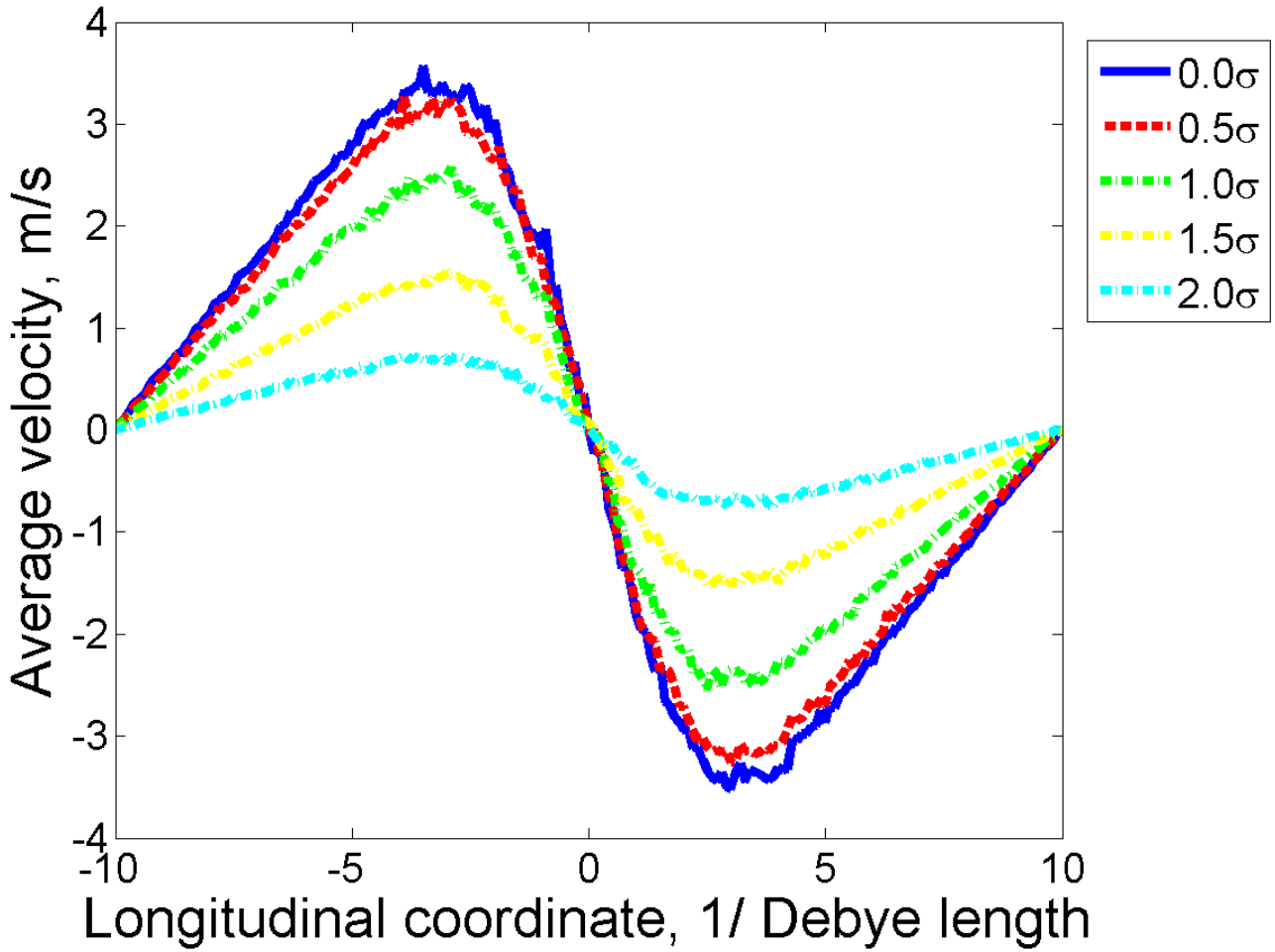
nz, various ion locations



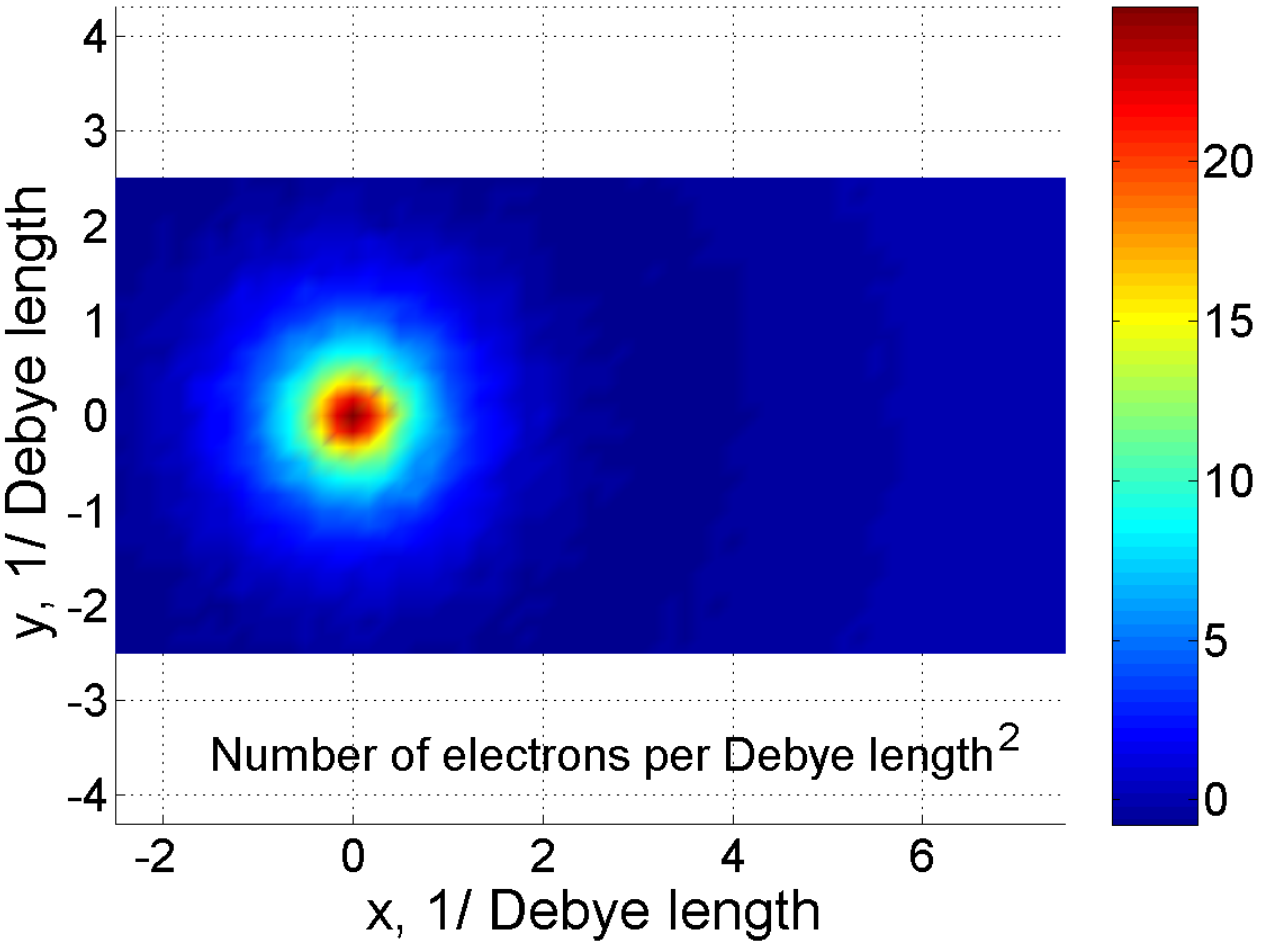
vz, various ion locations



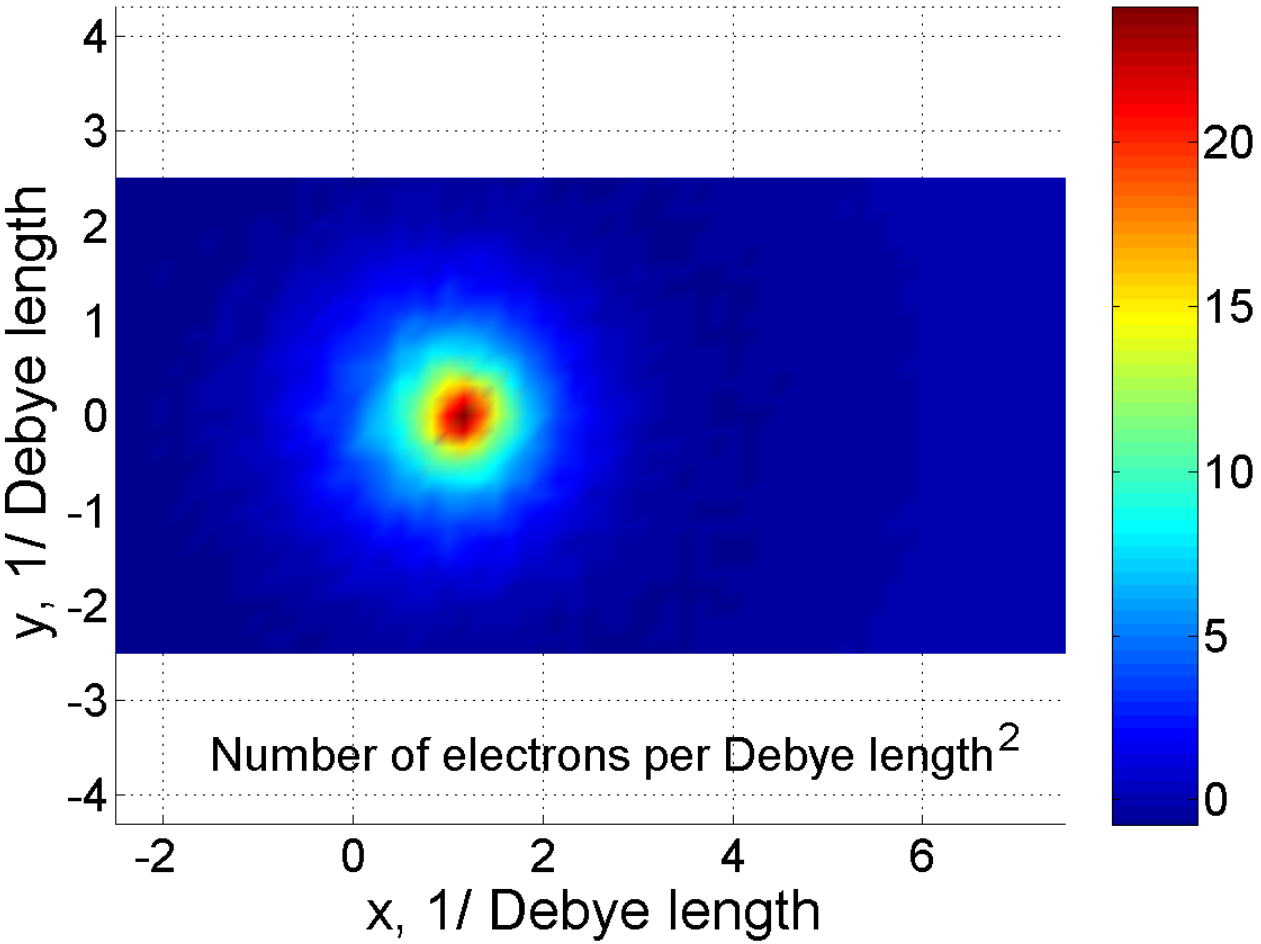
vz, various ion locations



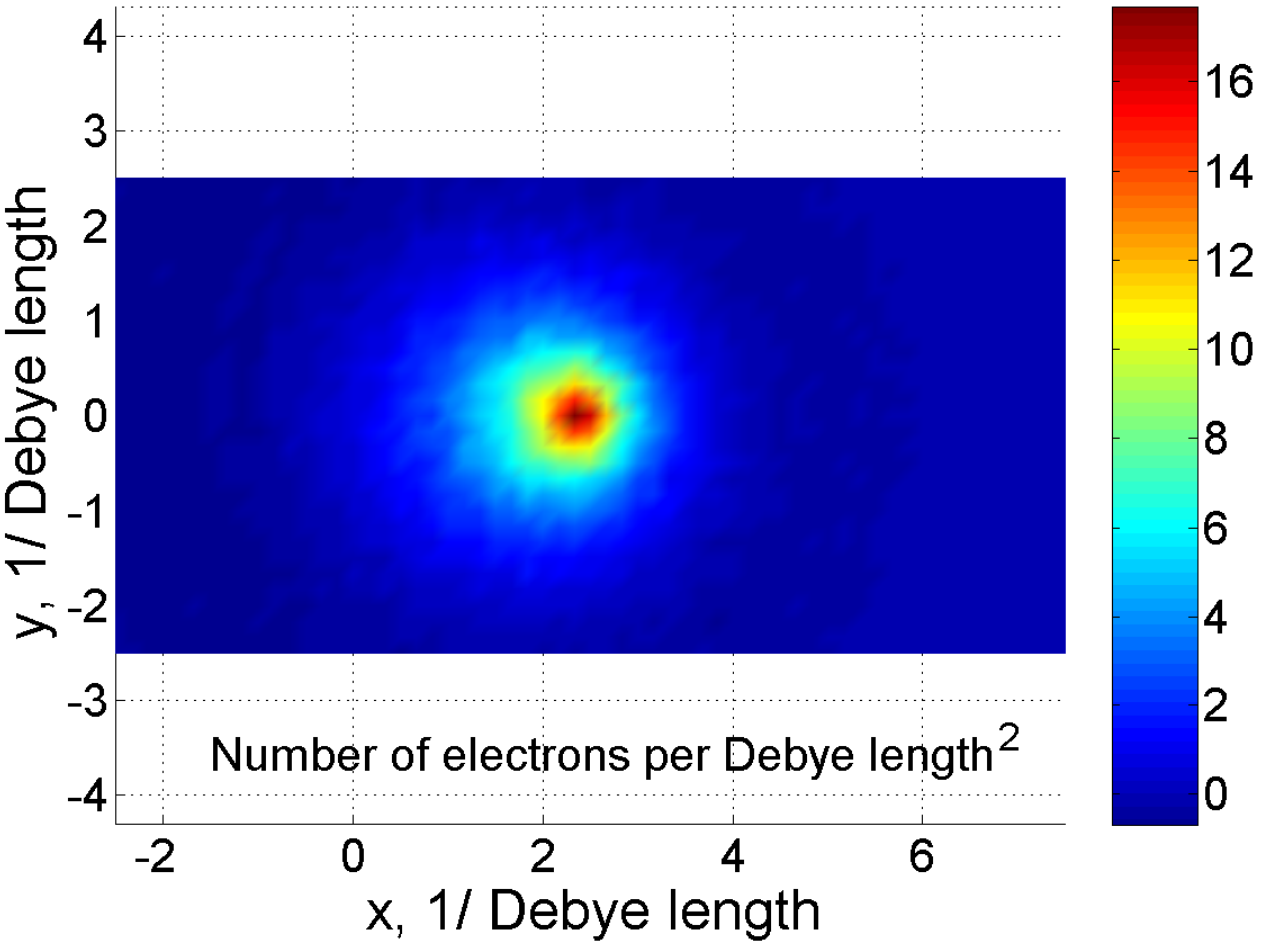
nxy, ion at center



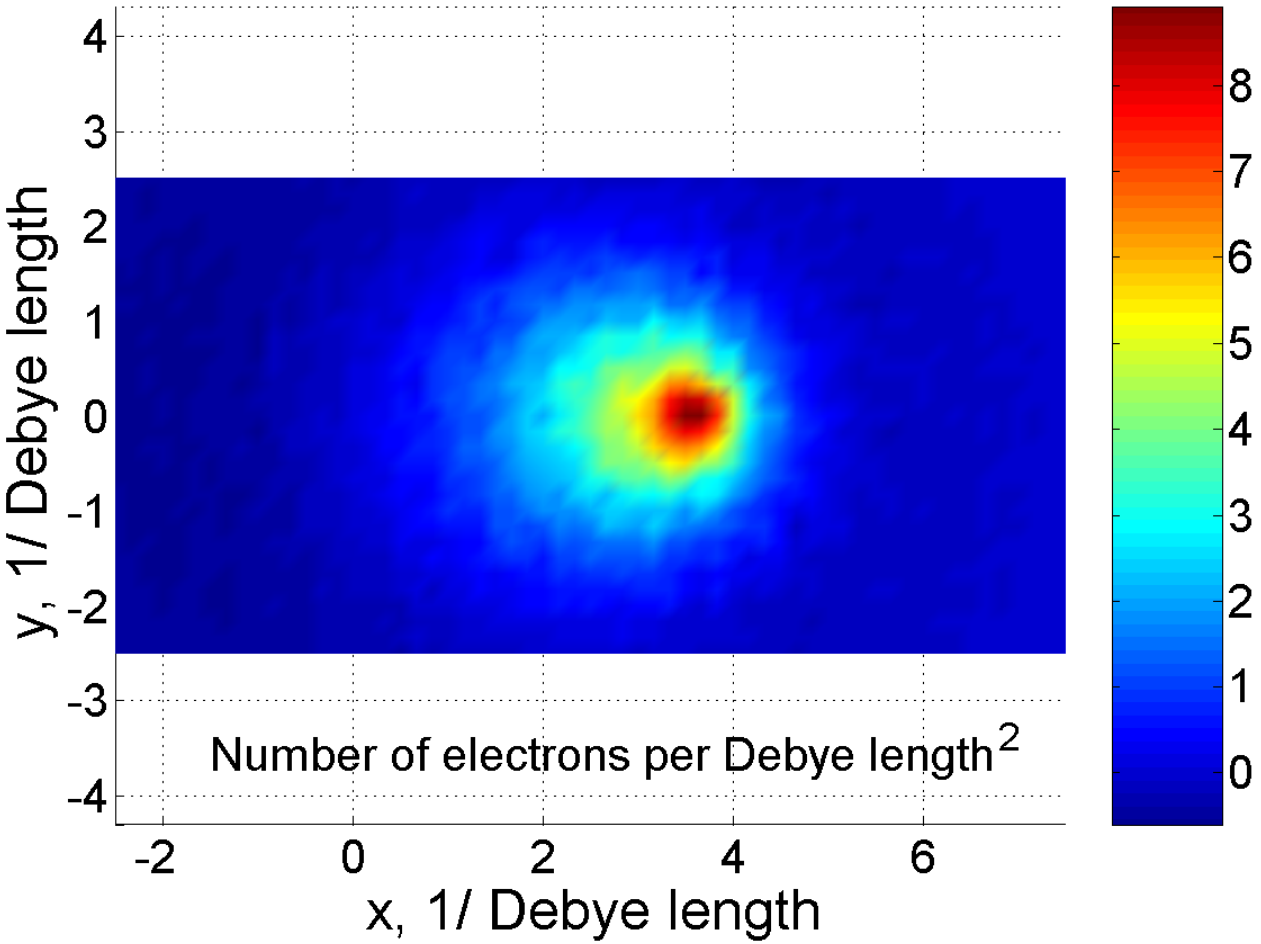
nxy, ion 0.5σ off center



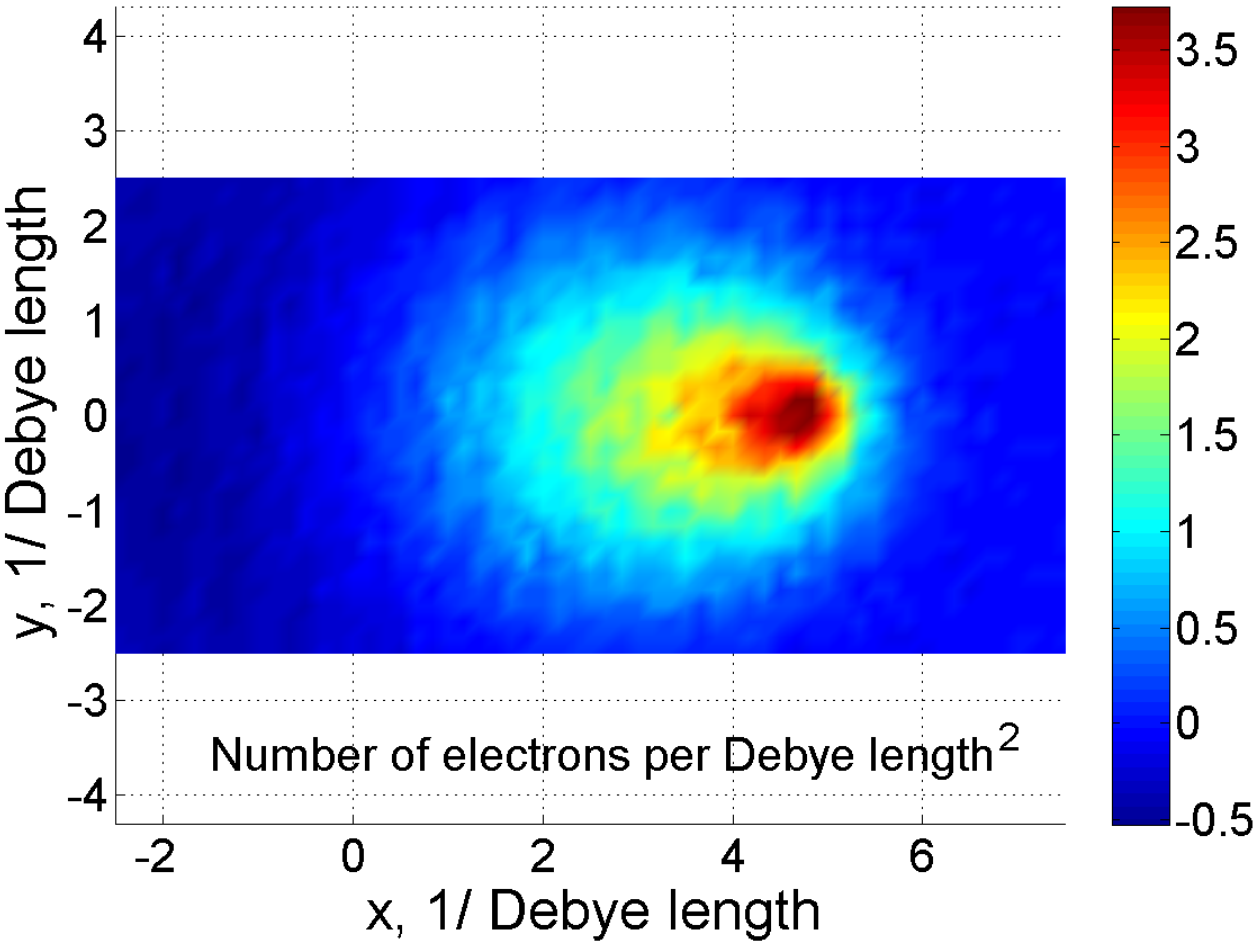
nxy, ion 1.0σ off center



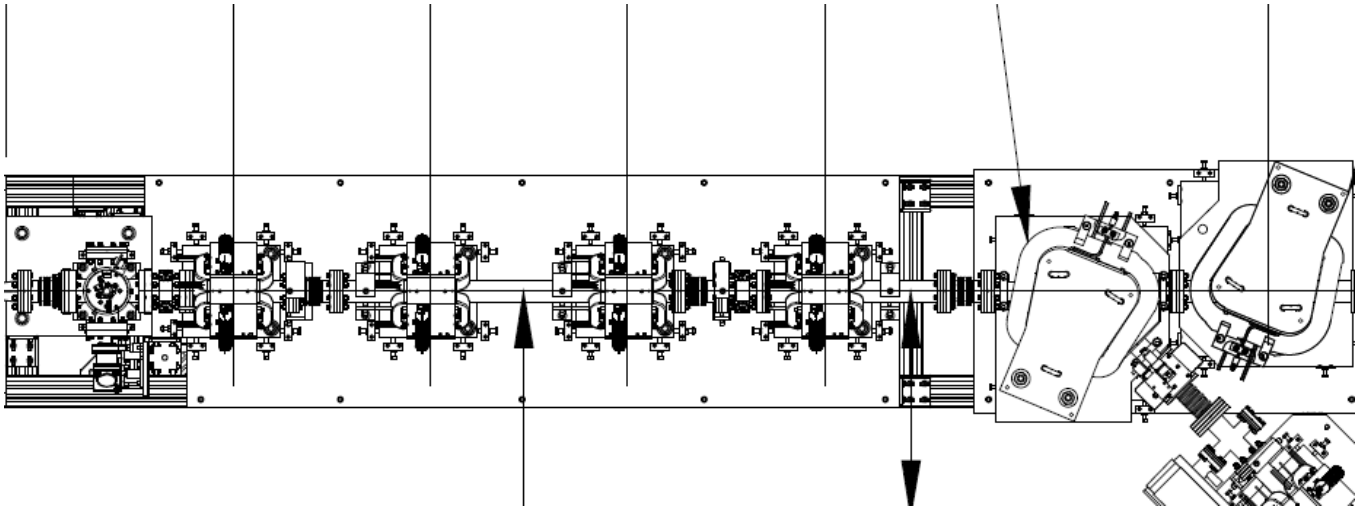
nxy, ion 1.5σ off center



nxy, ion 2.0σ off center



Layout



Hard edge

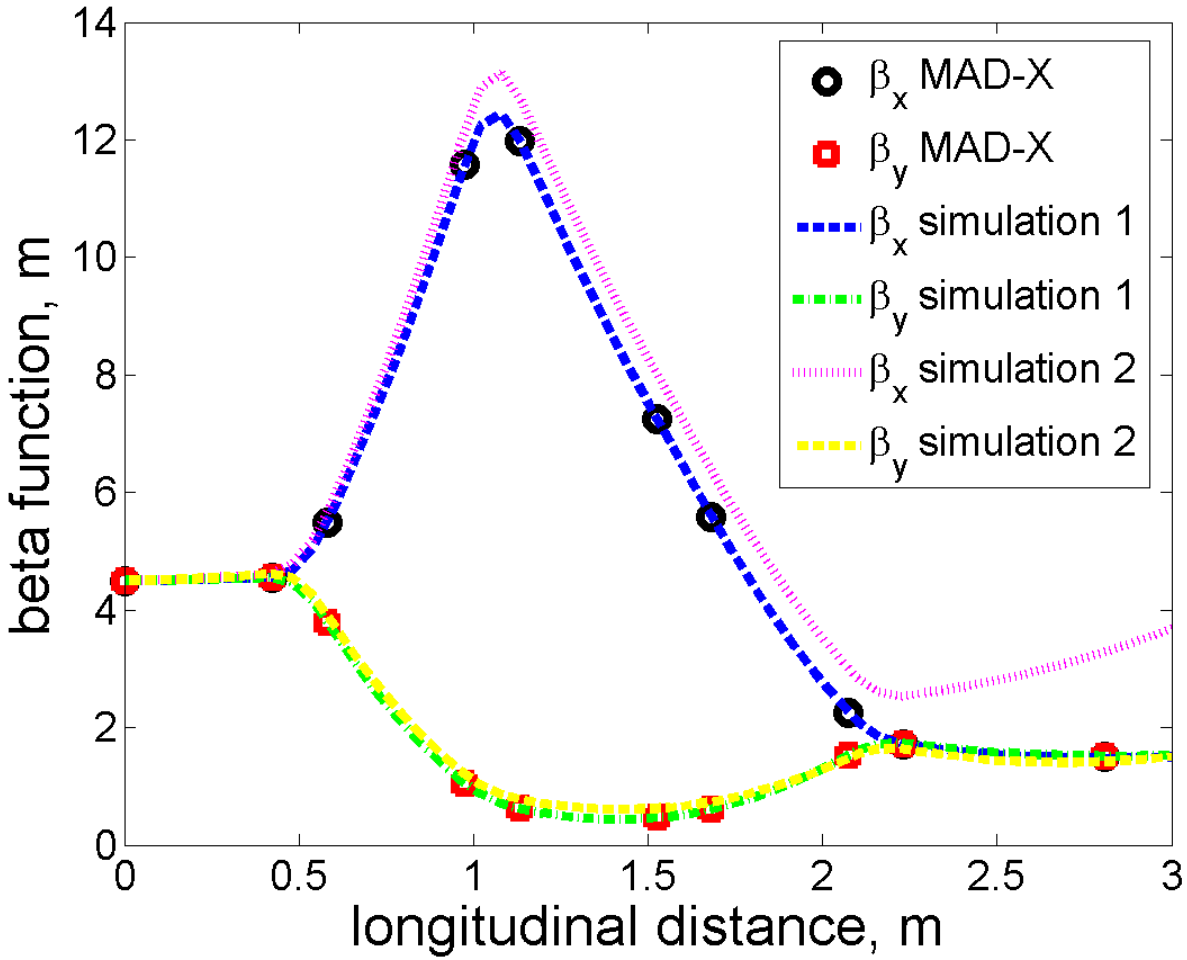
$$B_y = K \cdot x$$
$$B_x = K \cdot y$$

Soft edge

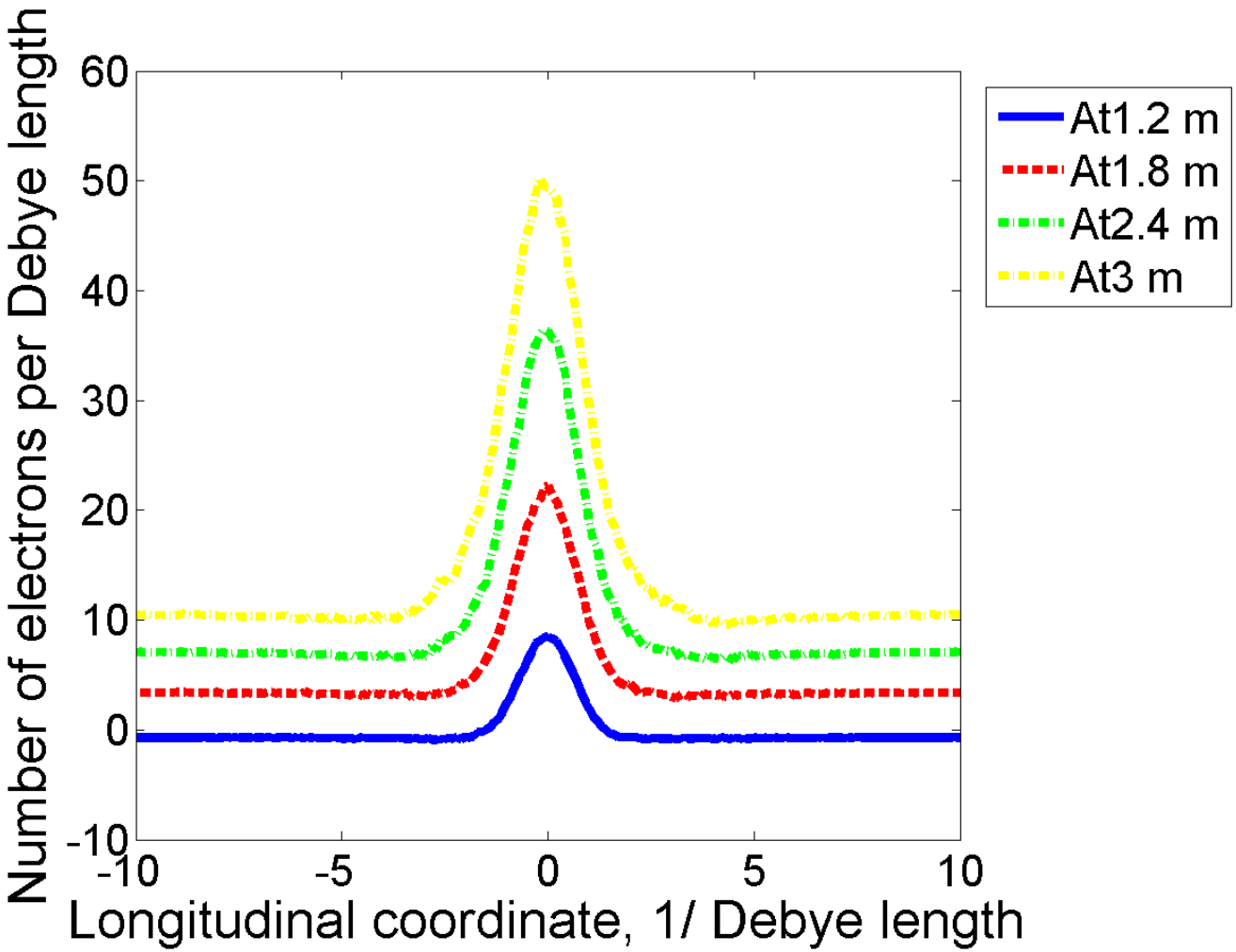
$$\begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix} = -\frac{\mathrm{G}}{\mathbf{b1}} \cdot \begin{pmatrix} B_{\mathrm{fringe,x}}\left(\mathbf{b1}\,x, \mathbf{b1}\,y, \mathbf{b1}\,(z-\frac{1}{2}L)\right) + B_{\mathrm{fringe,x}}\left(\mathbf{b1}\,x, \mathbf{b1}\,y, \mathbf{b1}\,(-z-\frac{1}{2}L)\right) \\ B_{\mathrm{fringe,y}}\left(\mathbf{b1}\,x, \mathbf{b1}\,y, \mathbf{b1}\,(z-\frac{1}{2}L)\right) + B_{\mathrm{fringe,y}}\left(\mathbf{b1}\,x, \mathbf{b1}\,y, \mathbf{b1}\,(-z-\frac{1}{2}L)\right) \\ B_{\mathrm{fringe,z}}\left(\mathbf{b1}\,x, \mathbf{b1}\,y, \mathbf{b1}\,(z-\frac{1}{2}L)\right) - B_{\mathrm{fringe,z}}\left(\mathbf{b1}\,x, \mathbf{b1}\,y, \mathbf{b1}\,(-z-\frac{1}{2}L)\right) \end{pmatrix}$$

$$\mathbf{B}_{\mathrm{fringe}}(x,y,z) = \frac{1}{4}\begin{pmatrix} -y - 2\arctan\left(-\dfrac{\sin(y)}{e^{-z}+\cos(y)}\right) + \dfrac{y\sinh(z)}{\cos(x)+\cosh(z)} \\ -x - 2\arctan\left(-\dfrac{\sin(x)}{e^{-z}+\cos(x)}\right) + \dfrac{x\sinh(z)}{\cos(y)+\cosh(z)} \\ \dfrac{y\sin(x)}{\cos(x)+\cosh(z)} + \dfrac{x\sin(y)}{\cos(y)+\cosh(z)} \end{pmatrix}$$

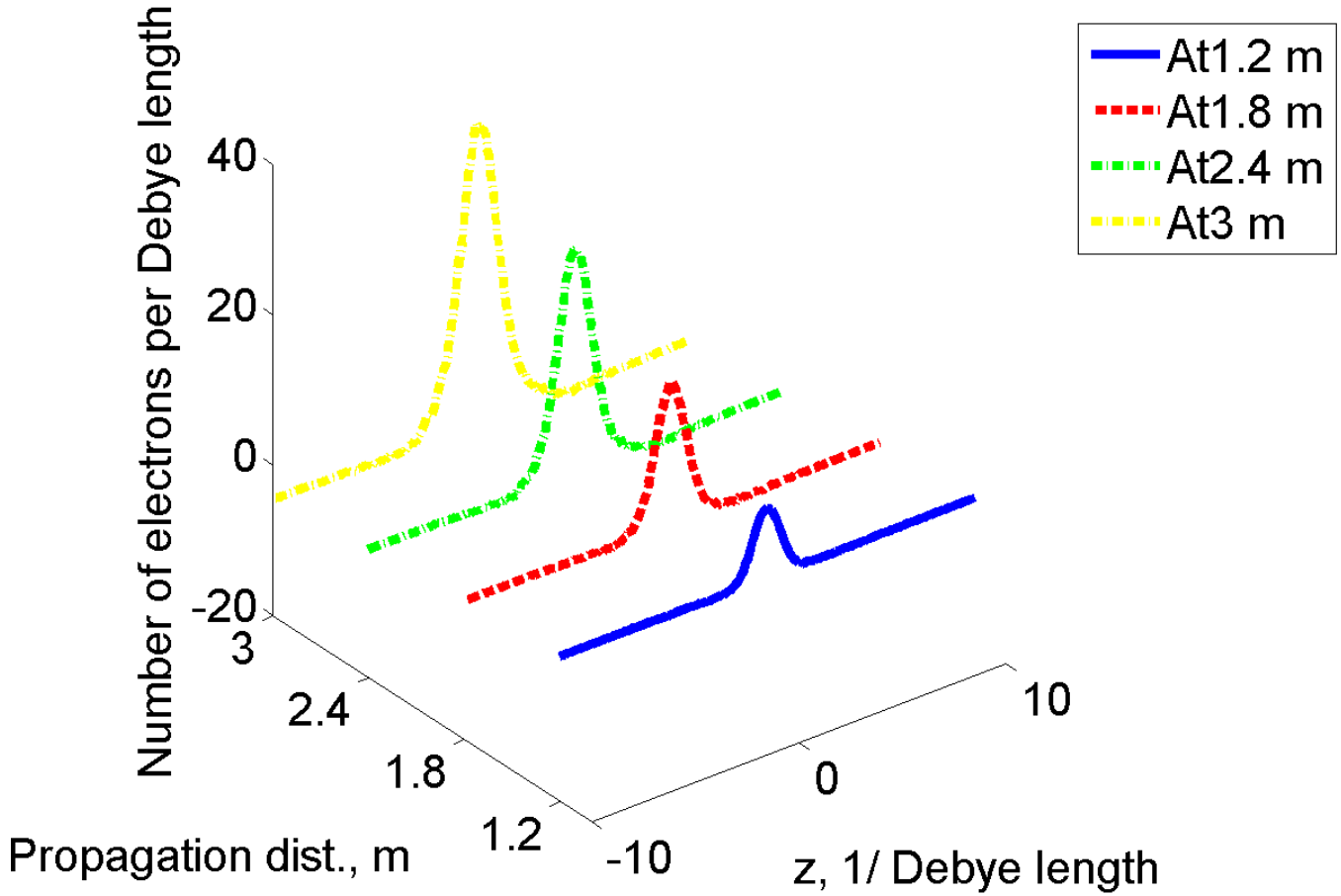
Beta function changes in quadrupoles fields



