1. The maximal gain happens at the detuning satisfying the following equation.

$$\frac{d}{d\hat{C}}\left[\hat{C}^{-3}\left(1-\cos\hat{C}-\frac{1}{2}\hat{C}\sin\hat{C}\right)\right]=\hat{C}^{-4}\left[3\cos\hat{C}+2\hat{C}\sin\hat{C}-\frac{1}{2}\hat{C}^{2}\cos\hat{C}-3\right]=0\,. \qquad (1)$$

Numerically finding the solutions of

$$3\cos\hat{C}+2\hat{C}\sin\hat{C}-\frac{1}{2}\hat{C}^{2}\cos\hat{C}-3=0 \qquad (2)$$

, which leads to maximal gain, gives

$$\hat{C}\approx 2.606\,. \qquad (3)$$

Inserting the definition of $\hat{C}$ into eq. (3) leads to

$$Cl_w=\left(k_w+k-\frac{\omega}{v_z\left(\varepsilon_0+\Delta\varepsilon\right)}\right)l_w\approx\frac{\omega l_w}{\gamma_z^2 c}\frac{\Delta\varepsilon}{\varepsilon_0}=2.606 \qquad (4)$$

, or

$$\frac{\Delta\varepsilon}{\varepsilon_0}=2.606\frac{\gamma_z^2\lambda_0}{2\pi l_w}=2.606\frac{\lambda_w}{4\pi l_w}=\frac{2.606}{4\pi}\frac{1}{N_w}, \qquad (5)$$

where $N_w=l_w/\lambda_w$ is the number of wiggler period.