Homework 6. Due October 4

Problem 1. 5 points. A short ultra-relativistic (1-v/c <<1) bunch with charge of 5 nC is passing through a 0.3 meter long 500 MHz pillbox accelerating cavity operating at the fundamental TM010 with peak accelerating field of 5 MV/m.

(1) Find the change of the cavity voltage ΔV/V (accelerating field) after the beam passes through it as function of the phase of the beam passing the cavity. What are the maximum and minimum ΔV/V?

(2) How the beam loading ΔV/V depends on the accelerating field? At what level of accelerating field it reaches ΔV/V 1%?

(a) Assume that beam does not change velocity in the cavity;
(b) Hint – use energy conservation law
(c) Assume that relative change of the voltage ΔV/V is small, e.g. the beam loading can be treated as a perturbation.

Problem 2. 5 points. Cavities filled with ferrite material are used for RF system requiring large frequency tuning range. The frequency is controlled by applying external magnetic field, Bext, to the ferrite material and by doing so to change it magenta permeability μ(Bext). A 300 m in circumference AGS synchrotron accelerates polarized protons from total energy of 2.5 GeV to 25 GeV.

(a) Calculate the range of the beam revolution frequency in AGS;
(b) Assuming 100% filling by ferrite, what should be ratio of μmax to μmin. Where μ should have maximum value?

Note: RF systems operate on a fixed integer harmonic of the revolution frequency.