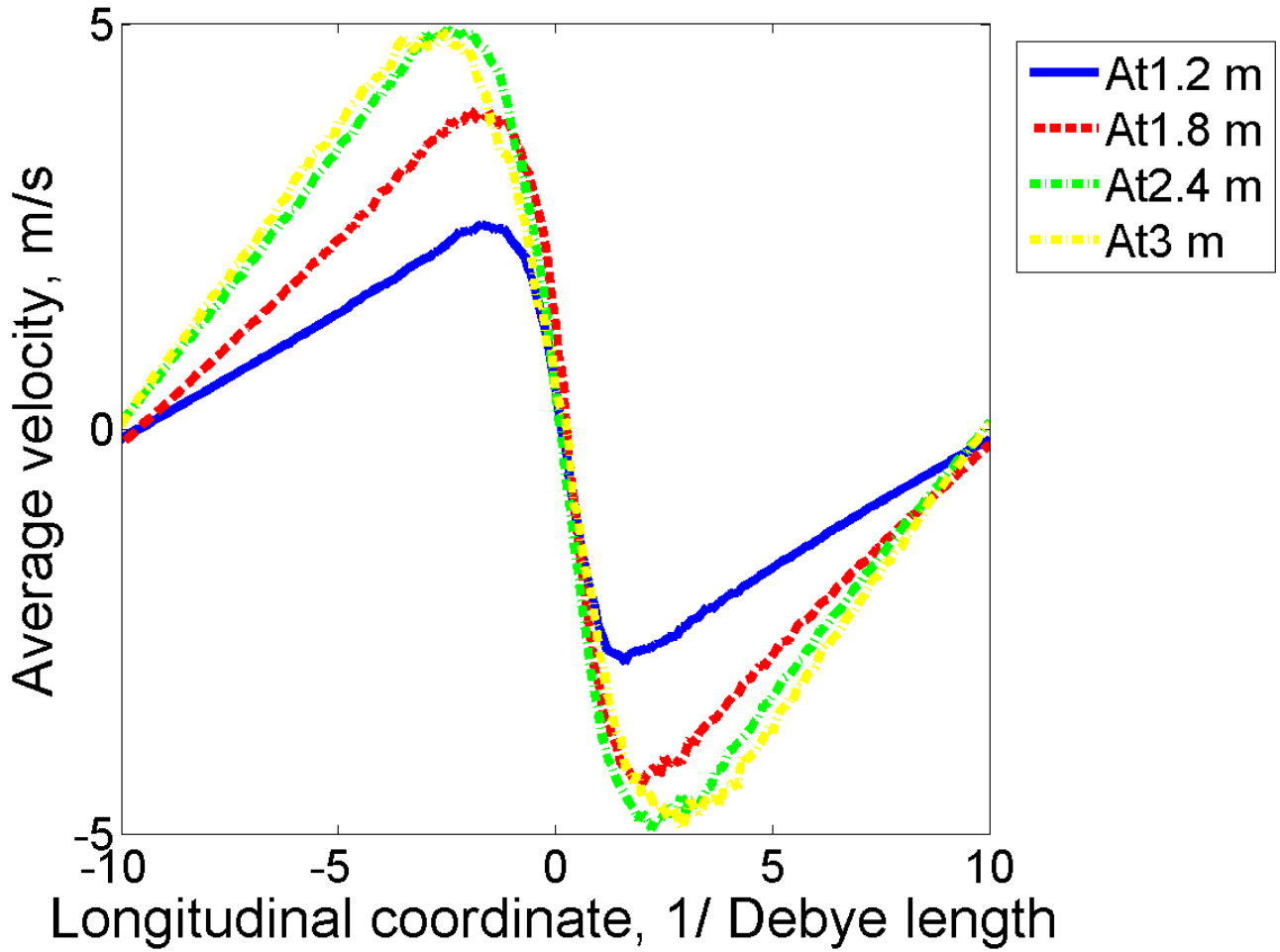
nz, ion at center



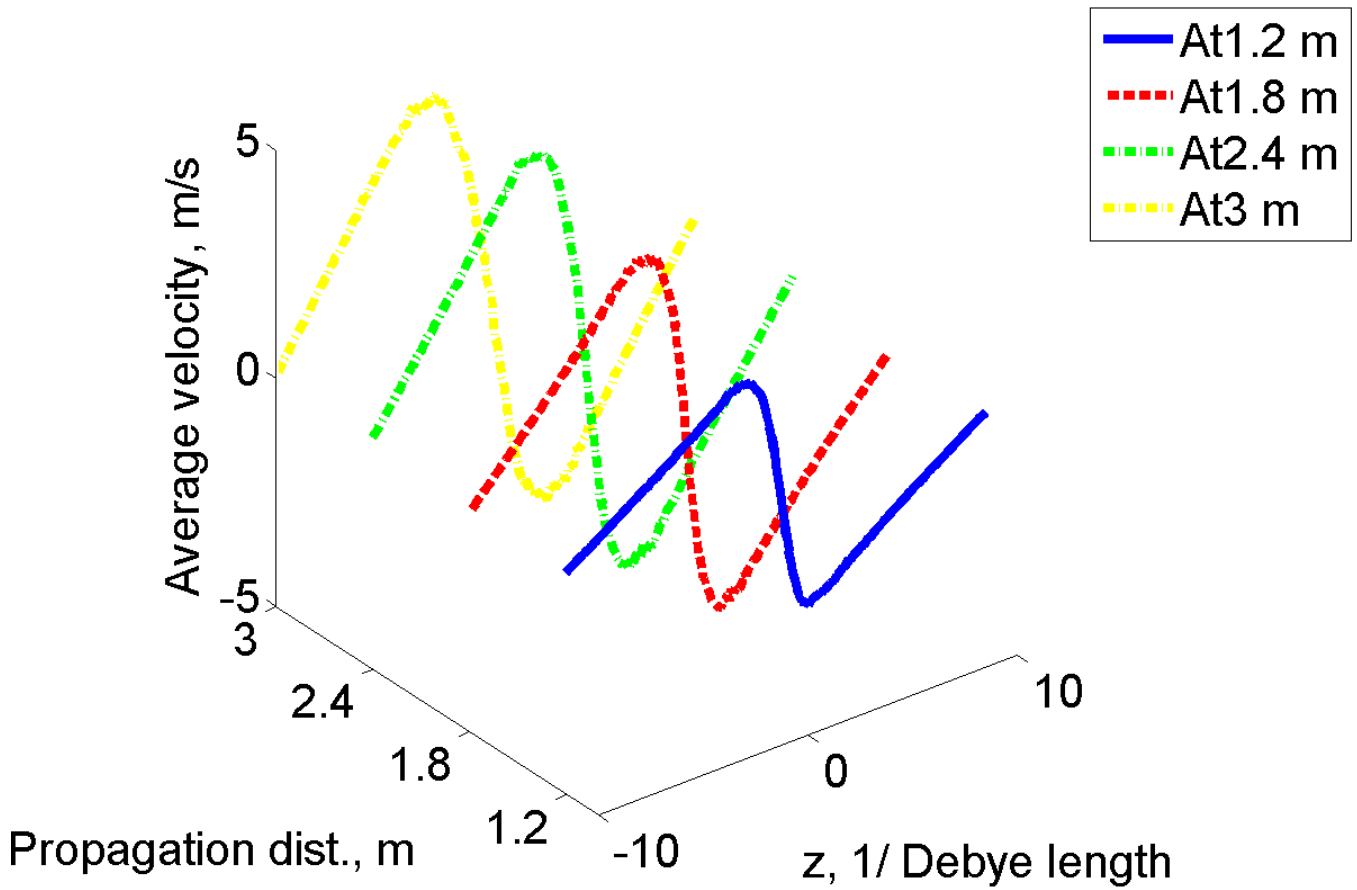
nz, ion at center



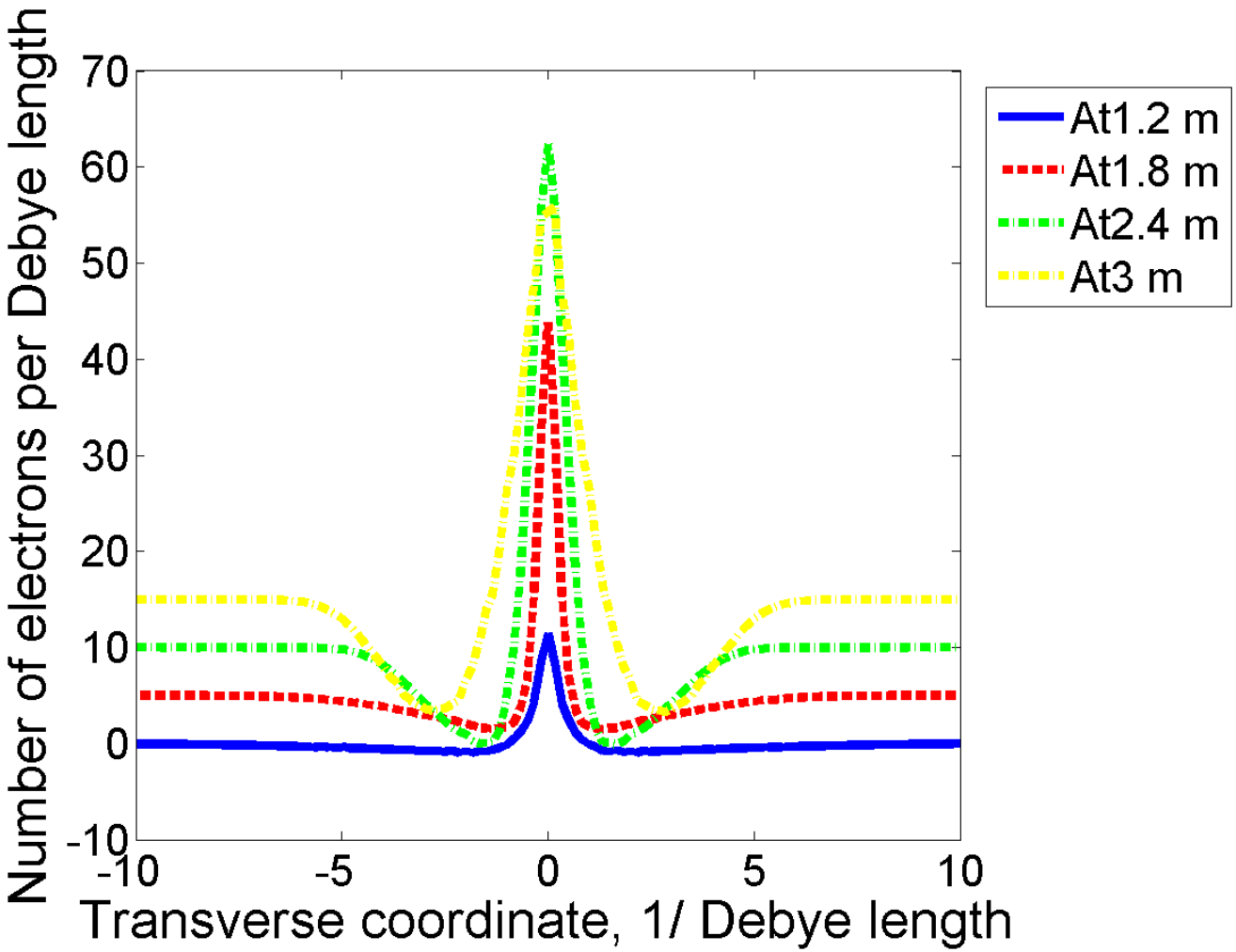
vz, ion at center



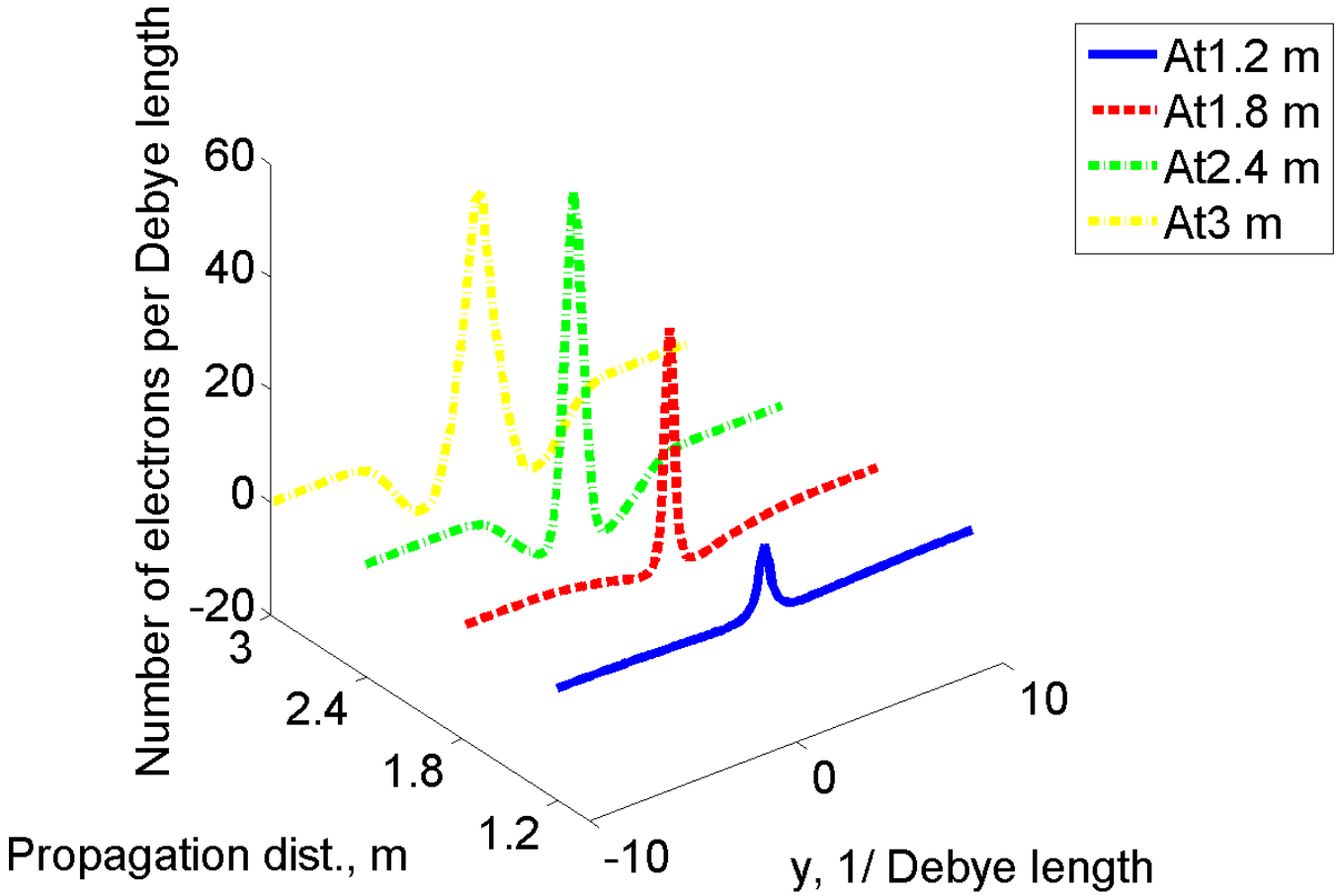
vz, ion at center



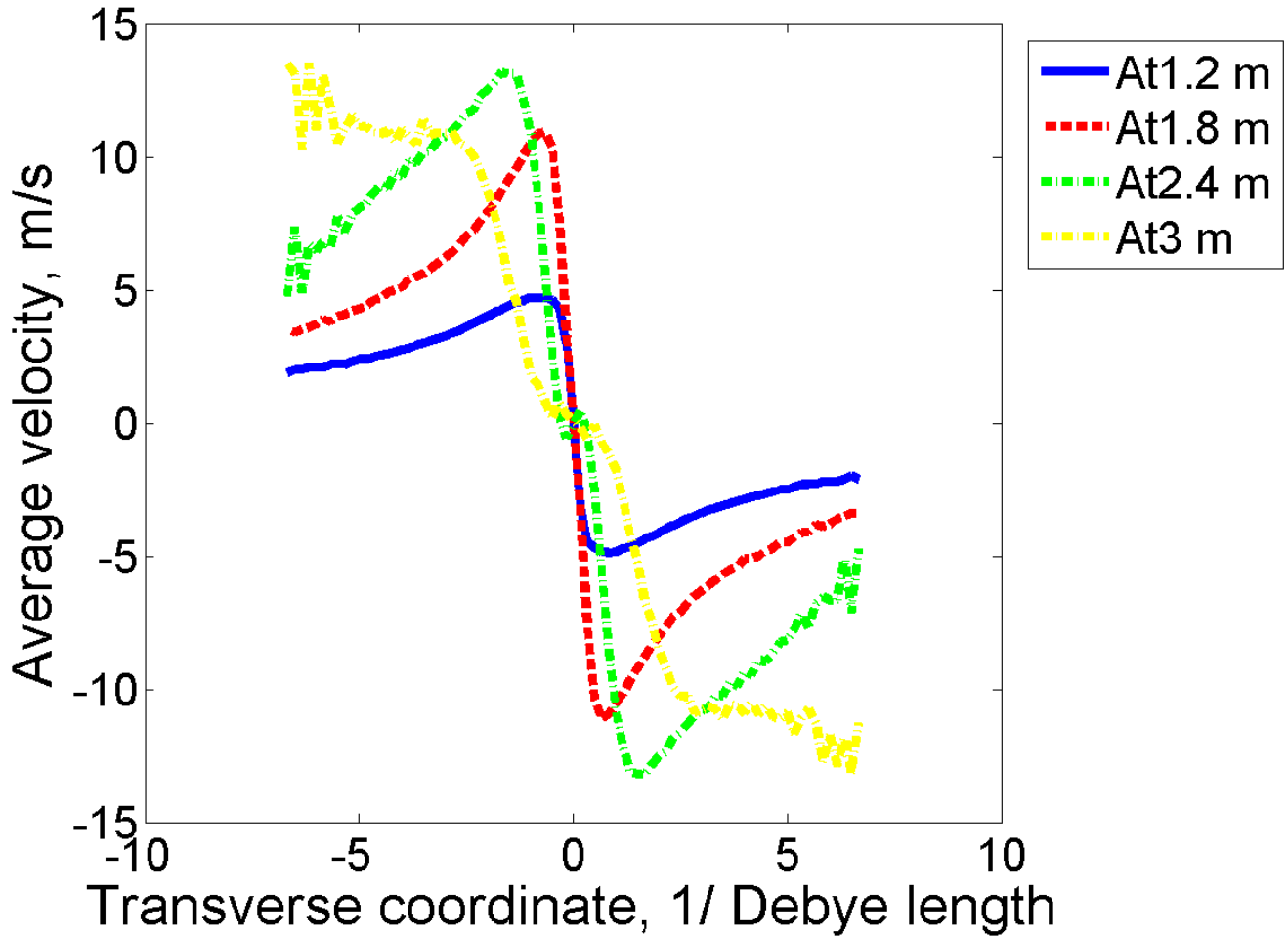
ny, ion at center



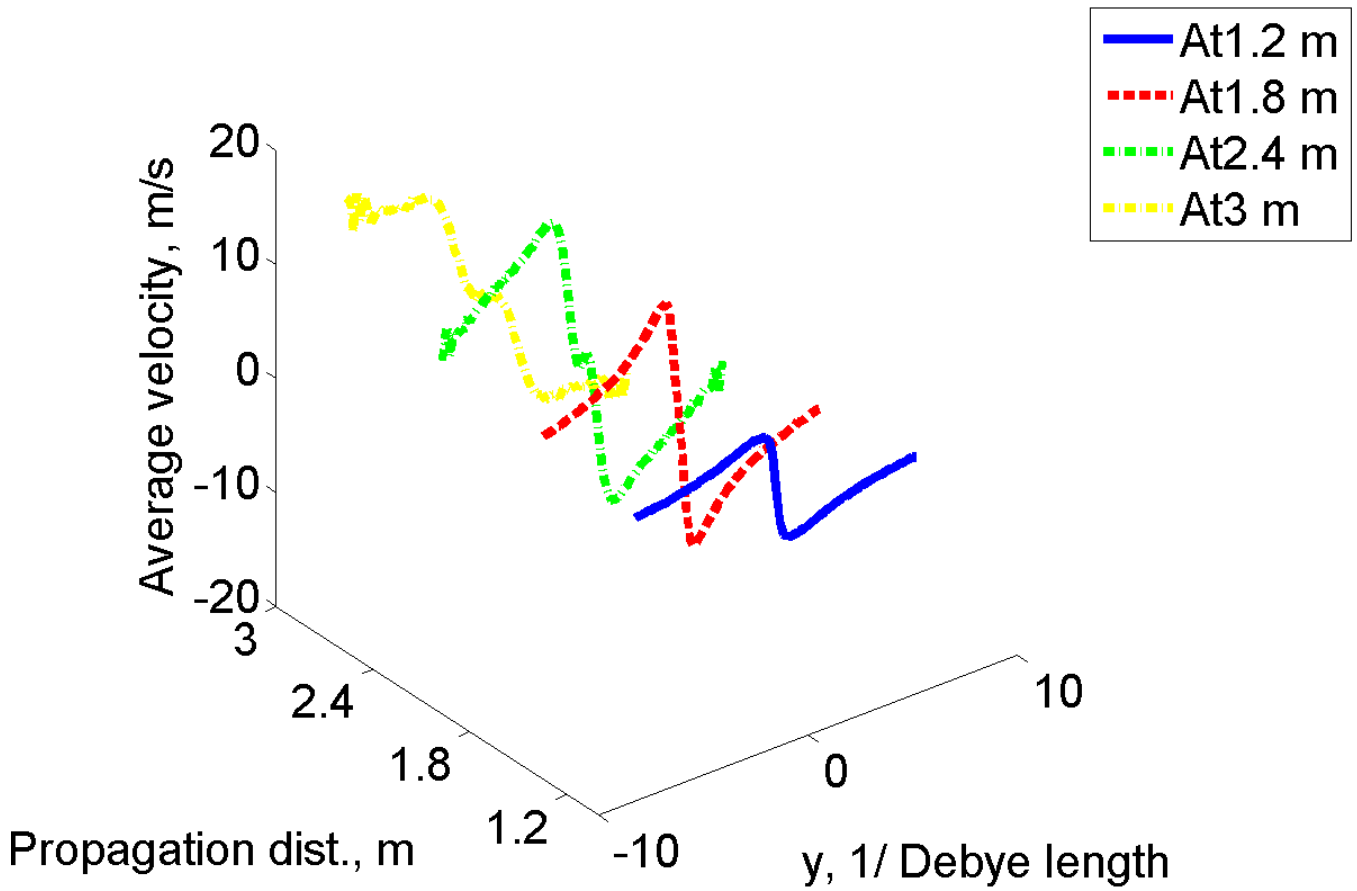
ny, ion at center



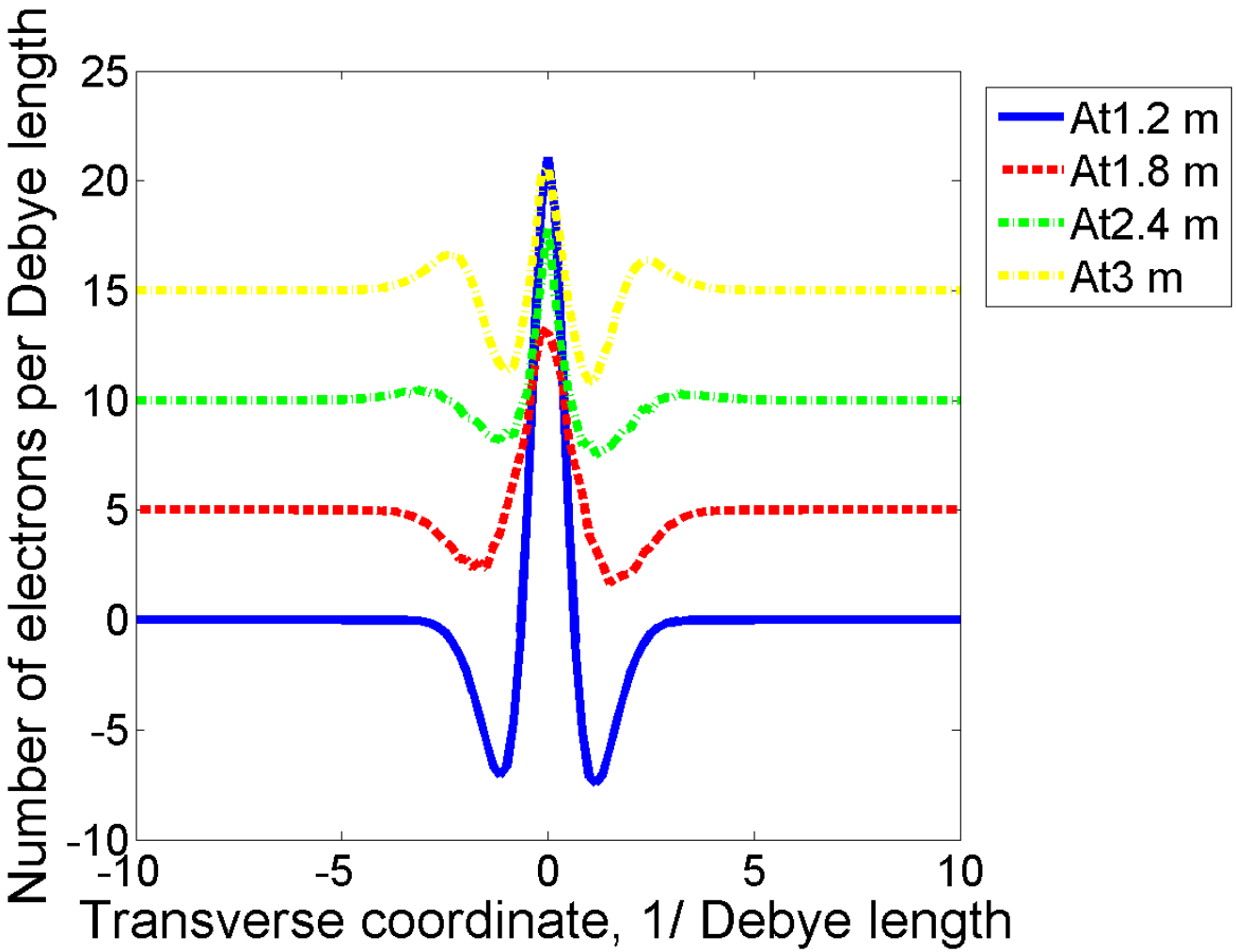
vy, ion at center



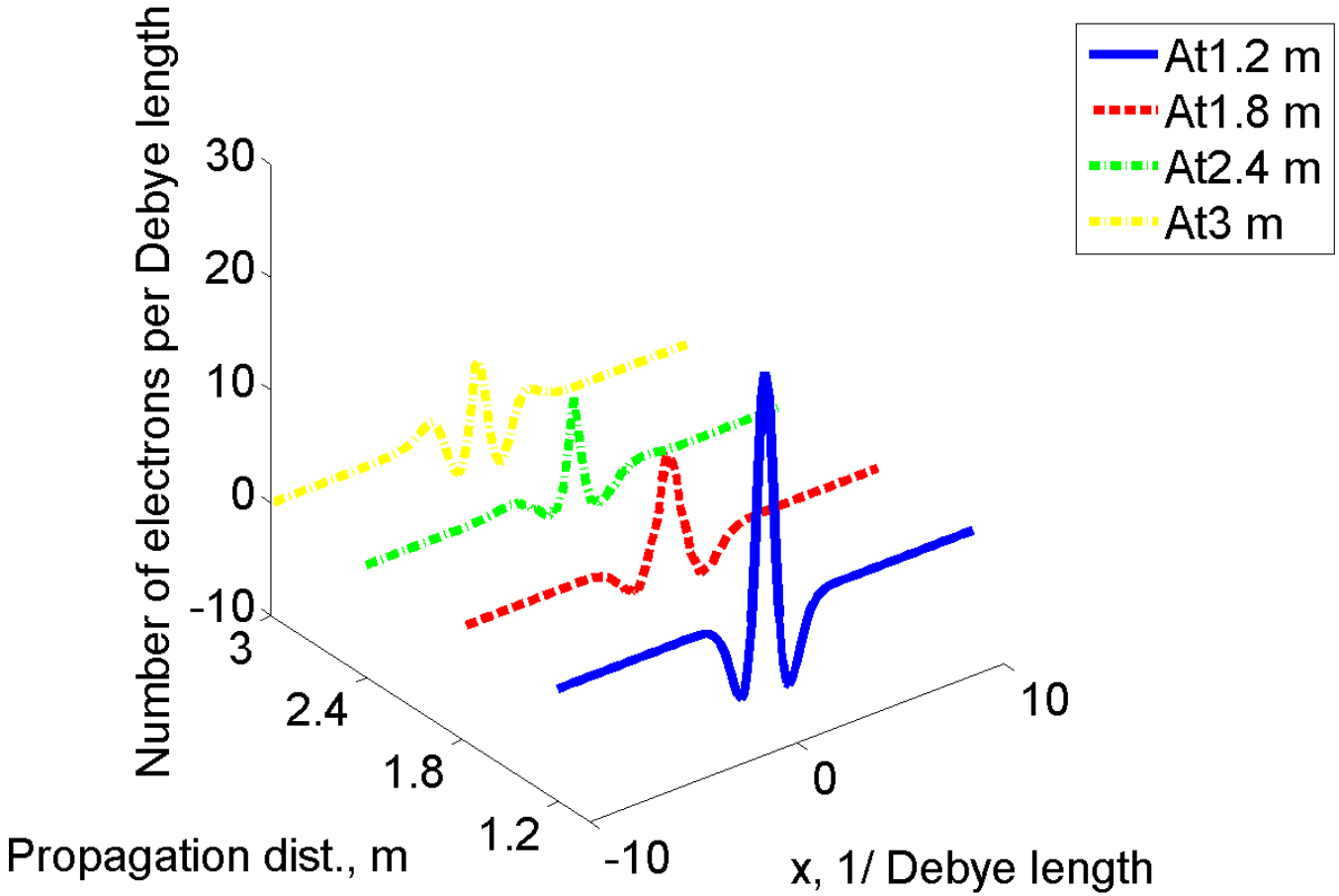
vy, ion at center



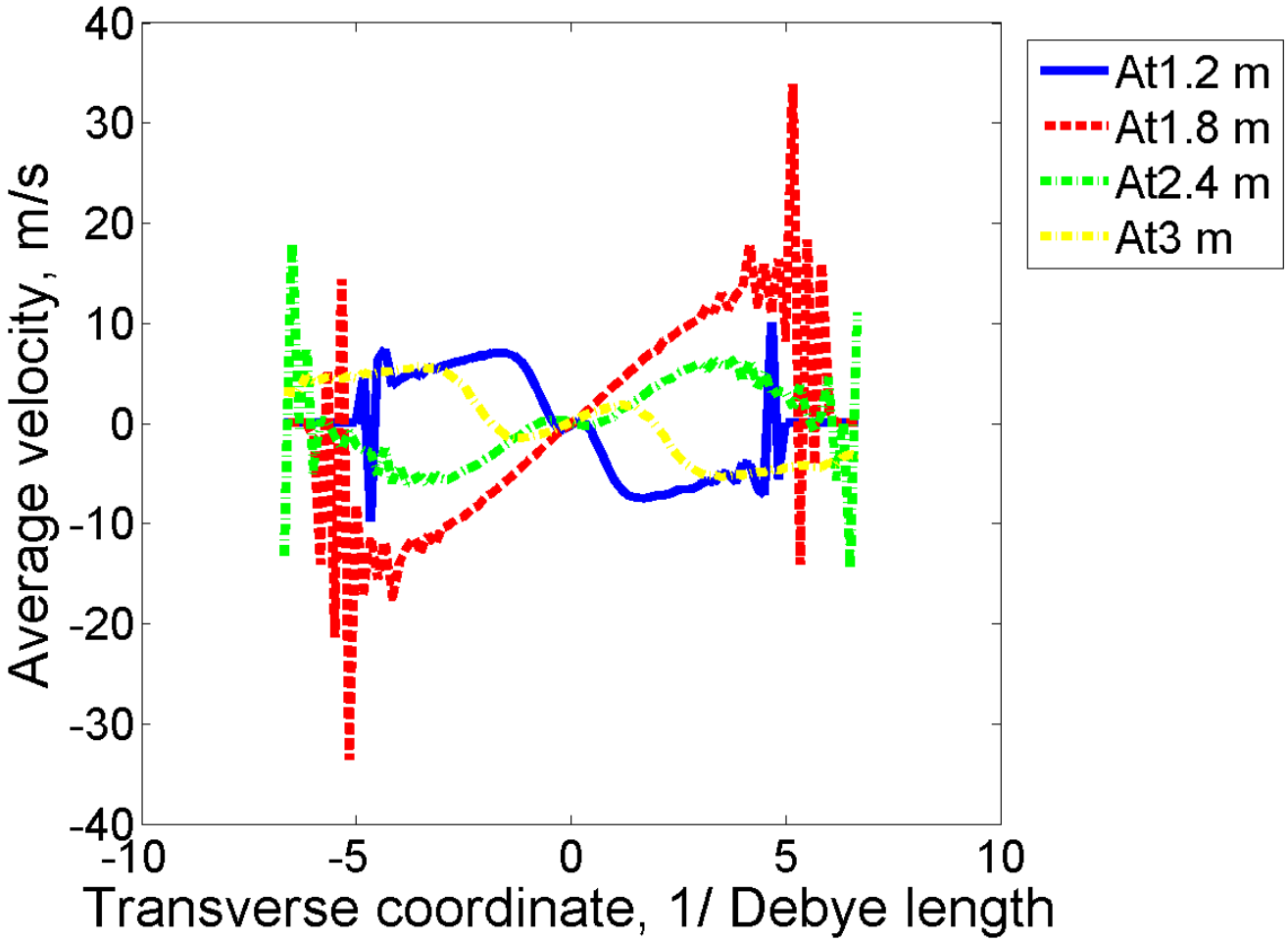
nx, ion at center



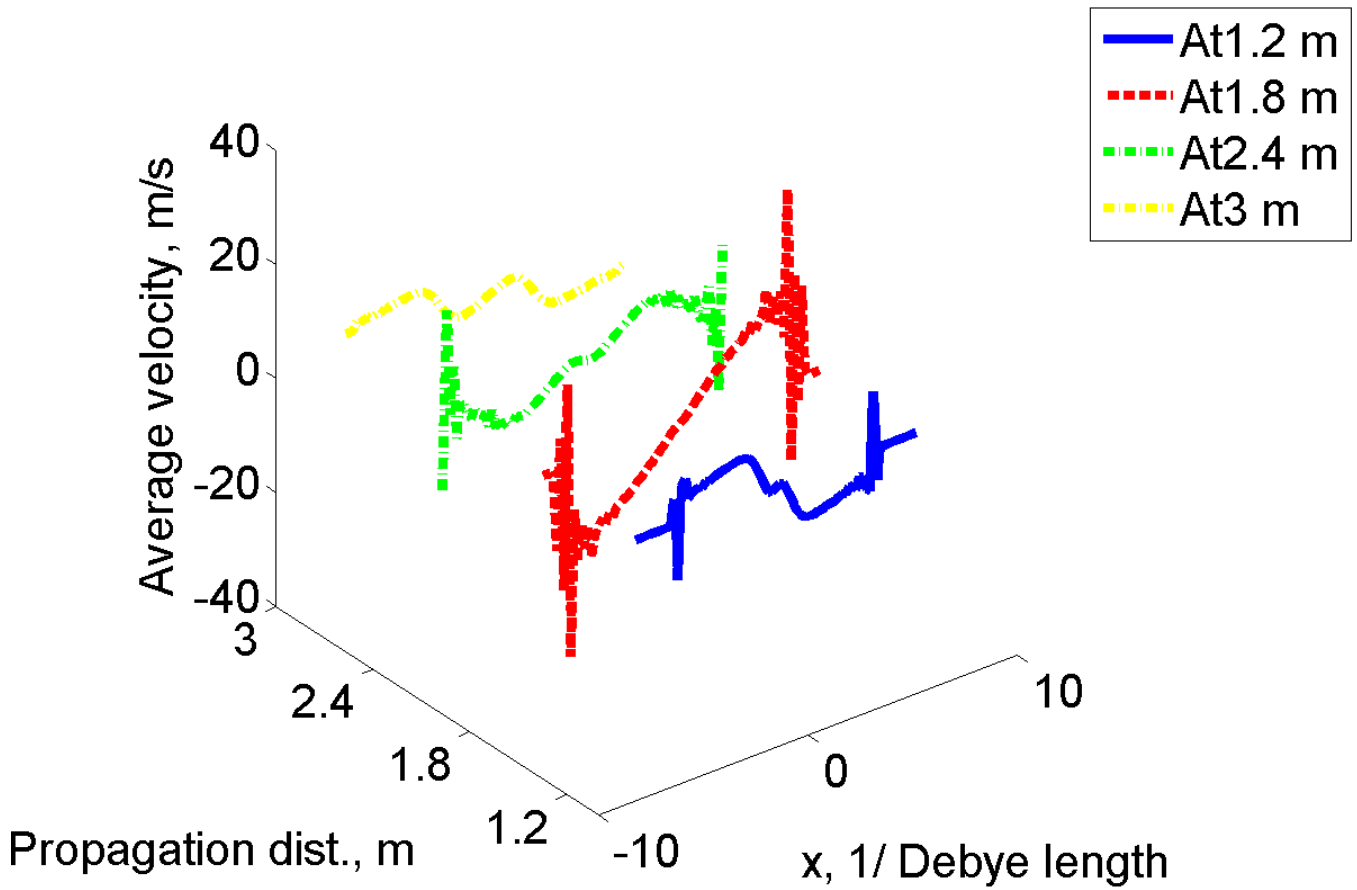
nx, ion at center



vx, ion at center



vx, ion at center



Quadrupoles makes phase advance

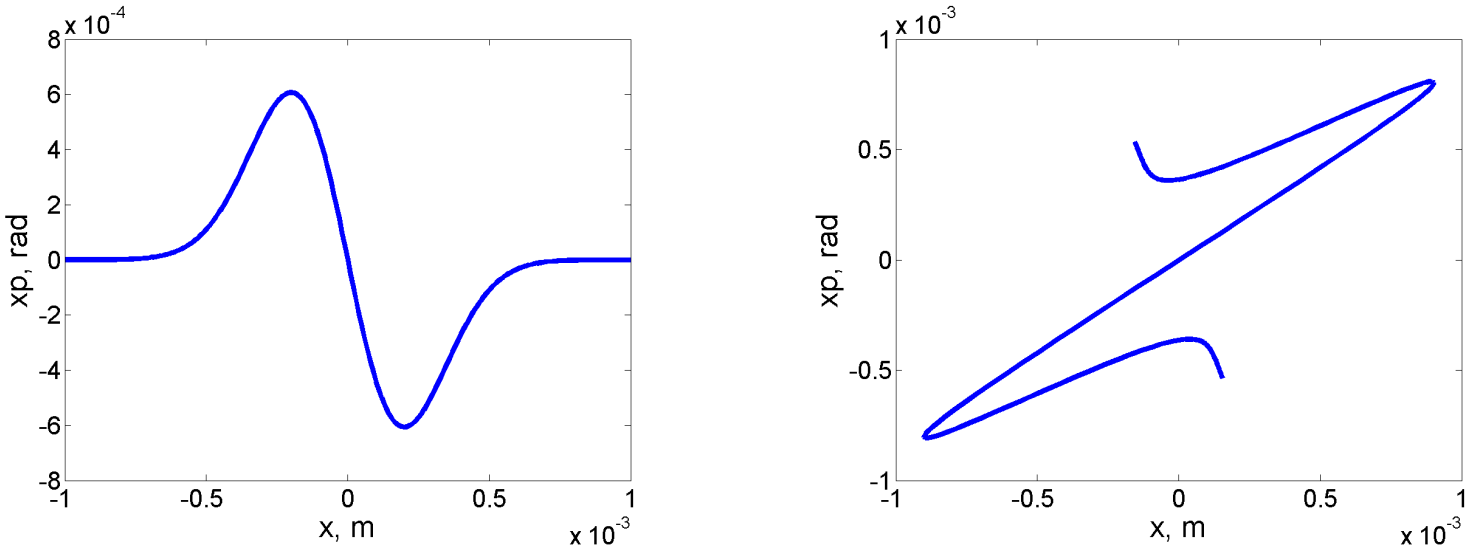
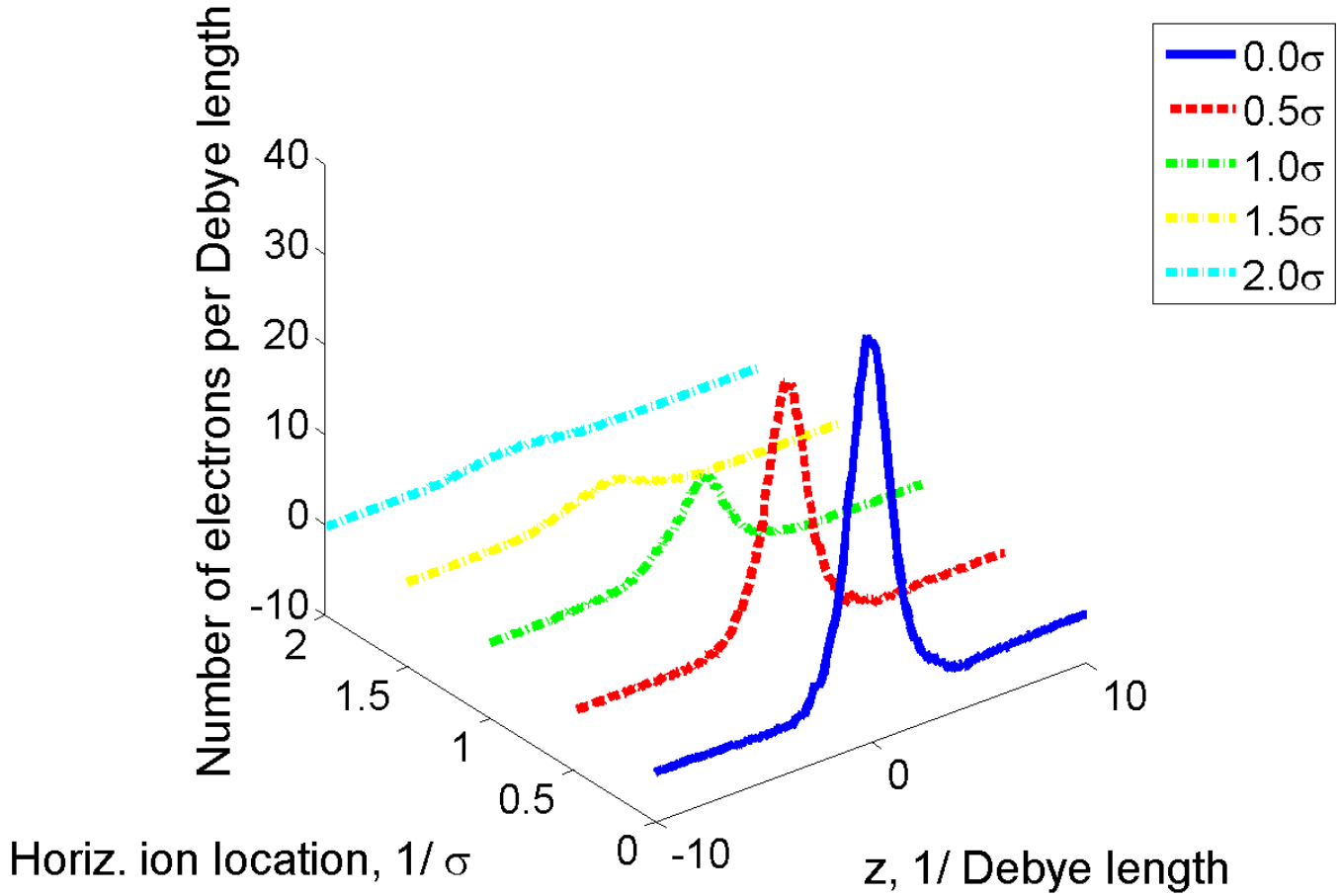
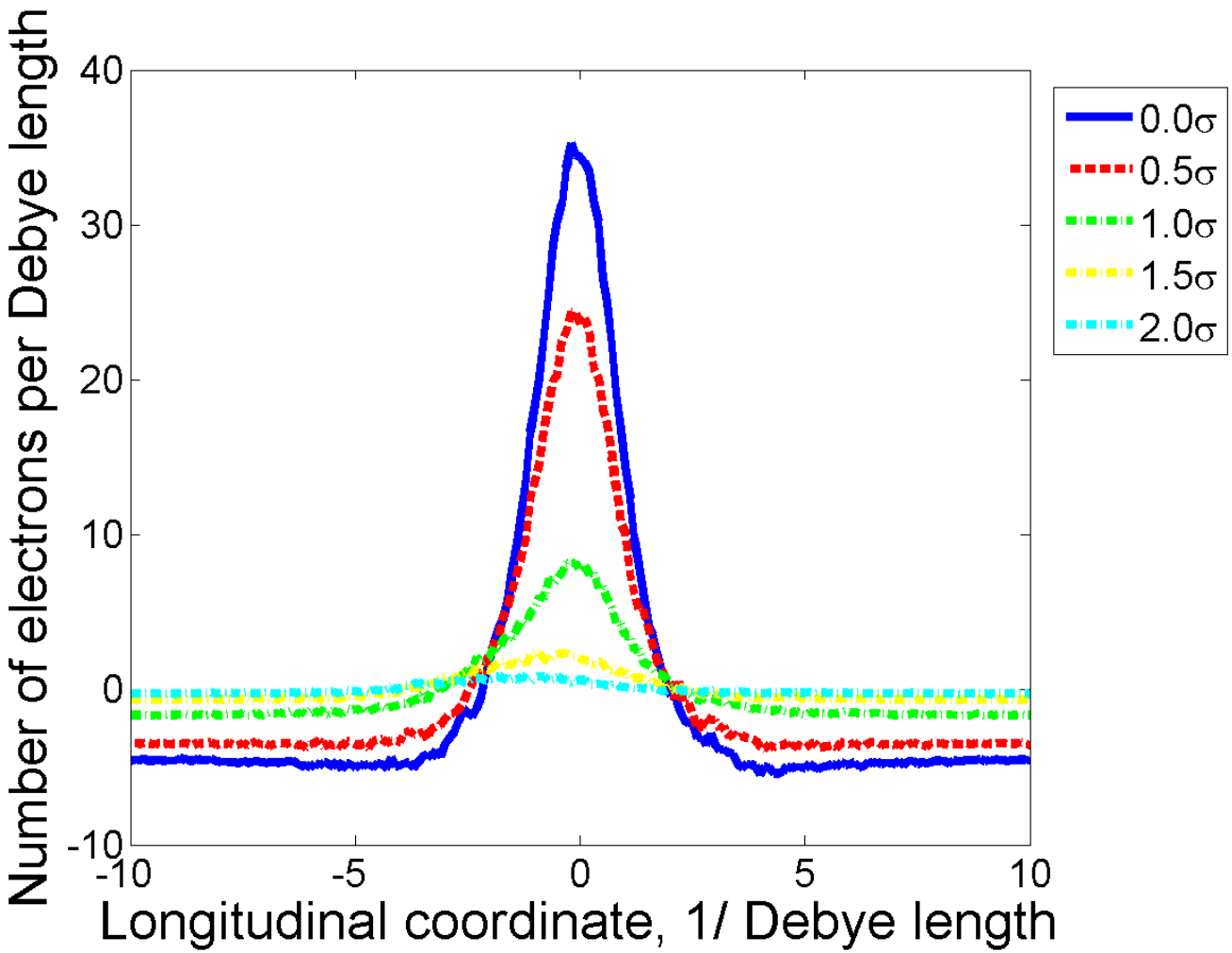


