1. The maximal gain happens at the detuning satisfying the following equation.

$$
\begin{equation*}
\frac{d}{d \hat{C}}\left[\hat{C}^{-3}\left(1-\cos \hat{C}-\frac{1}{2} \hat{C} \sin \hat{C}\right)\right]=\hat{C}^{-4}\left[3 \cos \hat{C}+2 \hat{C} \sin \hat{C}-\frac{1}{2} \hat{C}^{2} \cos \hat{C}-3\right]=0 \tag{1}
\end{equation*}
$$

Numerically finding the solutions of

$$
\begin{equation*}
3 \cos \hat{C}+2 \hat{C} \sin \hat{C}-\frac{1}{2} \hat{C}^{2} \cos \hat{C}-3=0 \tag{2}
\end{equation*}
$$

, which leads to maximal gain, gives

$$
\begin{equation*}
\hat{C} \approx 2.606 . \tag{3}
\end{equation*}
$$

Inserting the definition of $\hat{C}$ into eq. (3) leads to

$$
\begin{equation*}
C l_{w}=\left(k_{w}+k-\frac{\omega}{v_{z}\left(\varepsilon_{0}+\Delta \varepsilon\right)}\right) l_{w} \approx \frac{\omega l_{w}}{\gamma_{z}^{2} c} \frac{\Delta \varepsilon}{\varepsilon_{0}}=2.606 \tag{4}
\end{equation*}
$$

, or

$$
\begin{equation*}
\frac{\Delta \varepsilon}{\varepsilon_{0}}=2.606 \frac{\gamma_{z}^{2} \lambda_{0}}{2 \pi l_{w}}=2.606 \frac{\lambda_{w}}{4 \pi l_{w}}=\frac{2.606}{4 \pi} \frac{1}{N_{w}} \tag{5}
\end{equation*}
$$

where $N_{w}=l_{w} / \lambda_{w}$ is the number of wiggler period.

