Homework 4
Due: *Friday, March 4, 2020*

1. In class, we derived fluid equations by taking moments of the Vlasov equation

$$\frac{\partial f}{\partial t} + v_k \frac{\partial f}{\partial x_k} + a_k \frac{\partial f}{\partial v_k} = 0$$

(a) Derive the heat flux equation by taking the second moment of the Vlasov equation (lots of algebra!)

$$\frac{\partial}{\partial t} P_{ij} + V_k \frac{\partial}{\partial x_k} P_{ij} + P_{ij} \frac{\partial V_k}{\partial x_k} + P_{ik} \frac{\partial V_j}{\partial x_k} + P_{jk} \frac{\partial V_i}{\partial x_k} + \frac{\partial}{\partial x_k} Q_{ijk} = q \frac{B_m}{m} (\epsilon_{ilm} P_{jl} + \epsilon_{jlm} P_{il})$$

Where

$$V_i = \int (d^3v) v_i f$$
$$P_{ij} = m \int (d^3v) v_{ri} v_{rj} f$$
$$Q_{ijk} = m \int (d^3v) v_{ri} v_{rj} v_{rk} f$$

(b) For adiabatic processes there is no heat flux, so $\frac{\partial}{\partial x_k} Q_{ijk} = 0$. For this case, take the "trace" of the heat flux equation, i.e. let $i = j$ and assume that the pressure is isotropic, i.e. the same in all directions,

$P = [p]$ in 1D, $\begin{bmatrix} p & 0 \\ 0 & p \end{bmatrix}$ in 2D, and $\begin{bmatrix} p & 0 & 0 \\ 0 & p & 0 \\ 0 & 0 & p \end{bmatrix}$ in 3D

(c) Use the resulting equations to derive the adiabatic equations of states for N=1, 2, 3 dimensions.

2. The entropy of the gas is defined as $S = -k \int (d^3x)(d^3v)(\ln f)\, f$ where k is the Boltzmann's constant. If f satisfies the Vlasov equation, then what is dS/dt? Is this result expected?