Figure: Initial (left) and final (right) phase plots of modulation signal

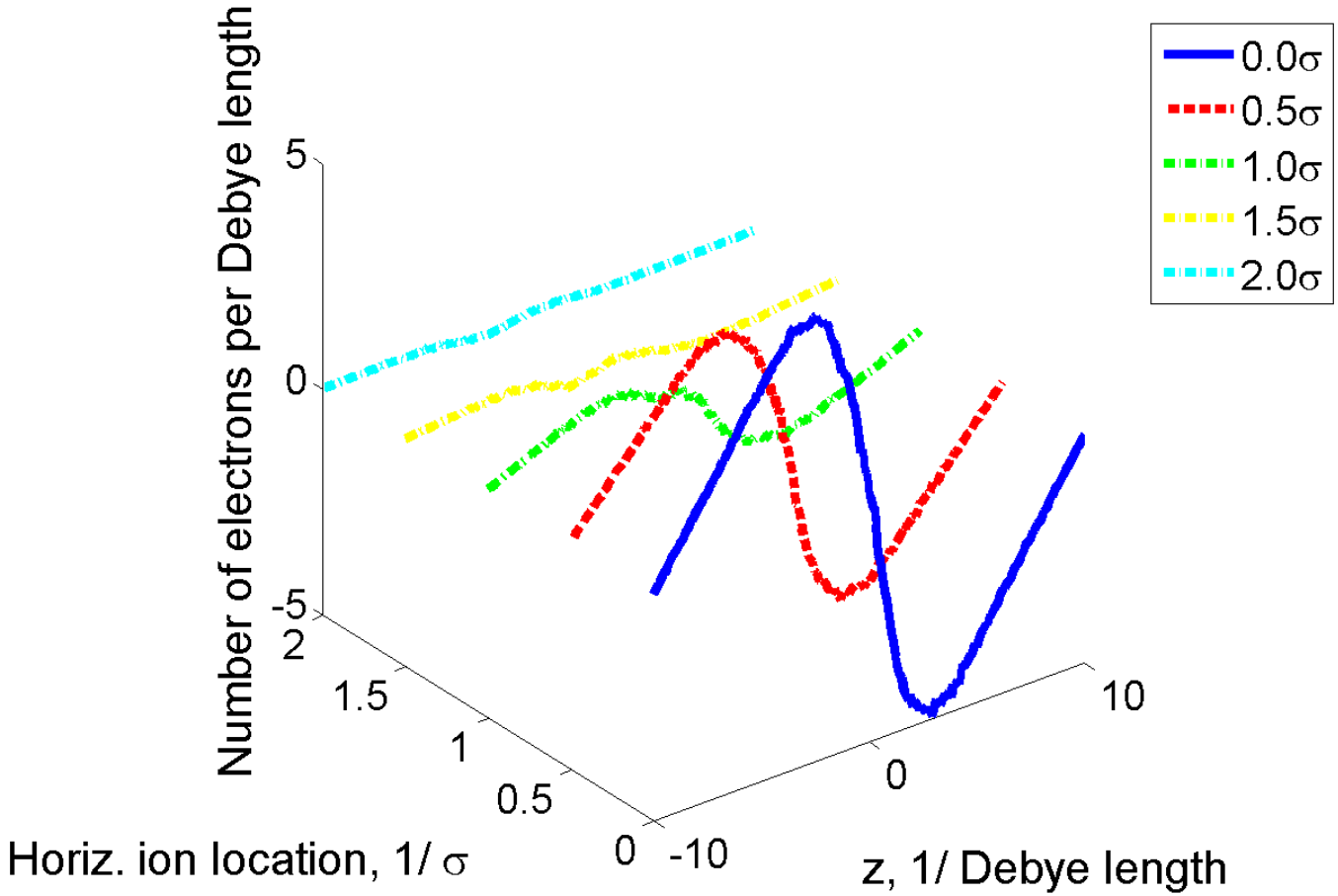
nz, various ion locations



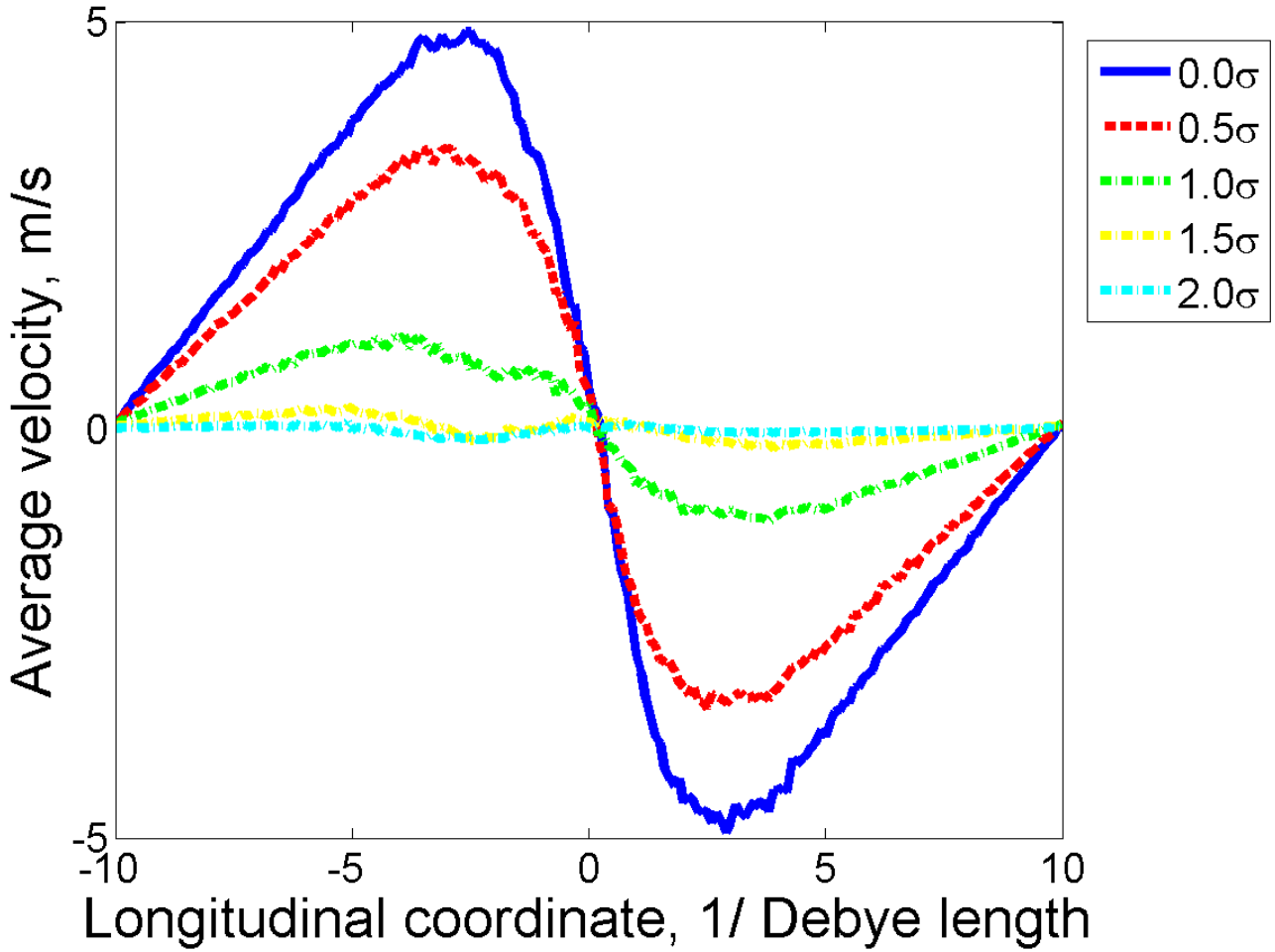
nz, various ion locations



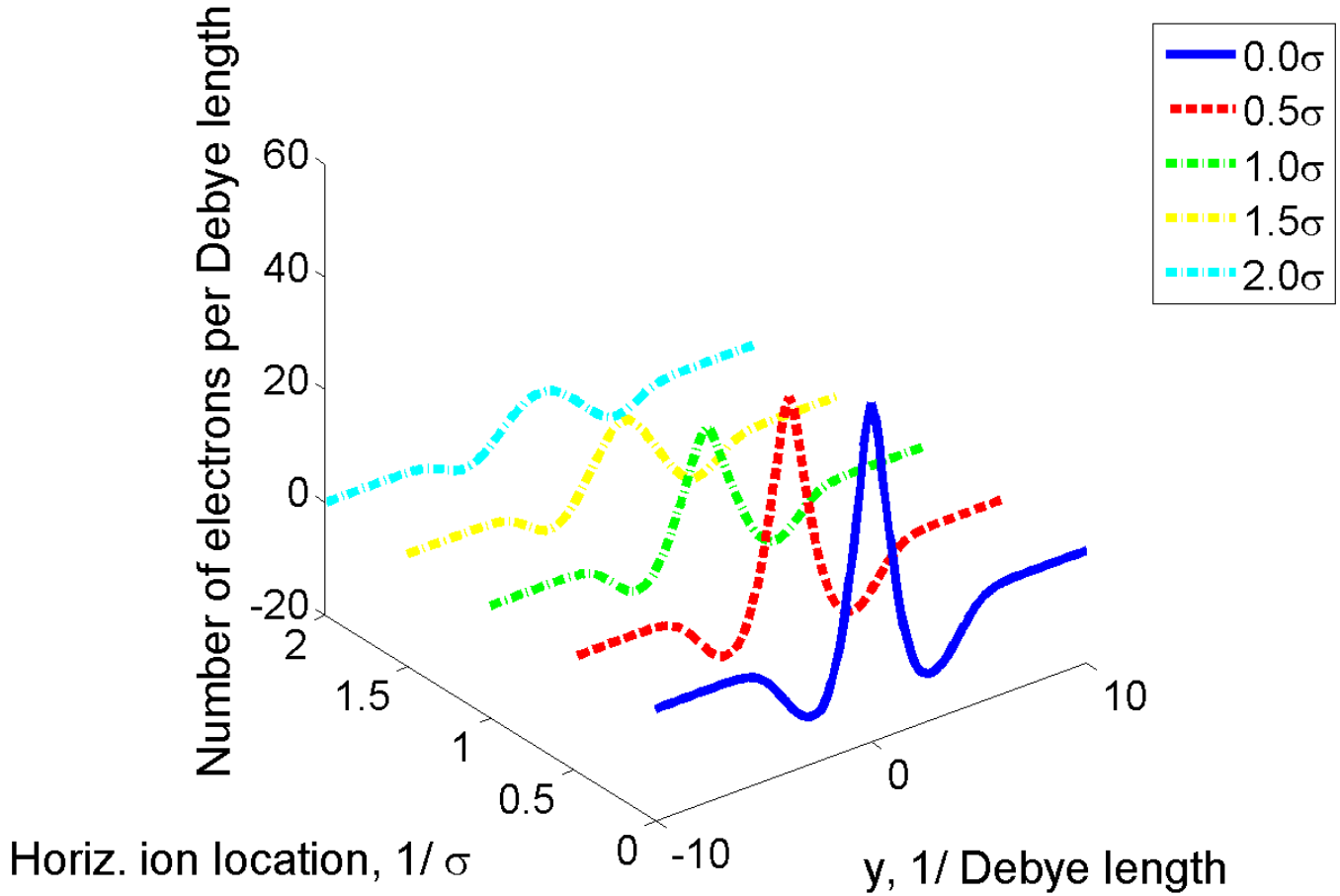
vz, various ion locations



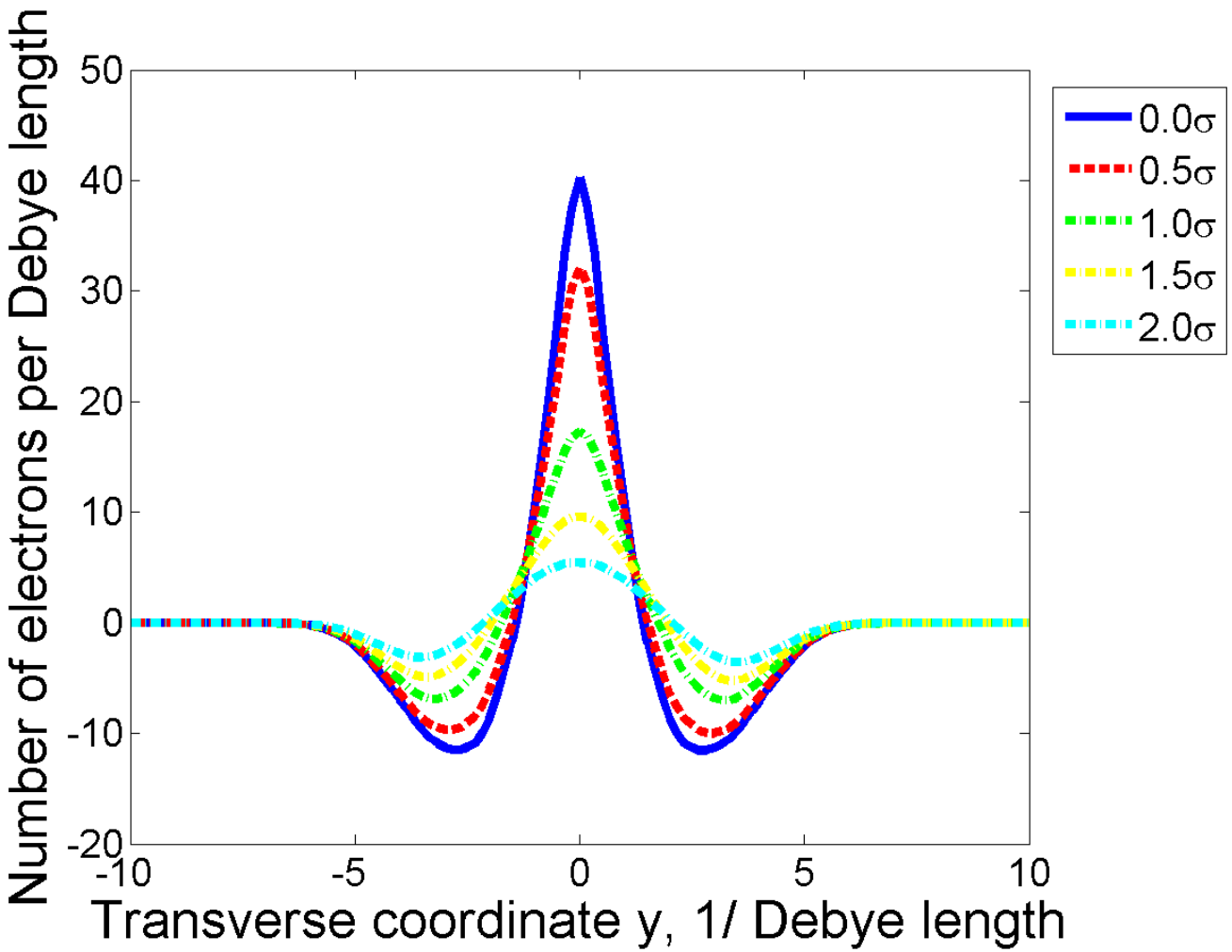
vz, various ion locations



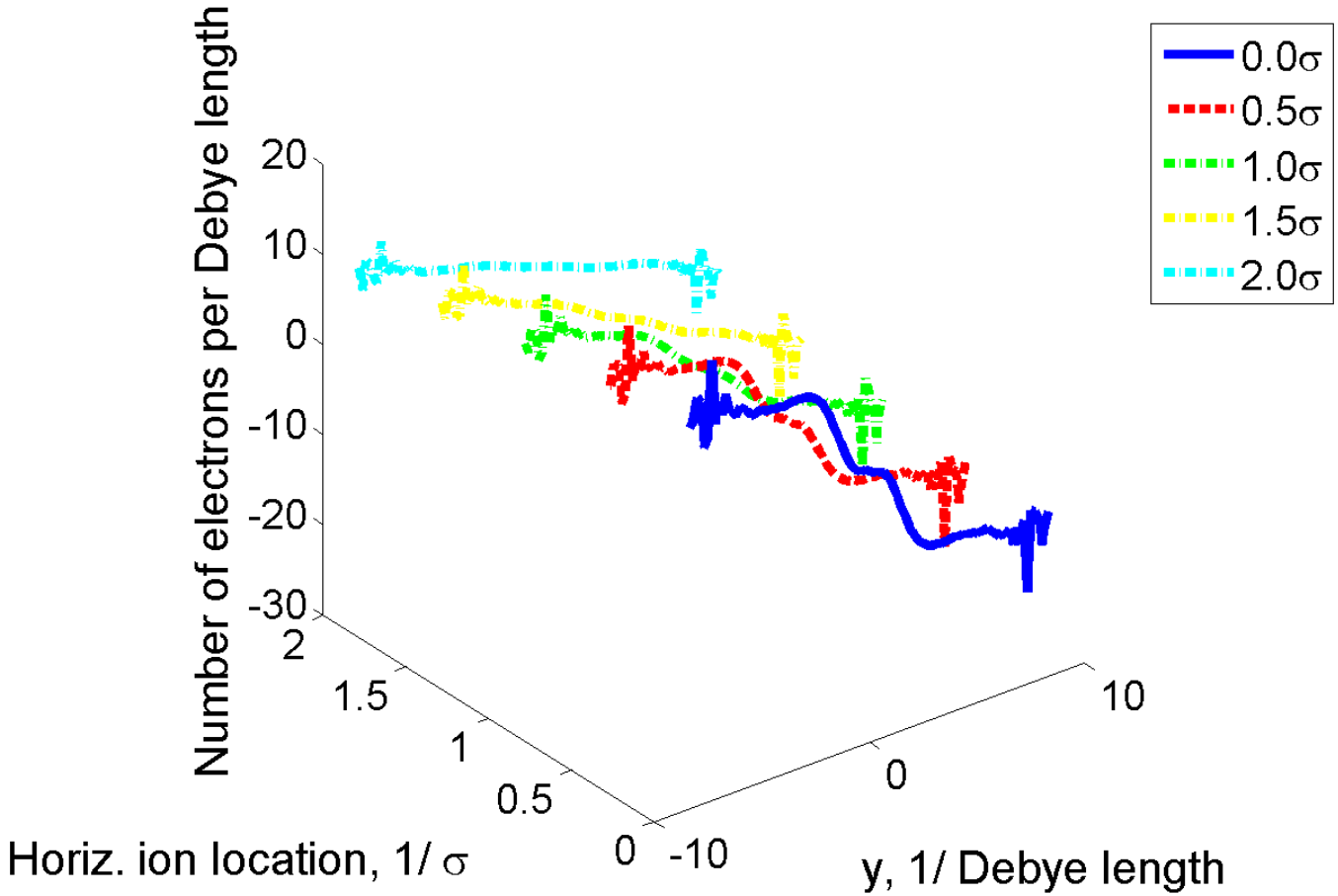
ny, various ion locations



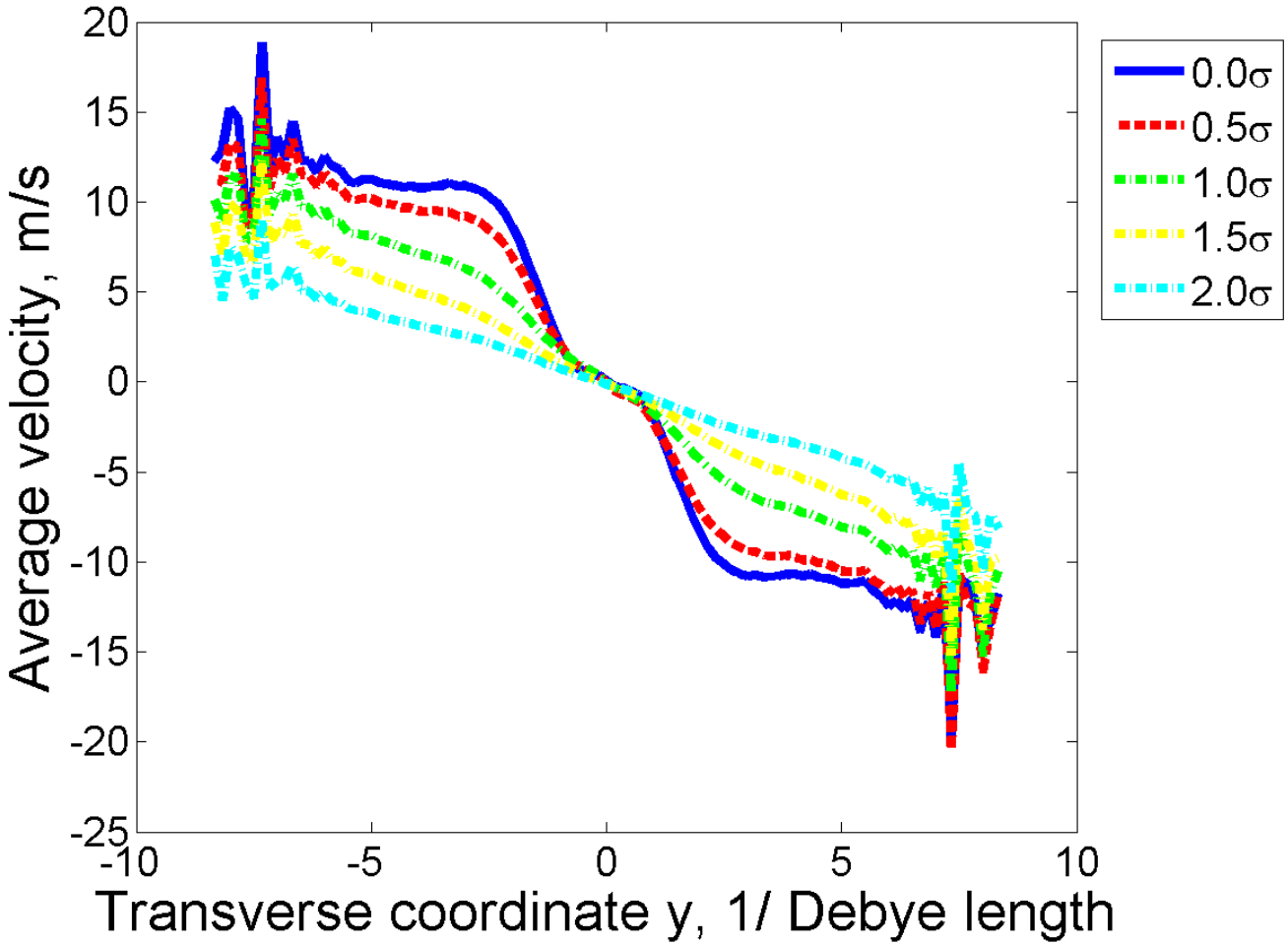
ny, various ion locations



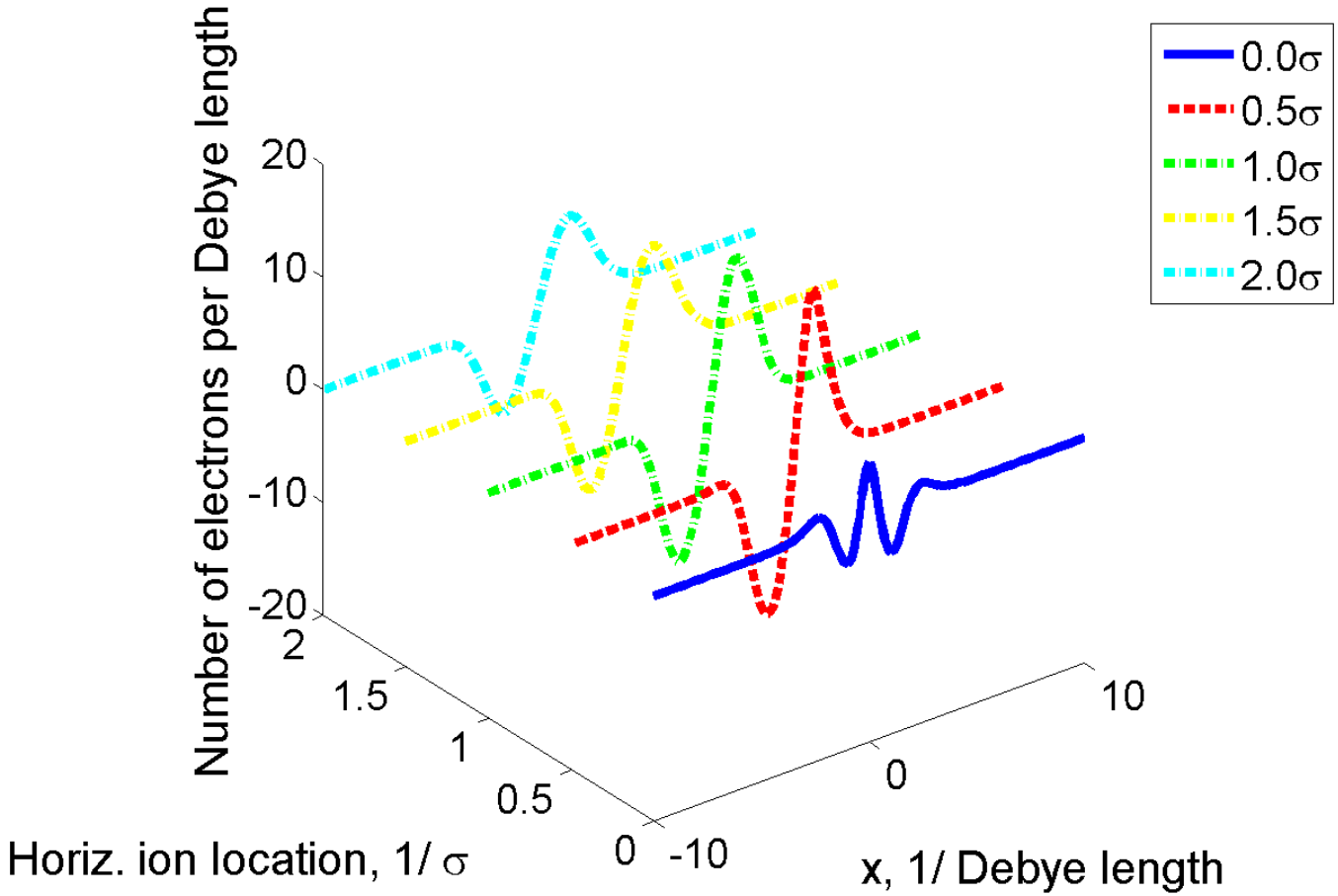
vy, various ion locations



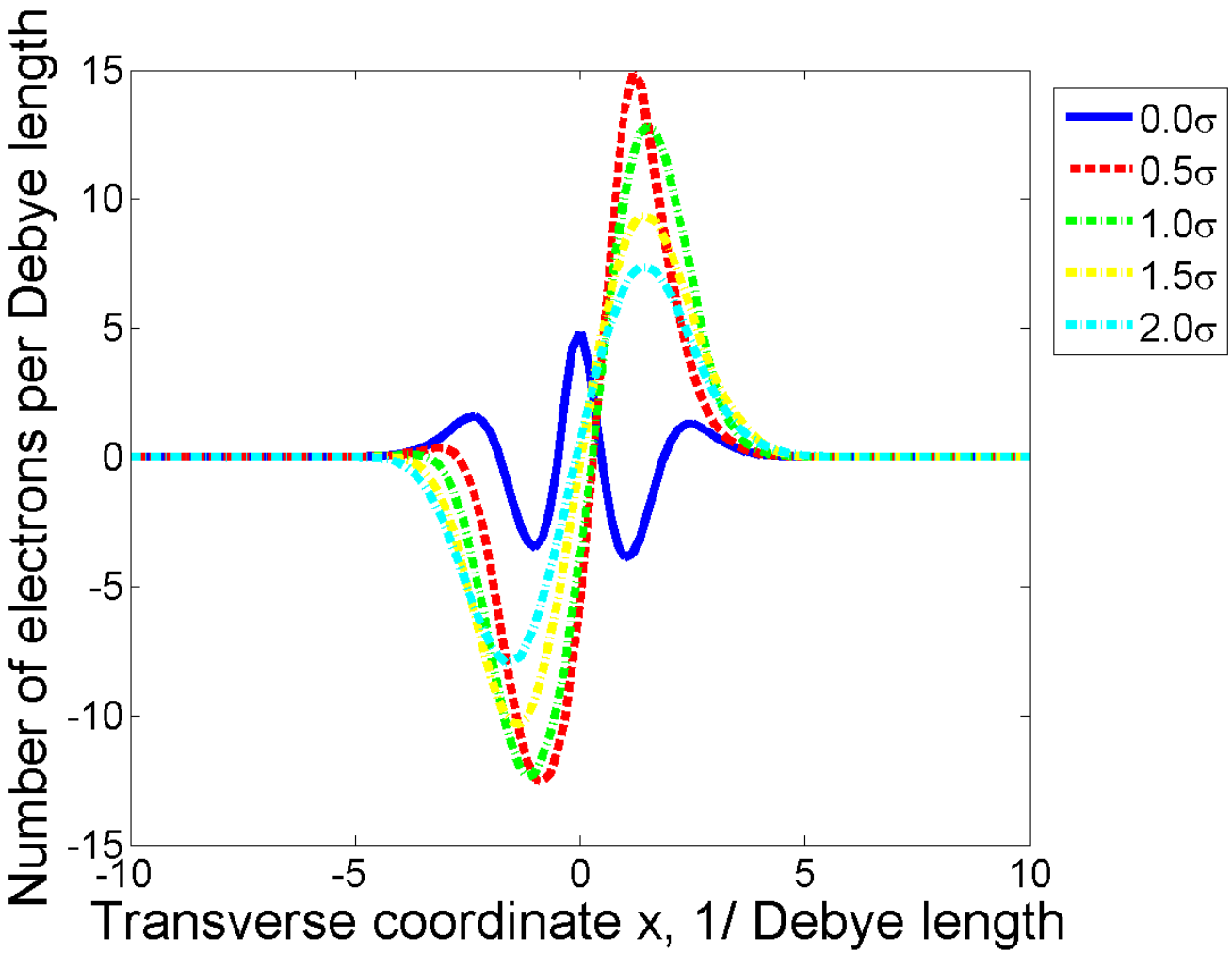
vy, various ion locations



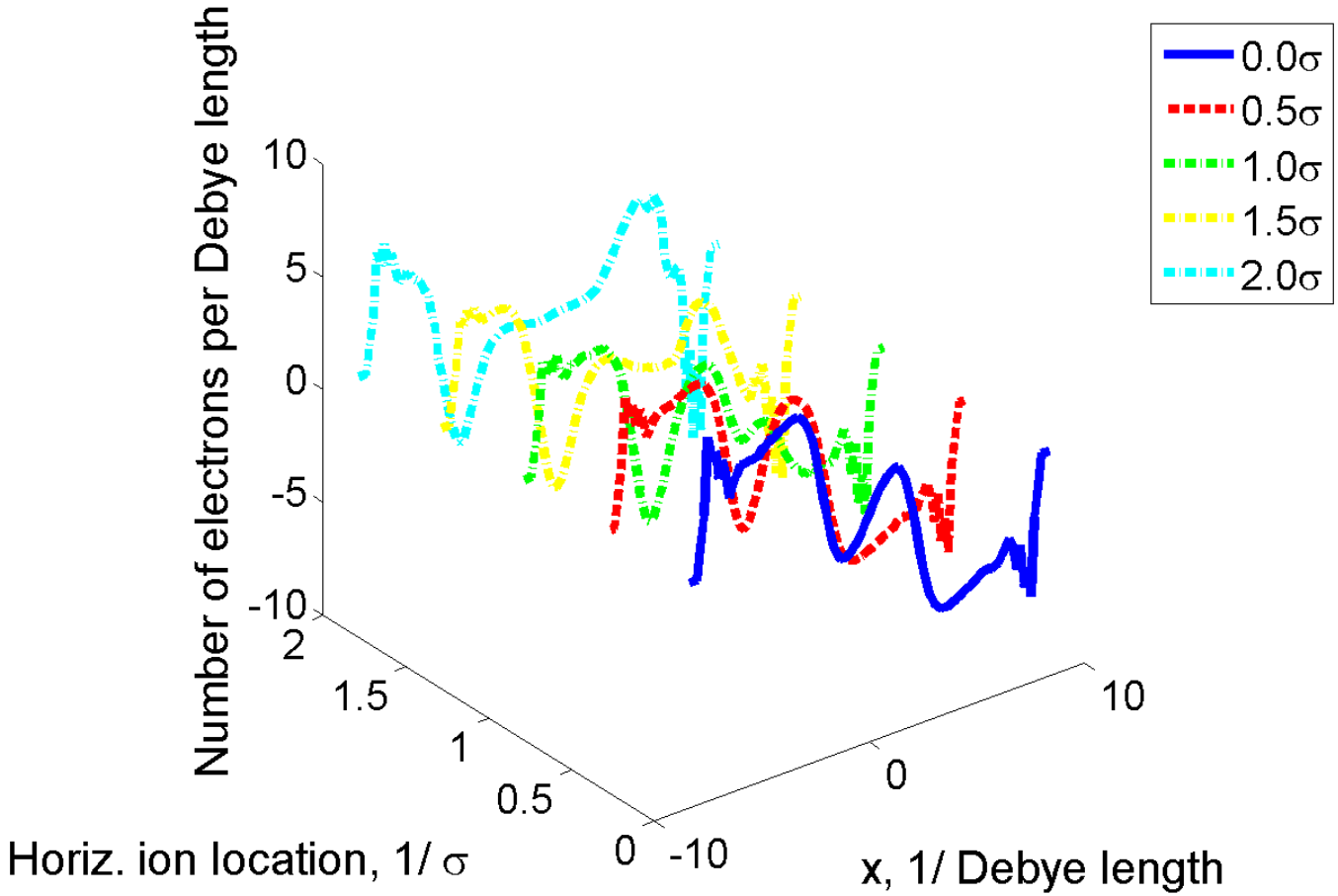
nx, various ion locations



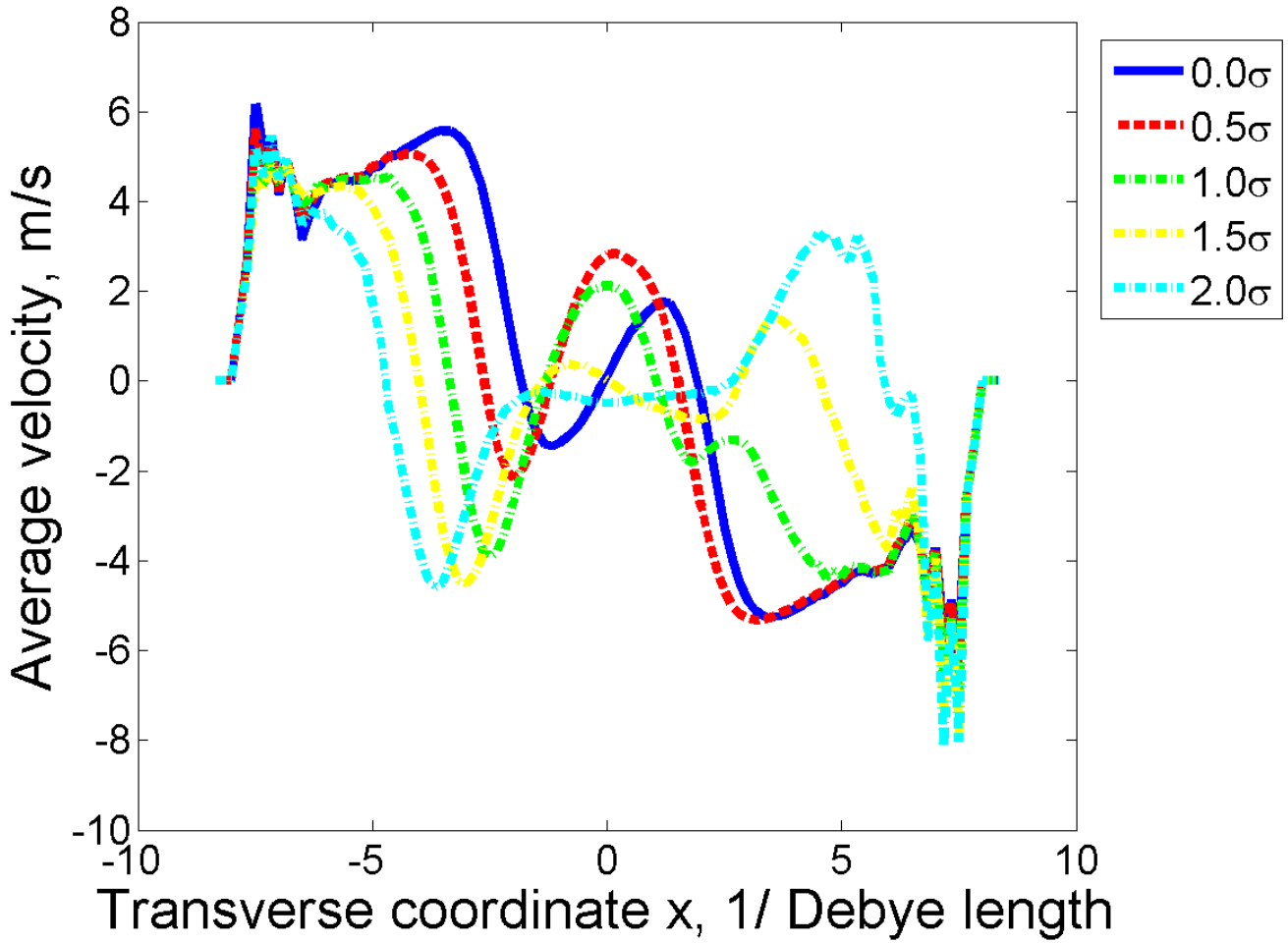
nx, various ion locations



vx, various ion locations



vx, various ion locations



More results

- More ion locations
- Moving ion

$$\text{Modulator(SPACE)} \Rightarrow \text{Amplifier(GENESIS)} \Rightarrow \text{Kicker(SPACE)}$$

GENESIS parameters

- Optical wavelength (slice length), 1.357e-5[m]
- Number of slices, 400
- Bunching factor of jth slice

$$b_j = \frac{1}{N} \sum_{k=1+jN}^{(j+1)N} e^{i\theta_k}$$

- Wiggler period, 4[cm]
- Number of wiggler period, 200 / 188 (8[m]/7.5[m])

Wavelength

$$\begin{aligned}
\lambda &= \frac{\lambda_u}{2\gamma^2}\left(1+\frac{K^2}{2}\right) \\
\lambda &= 1.357e-5[m] \\
\lambda_u &= 4e-2[m] \\
K &= 0.7 \\
K &= \frac{eB_0\lambda_u}{2\pi mc} \\
B_0 &= 0.2[T]
\end{aligned}$$

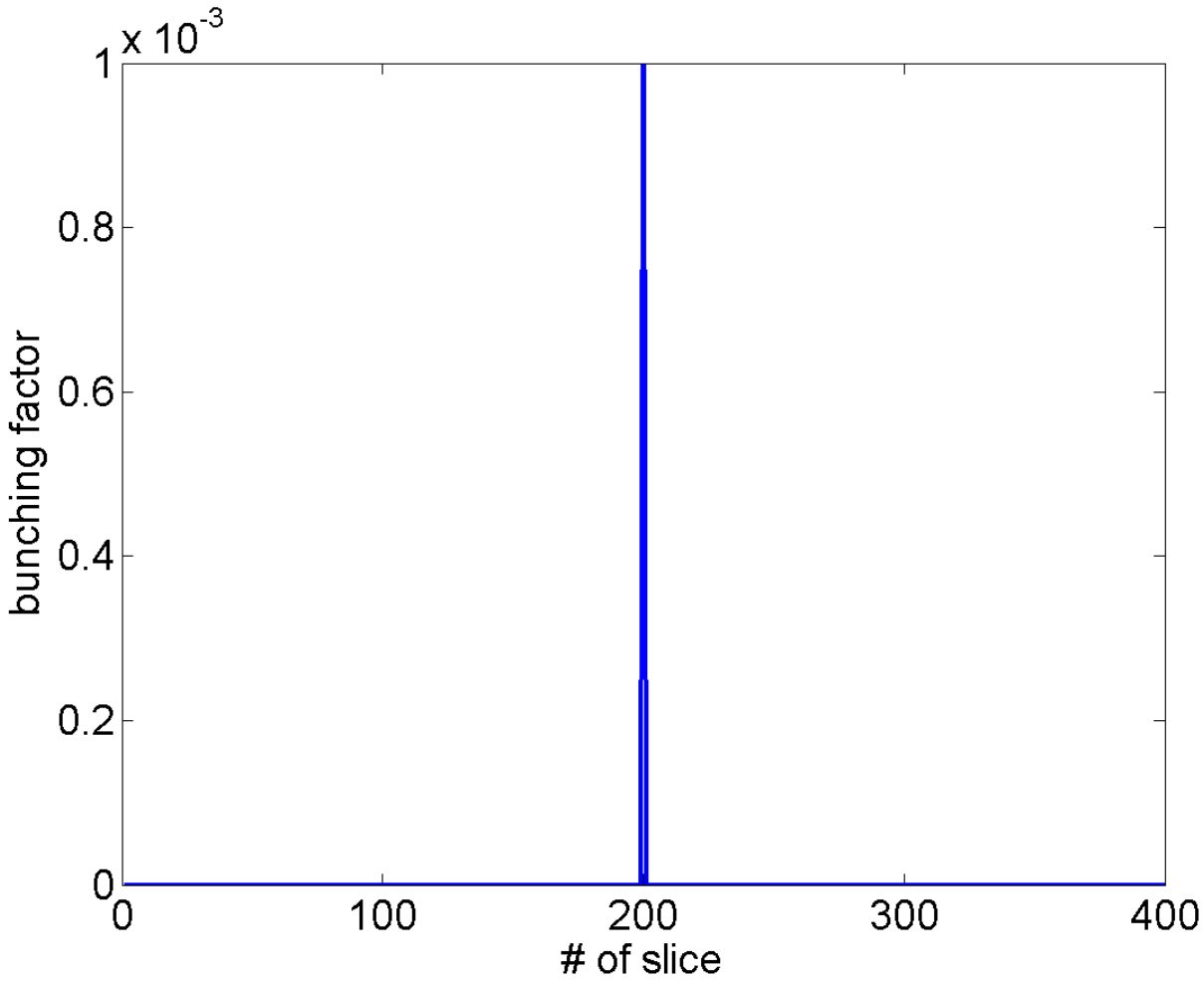
Modulator to FEL

- Let GENESIS generate particles for 400 slices
- Replace one slice with distribution from modulator simulations (background beam and modulated beam)
- Run GENESIS with replaced slice
- Take difference between background beam and modulated beam

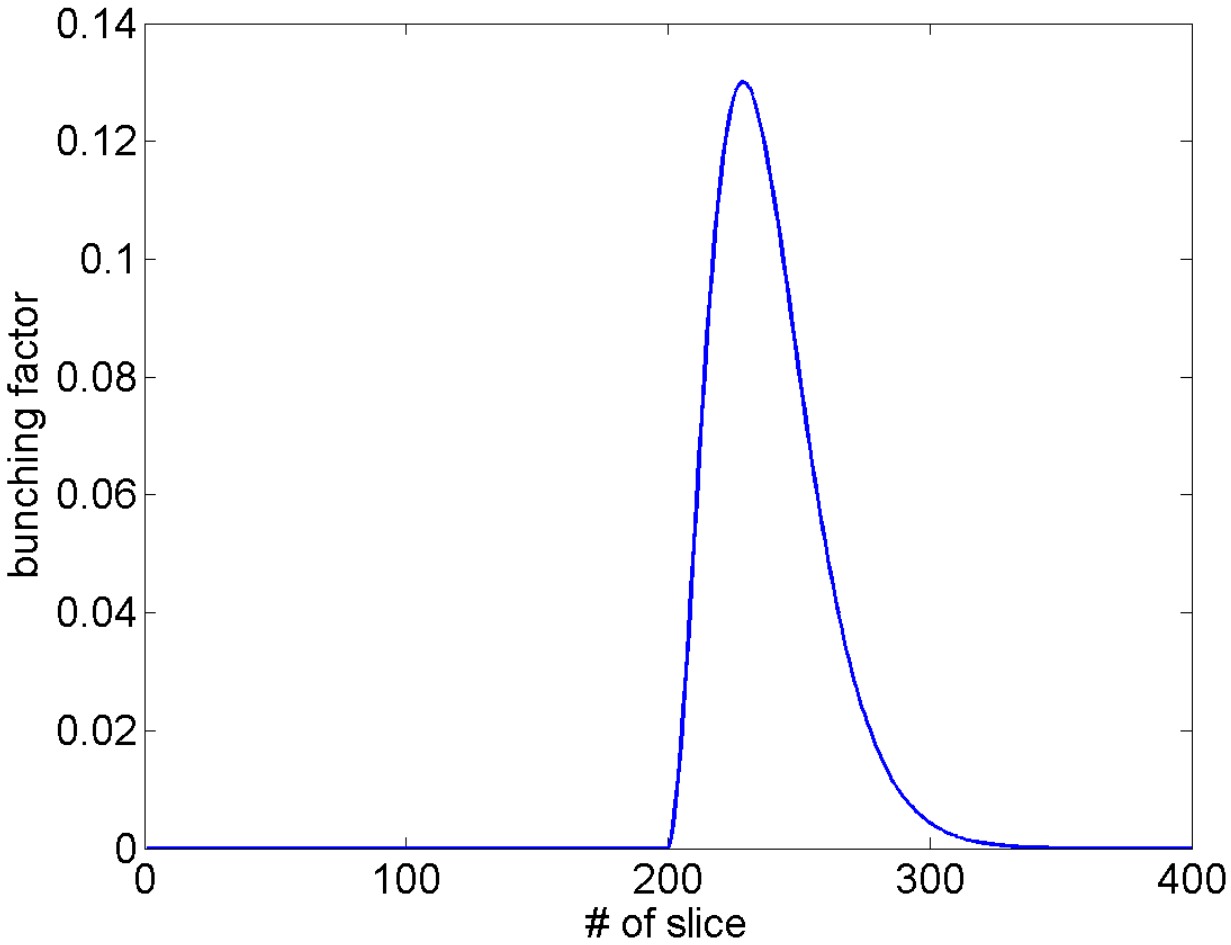
FEL to kicker

- Take the output of GENESIS as the input of kicker simulation
- Run kicker simulation

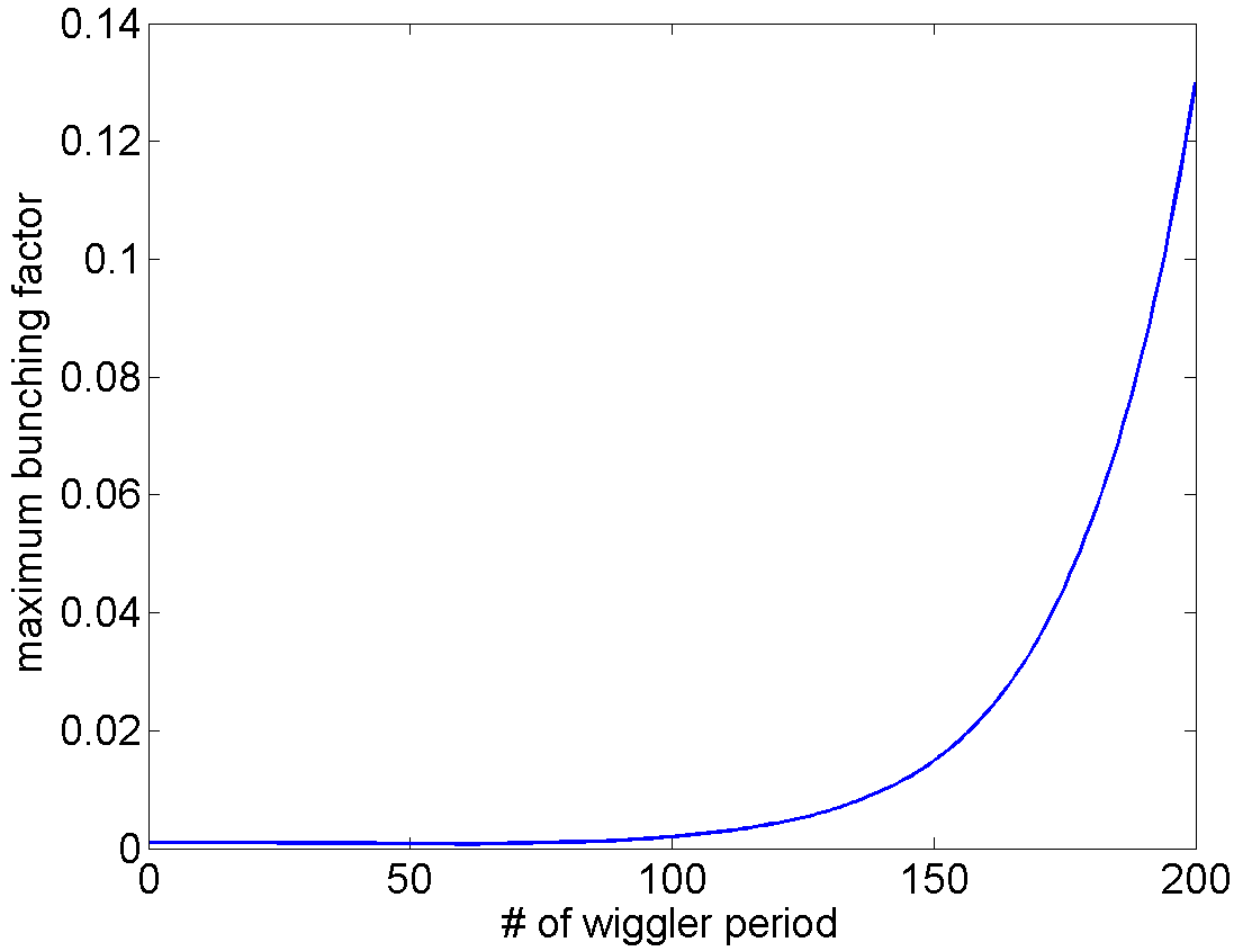
Initial bunching factor in FEL



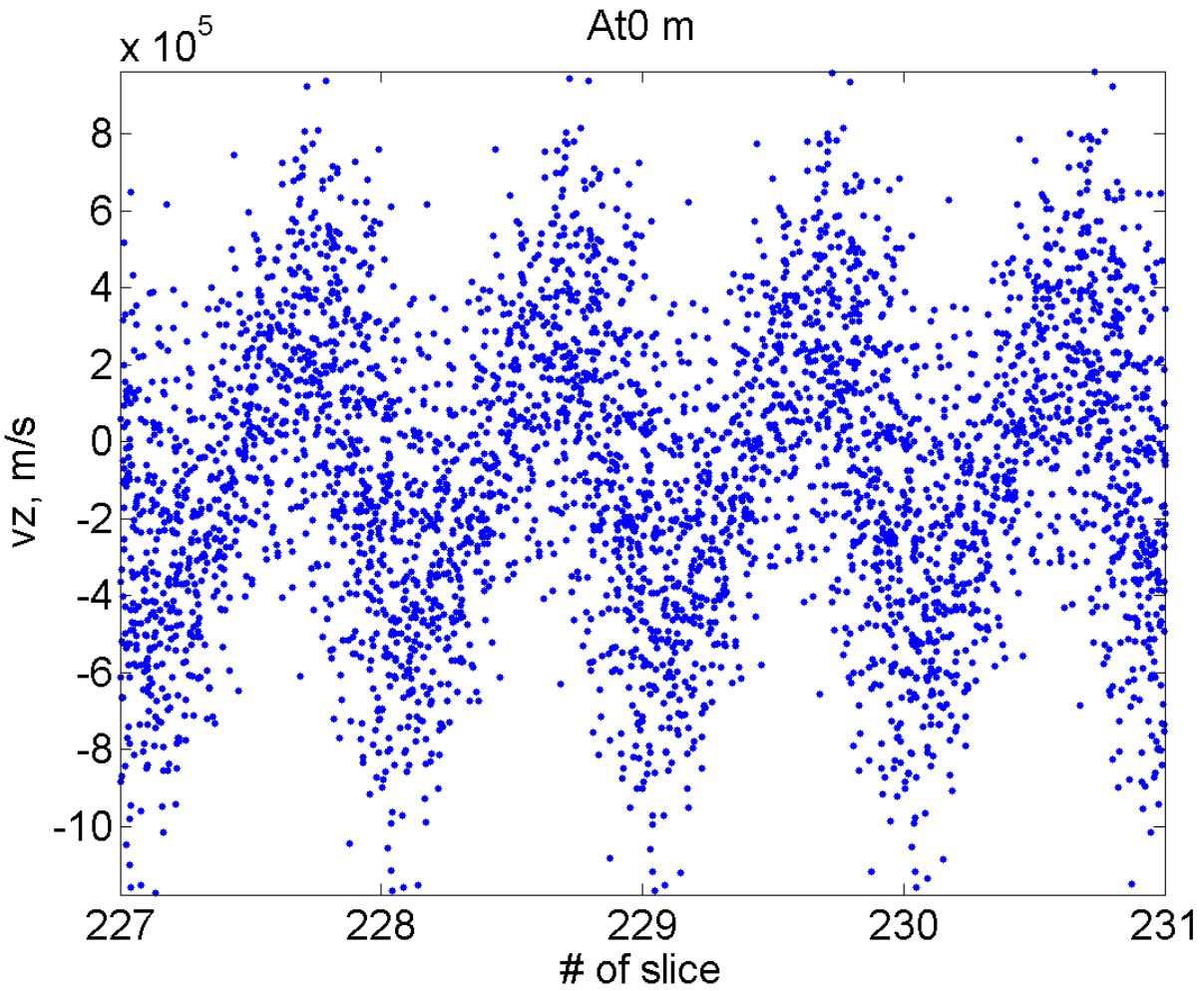
Final bunching factor in FEL



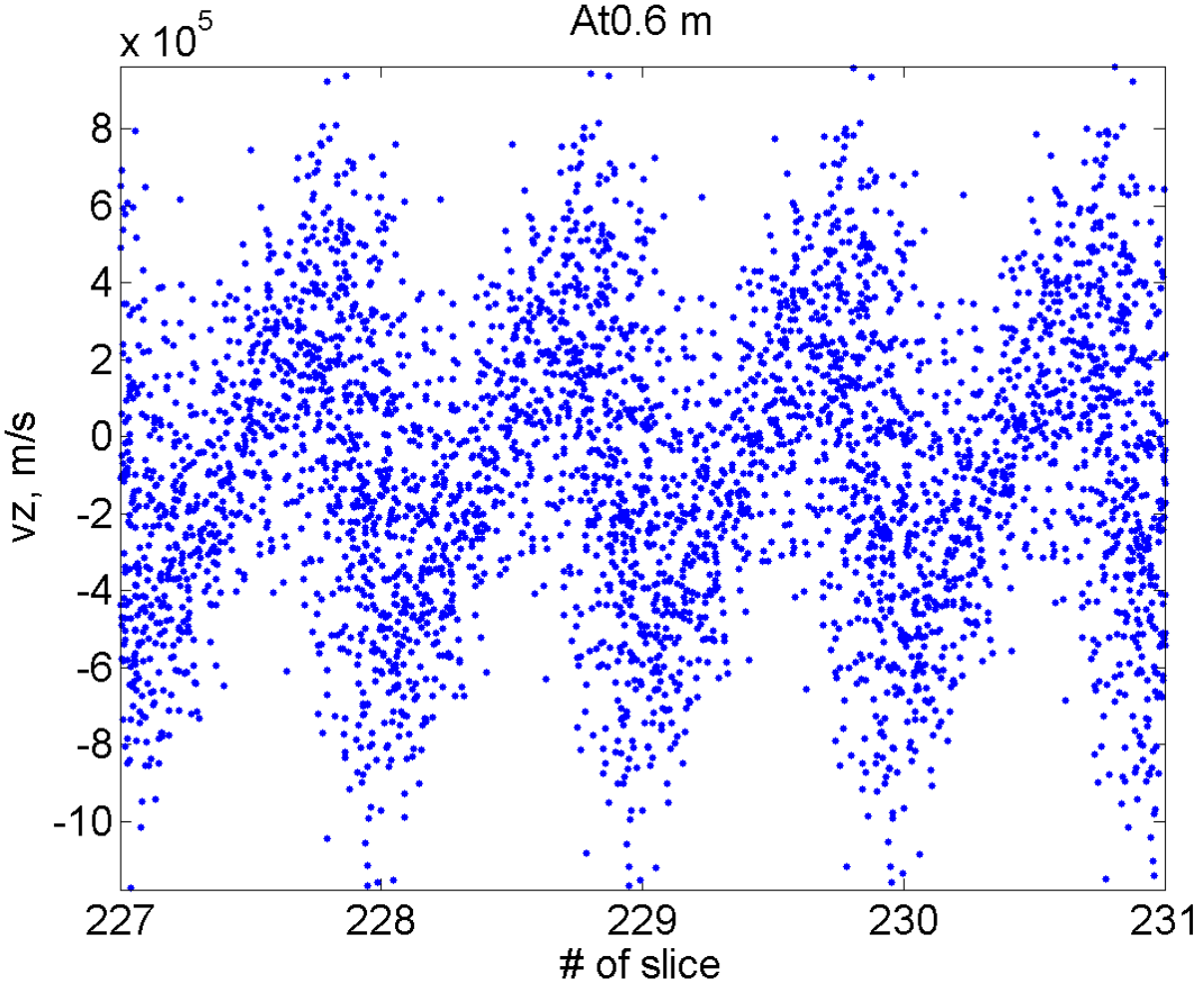
Bunching factor changes in FEL



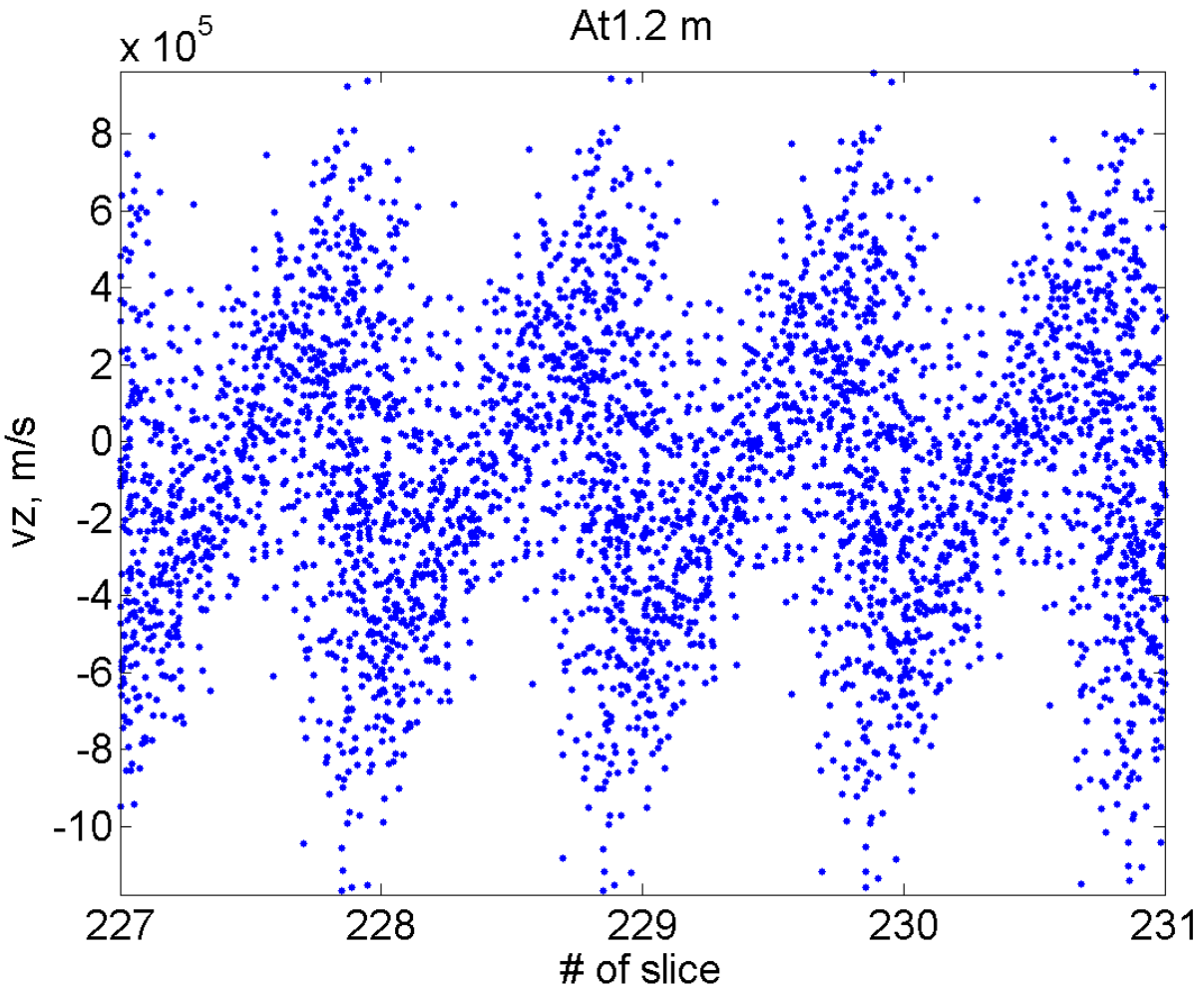
$z - v_z$ plot in kicker



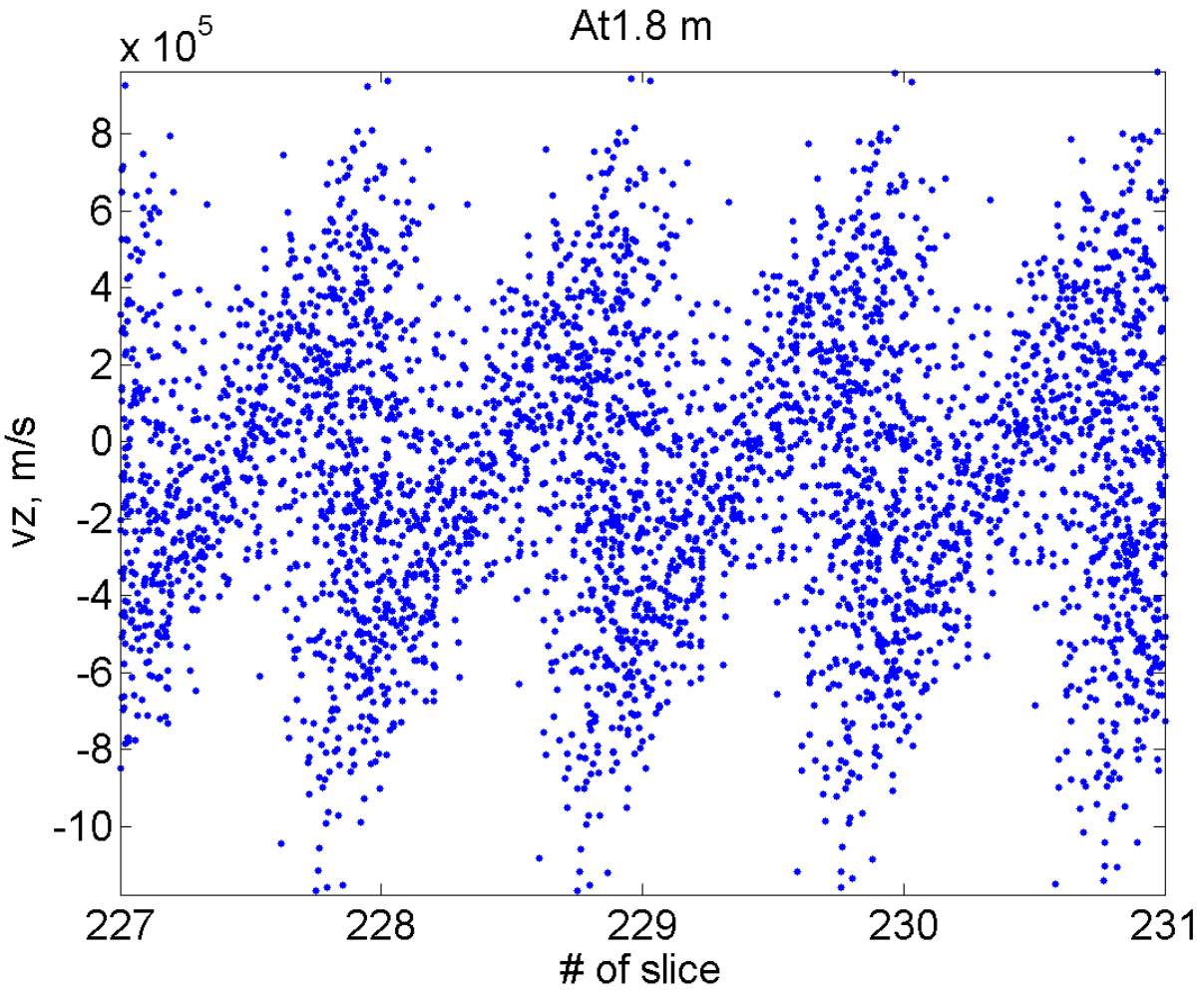
$z - v_z$ plot in kicker



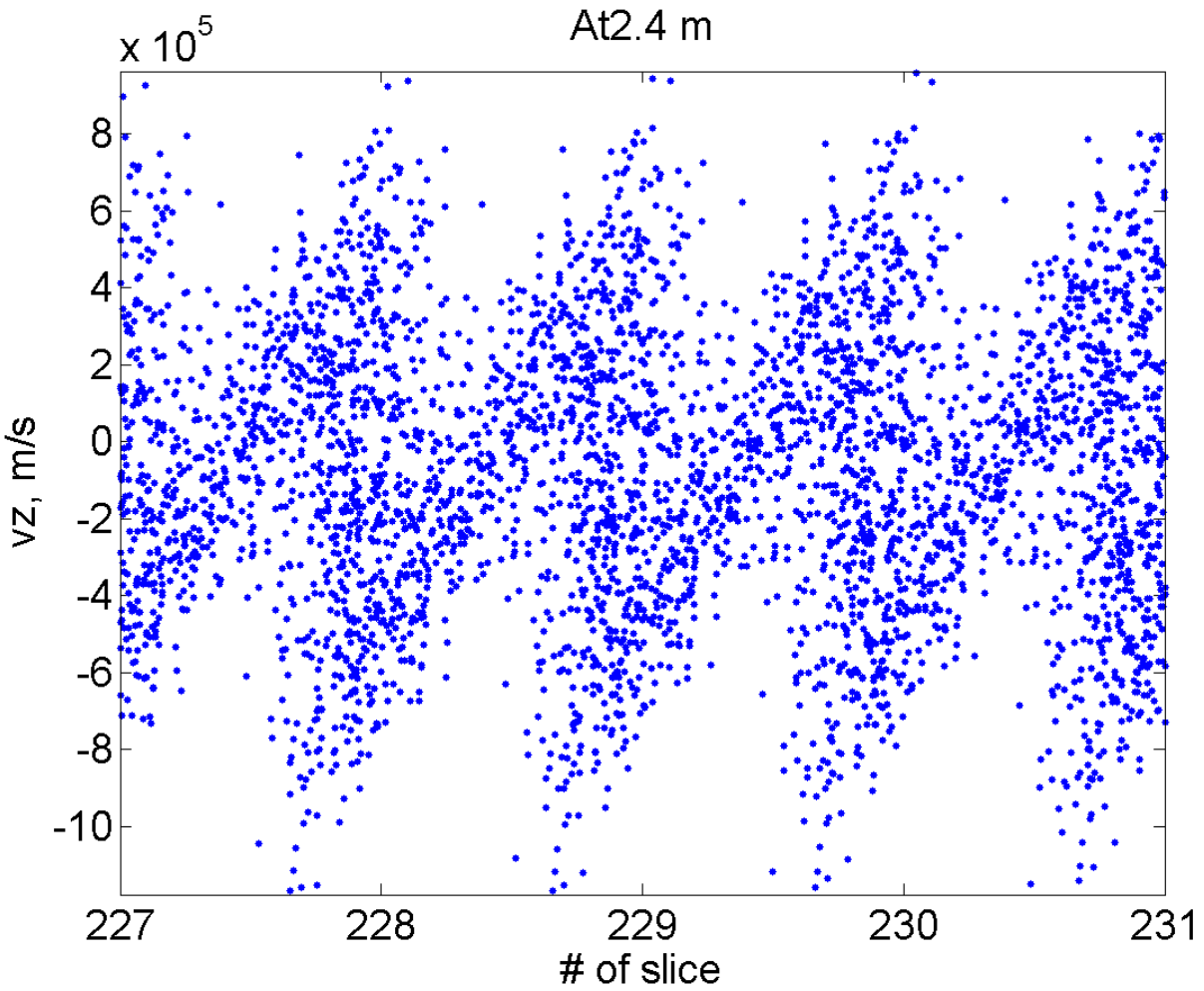
$z - v_z$ plot in kicker



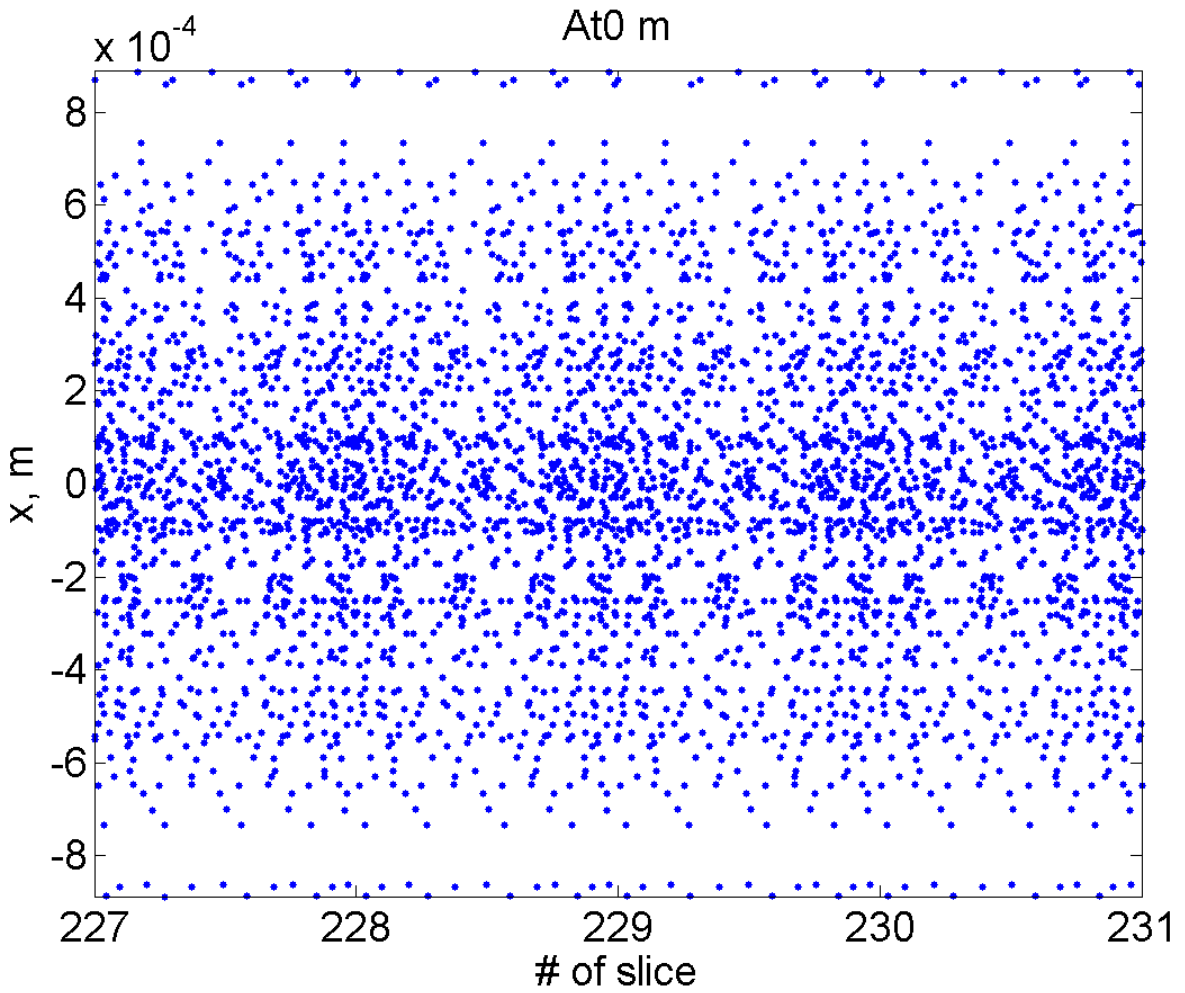
$z - v_z$ plot in kicker



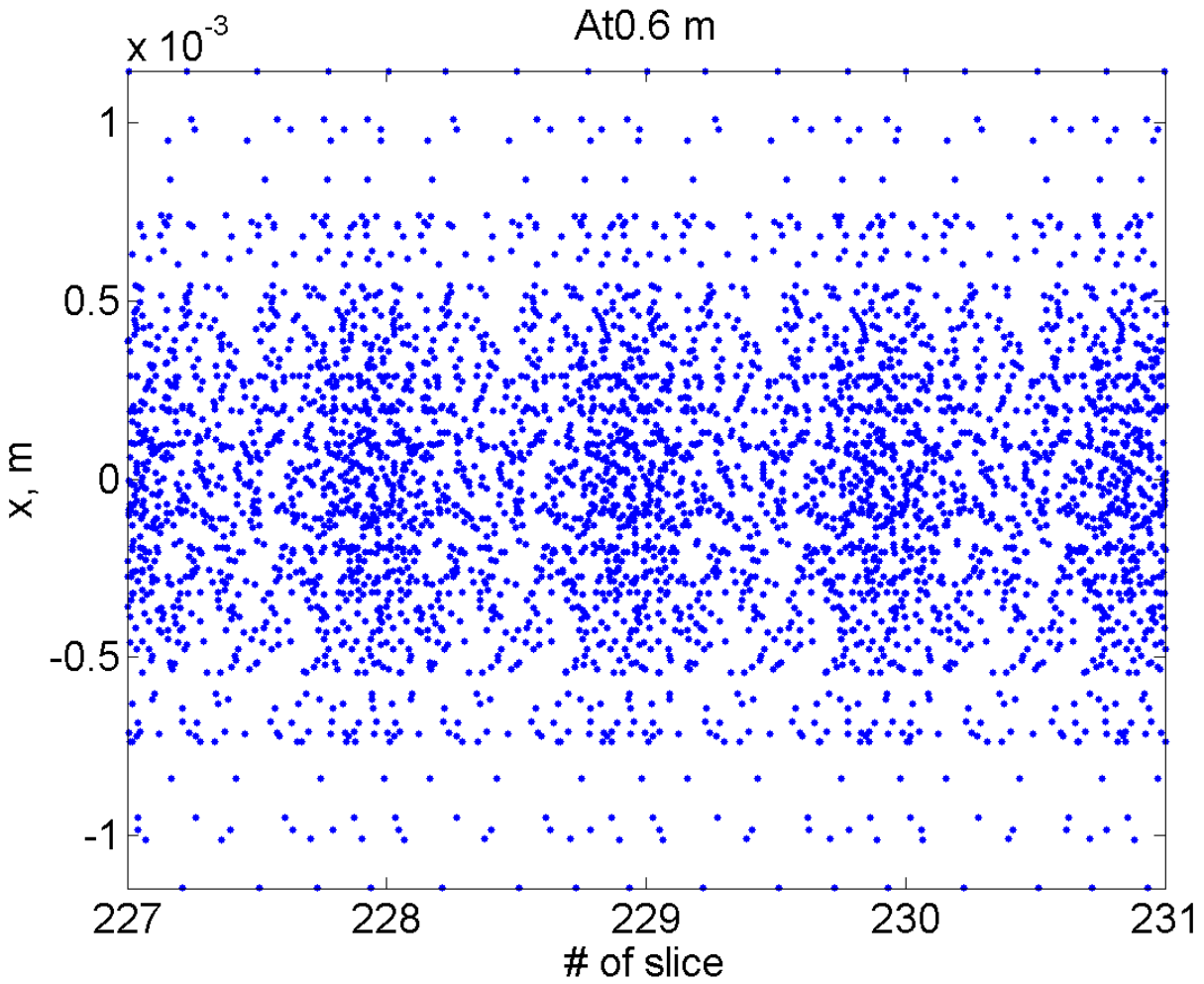
$z - v_z$ plot in kicker



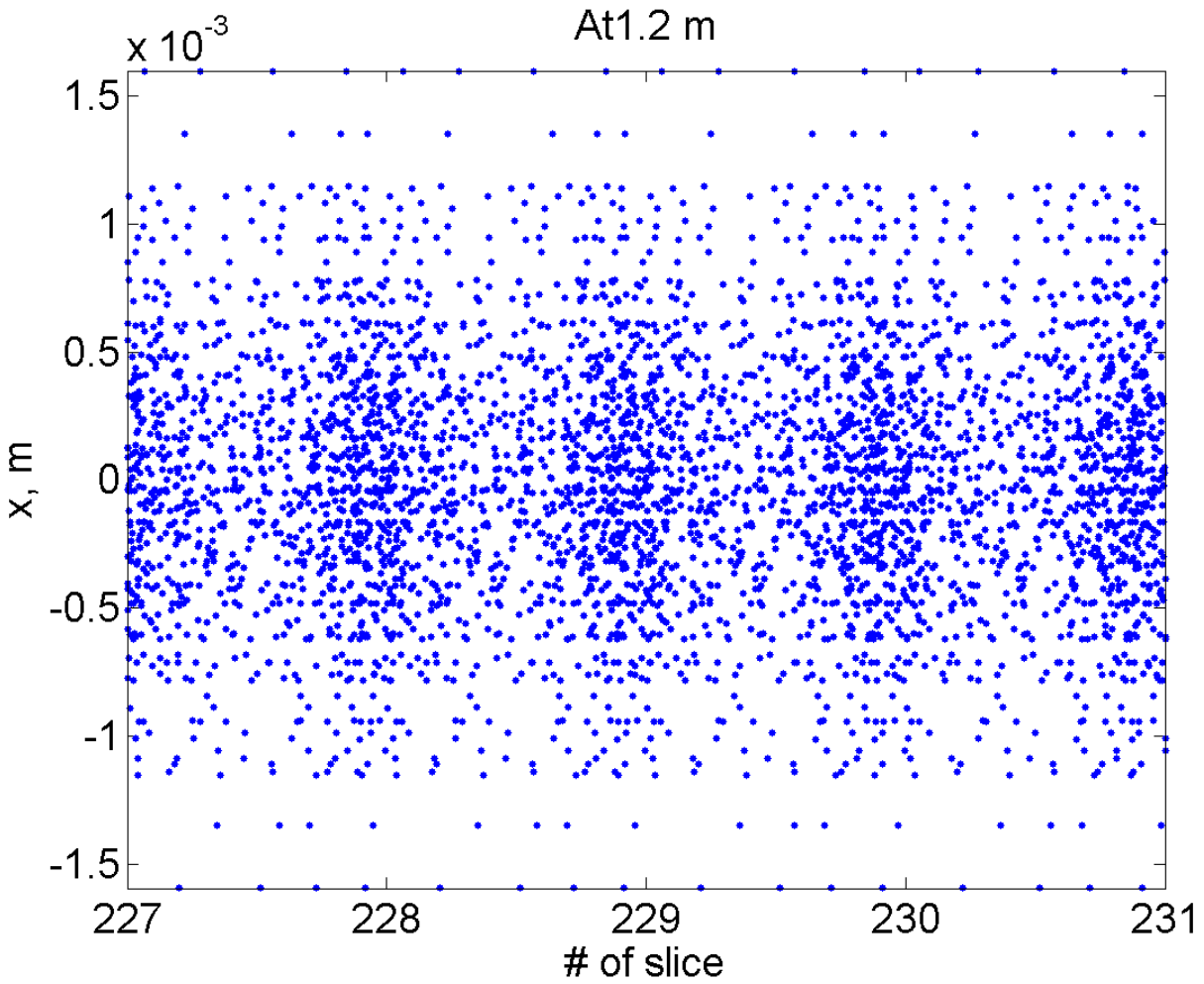
$z - x$ plot in kicker



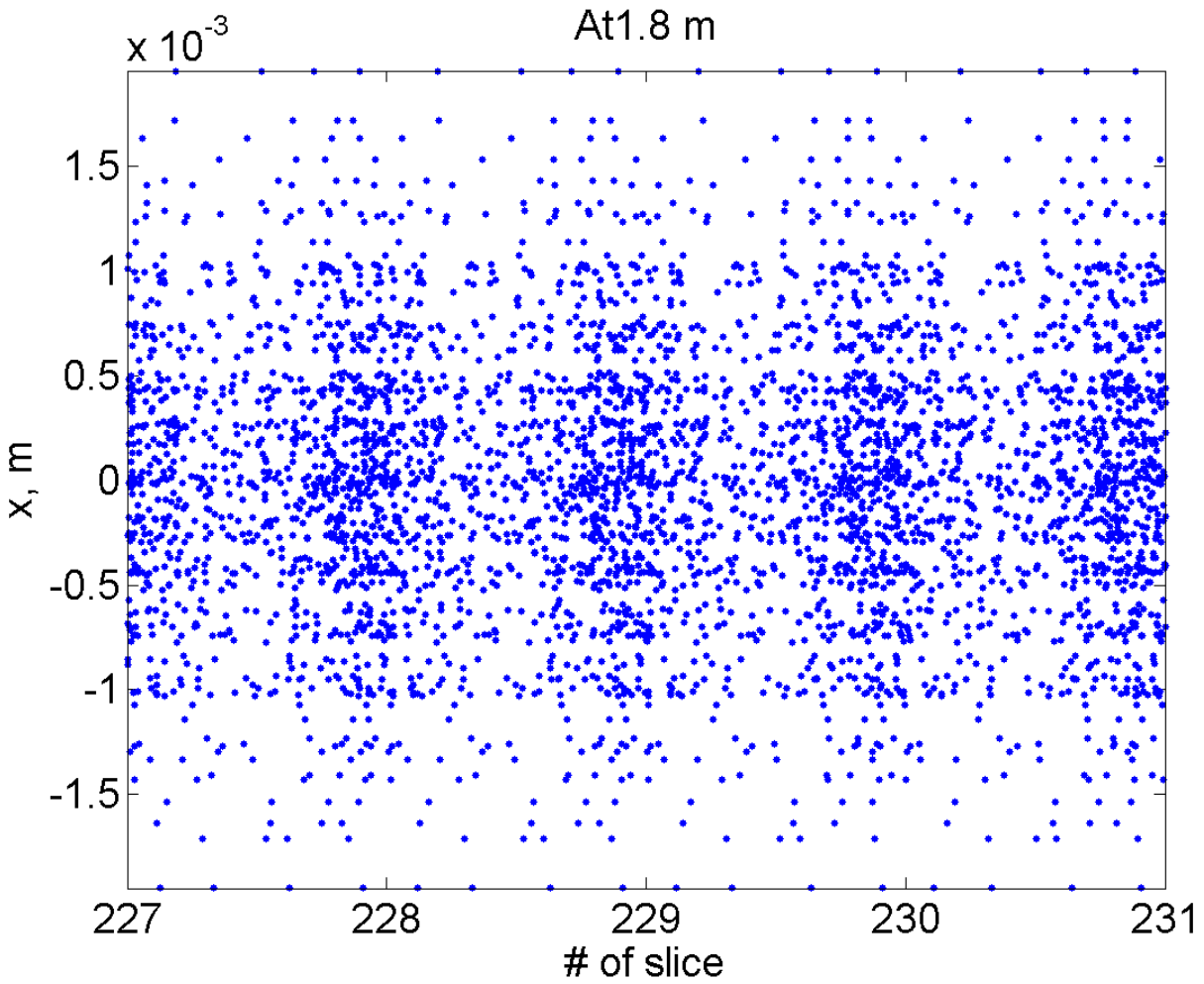
$z - x$ plot in kicker



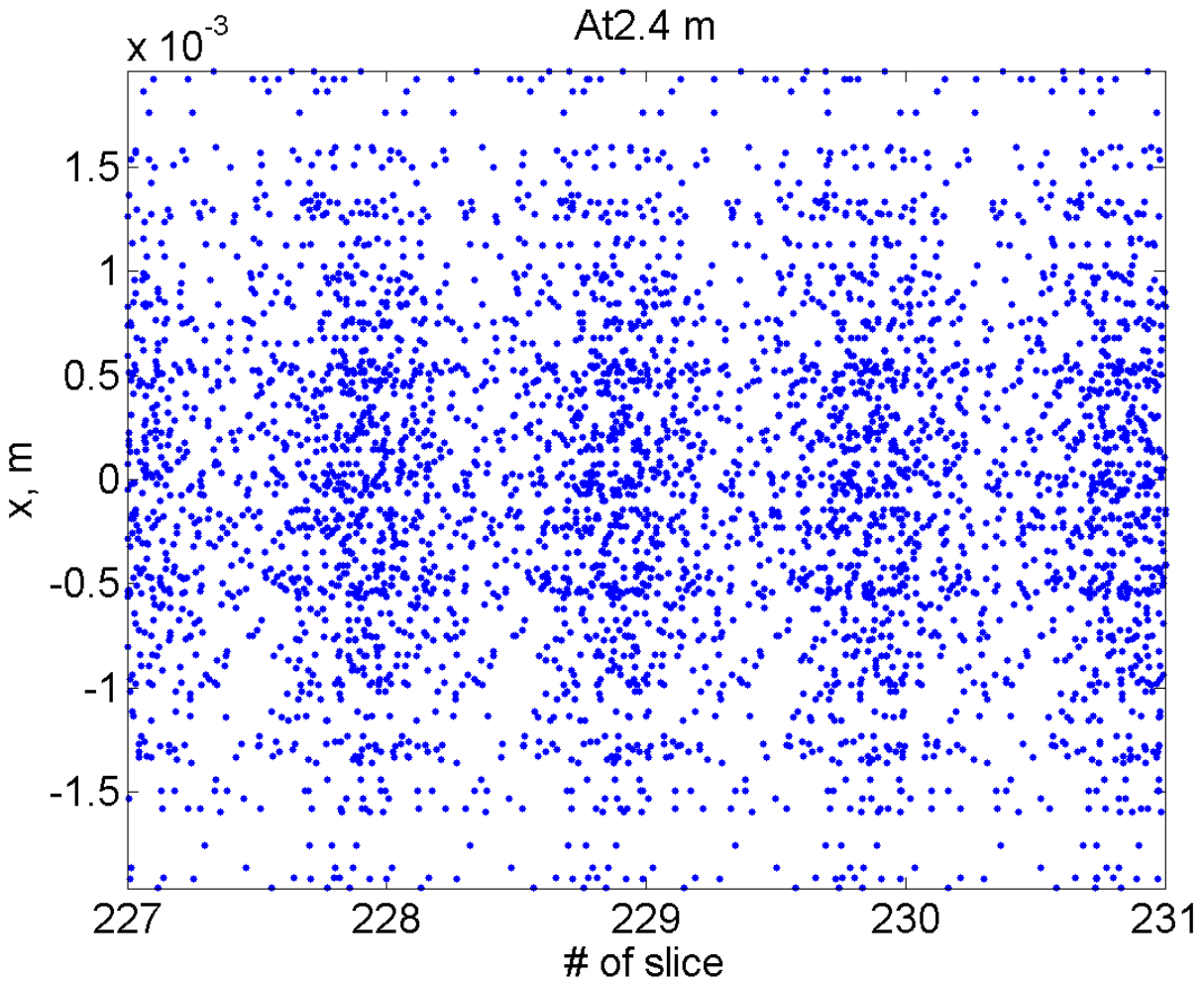
$z - x$ plot in kicker



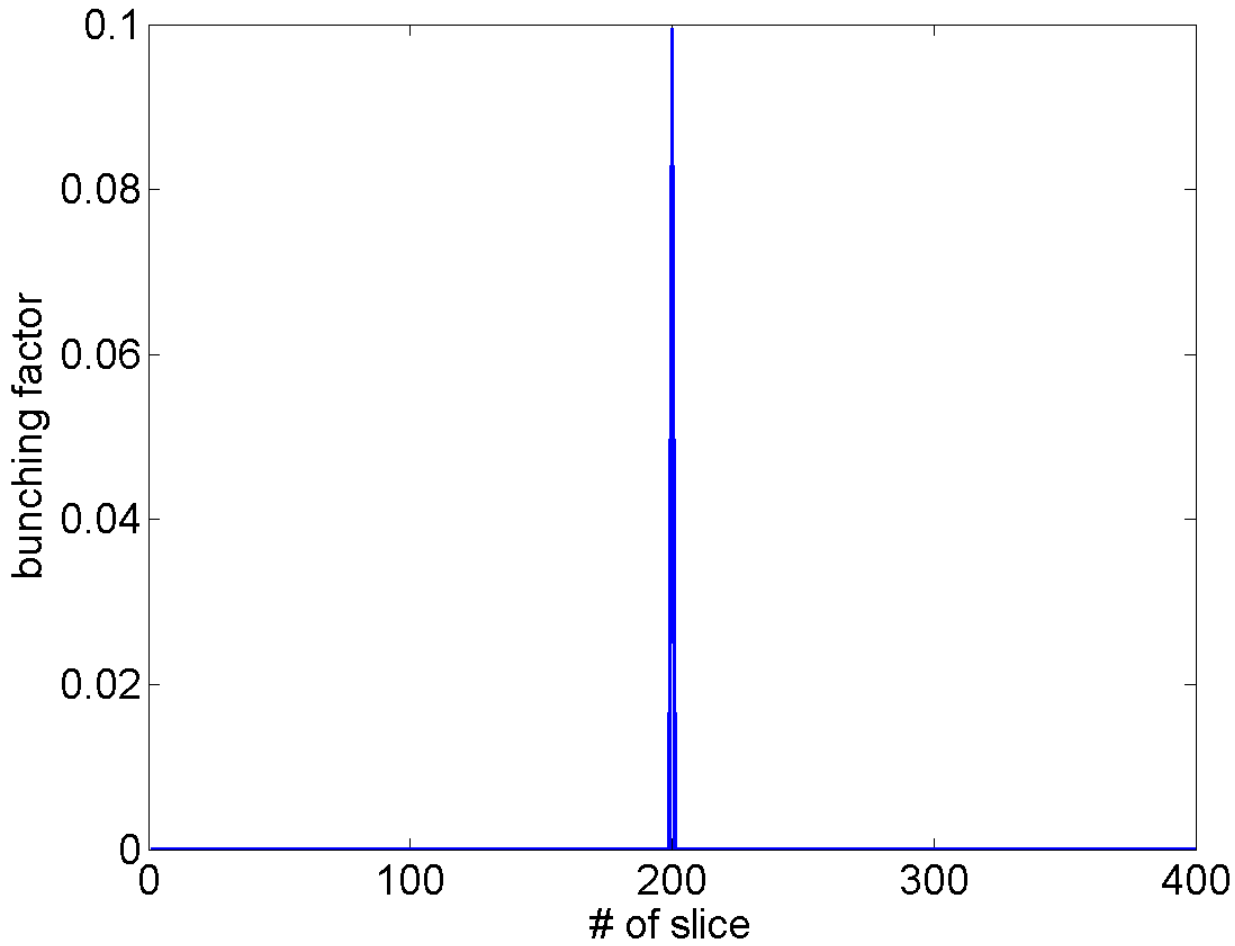
$z - x$ plot in kicker



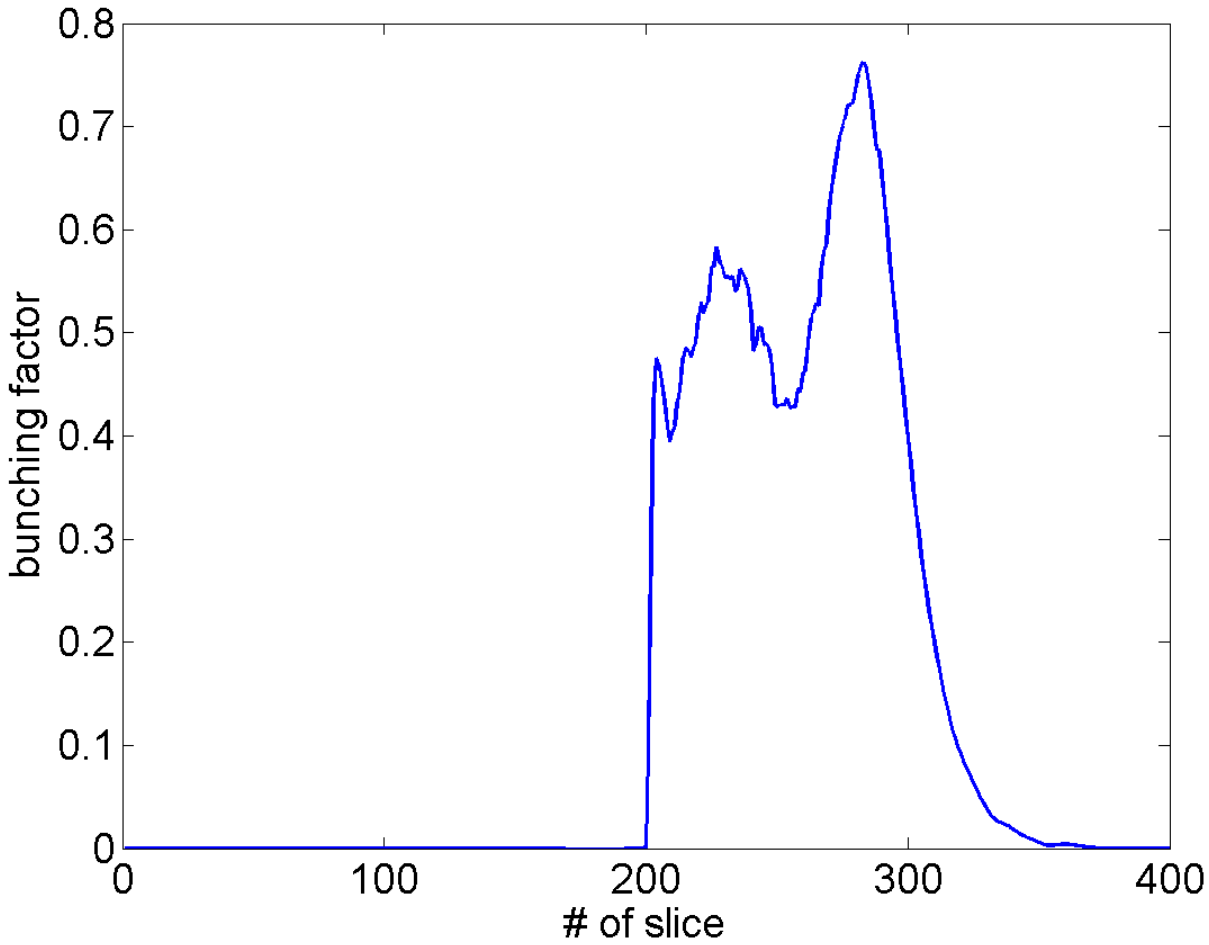
$z - x$ plot in kicker



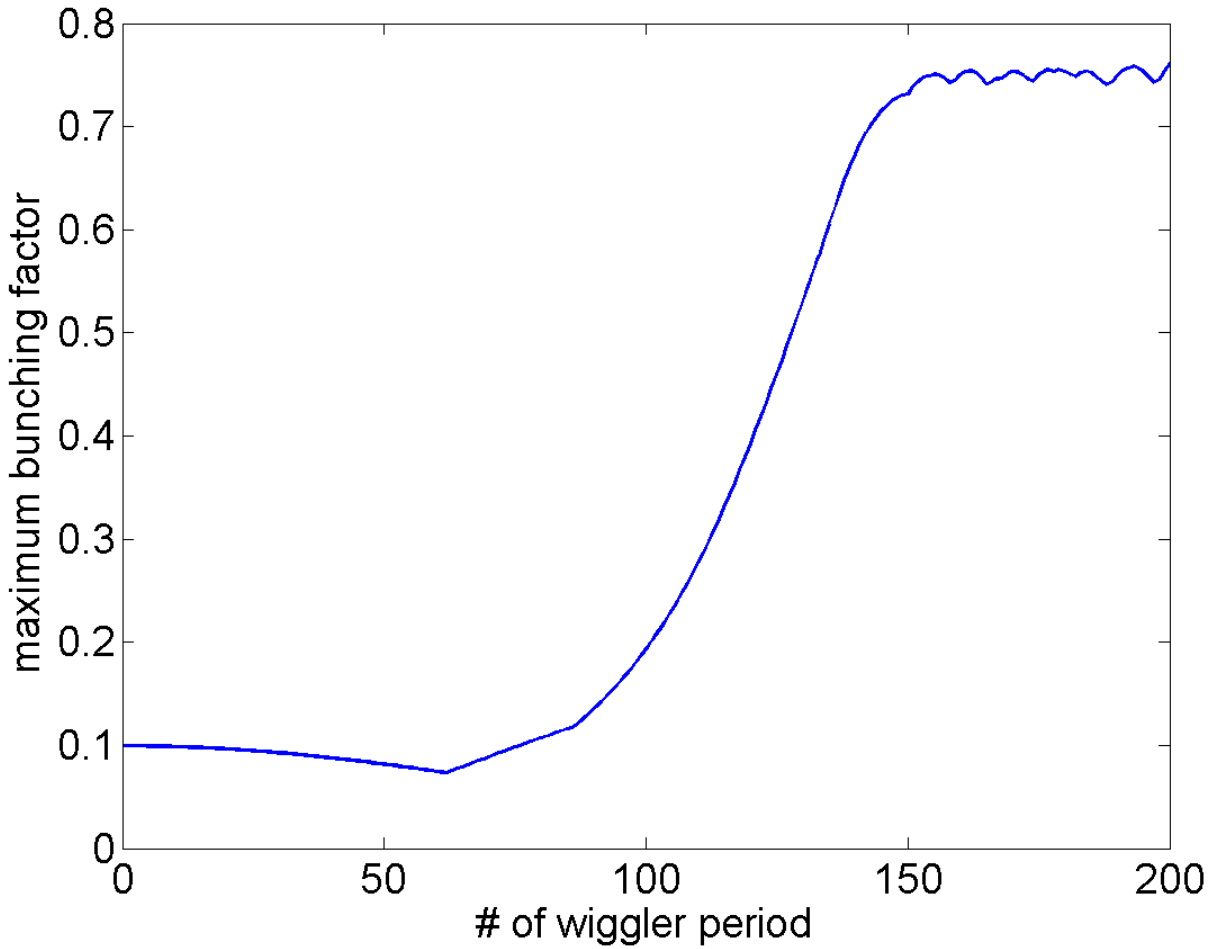
Initial bunching factor in FEL



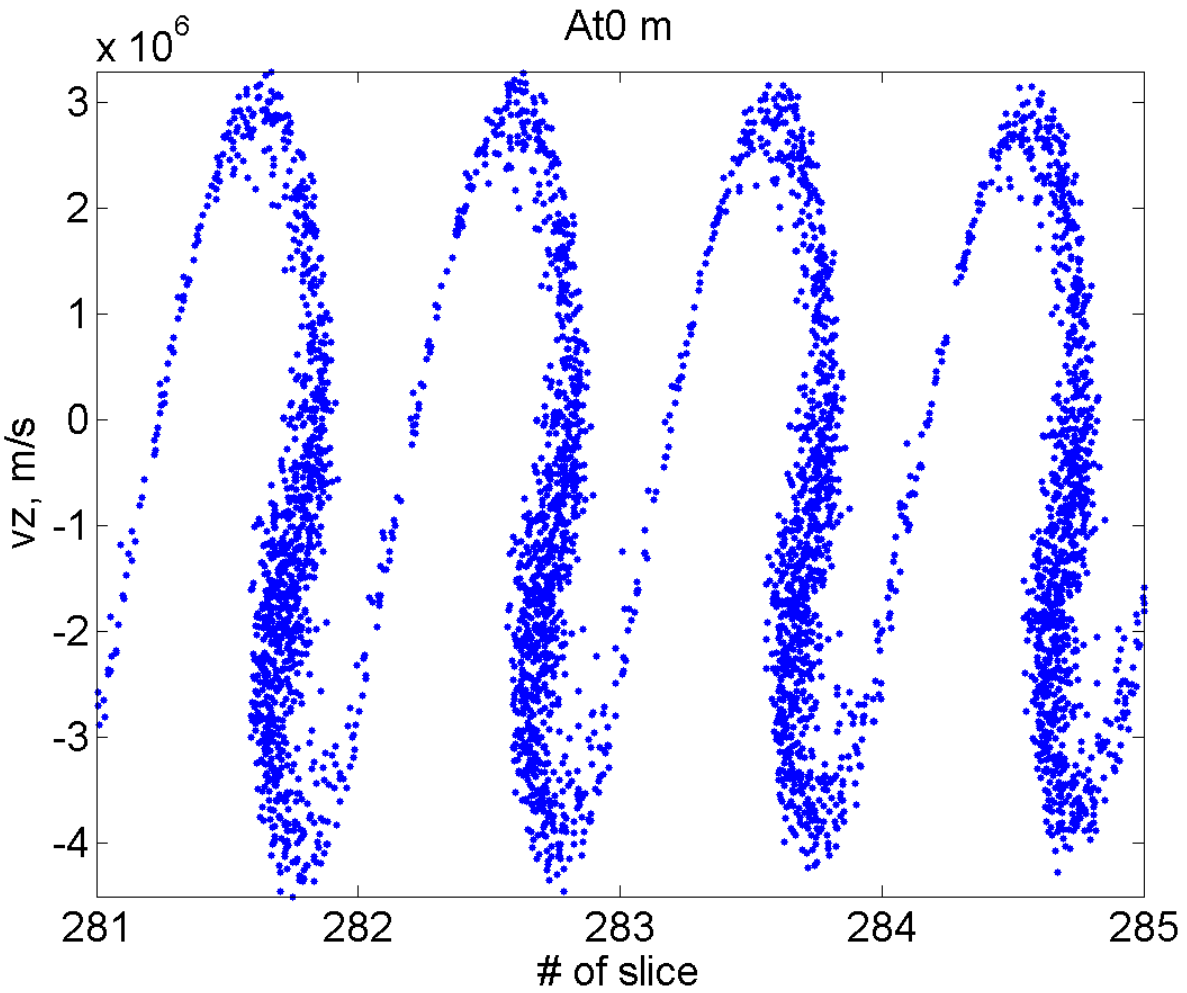
Final bunching factor in FEL



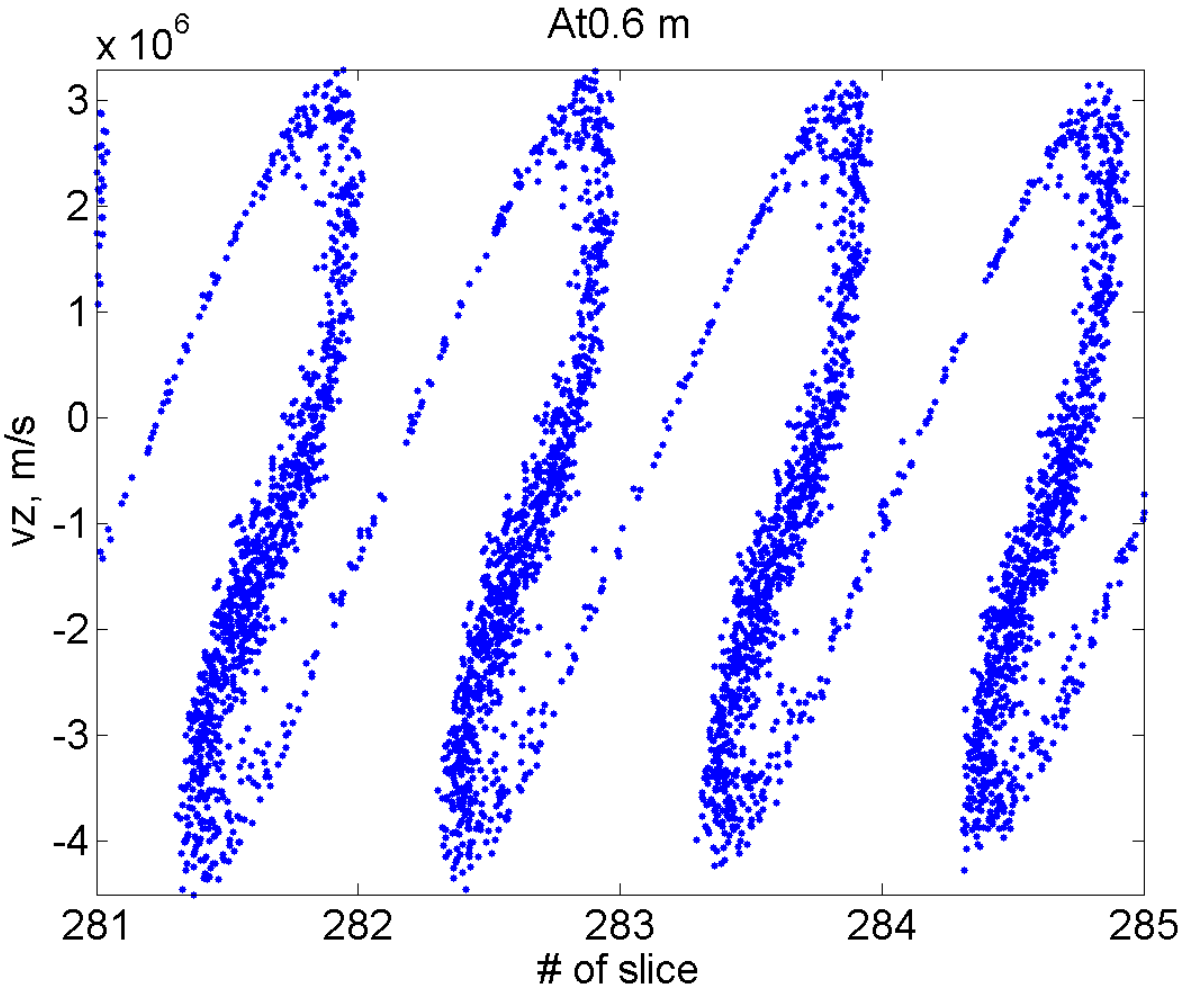
Bunching factor changes in FEL



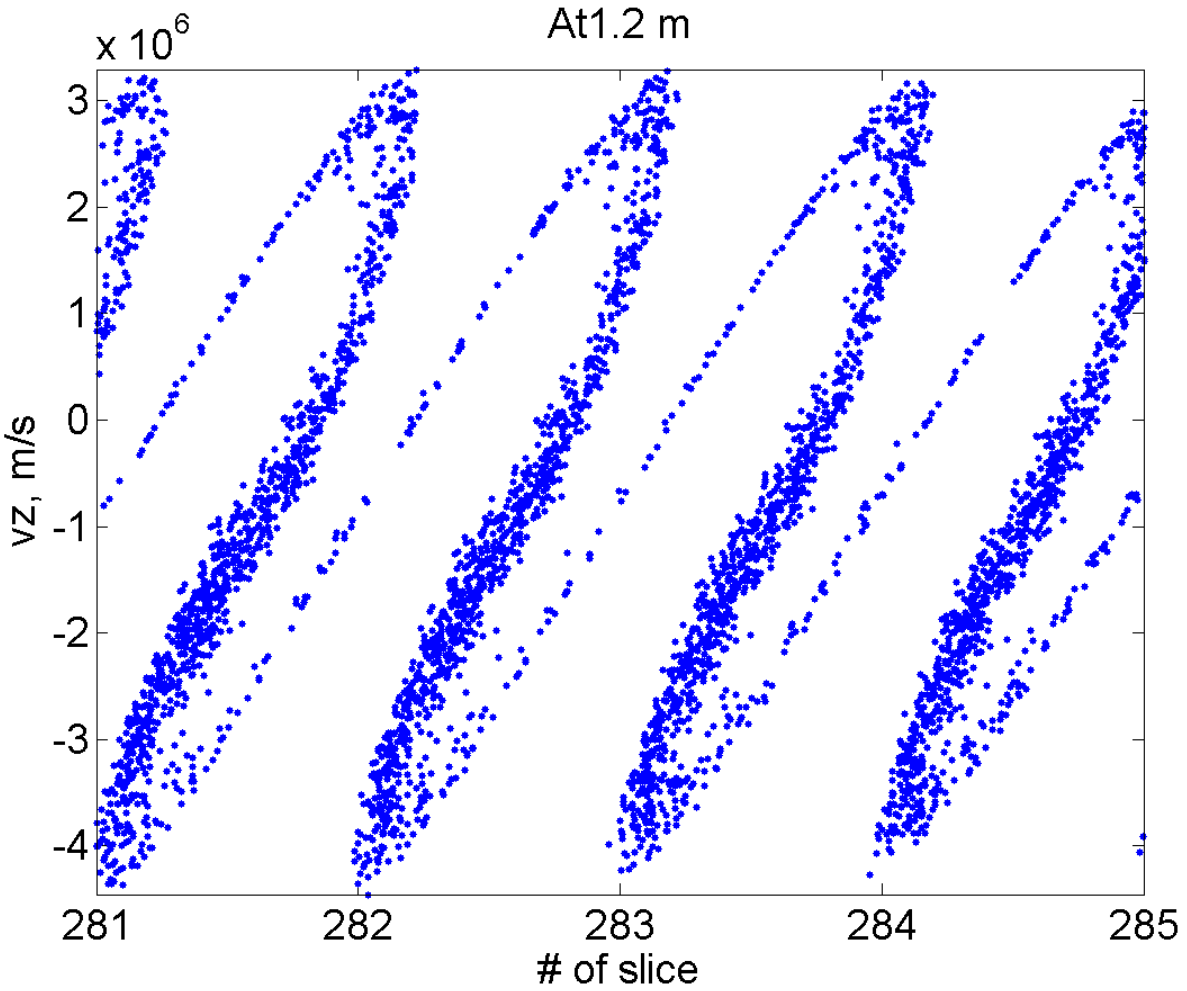
$z - v_z$ plot in kicker



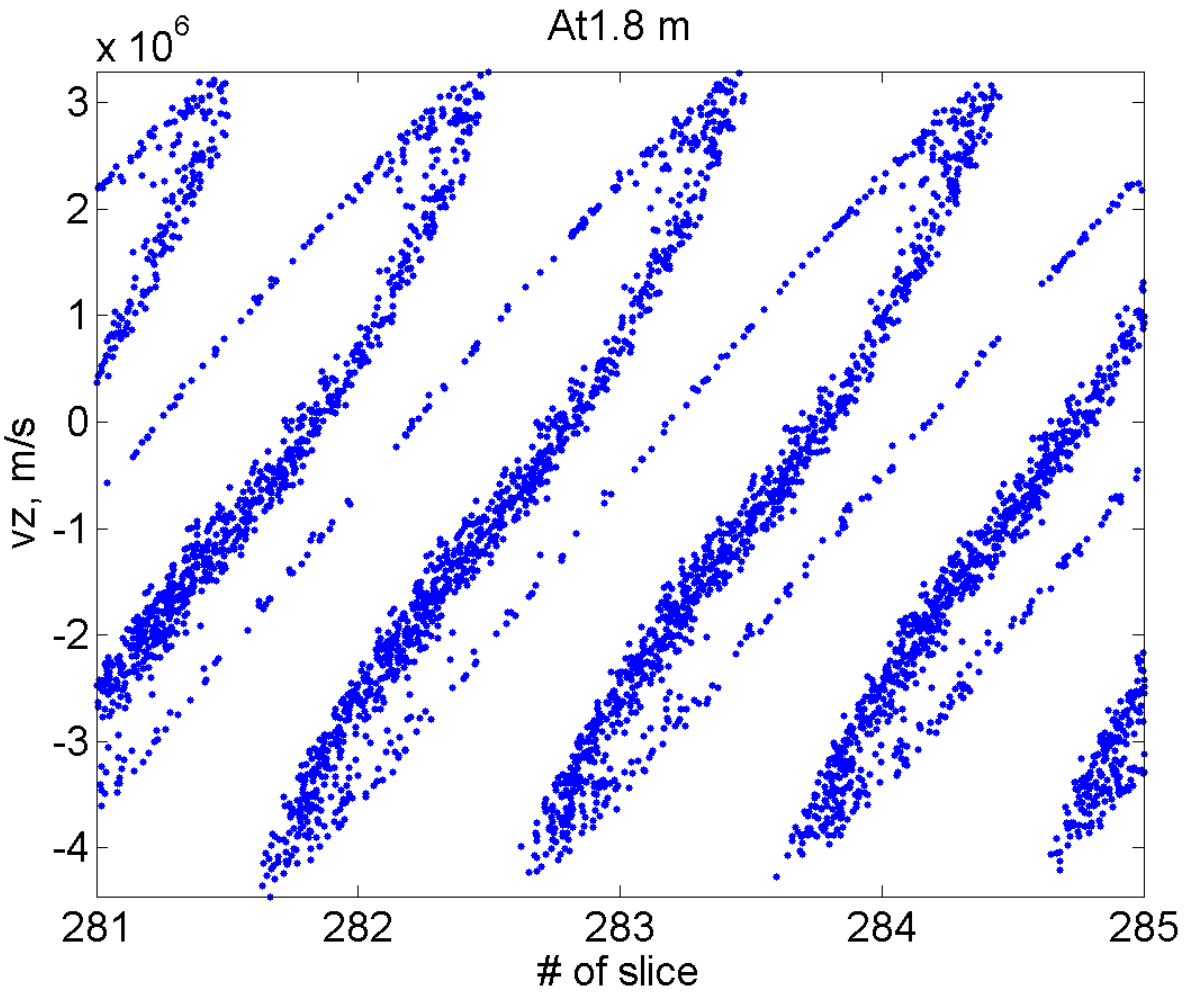
$z - v_z$ plot in kicker



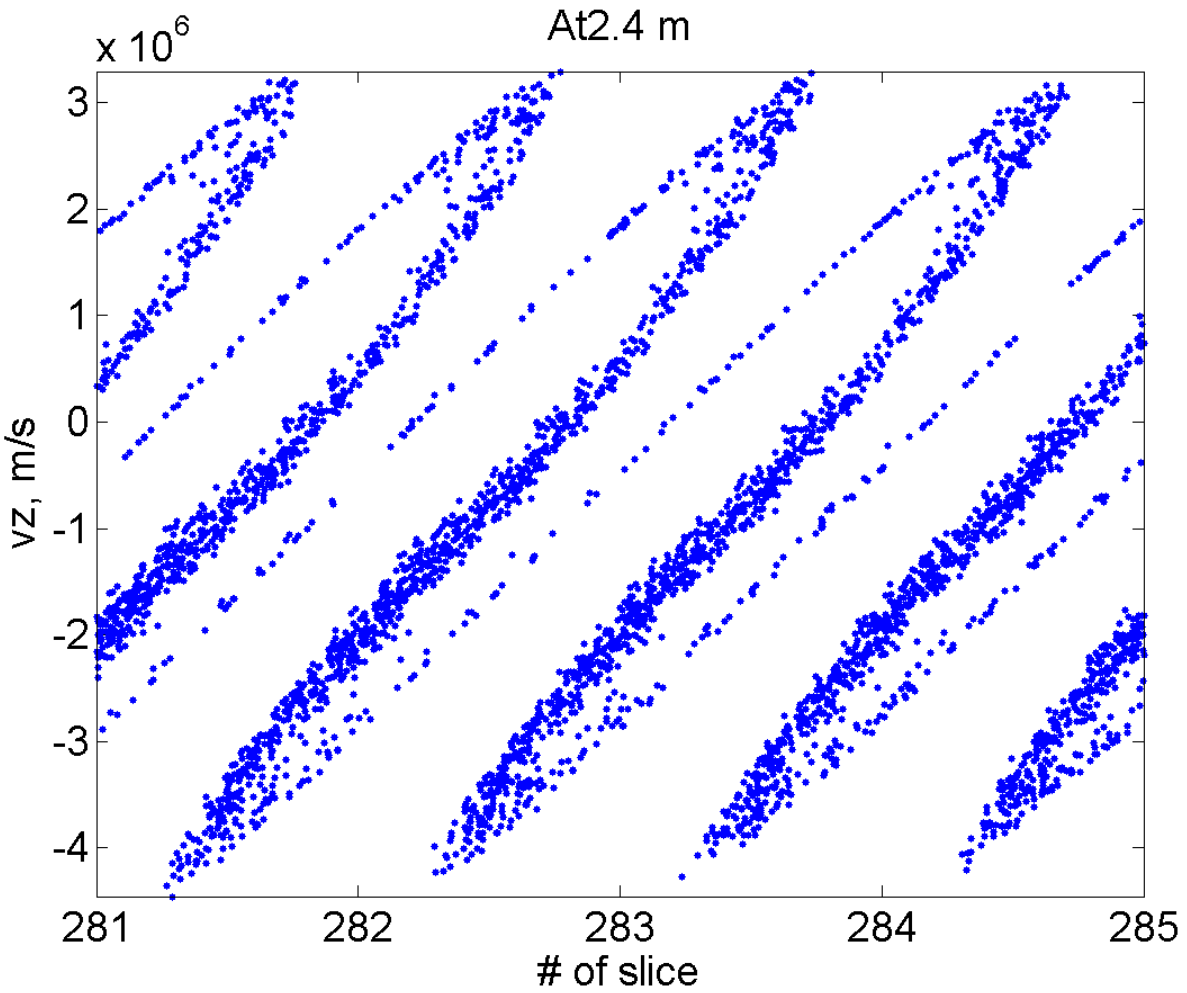
$z - v_z$ plot in kicker



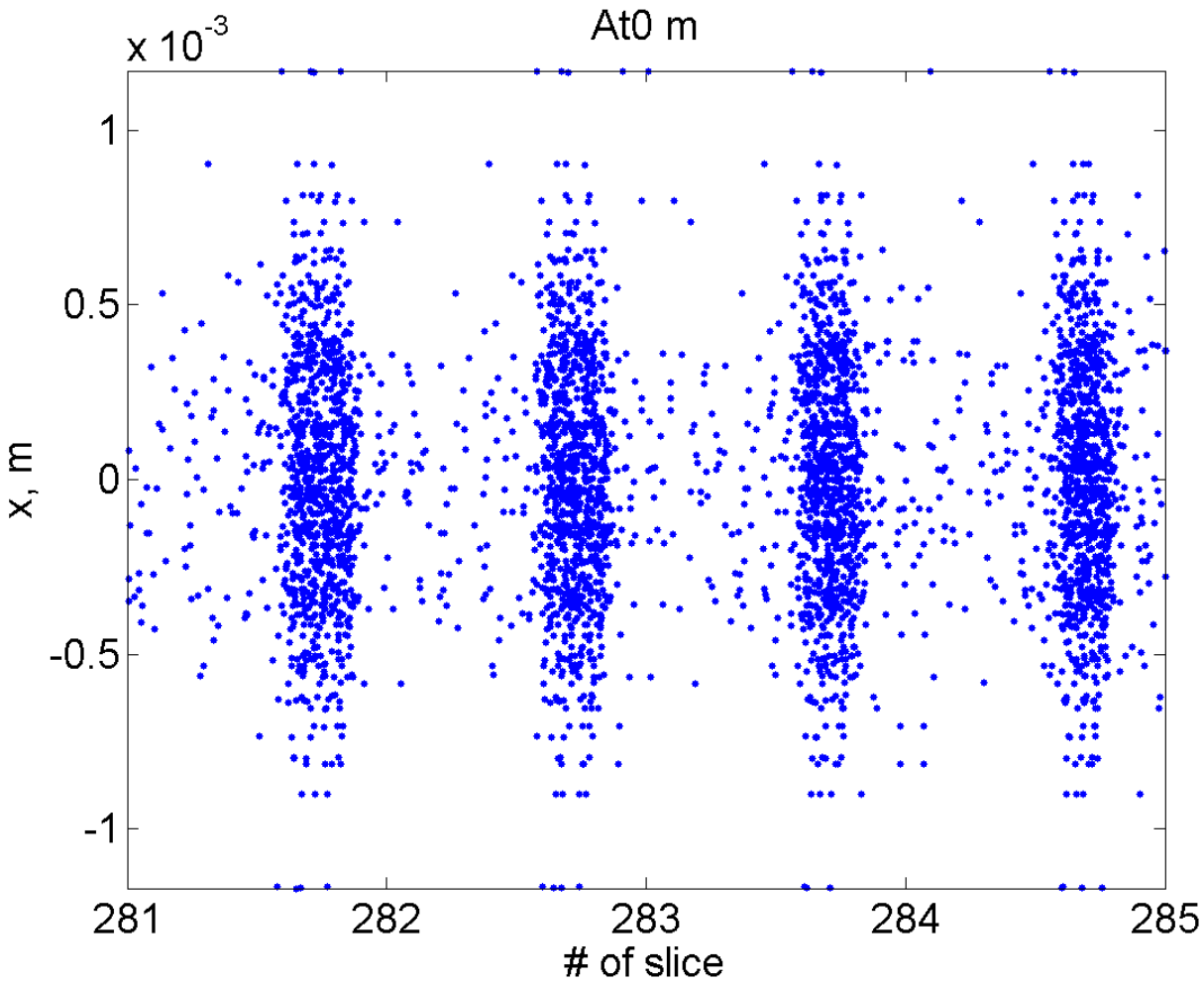
$z - v_z$ plot in kicker



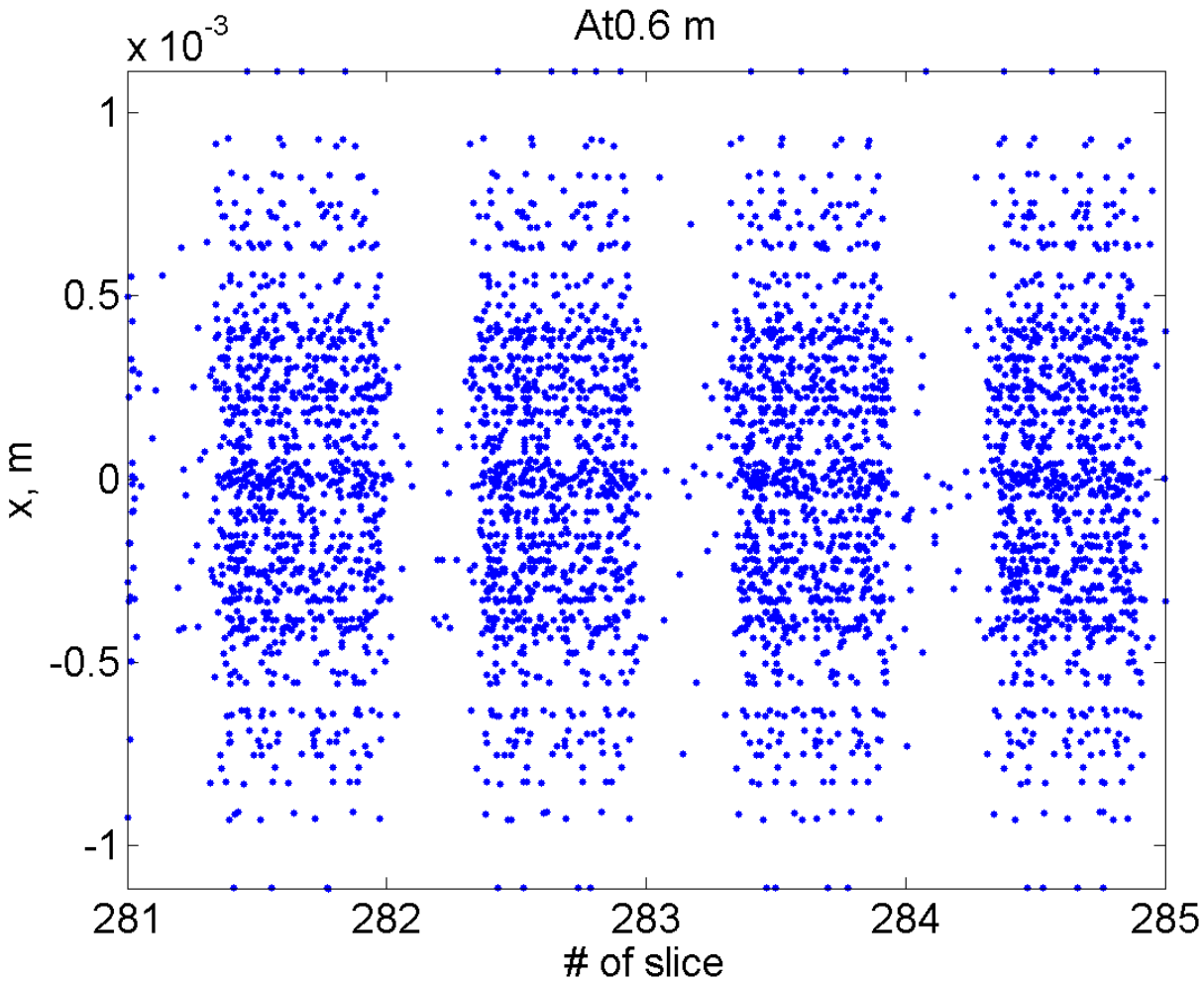
$z - v_z$ plot in kicker



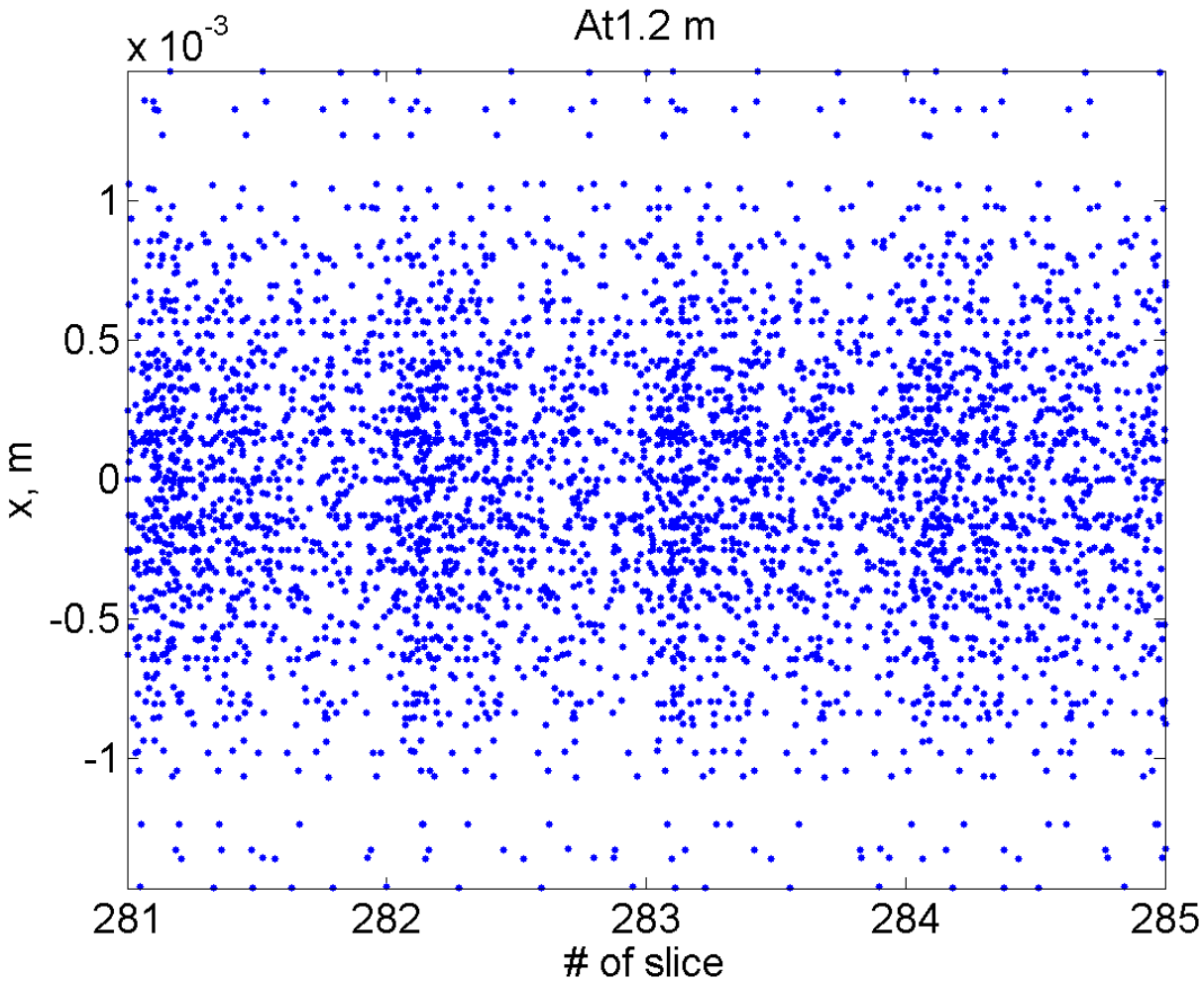
$z - x$ plot in kicker



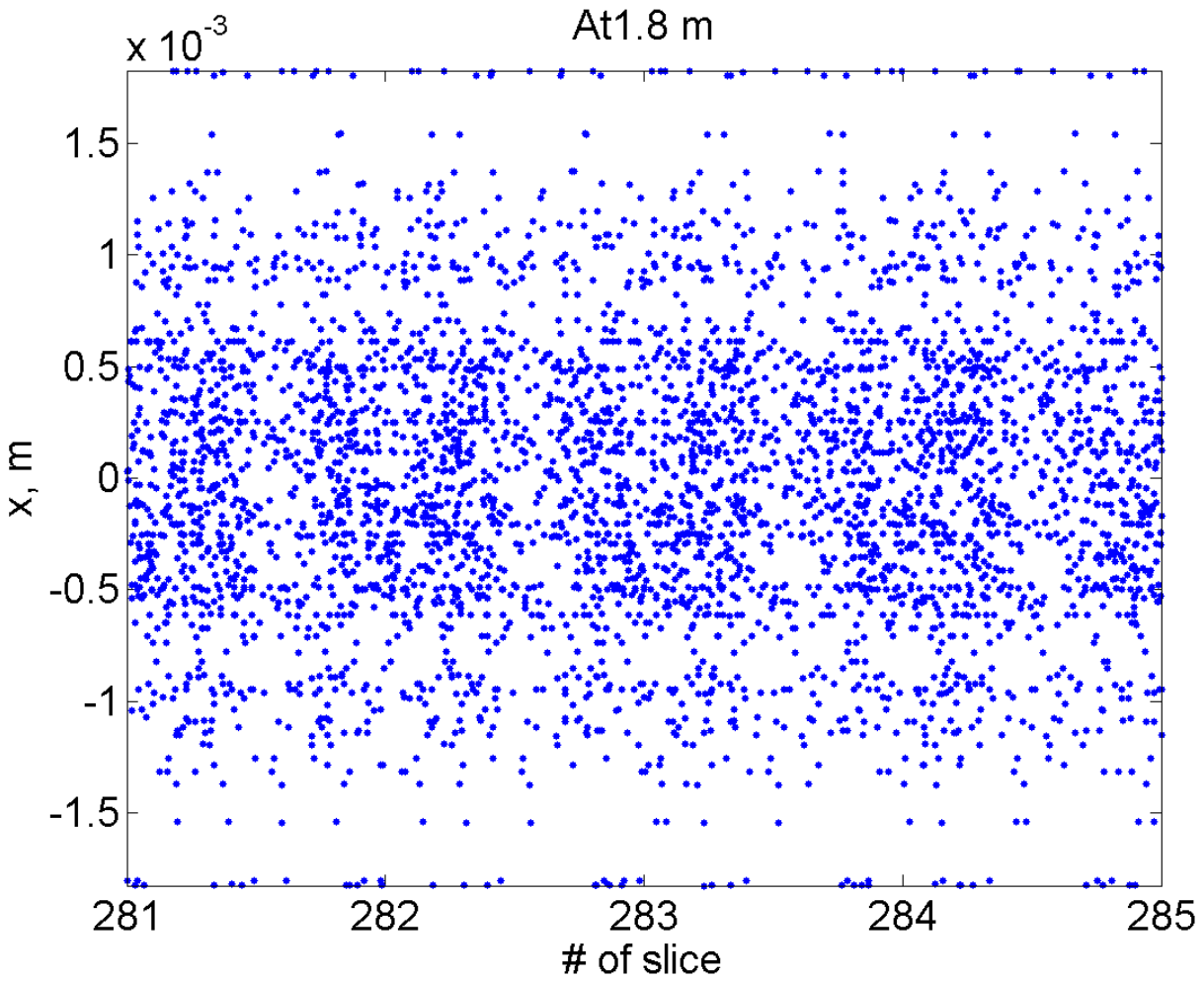
$z - x$ plot in kicker



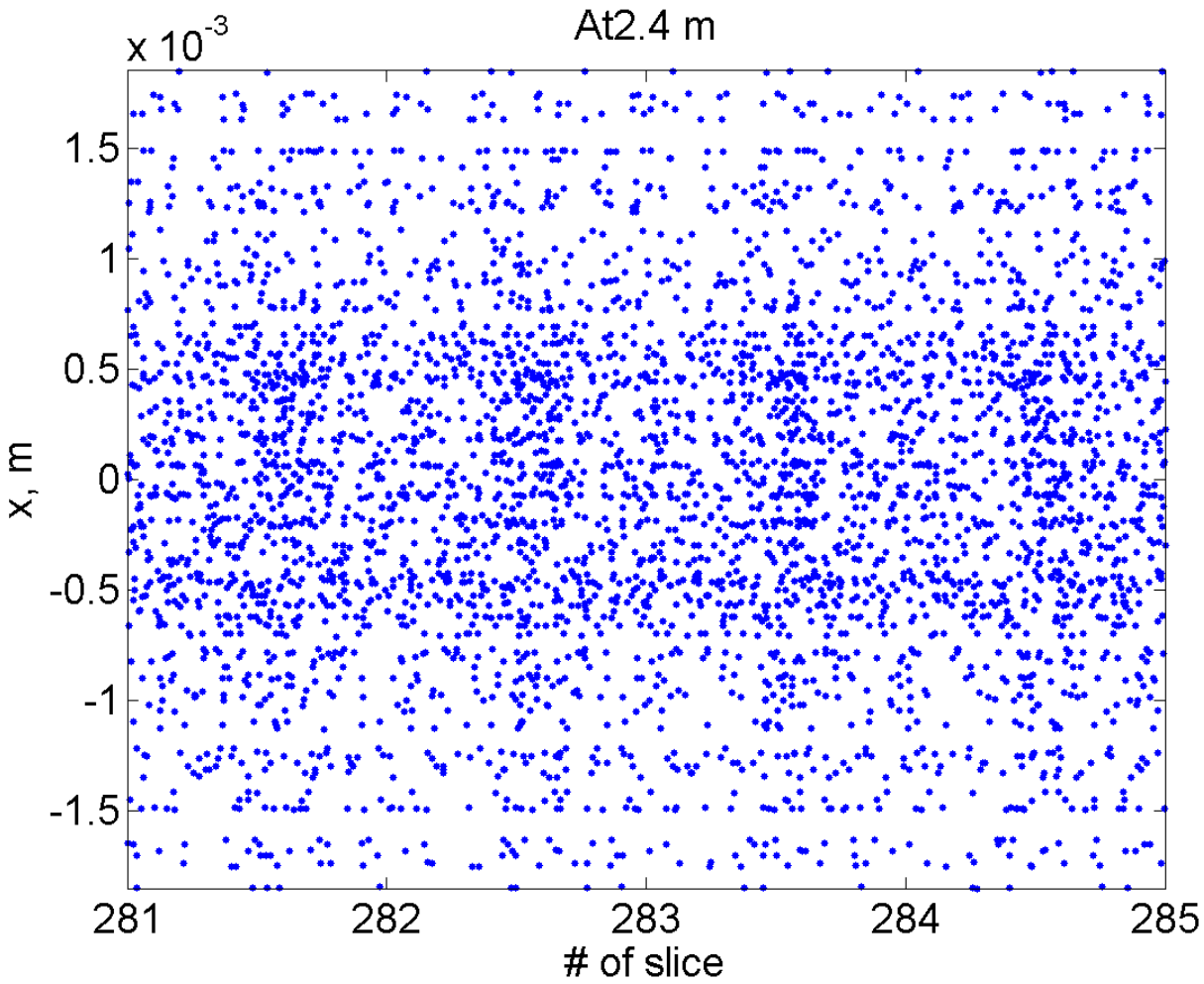
$z - x$ plot in kicker



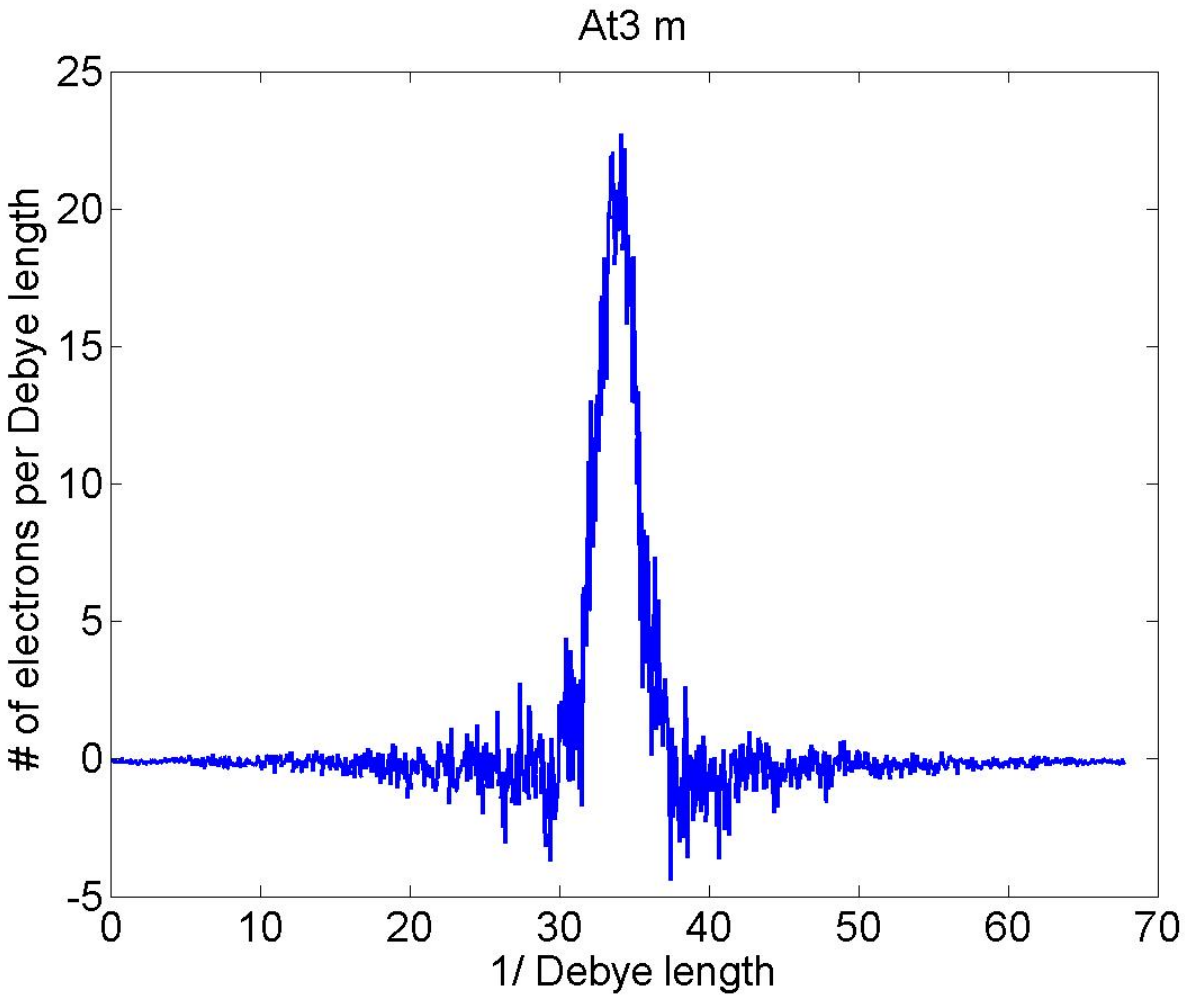
$z - x$ plot in kicker



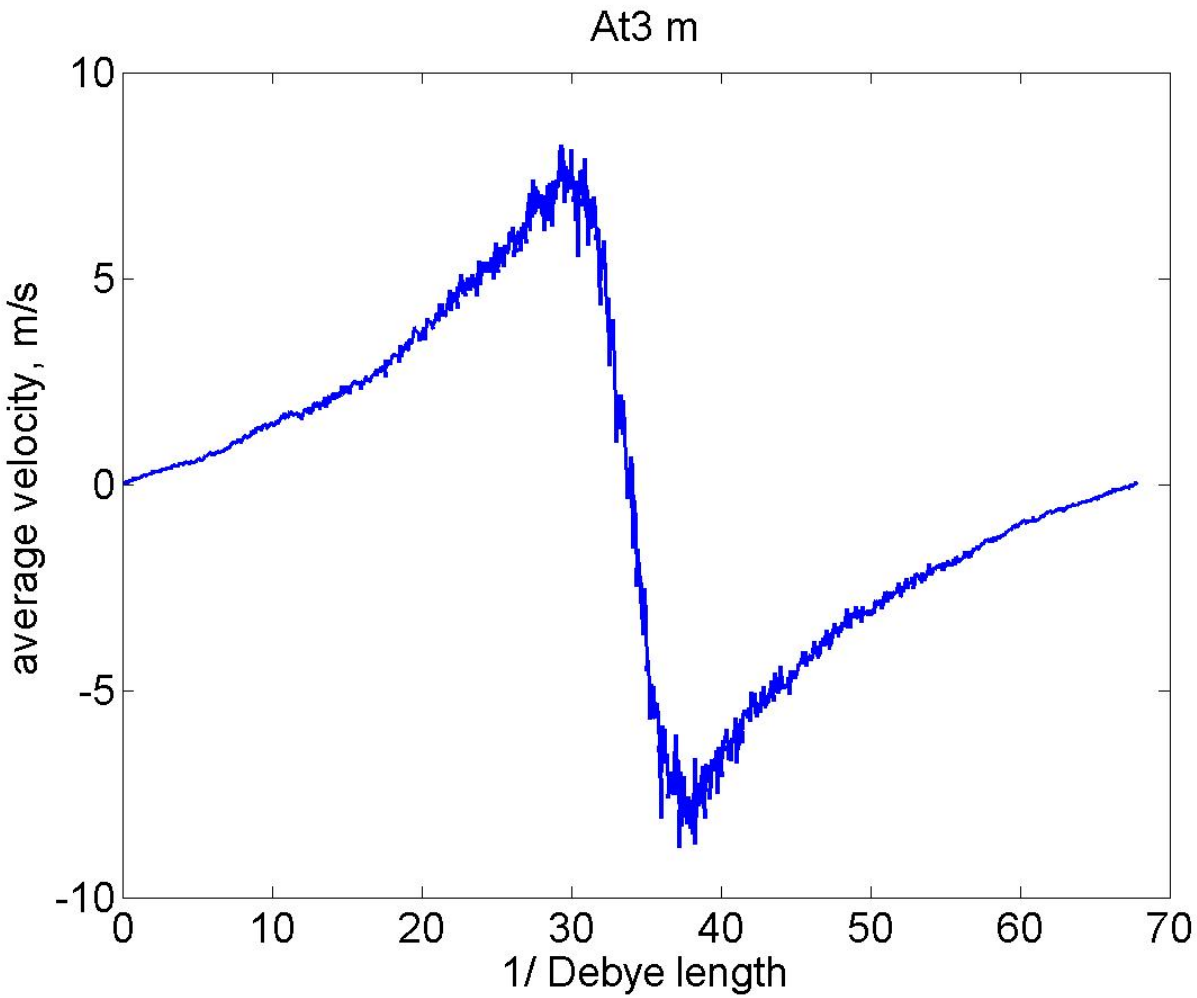
$z - x$ plot in kicker



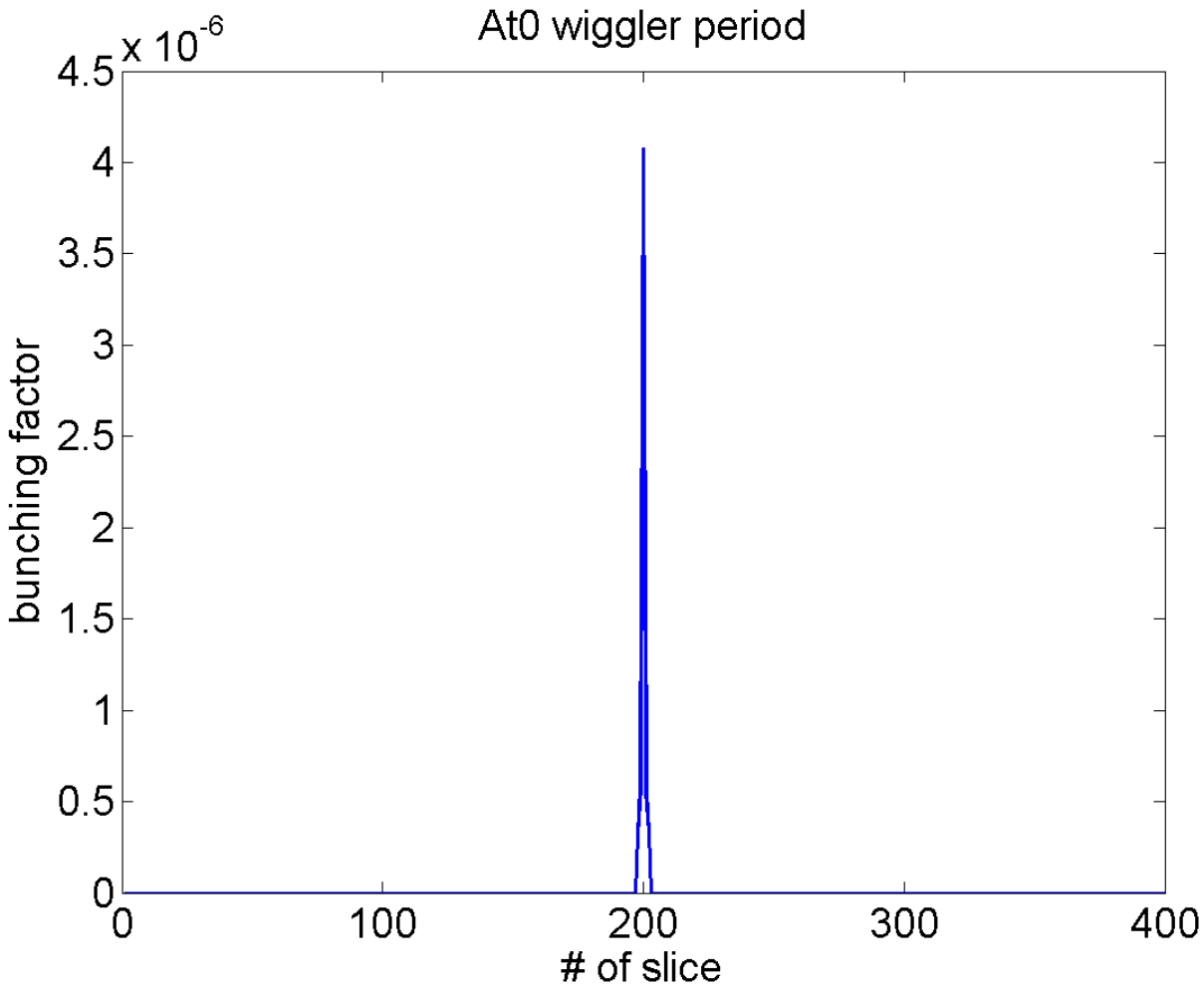
nz in modulator



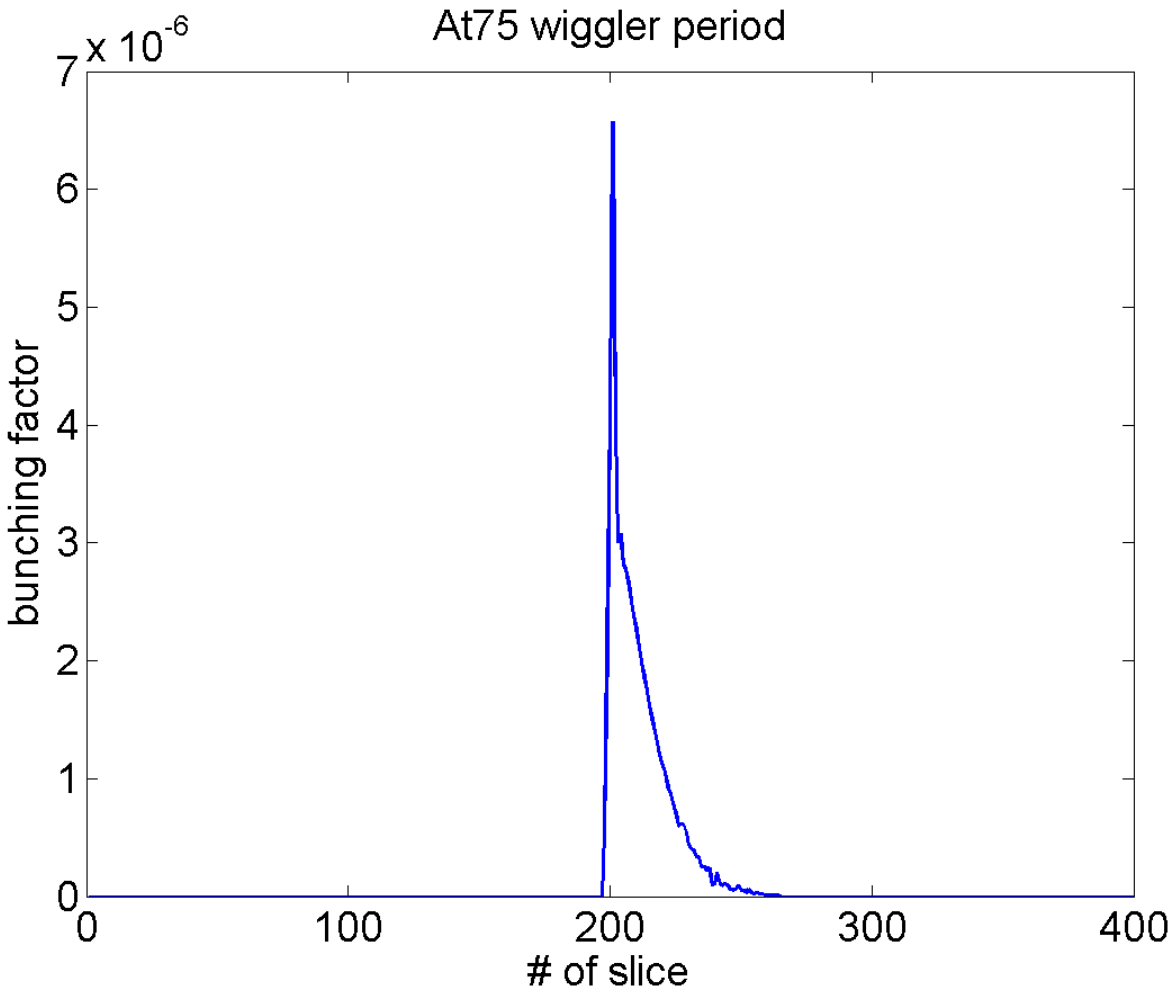
vz in modulator



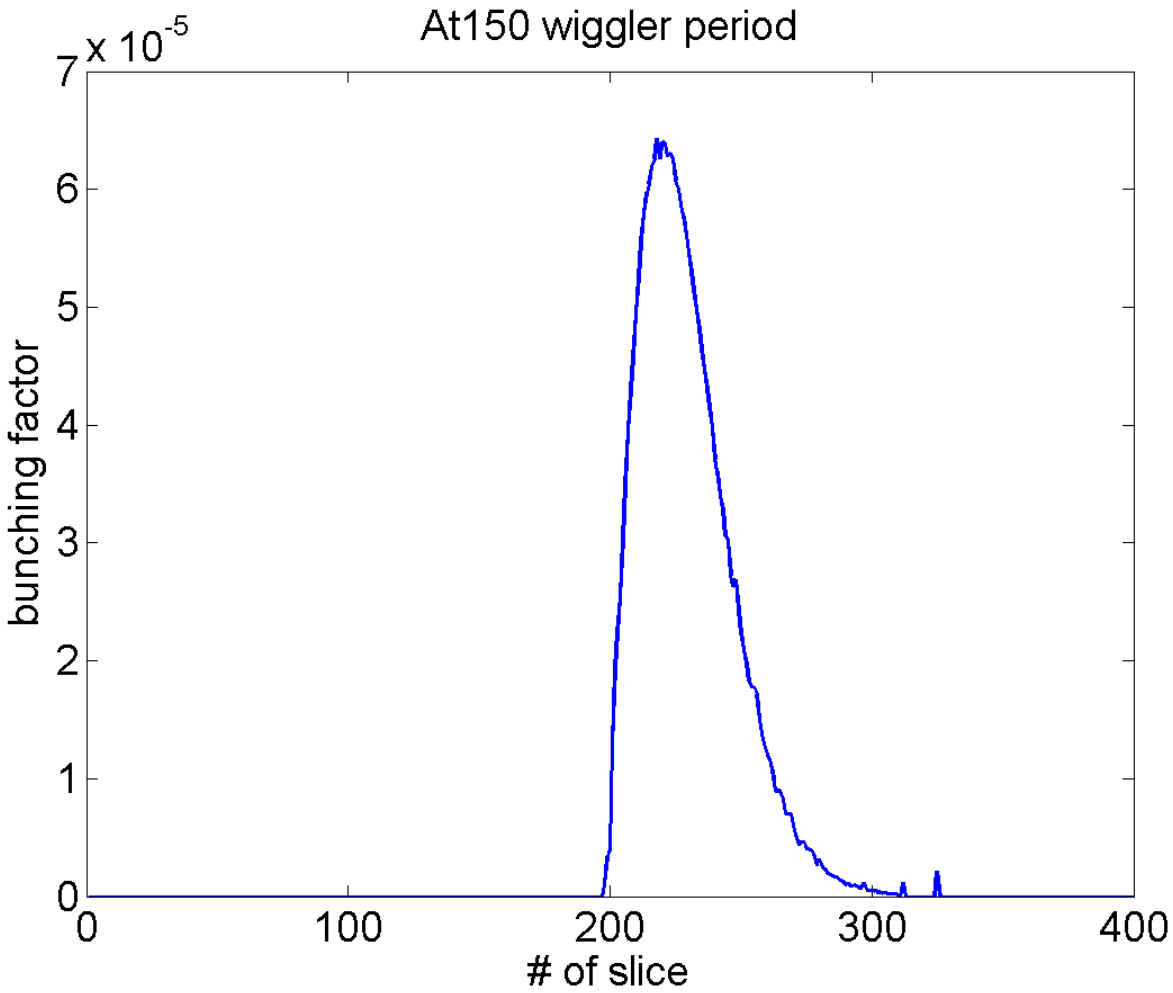
Bunching factor in FEL



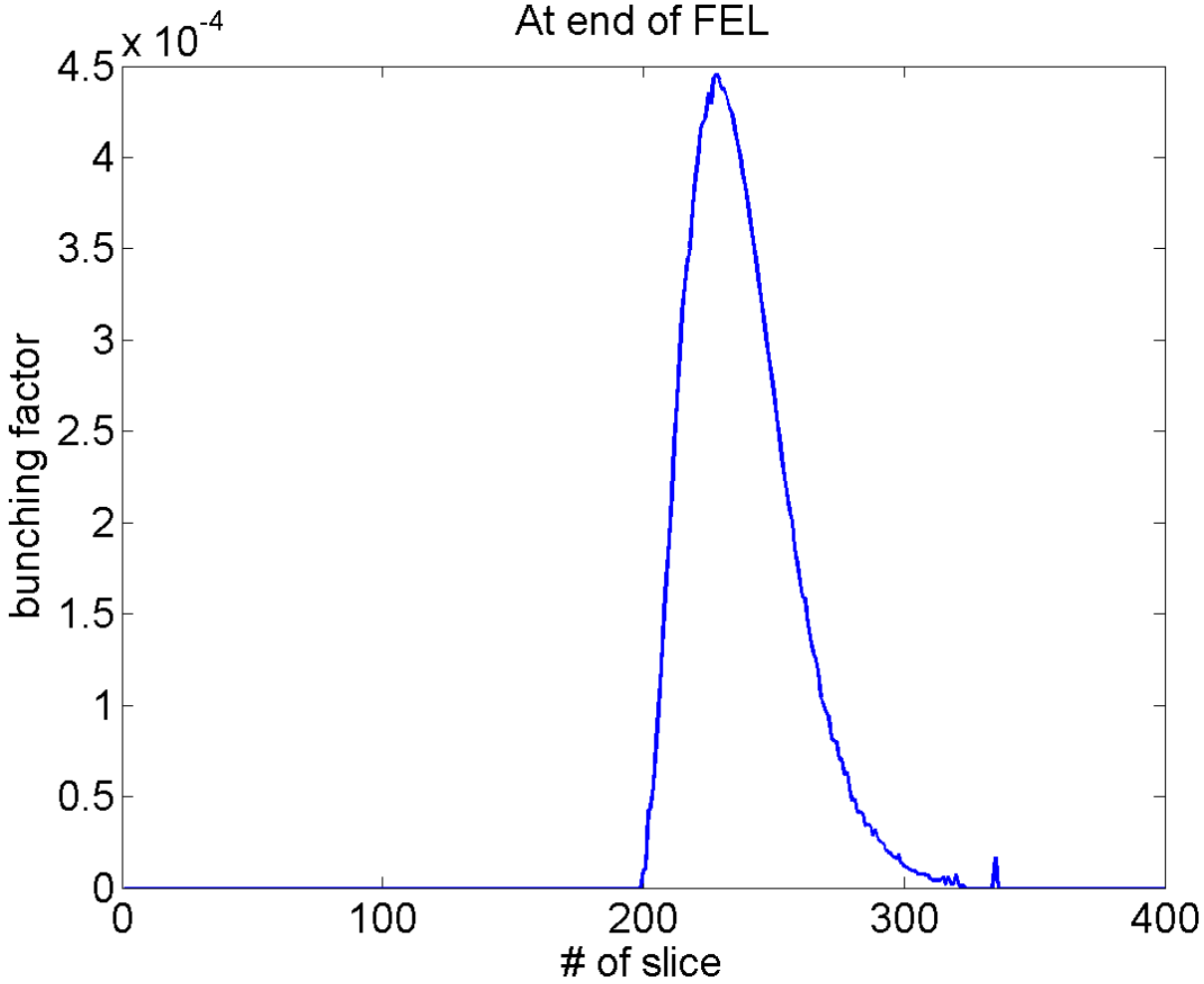
Bunching factor in FEL



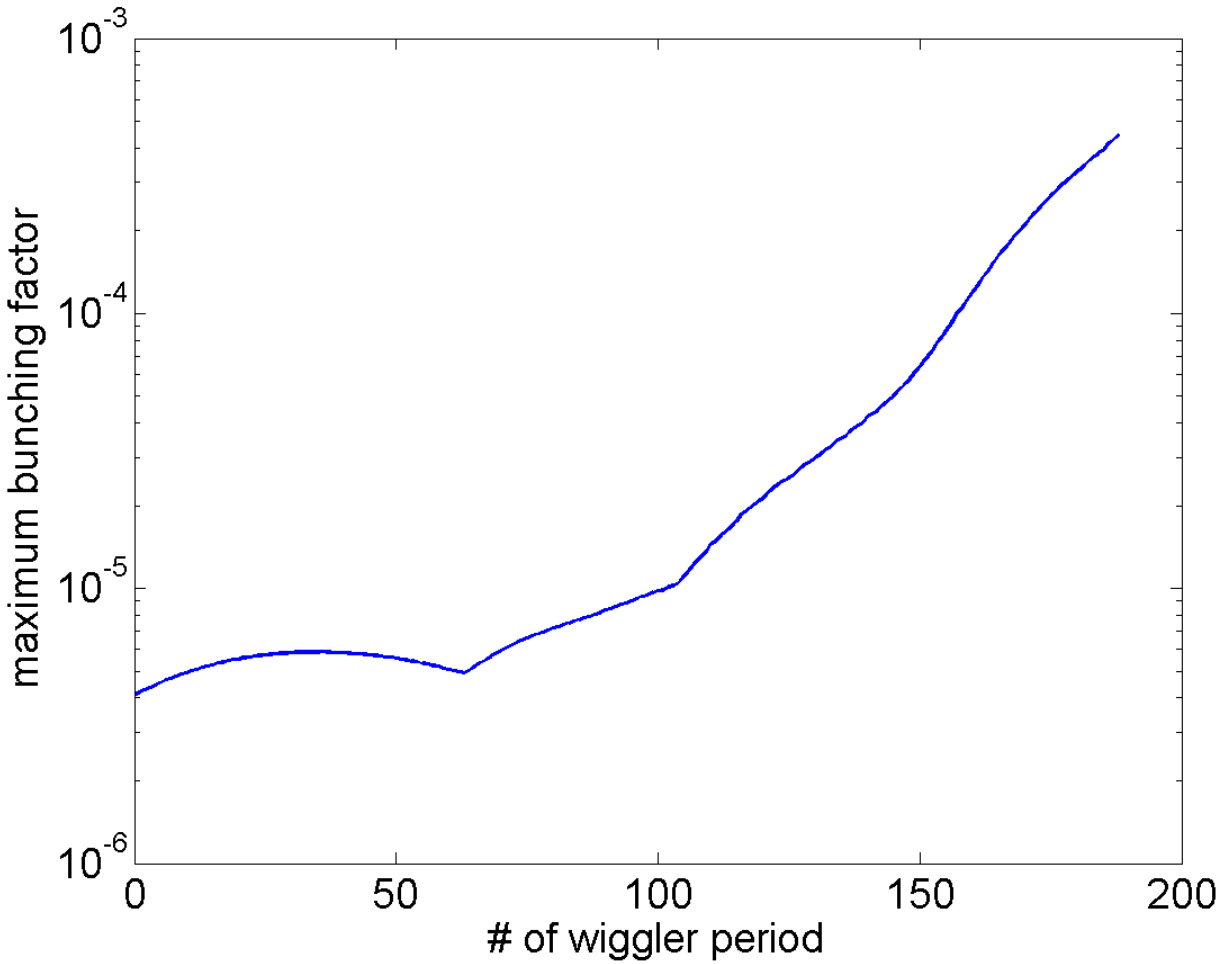
Bunching factor in FEL



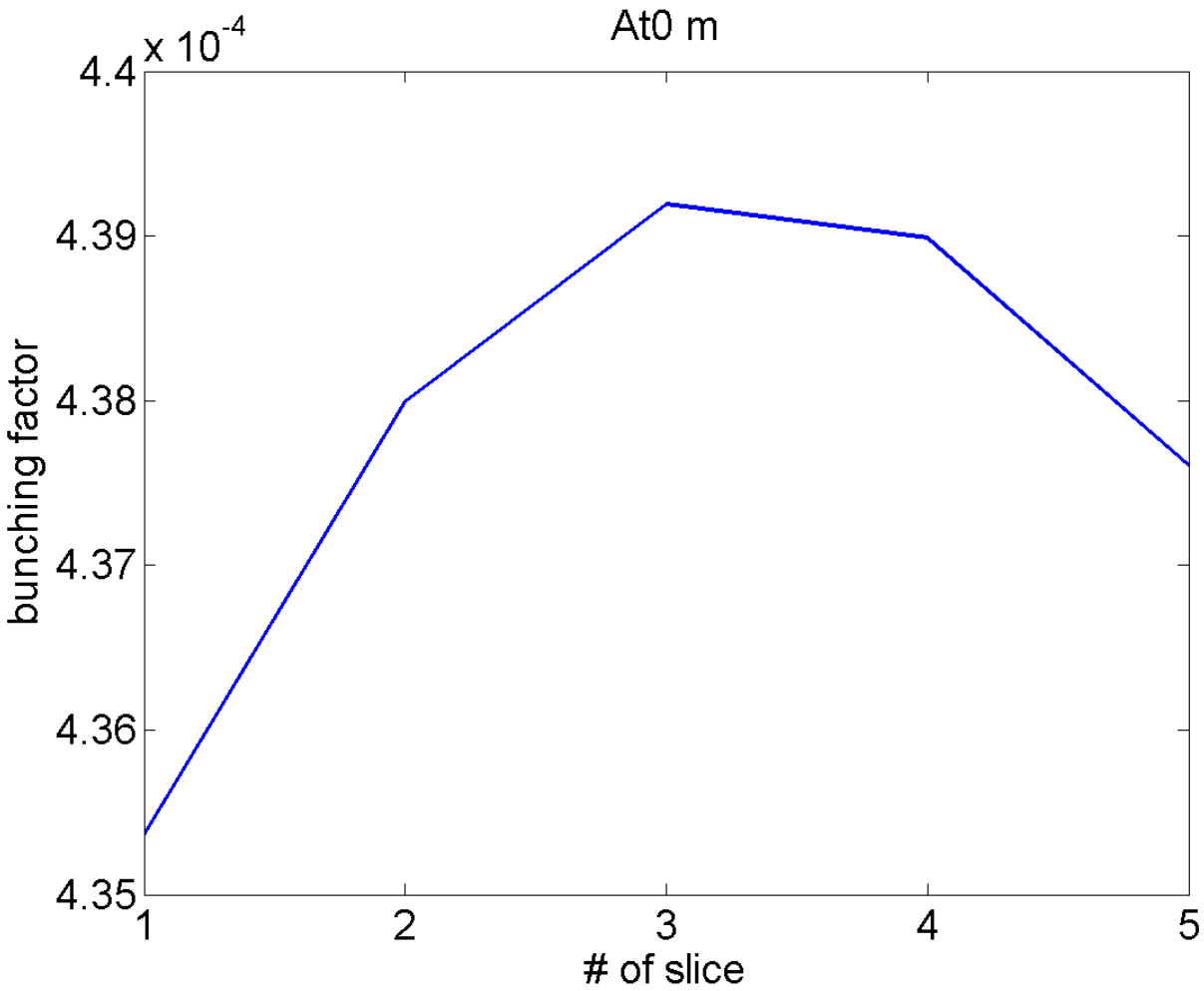
Bunching factor in FEL



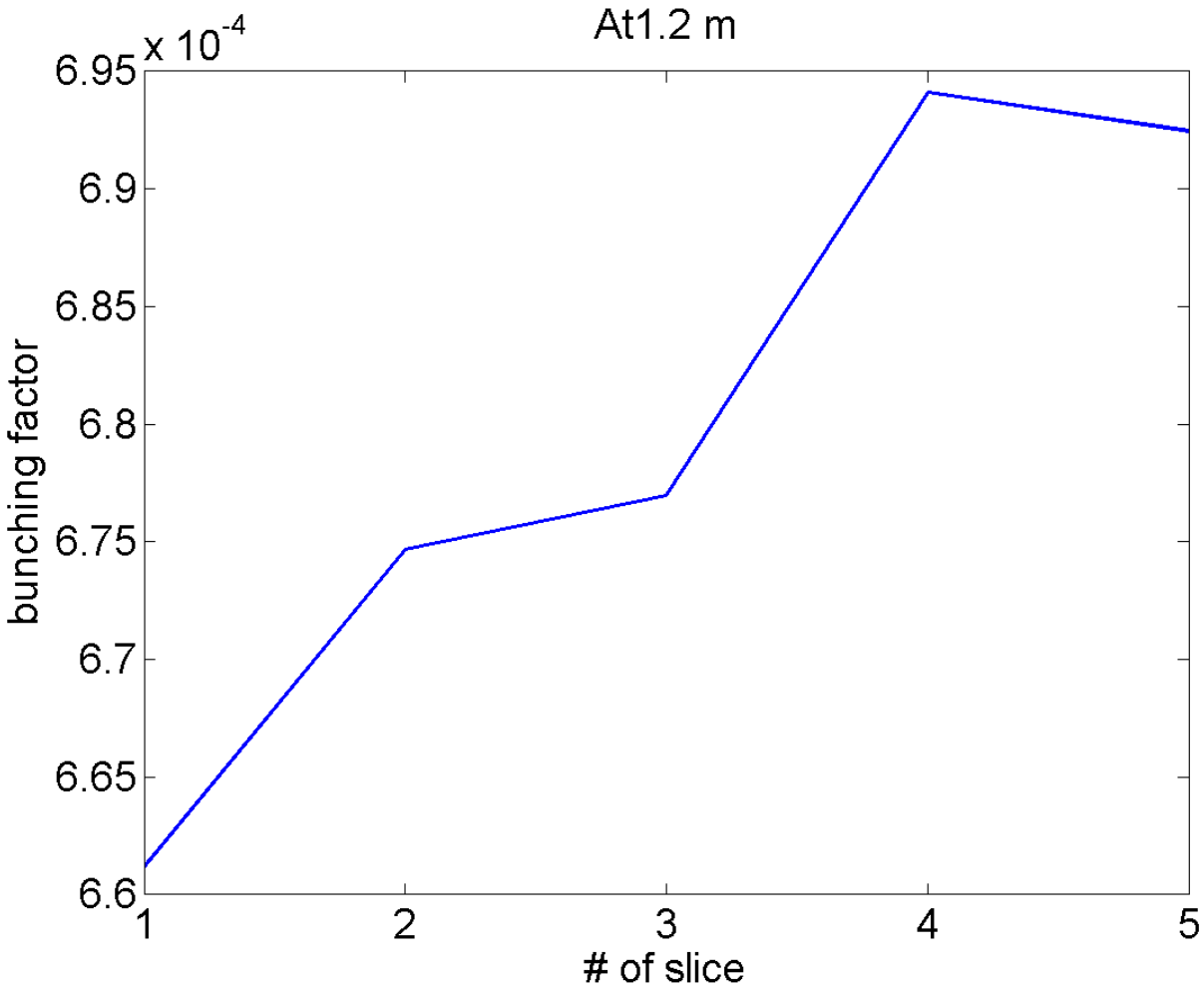
Bunching factor changes in FEL



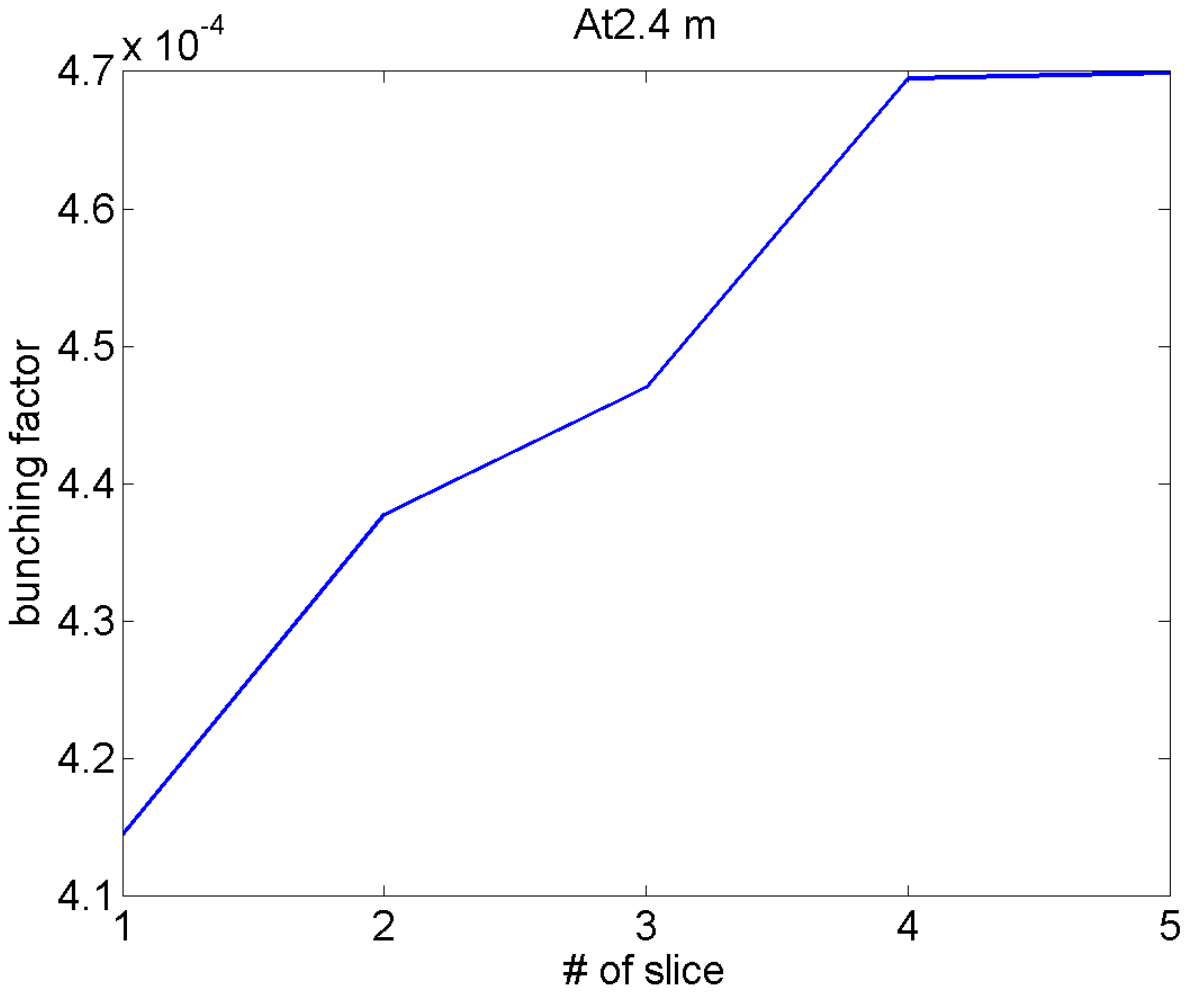
Bunching factor in kicker



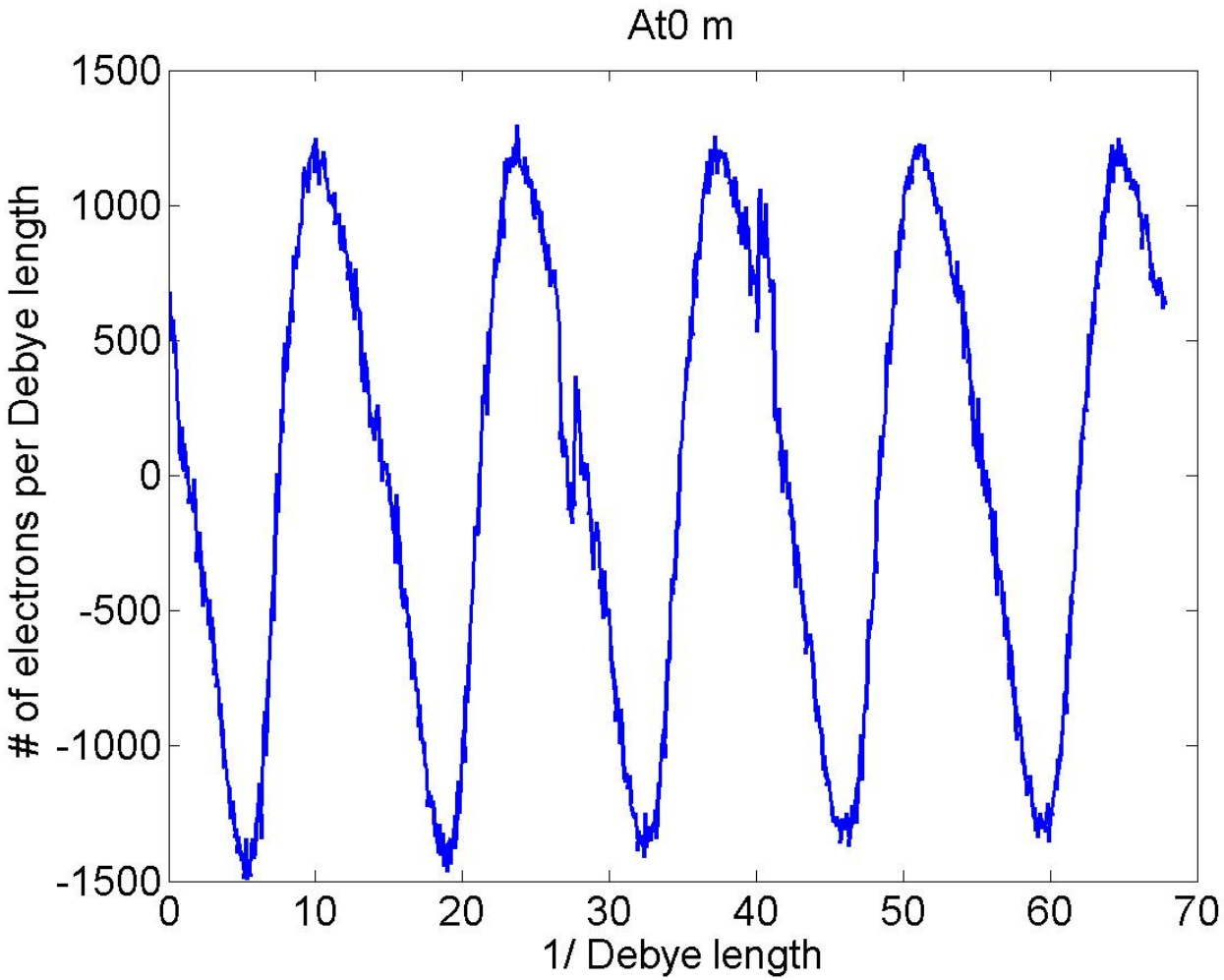
Bunching factor in kicker



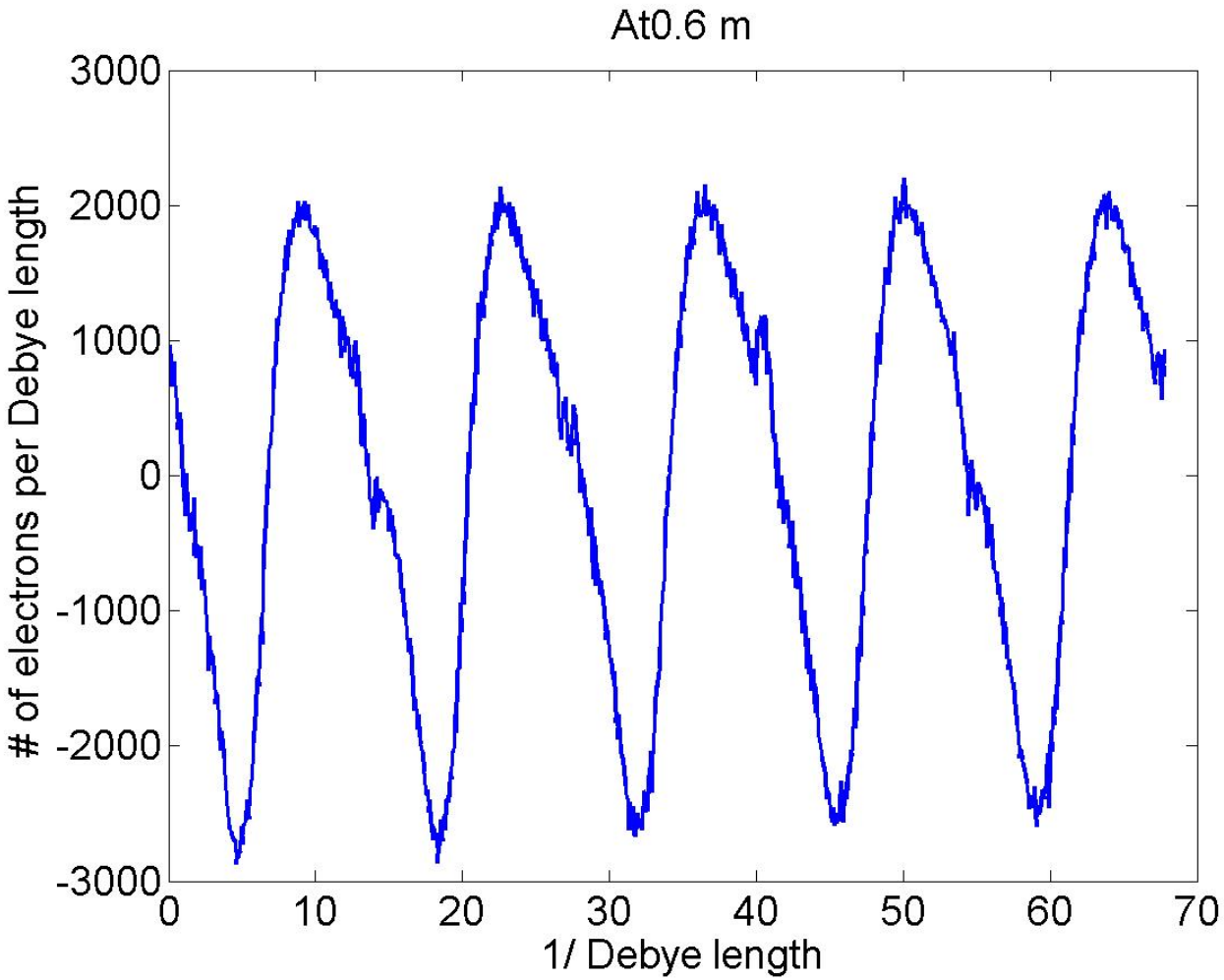
Bunching factor in kicker



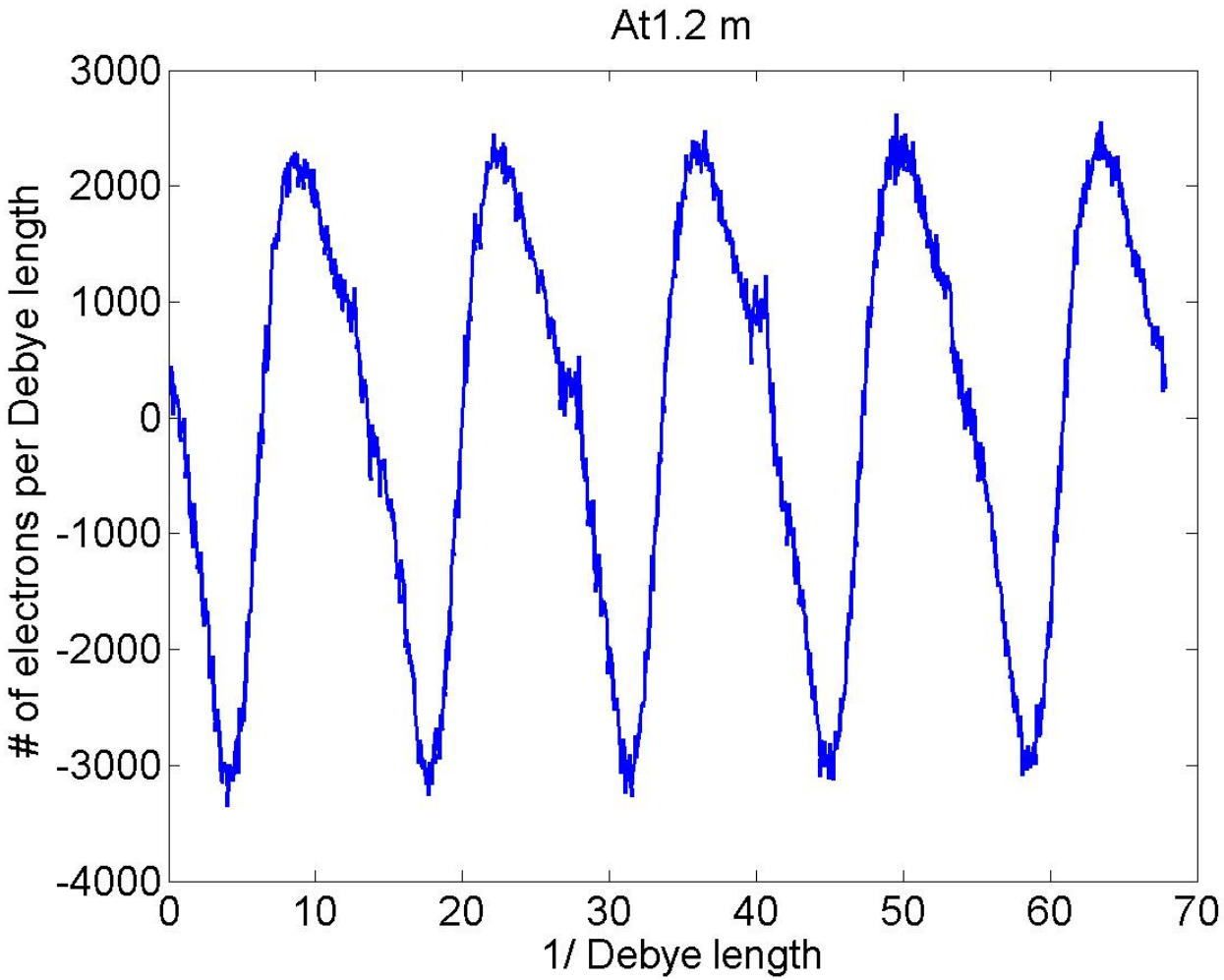
nz in kicker


At0 m
# of electrons per Debye length
1/ Debye length
1500
1000
500
0
-500
-1000
-1500
0
10
20
30
40
50
60
70

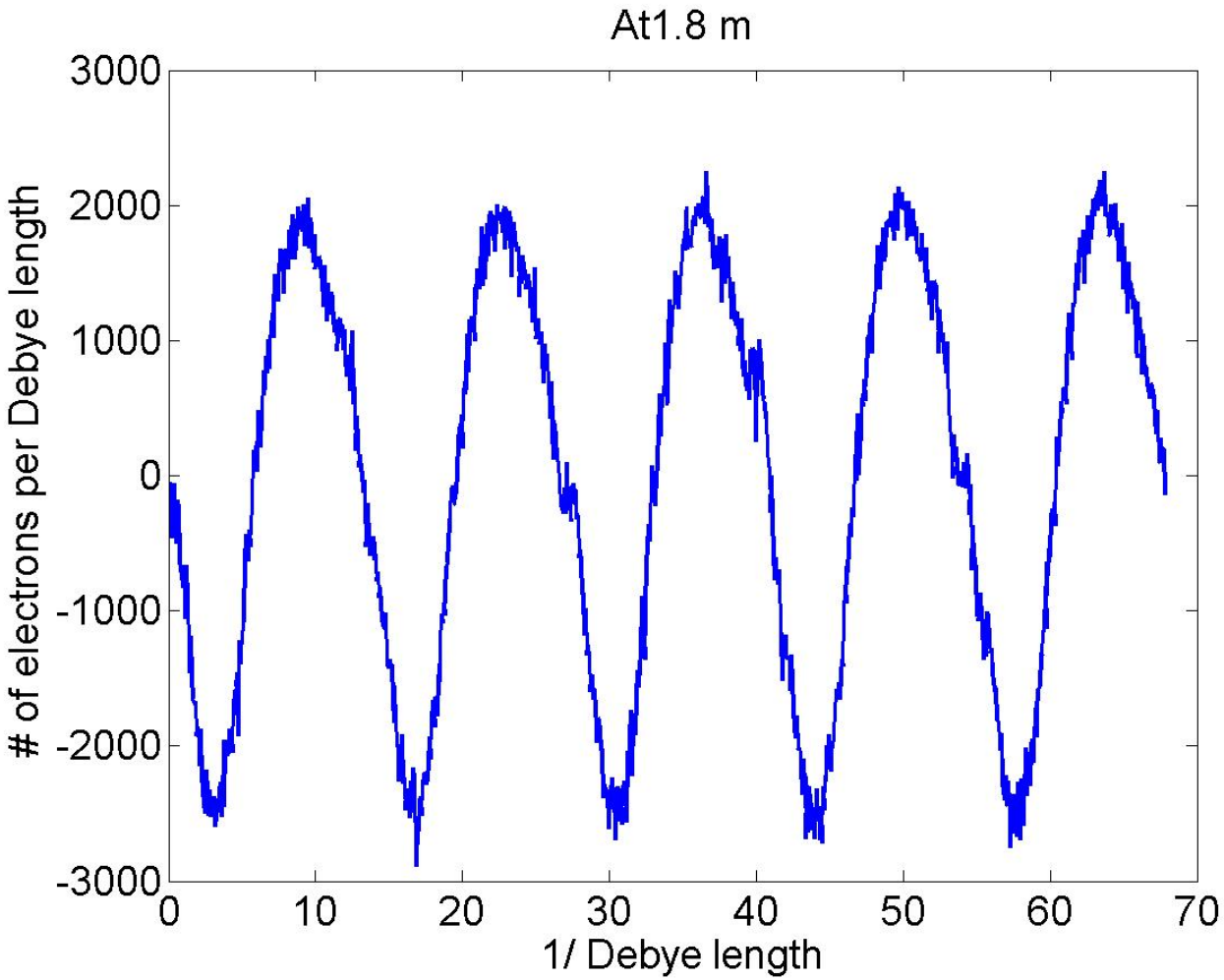
nz in kicker


At0.6 m
# of electrons per Debye length
1/ Debye length
3000
2000
1000
0
-1000
-2000
-3000
0
10
20
30
40
50
60
70

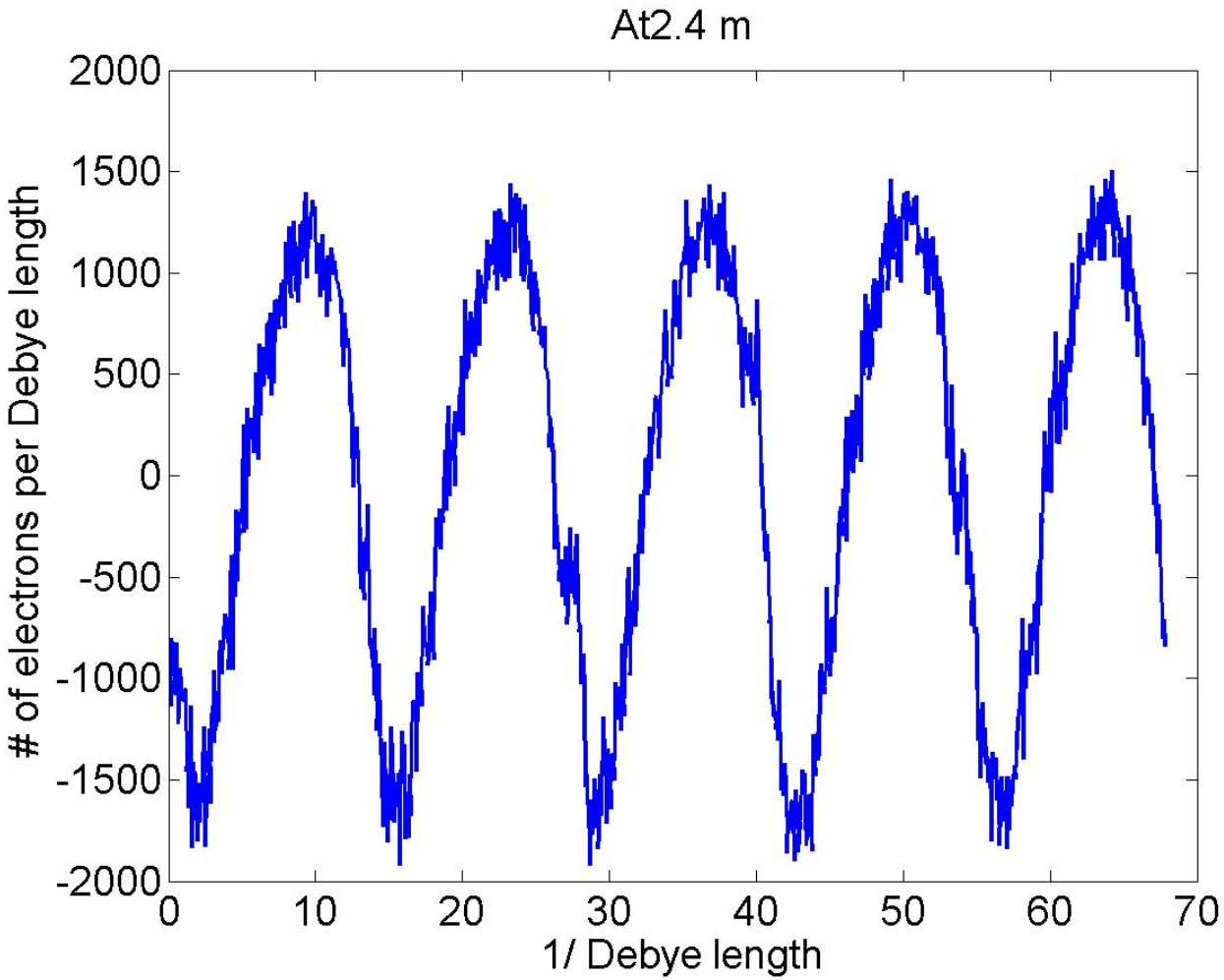
nz in kicker


At1.2 m
# of electrons per Debye length
1/ Debye length
3000
2000
1000
0
-1000
-2000
-3000
-4000
0
10
20
30
40
50
60
70

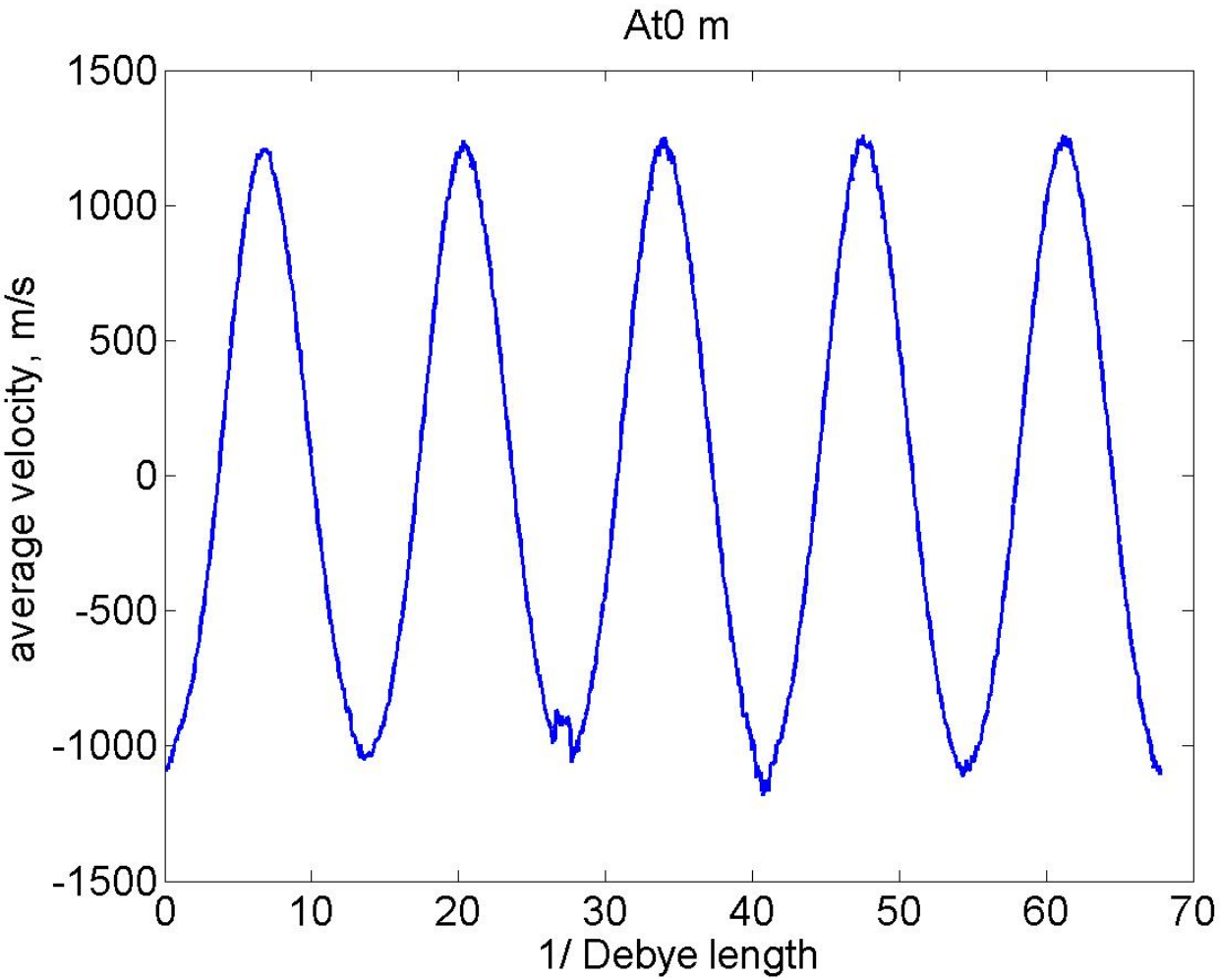
nz in kicker


At1.8 m
3000
2000
1000
0
-1000
-2000
-3000
# of electrons per Debye length
0
10
20
30
40
50
60
70
1/ Debye length

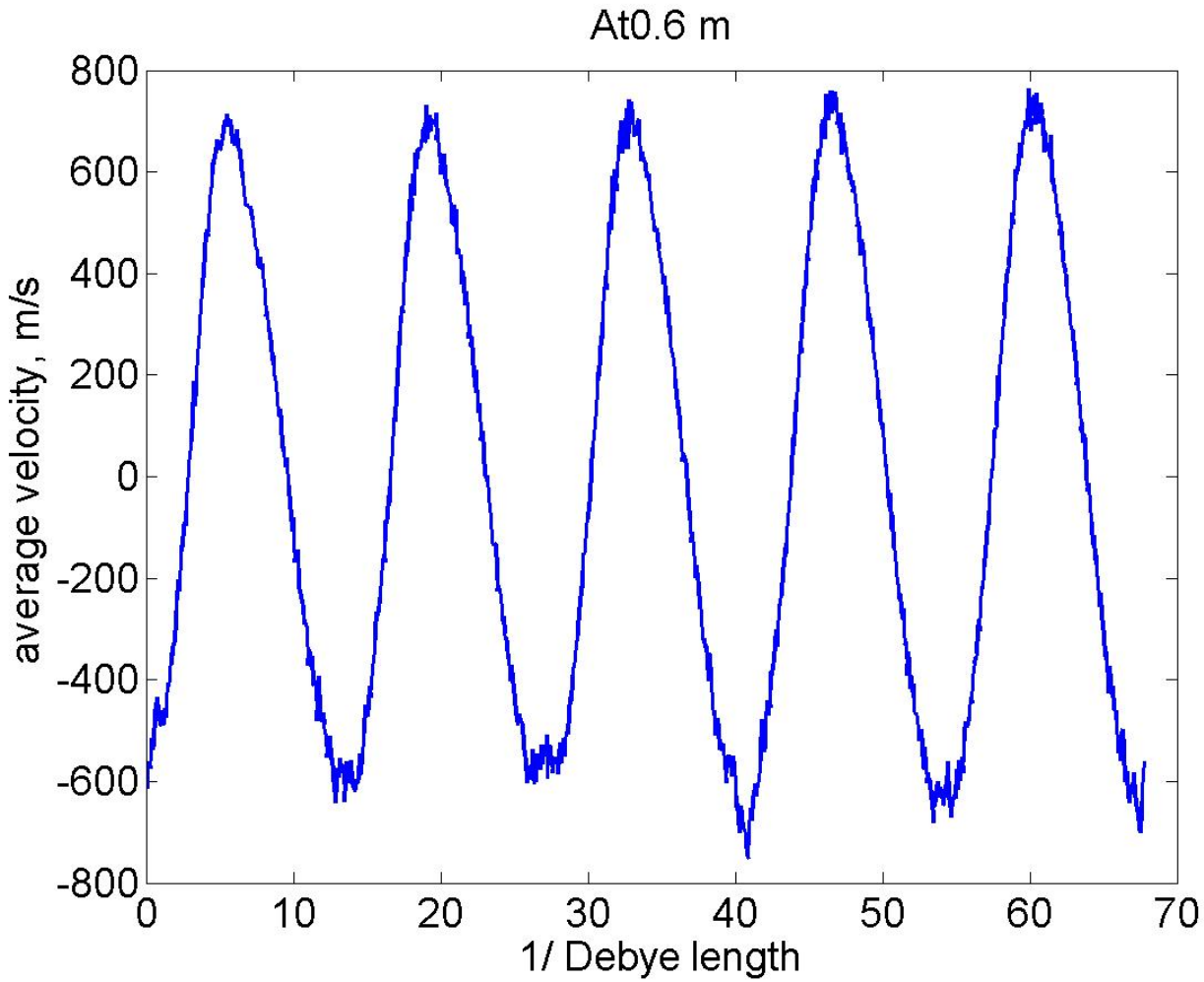
nz in kicker


At2.4 m
# of electrons per Debye length
1/ Debye length
2000
1500
1000
500
0
-500
-1000
-1500
-2000
0
10
20
30
40
50
60
70

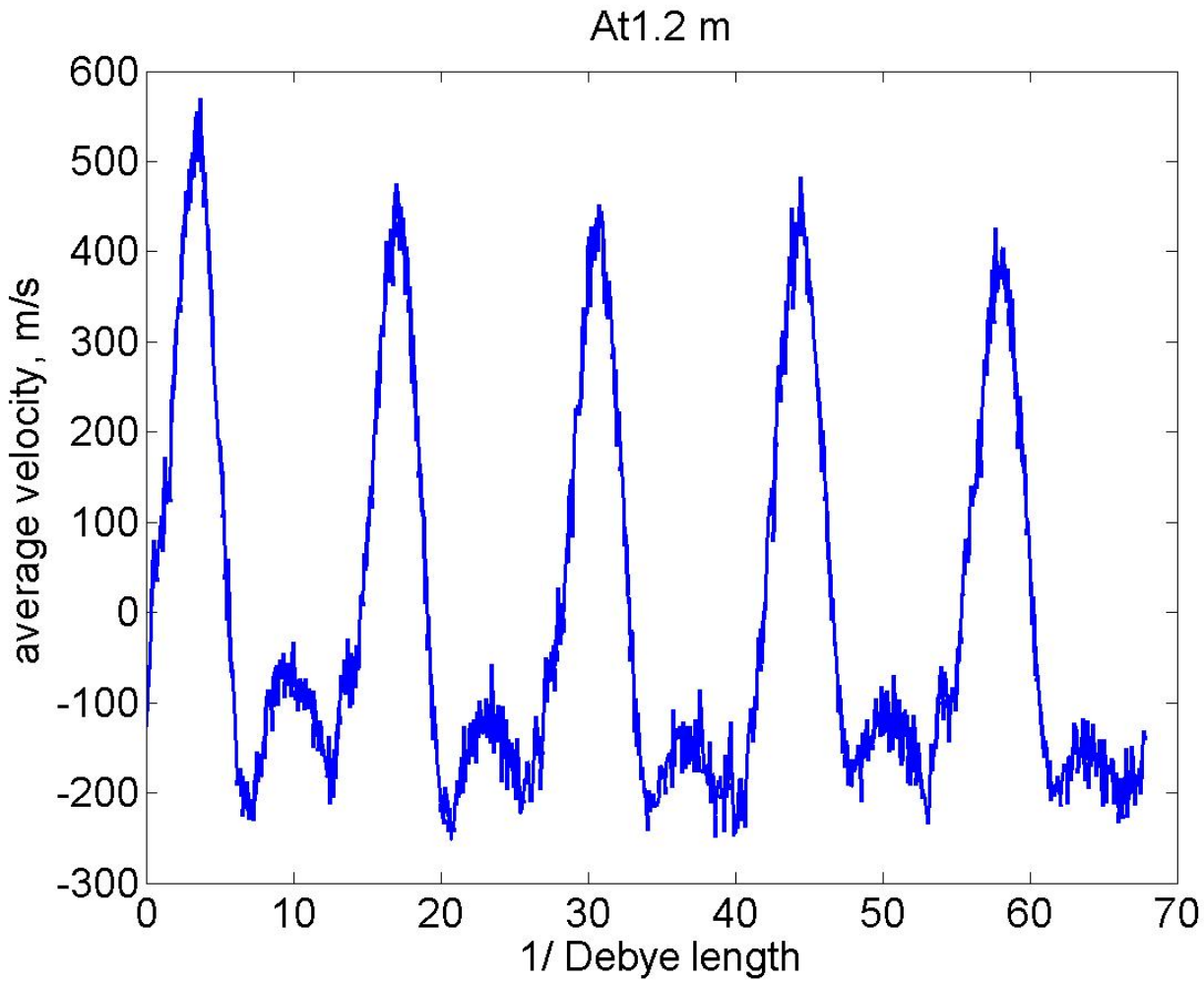
vz in kicker


At0 m
1500
1000
500
0
-500
-1000
-1500
average velocity, m/s
0
10
20
30
40
50
60
70
1/ Debye length

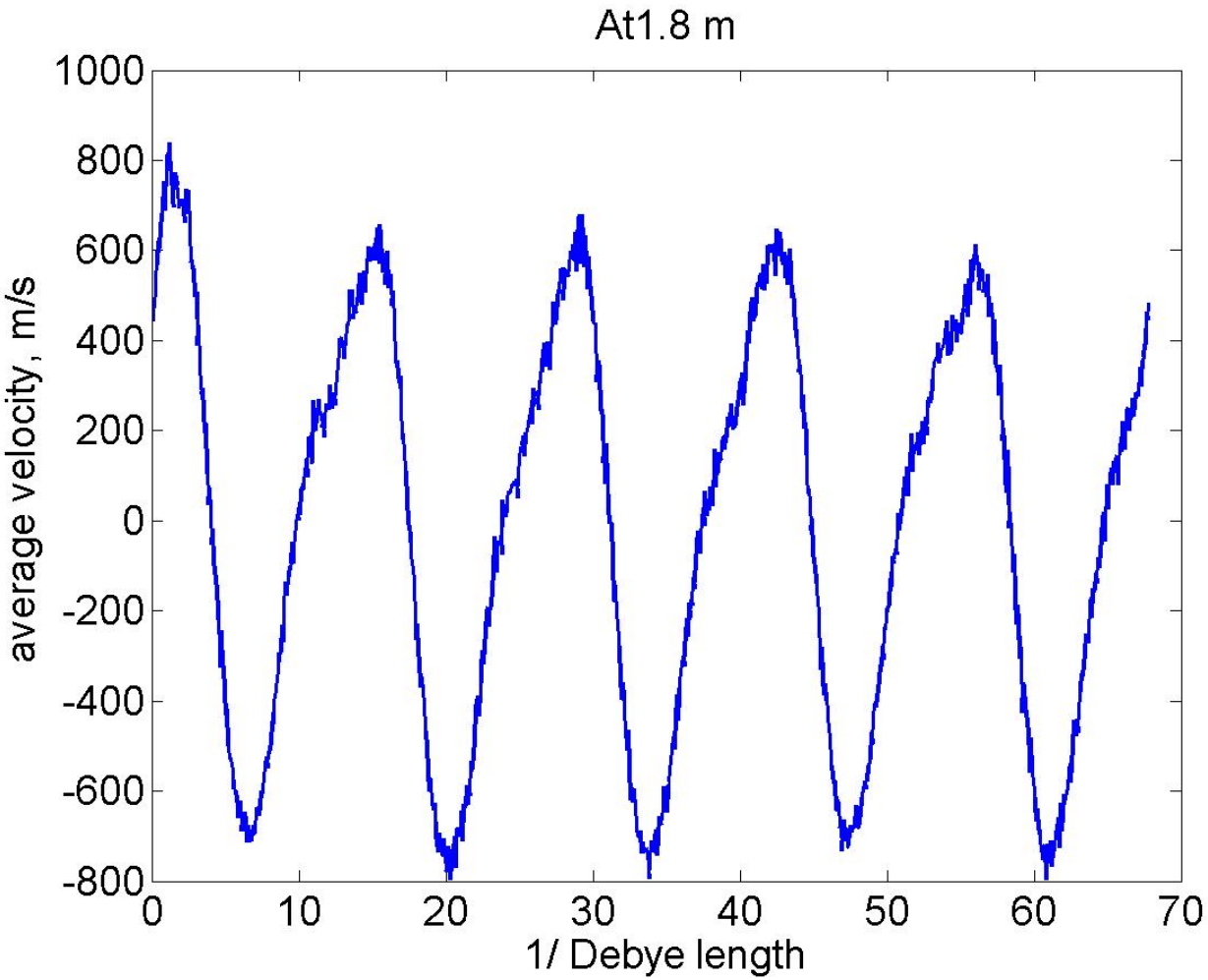
vz in kicker


At0.6 m
average velocity, m/s
800
600
400
200
0
-200
-400
-600
-800
0
10
20
30
40
50
60
70
1/ Debye length

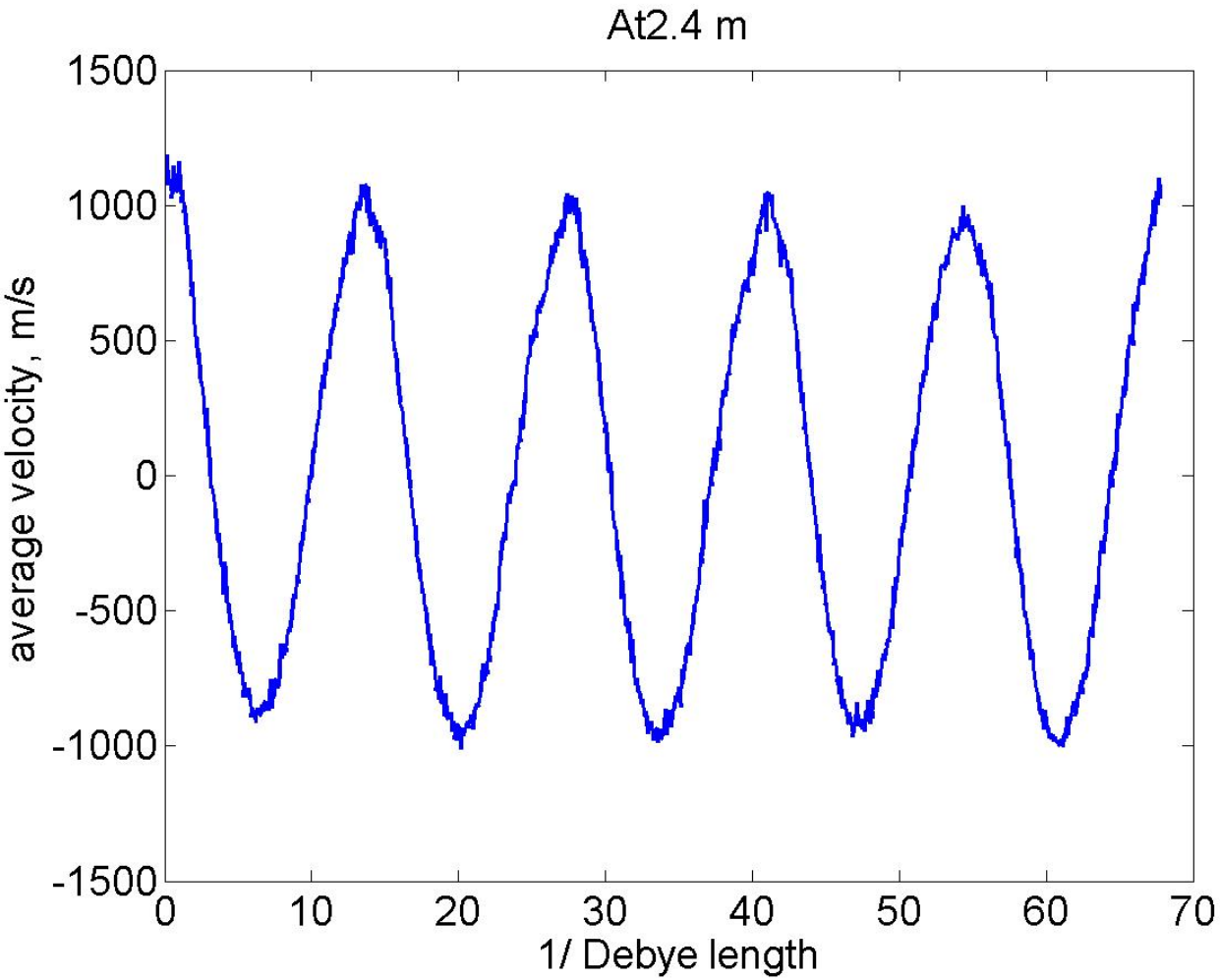
vz in kicker


At1.2 m
average velocity, m/s
1/ Debye length
600
500
400
300
200
100
0
-100
-200
-300
0
10
20
30
40
50
60
70

vz in kicker


At1.8 m
average velocity, m/s
1000
800
600
400
200
0
-200
-400
-600
-800
0
10
20
30
40
50
60
70
1/ Debye length

vz in kicker


At2.4 m
1500
1000
500
0
-500
-1000
-1500
average velocity, m/s
0
10
20
30
40
50
60
70
1/ Debye length

Conclusion

- Modulator simulations using uniform electron beam

  Obtain agreement with theory
- Modulator simulations using Gaussian electron beam and continuous focusing

  Better understand modulation process
- Modulator simulations using Gaussian electron beam and quadrupoles focusing Give predictions for CEC experiments
- Connecting SPACE and GENESIS

  Capable of doing start-to-end CEC simulations

Future work

- Use real beam distribution as input
- Use various locations and velocities of ion

References

V. N. Litvinenko et al., Proc. FEL07, BNL-79509-2007-CP.

V. N. Litvinenko, Proc. PAC09, FR1GRI01.

K. Yu et al., Proc. IPAC 2015, MOPMN012.

K. Yu et al., Proc. IPAC 2015, MOPMN013.

J. Ma et al., Proc. IPAC 2015, MOPMN015.

J. Ma et al., Proc. NA-PAC 2016, WEPOA55.

X. Wang et al., J. Comput. Phys., 316 (2016) 682 - 699.

G. Wang et al., Phys. Rev. E, 78 (2008) 026413.

A. Elizarov et al., Phys. Rev. Accel. Beams, 18 (2015) 044001.

G.I. Bell et al., arXiv:1404.2320, (2014).

Y. Jing et al., arXiv:1505.04735, (2015).

G. Bell et al., Proc. PAC 2011, MOP067.

G. Wang et al., Proc. IPAC 2013, MOPEA083.

Acknowledgement

CAD at BNL

Gang Wang

Vladimir Litvinenko

Yichao Jing

Yue Hao

CSI at BNL

Roman Samulyak

Kwang Min Yu

AMS at SBU

Xingyu Wang

THANK YOU !